CHAPTER 8

Rational and Radical Functions

Chapter Focus

- Apply algebraic reasoning to solve problems with rational and radical expressions.
- Make connections among multiple representations of rational and radical functions.

Race to the Finish

You can use rational expressions and functions to determine a bicyclist's average speed in a race with multiple stages.

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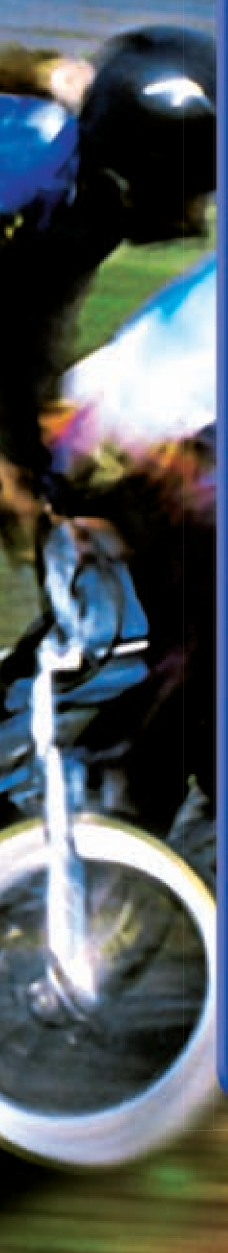
Chapter Project Online
KEYWORD: MB7 ChProj



ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. asymptote
2. rational number
3. reflection
4. translation
5. zero of a function

A. any number that can be expressed as a quotient of two integers, where the denominator is not zero

B. a transformation that flips a figure across a line

C. any number x such that $f(x) = 0$

D. a line that a curve approaches as the value of x or y becomes very large or very small

E. a whole number or its opposite

F. a transformation that moves each point in a figure the same distance in the same direction

Properties of Exponents

Simplify each expression. Assume that all variables are nonzero.

6. $\dfrac{x^{11}y^5}{x^4y^7}$
7. $\left(\dfrac{3x^2y}{z}\right)^4$
8. $\left(x^3\right)^{-2}$
9. $\left(3x^3y\right)\left(6xy^5\right)$
10. $\left(2x^{-4}\right)^3$
11. $12x^0$

Combine Like Terms

Simplify each expression.

12. $5x^2 + 10x - 4x + 6$
13. $3x + 12 - 10x$
14. $x^2 + x + 3x^2 - 4x$

Greatest Common Factor

Find the greatest common factor of each pair of expressions.

15. $3a^2$ and $12a$
16. c^2d and cd^2
17. $16x^4$ and $40x^3$

Factor Trinomials

Factor each trinomial.

18. $x^2 - 4x - 5$
19. $x^2 + 2x - 24$
20. $x^2 + 12x + 32$
21. $x^2 + 9x + 18$
22. $x^2 - 6x + 9$
23. $x^2 - 8x - 20$

Solve Quadratic Equations

Solve.

24. $5x^2 = 45$
25. $4x^2 - 7 = 93$
26. $2(x - 2)^2 = 32$

CHAPTER 8

Study Guide: Preview

Where You've Been

Previously, you

- solved problems with linear functions.
- simplified polynomial expressions.
- graphed functions with asymptotes.
- solved quadratic equations and inequalities.

In This Chapter

You will study

- solving problems with variation functions.
- simplifying rational and radical expressions.
- graphing rational and radical functions.
- solving rational and radical equations and inequalities.

Where You're Going

You can use the skills in this chapter

- in future math classes, including Precalculus.
- to solve problems in other classes, such as Chemistry, Physics, and Biology.
- outside of school to make predictions involving time, money, or speed.

Key Vocabulary/Vocabulario

complex fraction	fracción compleja
constant of variation	constante de variación
continuous function	función continua
direct variation	variación directa
discontinuous function	función discontinua
extraneous solutions	soluciones extrañas
hole (in a graph)	hoyo (en una gráfica)
inverse variation	variación inversa
radical equation	ecuación radical
radical function	función radical
rational equation	ecuación racional
rational exponent	exponente racional
rational function	función racional

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word *extraneous* contains the word *extra*. What does *extra* mean? What do you think an **extraneous solution** is?
2. The graph of a **continuous function** has no gaps or breaks. How do you think a **discontinuous function** differs from a continuous function?
3. Do you think a **hole** could occur in the graph of a *continuous function* or a *discontinuous function*? Why?
4. A rational number can be written as a ratio of two integers. What do you think a **rational exponent** is?

Reading and Writing Math

Study Strategy: Make Flash Cards

You can use flash cards to help you remember a sequence of steps, the definitions of vocabulary words, or important formulas and properties.

Use these hints to make useful flash cards:

- Write a vocabulary word or the name of a formula or property on one side of a card and the meaning on the other.
- When memorizing a sequence of steps, make a flash card for each step.
- Use examples or diagrams if needed.
- Label each card with a lesson number in case you need to look back at your textbook for more information.

From Lesson 7-4

Know it! Note

Quotient Property of Logarithms

For any positive numbers m, n, and b $(b \neq 1)$,

WORDS	NUMBERS	ALGEBRA
The logarithm of a quotient is the logarithm of the dividend minus the logarithm of the divisor.	$\log_b\left(\frac{16}{2}\right) = \log_b 16 - \log_b 2$	$\log_b \frac{m}{n} = \log_b m - \log_b n$

Sample Flash Card

Lesson 7-4

Quotient Property of Logarithms

$\log_b \frac{m}{n} = ?$

Front

$\log_b \frac{m}{n} = \log_b m - \log_b n$

example:

$\log_2\left(\frac{16}{2}\right) = \log_2 16 - \log_2 2$

Back

Try This

Make flash cards that can help you remember each piece of information.

1. The Product of Powers Property states that to multiply powers with the same base, add the exponents. (See Lesson 1-5.)

2. The quadratic formula, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, can be used to find the roots of an equation with the form $ax^2 + bx + c = 0$ $(a \neq 0)$. (See Lesson 5-6.)

8-1 Algebra LAB

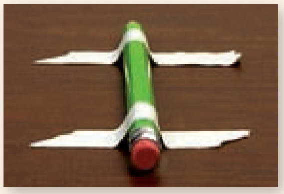
Model Inverse Variation

In this activity, you will explore the relationship between the mass of an object and the object's distance from the pivot point, or fulcrum, of a balanced lever.

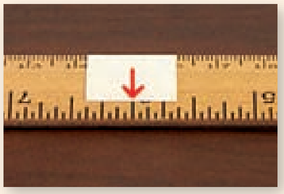
Use with Lesson 8-1

Activity

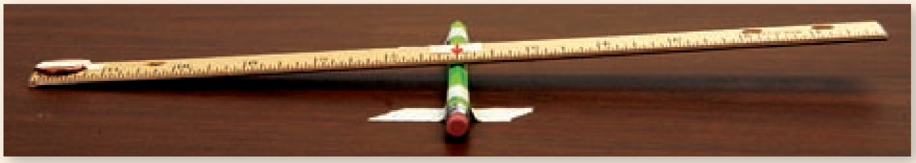
1 Secure a pencil to a tabletop with tape. The pencil will be the *fulcrum*.



2 Draw an arrow on a piece of tape, and use the arrow to mark the midpoint of a ruler. Then tape a penny to the end of the ruler.



3 Place the midpoint of the ruler on top of the pencil. The ruler is the *lever*.



4 Place one penny on the lever opposite the taped penny. If needed, move the untaped penny to a position that makes the lever balanced. Find the distance from the untaped penny to the fulcrum, and record the distance in a table like the one below. (Measure from the center of the penny.) Repeat this step with stacks of two to seven pennies.

Let x be the number of pennies and y be the distance from the fulcrum. Plot the points from your table on a graph. Then draw a smooth curve through the points.

Number of Pennies	1	2	3	4	5	6	7
Distance from Fulcrum (cm)							

Try This

1. Multiply the corresponding x- and y-values together. What do you notice?
2. Use your answer to Problem 1 to write an equation relating distance from the fulcrum to the number of pennies.
3. Would it be possible to balance a stack of 20 pennies on the lever? Use your equation from Problem 2 to justify your answer.
4. **Make a Conjecture** The relationship between the mass of an object on a balanced lever and the object's distance from the fulcrum can be modeled by an *inverse variation* function. Based on your data and graph, how are the variables in an inverse variation related?

8-1 Variation Functions

Objective
Solve problems involving direct, inverse, joint, and combined variation.

Vocabulary
direct variation
constant of variation
joint variation
inverse variation
combined variation

Why learn this?
You can use variation functions to determine how many people are needed to complete a task, such as building a home, in a given time. (See Example 5.)

In Chapter 2, you studied many types of linear functions. One special type of linear function is called *direct variation.* A **direct variation** is a relationship between two variables x and y that can be written in the form $y = kx$, where $k \neq 0$. In this relationship, k is the **constant of variation**. For the equation $y = kx$, y varies directly as x.

A direct variation equation is a linear equation in the form $y = mx + b$, where $b = 0$ and the constant of variation k is the slope. Because $b = 0$, the graph of a direct variation always passes through the origin.

EXAMPLE 1 Writing and Graphing Direct Variation

Given: y varies directly as x, and $y = 14$ when $x = 3.5$. Write and graph the direct variation function.

$y = kx$ — *y varies directly as x.*

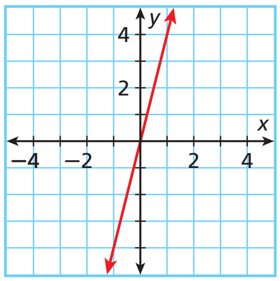
$14 = k(3.5)$ — *Substitute 14 for y and 3.5 for x.*

$4 = k$ — *Solve for the constant of variation k.*

$y = 4x$ — *Write the variation function by using the value of k.*

Graph the direct variation function.

The y-intercept is 0, and the slope is 4.



Check Substitute the original values of x and y into the equation.

$y = 4x$	
14	$4(3.5)$
14	14 ✔

Helpful Hint
If k is positive in a direct variation, the value of y increases as the value of x increases.

CHECK IT OUT! **1.** Given: y varies directly as x, and $y = 6.5$ when $x = 13$. Write and graph the direct variation function.

When you want to find specific values in a direct variation problem, you can solve for k and then use substitution or you can use the proportion derived below.

$$y_1 = kx_1 \rightarrow \frac{y_1}{x_1} = k \quad \text{and} \quad y_2 = kx_2 \rightarrow \frac{y_2}{x_2} = k \quad \text{so,} \quad \frac{y_1}{x_1} = \frac{y_2}{x_2}.$$

EXAMPLE 2 Solving Direct Variation Problems

Geometry

Reading Math

The phrases "y varies directly as x" and "y is directly proportional to x" have the same meaning.

The circumference of a circle C varies directly as the radius r, and $C = 7\pi$ ft when $r = 3.5$ ft. Find r when $C = 4.5\pi$ ft.

Method 1 Find k.

$C = kr$

$7\pi = k(3.5)$ *Substitute.*

$2\pi = k$ *Solve for k.*

Write the variation function.

$C = (2\pi)r$ *Use 2π for k.*

$4.5\pi = (2\pi)r$ *Substitute 4.5π for C.*

$2.25 = r$ *Solve for r.*

Method 2 Use a proportion.

$\frac{C_1}{r_1} = \frac{C_2}{r_2}$

$\frac{7\pi}{3.5} = \frac{4.5\pi}{r}$ *Substitute.*

$7\pi r = 15.75\pi$ *Find the cross products.*

$r = 2.25$ *Solve for r.*

The radius r is 2.25 ft.

CHECK IT OUT! **2.** The perimeter P of a regular dodecagon varies directly as the side length s, and $P = 18$ in. when $s = 1.5$ in. Find s when $P = 75$ in.

A **joint variation** is a relationship among three variables that can be written in the form $y = kxz$, where k is the constant of variation. For the equation $y = kxz$, y varies jointly as x and z.

EXAMPLE 3 Solving Joint Variation Problems

Geometry

The area A of a triangle varies jointly as the base b and the height h, and $A = 12\text{ m}^2$ when $b = 6$ m and $h = 4$ m. Find b when $A = 36\text{ m}^2$ and $h = 8$ m.

Step 1 Find k.

$A = kbh$ *Joint variation*

$12 = k(6)(4)$ *Substitute.*

$\frac{1}{2} = k$ *Solve for k.*

Step 2 Use the variation function.

$A = \frac{1}{2}bh$ *Use $\frac{1}{2}$ for k.*

$36 = \frac{1}{2}b(8)$ *Substitute.*

$9 = b$ *Solve for b.*

The base b is 9 m.

CHECK IT OUT! **3.** The lateral surface area L of a cone varies jointly as the base radius r and the slant height ℓ, and $L = 63\pi\text{ m}^2$ when $r = 3.5$ m and $\ell = 18$ m. Find r to the nearest tenth when $L = 8\pi\text{ m}^2$ and $\ell = 5$ m.

A third type of variation describes a situation in which one quantity increases and the other decreases. For example, the table shows that the time needed to drive 600 miles decreases as speed increases.

Speed (mi/h)	Time (h)	Distance (mi)
30	20	600
40	15	600
50	12	600

This type of variation is an inverse variation. An **inverse variation** is a relationship between two variables x and y that can be written in the form $y = \frac{k}{x}$, where $k \neq 0$. For the equation $y = \frac{k}{x}$, y varies inversely as x.

EXAMPLE 4

Writing and Graphing Inverse Variation

Given: y varies inversely as x, and $y = 3$ when $x = 8$. Write and graph the inverse variation function.

$y = \frac{k}{x}$ *y varies inversely as x.*

$3 = \frac{k}{8}$ *Substitute 3 for y and 8 for x.*

$k = 24$ *Solve for k.*

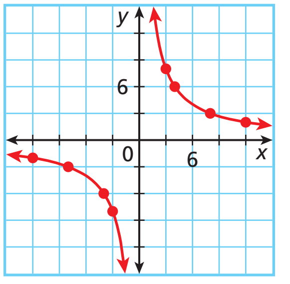
$y = \frac{24}{x}$ *Write the variation function.*

> **Helpful Hint**
> When graphing an inverse variation function, use values of x that are factors of k so that the y-values will be integers.

To graph, make a table of values for both positive and negative values of x. Plot the points, and connect them with two smooth curves. Because division by 0 is undefined, the function is undefined when $x = 0$.

x	y
−3	−8
−4	−6
−8	−3
−12	−2

x	y
3	8
4	6
8	3
12	2



CHECK IT OUT! **4.** Given: y varies inversely as x, and $y = 4$ when $x = 10$. Write and graph the inverse variation function.

When you want to find specific values in an inverse variation problem, you can solve for k and then use substitution or you can use the equation derived below.

$$y_1 = \frac{k}{x_1} \rightarrow y_1x_1 = k \quad \text{and} \quad y_2 = \frac{k}{x_2} \rightarrow y_2x_2 = k \quad \text{so,} \quad y_1x_1 = y_2x_2.$$

EXAMPLE 5

Community Service Application

The time t that it takes for a group of volunteers to construct a house varies inversely as the number of volunteers v. If 20 volunteers can build a house in 62.5 working hours, how many volunteers would be needed to build a house in 50 working hours?

Method 1 Find k.

$t = \frac{k}{v}$

$62.5 = \frac{k}{20}$ *Substitute.*

$1250 = k$ *Solve for k.*

$t = \frac{1250}{v}$ *Use 1250 for k.*

$50 = \frac{1250}{v}$ *Substitute 50 for t.*

$v = 25$ *Solve for v.*

Method 2 Use $t_1v_1 = t_2v_2$.

$t_1v_1 = t_2v_2$

$62.5(20) = 50v$ *Substitute.*

$1250 = 50v$ *Simplify.*

$25 = v$ *Solve for v.*

So 25 volunteers would be needed to build a home in 50 working hours.

CHECK IT OUT! **5. What if...?** How many working hours would it take 15 volunteers to build a house?

You can use algebra to rewrite variation functions in terms of k.

Direct Variation

$$y = kx \rightarrow k = \underbrace{\frac{y}{x}}_{\text{Constant ratio}}$$

Inverse Variation

$$y = \frac{k}{x} \rightarrow k = \underbrace{xy}_{\text{Constant product}}$$

Notice that in direct variation, the *ratio* of the two quantities is constant. In inverse variation, the *product* of the two quantities is constant.

EXAMPLE 6 Identifying Direct and Inverse Variation

Determine whether each data set represents a direct variation, an inverse variation, or neither.

A

x	3	8	10
y	9	24	30

In each case, $\frac{y}{x} = 3$. The ratio is constant, so this represents a direct variation.

B

x	4.5	12	2
y	8	3	18

In each case, $xy = 36$. The product is constant, so this represents an inverse variation.

CHECK IT OUT! **Determine whether each data set represents a direct variation, an inverse variation, or neither.**

6a.

x	3.75	15	5
y	12	3	9

6b.

x	1	40	26
y	0.2	8	5.2

A **combined variation** is a relationship that contains both direct and inverse variation. Quantities that vary directly appear in the numerator, and quantities that vary inversely appear in the denominator.

EXAMPLE 7 *Chemistry Application*

The volume V of a gas varies inversely as the pressure P and directly as the temperature T. A certain gas has a volume of 10 liters (L), a temperature of 300 kelvins (K), and a pressure of 1.5 atmospheres (atm). If the gas is compressed to a volume of 7.5 L and is heated to 350 K, what will the new pressure be?

Step 1 Find k.

$V = \frac{kT}{P}$ *Combined variation*

$10 = \frac{k(300)}{1.5}$ *Substitute.*

$0.05 = k$ *Solve for k.*

Step 2 Use the variation function.

$V = \frac{0.05T}{P}$ *Use 0.05 for k.*

$7.5 = \frac{0.05(350)}{P}$ *Substitute.*

$P = 2.\overline{3}$ *Solve for P.*

The new pressure will be $2.\overline{3}$, or $2\frac{1}{3}$, atm.

Helpful Hint

A kelvin (K) is a unit of temperature that is often used by chemists.

0°C = 273.15 K

100°C = 373.15 K

CHECK IT OUT! **7.** If the gas is heated to 400 K and has a pressure of 1 atm, what is its volume?

THINK AND DISCUSS

1. Explain why the graph of a direct variation is a line.

2. Describe the type of variation between the length and the width of a rectangular room with an area of 400 ft^2.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write the general variation equation, draw a graph, or give an example.

Type of Variation	Equation	Graph	Example
Direct			
Joint			
Inverse			

8-1 Exercises

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Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A variation function in which k is positive and one quantity decreases when the other increases is a(n) __?__. (*direct variation* or *indirect variation*)

SEE EXAMPLE 1 p. 569

Given: y varies directly as x. Write and graph each direct variation function.

2. $y = 6$ when $x = 3$ **3.** $y = 45$ when $x = -5$ **4.** $y = 54$ when $x = 4.5$

SEE EXAMPLE 2 p. 570

5. **Physics** The wavelength λ of a wave of a certain frequency varies directly as the velocity v of the wave, and $\lambda = 60$ ft when $v = 15$ ft/s. Find λ when $v = 3$ ft/s.

6. **Work** The dollar amount d that Julia earns varies directly as the number of hours t that she works, and $d = \$116.25$ when $t = 15$ h. Find t when $d = \$178.25$.

SEE EXAMPLE 3 p. 570

7. **Geometry** The volume V of a rectangular prism of a particular height varies jointly as the length ℓ and the width w, and $V = 224$ ft^3 when $\ell = 8$ ft and $w = 4$ ft. Find ℓ when $V = 210$ ft^3 and $w = 5$ ft.

8. **Economics** The total cost C of electricity for a particular light bulb varies jointly as the time t that the light bulb is used and the cost k per kilowatt-hour, and $C = 12¢$ when $t = 50$ h and $k = 6¢$ per kilowatt-hour. Find C to the nearest cent when $t = 30$ h and $k = 8¢$ per kilowatt-hour.

SEE EXAMPLE 4 p. 571

Given: y varies inversely as x. Write and graph each inverse variation function.

9. $y = 2$ when $x = 7$ **10.** $y = 8$ when $x = 4$ **11.** $y = \frac{1}{2}$ when $x = -10$

SEE EXAMPLE 5 p. 571

12. **Travel** The time t that it takes for a salesman to drive a certain distance d varies inversely as the average speed r. It takes the salesman 4.75 h to travel between two cities at 60 mi/h. How long would the drive take at 50 mi/h?

SEE EXAMPLE 6 p. 572

Determine whether each data set represents a direct variation, an inverse variation, or neither.

13.

x	2	5	9
y	3	6	4

14.

x	6	4	1
y	2	3	12

15.

x	24	4	12
y	30	5	15

SEE EXAMPLE 7
p. 572

16. **Cars** The power P that must be delivered by a car engine varies directly as the distance d that the car moves and inversely as the time t required to move that distance. To move the car 500 m in 50 s, the engine must deliver 147 kilowatts (kW) of power. How many kilowatts must the engine deliver to move the car 700 m in 30 s?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–21	2
22–23	3
24–26	4
27	5
28–30	6
31	7

Extra Practice

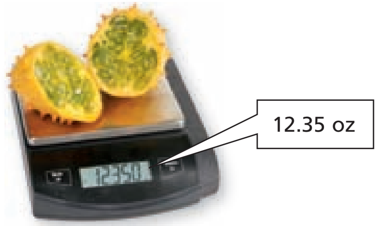
Skills Practice p. S18
Application Practice p. S39

Given: y varies directly as x. Write and graph each direct variation function.

17. $y = 4$ when $x = 8$ **18.** $y = 12$ when $x = 2$ **19.** $y = -15$ when $x = 5$

20. **Medicine** The dosage d of a drug that a physician prescribes varies directly as the patient's mass m, and $d = 100$ mg when $m = 55$ kg. Find d to the nearest milligram when $m = 70$ kg.

21. **Nutrition** The number of Calories C in a horned melon varies directly as its weight w, and $C = 25$ Cal when $w = 3.5$ oz. How many Calories are in the horned melon shown on the scale? Round to the nearest Calorie.



22. **Agriculture** The number of bags of soybean seeds N that a farmer needs varies jointly as the number of acres a to be planted and the pounds of seed needed per acre p, and $N = 980$ when $a = 700$ acres and $p = 70$ lb/acre. Find N when $a = 1000$ acres and $p = 75$ lb/acre.

23. **Physics** The heat Q required to raise the temperature of water varies jointly as the mass m of the water and the amount of temperature change T, and $Q = 20{,}930$ joules (J) when $m = 1$ kg and $T = 5°C$. Find m when $Q = 8372$ J and $T = 10°C$.

LINK

Entertainment

Broadway shows are plays or musicals presented in larger theaters near New York City's Times Square. In 2004, 11.3 million tickets were sold to Broadway shows for a total of almost $750 million.

Given: y varies inversely as x. Write and graph the inverse variation function.

24. $y = 1$ when $x = 0.8$ **25.** $y = 1.75$ when $x = 6$ **26.** $y = -2$ when $x = 3$

27. **Entertainment** The number of days it takes a theater crew to set up a stage for a musical varies inversely as the number of workers. If the stage can be set up in 3 days by 20 workers, how many days would it take if only 12 workers were available?

Determine whether each data set represents a direct variation, an inverse variation, or neither.

28.

x	5	6.25	10
y	5	4	2.5

29.

x	5	7	9
y	3	5	7

30.

x	8	14	24
y	12	21	36

31. **Chemistry** The volume V of a gas varies inversely as the pressure P and directly as the temperature T. A certain gas has a volume of 20 L, a temperature of 320 K, and a pressure of 1 atm. If the gas is compressed to a volume of 15 L and is heated to 330 K, what will the new pressure be?

Tell whether each statement is sometimes, always, or never true.

32. Direct variation is a linear function.

33. A linear function is a direct variation.

34. An inverse variation is a linear function.

35. In a direct variation, $x = 0$ when $y = 0$.

36. The graph of an inverse variation passes through the origin.

MULTI-STEP TEST PREP

37. This problem will prepare you for the Multi-Step Test Prep on page 608.

In an auto race, a car with an average speed of 200 mi/h takes an average of 31.5 s to complete one lap of the track.

a. Write an inverse variation function that gives the average speed s of a car in miles per hour as a function of the time t in seconds needed to complete one lap.

b. How many seconds does it take the car to complete one lap at an average speed of 210 mi/h?

38. Data Collection Use a graphing calculator, a motion detector, and a light detector to measure the intensity of light as distance from the light source increases. Position the detectors next to each other. Place a flashlight in front of the detectors, and then pull the flashlight away from them. Find an appropriate model for the intensity of the light as a function of the square of the distance from the light source.

39. Multi-Step Interest earned on a certificate of deposit (CD) at a certain rate varies jointly as the principal in dollars and the time in years.

a. Diane purchased a CD for $2500 that earned $12.50 simple interest in 3 months. Write a variation function for this data.

b. At which bank did Diane buy her CD?

c. How much interest would Diane earn in 6 months on a $3000 CD bought from the same bank?

Complete each table.

40. y varies jointly as x and z.

x	y	z
2		4
5	52.5	7
1.5	−36	
	1.38	23

41. y varies directly as x and inversely as z.

x	y	z
25	13.75	4
	1	11
17	18.7	
10		5

42. Estimation Shane swims 42 laps in 26 min 19 s. Without using a calculator, estimate how many minutes it would take Shane to swim 15 laps at the same average speed.

43. Critical Thinking Explain why only one point (x, y) is needed to write a direct variation function whose graph passes through this point.

44. Write About It Explain how to identify the type of variation from a list of ordered pairs.

TEST PREP

45. Which of the following would best be represented by an inverse variation function?

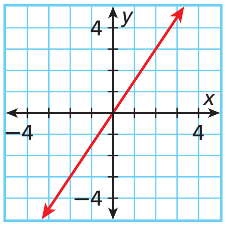
Ⓐ The distance traveled as a function of speed

Ⓑ The total cost as a function of the number of items purchased

Ⓒ The area of a circular swimming pool as a function of its radius

Ⓓ The number of posts in a 20-ft fence as a function of distance between posts

46. Which statement is best represented by the graph?



Ⓕ y varies directly as x^2.

Ⓖ y varies inversely as x.

Ⓗ y varies directly as x.

Ⓙ x varies inversely as y.

47. Which equation is best represented by the following statement: y varies directly as the square root of x?

Ⓐ $y = \frac{k}{\sqrt{x}}$ Ⓑ $y = \frac{k}{x^2}$ Ⓒ $k = \sqrt{xy}$ Ⓓ $y = k\sqrt{x}$

48. **Gridded Response** The cost per student of a ski trip varies inversely as the number of students who attend. It will cost each student \$250 if 24 students attend. How many students would have to attend to get the cost down to \$200?

CHALLENGE AND EXTEND

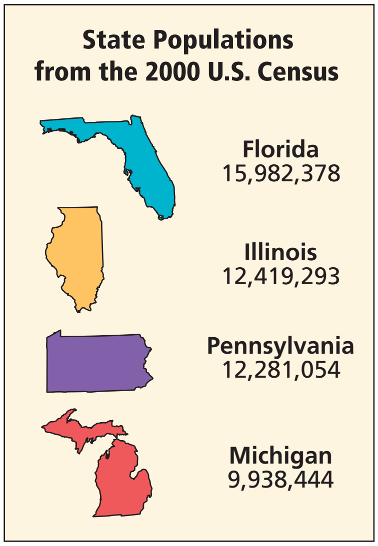
49. Given: y varies jointly as x and the square of z, and $y = 189$ when $x = 7$ and $z = 9$. Find y when $x = 2$ and $z = 6$.

50. **Government** The number of U.S. Representatives that each state receives can be approximated with a direct variation function where the number of representatives (rounded to the nearest whole number) varies directly with the state's population.

a. Given that Pennsylvania has 19 representatives, find k to eight decimal places and write the direct variation function.

b. Find the number of representatives for each state shown.

c. Given that Texas had 32 U.S. representatives in the year 2000, estimate the state's population in that year.



51. **Estimation** Given: y is inversely proportional to x, directly proportional to z^2, and the constant of variation is 7π. Estimate the value of y when $x = 12$ and $z = 2$.

SPIRAL REVIEW

52. **Architecture** Brad stands next to the Eiffel Tower. He is 6 ft 8 in. tall and casts a shadow of 9 ft 4 in. The Eiffel Tower is 985 ft tall. How long is the Eiffel Tower's shadow, in feet? *(Lesson 2-2)*

Write an equation of the line that includes the points in the table. *(Lesson 2-4)*

53.

x	1	3	5	7	9
y	1.5	4	6.5	9	11.5

54.

x	−4	−2	0	2	4
y	10	9	8	7	6

Make a table of values, and graph the exponential function. Describe the asymptote. Tell how the graph is transformed from the graph of $f(x) = 4^x$. *(Lesson 7-7)*

55. $g(x) = \frac{1}{2}(4^x) - 2$

56. $h(x) = (4^{x-1}) + 1$

8-2 Multiplying and Dividing Rational Expressions

Objectives
Simplify rational expressions.

Multiply and divide rational expressions.

Vocabulary
rational expression

Why learn this?
You can simplify rational expressions to determine the probability of hitting an archery target. (See Exercise 35.)

In Lesson 8-1, you worked with inverse variation functions such as $y = \frac{5}{x}$. The expression on the right side of this equation is a *rational expression*. A **rational expression** is a quotient of two polynomials. Other examples of rational expressions include the following:

$$\frac{x^2 - 4}{x + 2} \qquad \frac{10}{x^2 - 6} \qquad \frac{x + 3}{x - 7}$$

Because rational expressions are ratios of polynomials, you can simplify them the same way as you simplify fractions. Recall that to write a fraction in simplest form, you can divide out common factors in the numerator and denominator.

$$\frac{9}{24} = \frac{3 \cdot \cancel{3}}{8 \cdot \cancel{3}} = \frac{3}{8}$$

EXAMPLE 1 Simplifying Rational Expressions

Simplify. Identify any x-values for which the expression is undefined.

A $\frac{3x^7}{2x^4}$

$$\frac{3}{2}x^{7-4} = \frac{3}{2}x^3 \qquad \textit{Quotient of Powers Property}$$

The expression is undefined at $x = 0$ because this value of x makes $2x^4$ equal 0.

B $\frac{x^2 - 2x - 3}{x^2 + 5x + 4}$

$$\frac{(x-3)\cancel{(x+1)}}{\cancel{(x+1)}(x+4)} = \frac{(x-3)}{(x+4)} \qquad \textit{Factor; then divide out common factors.}$$

The expression is undefined at $x = -1$ and $x = -4$ because these values of x make the factors $(x + 1)$ and $(x + 4)$ equal 0.

Check Substitute $x = -1$ and $x = -4$ into the original expression.

$$\frac{(-1)^2 - 2(-1) - 3}{(-1)^2 + 5(-1) + 4} = \frac{0}{0} \qquad \frac{(-4)^2 - 2(-4) - 3}{(-4)^2 + 5(-4) + 4} = \frac{21}{0}$$

Both values of x result in division by 0, which is undefined.

Caution!
When identifying values for which a rational expression is undefined, identify the values of the variable that make the original denominator equal to 0.

CHECK IT OUT! **Simplify. Identify any x-values for which the expression is undefined.**

1a. $\frac{16x^{11}}{8x^2}$ **1b.** $\frac{3x + 4}{3x^2 + x - 4}$ **1c.** $\frac{6x^2 + 7x + 2}{6x^2 - 5x - 6}$

EXAMPLE 2 Simplifying by Factoring −1

Simplify $\frac{2x - x^2}{x^2 - x - 2}$. Identify any x-values for which the expression is undefined.

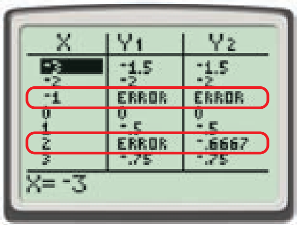
$$\frac{-1(x^2 - 2x)}{x^2 - x - 2}$$ *Factor out –1 in the numerator so that x^2 is positive, and reorder the terms.*

$$\frac{-1(x)\cancel{(x - 2)}}{\cancel{(x - 2)}(x + 1)}$$ *Factor the numerator and denominator. Divide out common factors.*

$$\frac{-x}{x + 1}$$ *Simplify.*

The expression is undefined at $x = 2$ and $x = -1$.

Check The calculator screens suggest that $\frac{2x - x^2}{x^2 - x - 2} = \frac{-x}{x + 1}$ except when $x = 2$ or $x = -1$.



CHECK IT OUT! **Simplify. Identify any x-values for which the expression is undefined.**

2a. $\frac{10 - 2x}{x - 5}$ **2b.** $\frac{-x^2 + 3x}{2x^2 - 7x + 3}$

You can multiply rational expressions the same way that you multiply fractions.

Know it! Note

Multiplying Rational Expressions
1. Factor all numerators and denominators completely.
2. Divide out common factors of the numerators and denominators.
3. Multiply numerators. Then multiply denominators.
4. Be sure the numerator and denominator have no common factors other than 1.

EXAMPLE 3 Multiplying Rational Expressions

Multiply. Assume that all expressions are defined.

A $\frac{2x^4y^5}{3x^2} \cdot \frac{15x^2}{8x^3y^2}$

$$\frac{2\cancel{x^4}^{1}\cancel{y^5}^{3}}{\cancel{3}\cancel{x^2}} \cdot \frac{^{5}\cancel{15}\cancel{x^2}}{_{4}\cancel{8}\cancel{x^3}\cancel{y^2}}$$

$$\frac{5xy^3}{4}$$

B $\frac{x + 2}{3x + 12} \cdot \frac{x + 4}{x^2 - 4}$

$$\frac{\cancel{x + 2}}{3\cancel{(x + 4)}} \cdot \frac{\cancel{x + 4}}{\cancel{(x + 2)}(x - 2)}$$

$$\frac{1}{3(x - 2)} \text{ or } \frac{1}{3x - 6}$$

CHECK IT OUT! **Multiply. Assume that all expressions are defined.**

3a. $\frac{x}{15} \cdot \frac{x^7}{2x} \cdot \frac{20}{x^4}$ **3b.** $\frac{10x - 40}{x^2 - 6x + 8} \cdot \frac{x + 3}{5x + 15}$

You can also divide rational expressions. Recall that to divide by a fraction, you multiply by its reciprocal.

$$\frac{1}{2} \div \frac{3}{4} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{4}^{2}}{3} = \frac{2}{3}$$

EXAMPLE 4 Dividing Rational Expressions

Divide. Assume that all expressions are defined.

A $\dfrac{4x^3}{9x^2y} \div \dfrac{16}{9y^5}$

$$\frac{4x^3}{9x^2y} \cdot \frac{9y^5}{16}$$

Rewrite as multiplication by the reciprocal.

$$\frac{\cancel{4}x^{\cancel{3}1}}{\cancel{9}\cancel{x^2}y} \cdot \frac{\cancel{9}y^{\cancel{5}4}}{\cancel{16}_4}$$

$$\frac{xy^4}{4}$$

B $\dfrac{x^5 - 4x^3}{x^2 - x - 2} \div \dfrac{x^5 - x^4 - 2x^3}{x^2 - 1}$

$$\frac{x^5 - 4x^3}{x^2 - x - 2} \cdot \frac{x^2 - 1}{x^5 - x^4 - 2x^3}$$

$$\frac{x^3(x^2 - 4)}{x^2 - x - 2} \cdot \frac{x^2 - 1}{x^3(x^2 - x - 2)}$$

$$\frac{\cancel{x^3}\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} \cdot \frac{(x-1)\cancel{(x+1)}}{\cancel{x^3}(x-2)\cancel{(x+1)}}$$

$$\frac{(x+2)(x-1)}{(x+1)(x-2)} \text{ or } \frac{x^2 + x - 2}{x^2 - x - 2}$$

CHECK IT OUT! **Divide. Assume that all expressions are defined.**

4a. $\dfrac{x^2}{4} \div \dfrac{x^4y}{12y^2}$ **4b.** $\dfrac{2x^2 - 7x - 4}{x^2 - 9} \div \dfrac{4x^2 - 1}{8x^2 - 28x + 12}$

EXAMPLE 5 Solving Simple Rational Equations

Solve. Check your solution.

A $\dfrac{x^2 - 9}{x + 3} = 7$

$$\frac{(x-3)\cancel{(x+3)}}{\cancel{x+3}} = 7$$

Note that $x \neq -3$.

$$x - 3 = 7$$

$$x = 10$$

Check

$\dfrac{x^2 - 9}{x + 3}$	$= 7$
$\dfrac{(10)^2 - 9}{10 + 3}$	7
$\dfrac{91}{13}$	7
7	7 ✔

B $\dfrac{x^2 + 3x - 4}{x - 1} = 5$

$$\frac{\cancel{(x-1)}(x+4)}{\cancel{x-1}} = 5$$

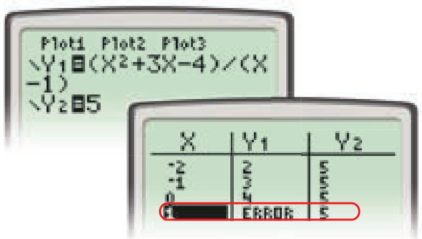
Note that $x \neq 1$.

$$x + 4 = 5$$

$$x = 1$$

Because the left side of the original equation is undefined when $x = 1$, there is no solution.

Check A graphing calculator shows that 1 is not a solution.



Caution!

As you simplify a rational expression, take note of values that must be excluded. The excluded values are those that make the rational expression undefined.

CHECK IT OUT! **Solve. Check your solution.**

5a. $\dfrac{x^2 + x - 12}{x + 4} = -7$ **5b.** $\dfrac{4x^2 - 9}{(2x + 3)} = 5$

THINK AND DISCUSS

1. Explain how you find undefined values for a rational expression.

2. Explain why it is important to check solutions to rational equations.

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write a worked-out example.

	Numerical Fractions	Rational Expressions
Simplifying		
Multiplying		
Dividing		

8-2 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 8-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How can you tell if an algebraic expression is a *rational expression*?

SEE EXAMPLE 1 p. 577

Simplify. Identify any x-values for which the expression is undefined.

2. $\dfrac{4x^6}{2x-6}$ **3.** $\dfrac{6x^2+13x-5}{6x^2-23x+7}$ **4.** $\dfrac{x+4}{3x^2+11x-4}$

SEE EXAMPLE 2 p. 578

5. $\dfrac{-x-4}{x^2-x-20}$ **6.** $\dfrac{6x^2+7x-3}{-3x^2+x}$ **7.** $\dfrac{6x^3+6x}{x^2+1}$

SEE EXAMPLE 3 p. 578

Multiply. Assume that all expressions are defined.

8. $\dfrac{x-2}{2x-3}\cdot\dfrac{4x-6}{x^2-4}$ **9.** $\dfrac{x-2}{x-3}\cdot\dfrac{2x-6}{x+5}$ **10.** $\dfrac{x^2-16}{x^2-4x+4}\cdot\dfrac{x-2}{x^2+6x+8}$

SEE EXAMPLE 4 p. 579

Divide. Assume that all expressions are defined.

11. $\dfrac{x^5y^4}{3xy}\div\dfrac{1}{x^3y}$ **12.** $\dfrac{x+3}{x^2-2x+1}\div\dfrac{x+3}{x-1}$

13. $\dfrac{x^2-25}{2x^2+5x-12}\div\dfrac{x^2-3x-10}{x^2+9x+20}$ **14.** $\dfrac{x^2+2x+1}{x^2-3x-18}\div\dfrac{x^2-1}{x^2-7x+6}$

SEE EXAMPLE 5 p. 579

Solve. Check your solution.

15. $\dfrac{16x^2-9}{4x+3}=-6$ **16.** $\dfrac{2x^2+7x-15}{2x-3}=10$ **17.** $\dfrac{x^2-4}{x-2}=1$

PRACTICE AND PROBLEM SOLVING

Simplify. Identify any x-values for which the expression is undefined.

18. $\dfrac{4x-8}{x^2-2x}$ **19.** $\dfrac{8x-4}{2x^2+9x-5}$ **20.** $\dfrac{x^2-36}{x^2-12x+36}$

21. $\dfrac{3x+18}{24-2x-x^2}$ **22.** $\dfrac{-2x^2-9x}{4x^2-81}$ **23.** $\dfrac{4x+20}{-5-x}$

Independent Practice

For Exercises	See Example
18–20	1
21–23	2
24–27	3
28–31	4
32–34	5

Extra Practice

Skills Practice p. S18
Application Practice p. S39

Multiply. Assume that all expressions are defined.

24. $\frac{x^2y}{4xy} \cdot \frac{x}{6} \cdot \frac{3y^5}{x^4}$

25. $\frac{x-4}{x-3} \cdot \frac{2x-1}{x+4}$

26. $\frac{x^2-2x-8}{9x^2-16} \cdot \frac{3x^2+10x+8}{x^2-16}$

27. $\frac{4x^2-20x+25}{x^2-4x} \cdot \frac{3x-12}{2x-5}$

Divide. Assume that all expressions are defined.

28. $\frac{4x^2+15x+9}{8x^2+10x+3} \div \frac{x^2+4x}{2x+1}$

29. $\frac{x^2-4x-5}{x^2-3x+2} \div \frac{x^2-3x-10}{x^2-4}$

30. $\frac{x+2}{x-4} \div \frac{1}{3x-12}$

31. $\frac{x^2-2x-3}{x^2-x-2} \div \frac{x^2+2x-15}{x^2+x-6}$

Solve. Check your solution.

32. $\frac{3x^2+10x+8}{-x-2} = -2$

33. $\frac{x^2-9}{x-3} = 5$

34. $\frac{x^2+3x-28}{(x+7)(x-4)} = -11$

LINK

Archery

Archery was practiced in ancient times on every inhabited continent except Australia. The painting in the photo above dates from about 1400 B.C.E. and shows archers from ancient Egypt.

35. Archery An archery target consists of an inner circle and four concentric rings. The width of each ring is equal to the radius r of the inner circle. Write a rational expression in terms of r that represents the probability that an arrow hitting the target at random will land in the inner circle. Then simplify the expression.

Multiply or divide. Assume that all expressions are defined.

36. $\frac{2x}{3} \cdot \frac{x^3}{6x-8}$

37. $\frac{4x^2-3x}{4x^2-1} \cdot \frac{2x+1}{x}$

38. $\frac{1}{25x^2-49} \div \frac{x}{10x-14}$

39. $2xy \cdot \frac{2x^2}{y} \cdot \frac{y^2}{2x}$

40. $\frac{14x^4}{xy} \cdot \frac{x^3}{6y^3} \div \frac{5x^2}{12y^5}$

41. $(y+4) \div \frac{4x+4+xy+y}{3}$

42. Critical Thinking What polynomial completes the equation $\frac{x-5}{x-2} \cdot \frac{\blacksquare}{x-5} = x+1$?

43. Geometry Use the table to determine the following.

	Square Prism	Cylinder
Area of Base	s^2	πr^2
Volume	s^2h	$\pi r^2 h$
Surface Area	$2s^2 + 4sh$	$2\pi r^2 + 2\pi rh$

a. For each figure, find the ratio of the volume to the area of the base.

b. For each figure, find the ratio of the surface area to the volume.

c. What if...? If the radius and the height of a cylinder were doubled, what effect would this have on the ratio of the cylinder's surface area to its volume?

MULTI-STEP TEST PREP

44. This problem will prepare you for the Multi-Step Test Prep on page 608.

For a car moving with initial speed v_0 and acceleration a, the distance d that the car travels in time t is given by $d = v_0t + \frac{1}{2}at^2$.

a. Write a rational expression in terms of t for the average speed of the car during a period of acceleration. Simplify the expression.

b. During a race, a driver accelerates for 3 s at a rate of 10 ft/s^2 in order to pass another car. The driver's initial speed was 264 ft/s. What was the driver's average speed during the acceleration?

45. **ERROR ANALYSIS** Two students simplified the same expression. Which is incorrect? Explain the error.

A

$$\frac{x+9}{x^2-81}$$
$$\frac{\cancel{x+9}}{(x-9)\cancel{(x+9)}}$$
$$x-9$$

B

$$\frac{x+9}{x^2-81}$$
$$\frac{\cancel{x+9}}{(x-9)\cancel{(x+9)}}$$
$$\frac{1}{x-9}$$

46. **Write About It** You can use polynomial division to find that $\frac{x^3-7x+6}{x-2} = x^2+2x-3$. Is this equation true for all values of x? Explain.

TEST PREP

47. For which values of x is the expression $\dfrac{x^2-x-12}{x^2+x-2}$ undefined?

Ⓐ 0 and 1 Ⓑ 1 and 2 Ⓒ −1 and 2 Ⓓ −2 and 1

48. Assume that all expressions are defined. Which expression is equivalent to $\frac{x^2+7x+10}{x^2-6x} \div \frac{x^3-4x}{x^2-8x+12}$?

Ⓕ $\dfrac{x+5}{x^2}$ Ⓖ $\dfrac{x^2}{x+5}$ Ⓗ $\dfrac{(x+5)(x+2)^2}{(x-6)^2}$ Ⓙ $\dfrac{(x-6)^2}{(x+5)(x+2)^2}$

49. The area of a rectangle is equal to $x^2+13x+36$ square units. If the length of the rectangle is equal to $x+9$ units, which expression represents its width?

Ⓐ $x+4$ Ⓑ $x+27$ Ⓒ x^2+4 Ⓓ x^2+27

CHALLENGE AND EXTEND

Multiply or divide. Assume that all expressions are defined.

50. $\dfrac{8x^3-1}{x+2} \cdot \dfrac{x^2-4}{2x^2-5x+2}$

51. $\dfrac{2x^2-50}{x^3+125} \cdot \dfrac{x^3-125}{x^2-10x+25}$

52. $\dfrac{x^2-16}{x-3} \div \left(\dfrac{x^2-9}{x+4}\right)^{-1}$

53. $\dfrac{x^5-4x^3-x^2+4}{x^3-2x^2+x-2} \div \dfrac{3x^3+3x^2+3x}{x^2-1} \cdot \dfrac{6x}{x^2-2x+1}$

SPIRAL REVIEW

Find each product. *(Lesson 6-2)*

54. $6x^2(x^4-2)$

55. $(x+5)(3x^2-7x-1)$

56. $8x^2y^3(xy^2-4x+7y)$

57. **Biology** The table shows the number of births in a population of mice over a 5-year period. Find an exponential model for the data. Use the model to estimate the number of births in the 8th year. *(Lesson 7-8)*

Years	1	2	3	4	5
Births	70,000	120,000	170,000	220,400	271,100

Given: y varies directly as x. Write and graph each direct variation function. *(Lesson 8-1)*

58. $y = 7$ when $x = 14$

59. $y = 8$ when $x = -2$

8-3 Adding and Subtracting Rational Expressions

Objectives
Add and subtract rational expressions.
Simplify complex fractions.

Vocabulary
complex fraction

Why learn this?
You can add and subtract rational expressions to estimate a train's average speed. (See Example 6.)

Adding and subtracting rational expressions is similar to adding and subtracting fractions. To add or subtract rational expressions with like denominators, add or subtract the numerators and use the same denominator.

$$\frac{1}{5} + \frac{3}{5} = \frac{4}{5} \qquad \frac{6}{7} - \frac{4}{7} = \frac{2}{7}$$

EXAMPLE 1 Adding and Subtracting Rational Expressions with Like Denominators

Add or subtract. Identify any x-values for which the expression is undefined.

A $\dfrac{3x-4}{x+3} + \dfrac{2x+5}{x+3}$

$\dfrac{3x-4+2x+5}{x+3}$ *Add the numerators.*

$\dfrac{5x+1}{x+3}$ *Combine like terms.*

The expression is undefined at $x = -3$ because this value makes $x + 3$ equal 0.

B $\dfrac{2x-1}{x^2+2} - \dfrac{4x+4}{x^2+2}$

$\dfrac{2x-1-(4x+4)}{x^2+2}$ *Subtract the numerators.*

$\dfrac{2x-1-4x-4}{x^2+2}$ *Distribute the negative sign.*

$\dfrac{-2x-5}{x^2+2}$ *Combine like terms.*

There is no real value of x for which $x^2 + 2 = 0$; the expression is always defined.

CHECK IT OUT! **Add or subtract. Identify any x-values for which the expression is undefined.**

1a. $\dfrac{6x+5}{x^2-3} + \dfrac{3x-1}{x^2-3}$ **1b.** $\dfrac{3x^2-5}{3x-1} - \dfrac{2x^2-3x-2}{3x-1}$

To add or subtract rational expressions with unlike denominators, first find the least common denominator (LCD). The LCD is the least common multiple of the polynomials in the denominators.

Know it! Note

Least Common Multiple (LCM) of Polynomials
To find the LCM of polynomials:
1. Factor each polynomial completely. Write any repeated factors as powers. For example, $x^3 + 6x^2 + 9x = x(x+3)^2$.
2. List the different factors. If the polynomials have common factors, use the highest power of each common factor.

EXAMPLE 2 Finding the Least Common Multiple of Polynomials

Find the least common multiple for each pair.

A $2x^3y^4$ and $3x^5y^3$

$$2x^3y^4 = 2 \cdot x^3 \cdot y^4$$
$$3x^5y^3 = 3 \cdot x^5 \cdot y^3$$

The LCM is $2 \cdot 3 \cdot x^5 \cdot y^4$, or $6x^5y^4$.

B $x^2 + 3x - 4$ and $x^2 - 3x + 2$

$$x^2 + 3x - 4 = (x+4)(x-1)$$
$$x^2 - 3x + 2 = (x-2)(x-1)$$

The LCM is $(x+4)(x-1)(x-2)$.

CHECK IT OUT! **Find the least common multiple for each pair.**

2a. $4x^3y^7$ and $3x^5y^4$ **2b.** $x^2 - 4$ and $x^2 + 5x + 6$

To add rational expressions with unlike denominators, rewrite both expressions with the LCD. This process is similar to adding fractions.

$$\frac{2}{6} + \frac{3}{10} = \frac{2}{2 \cdot 3}\left(\frac{5}{5}\right) + \frac{3}{2 \cdot 5}\left(\frac{3}{3}\right)$$
$$= \frac{10}{30} + \frac{9}{30} = \frac{19}{30}$$

EXAMPLE 3 Adding Rational Expressions

Add. Identify any x-values for which the expression is undefined.

A $\dfrac{x-1}{x^2+3x+2} + \dfrac{x}{x+1}$

$$\frac{x-1}{(x+2)(x+1)} + \frac{x}{x+1}$$ *Factor the denominators.*

$$\frac{x-1}{(x+2)(x+1)} + \frac{x}{x+1}\left(\frac{x+2}{x+2}\right)$$ *The LCD is $(x+2)(x+1)$, so multiply $\frac{x}{x+1}$ by $\frac{x+2}{x+2}$.*

$$\frac{x-1+x(x+2)}{(x+2)(x+1)}$$ *Add the numerators.*

$$\frac{x^2+3x-1}{(x+2)(x+1)}$$ *Simplify the numerator.*

$$\frac{x^2+3x-1}{(x+2)(x+1)} \text{ or } \frac{x^2+3x-1}{x^2+3x+2}$$ *Write the sum in factored or expanded form.*

The expression is undefined at $x = -2$ and $x = -1$ because these values of x make the factors $(x+2)$ and $(x+1)$ equal 0.

Remember!

Multiplying by 1 or a form of 1, such as $\frac{x+2}{x+2}$, does not change the value of an expression.

Add. Identify any x-values for which the expression is undefined.

B $\frac{x}{x+3} + \frac{-18}{x^2-9}$

$$\frac{x}{x+3} + \frac{-18}{(x+3)(x-3)}$$ *Factor the denominators.*

$$\frac{x}{x+3}\left(\frac{x-3}{x-3}\right) + \frac{-18}{(x+3)(x-3)}$$ *The LCD is $(x+3)(x-3)$, so multiply $\frac{x}{x+3}$ by $\frac{x-3}{x-3}$.*

$$\frac{x(x-3)+(-18)}{(x+3)(x-3)}$$ *Add the numerators.*

$$\frac{x^2-3x-18}{(x+3)(x-3)}$$ *Write the numerator in standard form.*

$$\frac{\cancel{(x+3)}(x-6)}{\cancel{(x+3)}(x-3)}$$ *Factor the numerator.*

$$\frac{x-6}{x-3}$$ *Divide out common factors.*

The expression is undefined at $x = -3$ and $x = 3$ because these values of x make the factors $(x+3)$ and $(x-3)$ equal 0.

Helpful Hint

Always simplify the sum or difference of rational expressions. A rational expression can be simplified if its numerator and denominator contain the same factor.

CHECK IT OUT! **Add. Identify any x-values for which the expression is undefined.**

3a. $\frac{3x}{2x-2} + \frac{3x-2}{3x-3}$ **3b.** $\frac{x}{x+3} + \frac{2x+6}{x^2+6x+9}$

EXAMPLE 4 Subtracting Rational Expressions

Subtract $\frac{2x^2-16}{x^2-4} - \frac{x+4}{x+2}$. Identify any x-values for which the expression is undefined.

$$\frac{2x^2-16}{(x-2)(x+2)} - \frac{x+4}{x+2}$$ *Factor the denominators.*

$$\frac{2x^2-16}{(x-2)(x+2)} - \frac{x+4}{x+2}\left(\frac{x-2}{x-2}\right)$$ *The LCD is $(x-2)(x+2)$, so multiply $\frac{x+4}{x+2}$ by $\frac{x-2}{x-2}$.*

$$\frac{2x^2-16-(x+4)(x-2)}{(x-2)(x+2)}$$ *Subtract the numerators.*

$$\frac{2x^2-16-(x^2+2x-8)}{(x-2)(x+2)}$$ *Multiply the binomials in the numerator.*

$$\frac{2x^2-16-x^2-2x+8}{(x-2)(x+2)}$$ *Distribute the negative sign.*

$$\frac{x^2-2x-8}{(x-2)(x+2)}$$ *Write the numerator in standard form.*

$$\frac{(x-4)\cancel{(x+2)}}{(x-2)\cancel{(x+2)}}$$ *Factor the numerator.*

$$\frac{x-4}{x-2}$$ *Divide out common factors.*

The expression is undefined at $x = 2$ and $x = -2$ because these values of x make the factors $(x-2)$ and $(x+2)$ equal 0.

Subtract. Identify any x-values for which the expression is undefined.

4a. $\frac{3x-2}{2x+5} - \frac{2}{5x-2}$ **4b.** $\frac{2x^2+64}{x^2-64} - \frac{x-4}{x+8}$

Some rational expressions are *complex fractions*. A **complex fraction** contains one or more fractions in its numerator, its denominator, or both. Examples of complex fractions are shown below.

$$\frac{x+2}{\frac{3}{x}} \qquad \frac{1+\frac{1}{x}}{4x+5} \qquad \frac{\frac{x+3}{x}}{\frac{x+4}{7x}}$$

Recall that the bar in a fraction represents division. Therefore, you can rewrite a complex fraction as a division problem and then simplify. You can also simplify complex fractions by using the LCD of the fractions in the numerator and denominator.

EXAMPLE 5 Simplifying Complex Fractions

Simplify $\frac{\frac{2}{x}+\frac{x}{4}}{\frac{x+1}{x}}$. Assume that all expressions are defined.

Method 1 Write the complex fraction as division.

$\left(\frac{2}{x}+\frac{x}{4}\right) \div \frac{x+1}{x}$ *Write as division.*

$\left(\frac{2}{x}+\frac{x}{4}\right) \cdot \frac{x}{x+1}$ *Multiply by the reciprocal.*

$\left[\frac{2}{x}\left(\frac{4}{4}\right)+\frac{x}{4}\left(\frac{x}{x}\right)\right] \cdot \frac{x}{x+1}$ *The LCD is 4x.*

$\left[\frac{2(4)+x(x)}{4x}\right] \cdot \frac{x}{x+1}$ *Add the numerators.*

$\frac{8+x^2}{4\cancel{x}} \cdot \frac{\cancel{x}}{x+1}$ *Simplify and divide out common factors.*

$\frac{8+x^2}{4(x+1)}$ or $\frac{x^2+8}{4x+4}$ *Multiply.*

Method 2 Multiply the numerator and denominator of the complex fraction by the LCD of the fractions in the numerator and denominator.

$\frac{\frac{2}{\cancel{x}}(4\cancel{x})+\frac{x}{\cancel{4}}(\cancel{4}x)}{\frac{x+1}{\cancel{x}}(4\cancel{x})}$ *The LCD is 4x.*

$\frac{(2)(4)+(x)(x)}{(x+1)(4)}$ *Divide out common factors.*

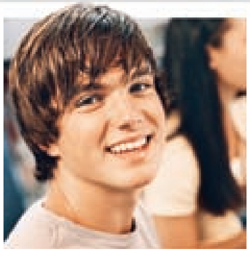
$\frac{8+x^2}{(x+1)(4)}$ or $\frac{x^2+8}{4x+4}$ *Simplify.*

Simplify. Assume that all expressions are defined.

5a. $\frac{\frac{x+1}{x^2-1}}{\frac{x}{x-1}}$ **5b.** $\frac{\frac{20}{x-1}}{\frac{6}{3x-3}}$ **5c.** $\frac{\frac{1}{x}+\frac{1}{2x}}{\frac{x+4}{x-2}}$

Student to Student

Simplifying Complex Fractions



Brian Carr
Riverside High School

When I simplify complex fractions, I draw arrows connecting the outermost and innermost numbers.

Then I make a new fraction by writing the product of the outermost numbers as the numerator and the product of the innermost numbers as the denominator.

$$\frac{\frac{3}{5}}{\frac{1}{2}} = \frac{6}{5} = 1\tfrac{1}{5}$$

It also works for rational expressions.

$$\frac{\frac{x^2}{x+1}}{\frac{x}{2}} = \frac{2x^2}{x(x+1)} = \frac{2x}{x+1}$$

EXAMPLE 6 ***Transportation Application***

A freight train averages 30 mi/h traveling to its destination with full cars and 40 mi/h on the return trip with empty cars. What is the train's average speed for the entire trip? Round to the nearest tenth.

Total distance: $2d$ — *Let d represent the one-way distance.*

Total time: $\frac{d}{30} + \frac{d}{40}$ — *Use the formula $t = \frac{d}{r}$.*

Average speed: $\dfrac{2d}{\frac{d}{30} + \frac{d}{40}}$ — *The average speed is $\frac{\text{total distance}}{\text{total time}}$.*

$\dfrac{2d(120)}{\frac{d}{30}(120) + \frac{d}{40}(120)}$ — *The LCD of the fractions in the denominator is 120.*

$\dfrac{240d}{4d + 3d}$ — *Simplify.*

$\dfrac{240\cancel{d}}{7\cancel{d}} \approx 34.3$ — *Combine like terms and divide out common factors.*

The train's average speed is 34.3 mi/h.

CHECK IT OUT!

6. Justin's average speed on his way to school is 40 mi/h, and his average speed on the way home is 45 mi/h. What is Justin's average speed for the entire trip? Round to the nearest tenth.

THINK AND DISCUSS

Know it! Note

1. Explain how to find the LCD of two rational expressions.

2. GET ORGANIZED Copy and complete the graphic organizer. In each box, write an example and show how to simplify it.

Rational Expressions

- Adding (like denominators)
- Subtracting (unlike denominators)
- Simplifying a complex fraction

8-3 Exercises

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Homework Help Online
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Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How does a *complex fraction* differ from other types of fractions?

SEE EXAMPLE 1 p. 583

Add or subtract. Identify any x-values for which the expression is undefined.

2. $\frac{2x-3}{4x-1}+\frac{3x+4}{4x-1}$
3. $\frac{3x-4}{4x+5}-\frac{5x+3}{4x+5}$
4. $\frac{4x-3}{2x-5}-\frac{4x+3}{2x-5}$

SEE EXAMPLE 2 p. 584

Find the least common multiple for each pair.

5. $4x^2y^3$ and $16x^4y$
6. x^2-25 and $x^2+10x+25$

SEE EXAMPLE 3 p. 584

Add or subtract. Identify any x-values for which the expression is undefined.

7. $\frac{3x-2}{x+6}+\frac{2x-3}{2x-1}$
8. $\frac{4x-5}{12x+4}+\frac{3x-1}{3x+1}$
9. $\frac{3x-4}{x^2-9}+\frac{2x-1}{x+3}$

SEE EXAMPLE 4 p. 585

10. $\frac{3x-5}{2x-5}-\frac{2x-5}{3x+1}$
11. $\frac{2x+8}{x^2-16}-\frac{3}{x-4}$
12. $\frac{x+2}{x^2+4x+3}-\frac{x+1}{x+3}$

SEE EXAMPLE 5 p. 586

Simplify. Assume that all expressions are defined.

13. $\dfrac{\frac{2x-3}{x-2}}{\frac{4x-3}{x^2-4}}$
14. $\dfrac{\frac{3x-7}{4x+5}}{\frac{6x-1}{5x-6}}$
15. $\dfrac{\frac{2}{x}+\frac{1}{x}}{\frac{2x}{x+2}}$

SEE EXAMPLE 6 p. 587

16. **Track** Yvette ran at an average speed of 6.20 ft/s during the first two laps of a race and an average speed of 7.75 ft/s during the second two laps of a race. What was Yvette's average speed for the entire race? Round to the nearest tenth.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–21	2
22–24	3
25–27	4
28–30	5
31	6

Extra Practice
Skills Practice p. S18
Application Practice p. S39

Add or subtract. Identify any x-values for which the expression is undefined.

17. $\frac{2x-3}{4x-7}+\frac{2x-3}{4x-7}$
18. $\frac{x-5}{3x+4}-\frac{3x-5}{3x+4}$
19. $\frac{x^2-3}{2x+7}-\frac{2x-5}{2x+7}$

Find the least common multiple for each pair.

20. $12x^2y^3$ and $14x^3y^2$
21. $16x^2-25$ and $4x^2-x-5$

Add or subtract. Identify any x-values for which the expression is undefined.

22. $\frac{3x-2}{x+2}+\frac{2x}{4x-1}$
23. $\frac{2x-7}{x-2}+\frac{8x}{3x-6}$
24. $\frac{5x}{4x^2}+\frac{7}{x+1}$
25. $\frac{4x-3}{x^2-9}-\frac{2x-3}{x-3}$
26. $\frac{x}{2x+3}-\frac{2x+1}{2x-3}$
27. $\frac{1}{x-4}-\frac{2}{x^2-6x+8}$

Simplify. Assume that all expressions are defined.

28. $\dfrac{\frac{2x-5}{x^2-9}}{\frac{3x-1}{x+3}}$
29. $\dfrac{\frac{3x-2}{x^2-4}}{\frac{5x+1}{x^2+x-6}}$
30. $\dfrac{\frac{x}{x+1}}{x+\frac{x}{3}}$

31. **Chemistry** A solution is heated from 0°C to 100°C. Between 0°C and 50°C, the rate of temperature increase is 1.5°C/min. Between 50°C and 100°C, the rate of temperature increase is 0.4°C/min. What is the average rate of temperature increase during the entire heating process? Round to the nearest tenth.

MULTI-STEP TEST PREP

32. This problem will prepare you for the Multi-Step Test Prep on page 608.

An auto race consists of 8 laps. A driver completes the first 3 laps at an average speed of 185 mi/h and the remaining laps at an average speed of 200 mi/h.

a. Let d represent the length of one lap. Write an expression in terms of d that represents the time in hours that it takes the driver to complete the race.

b. What is the driver's average speed during the race to the nearest mile per hour?

Add or subtract. Identify any x-values for which the expression is undefined.

33. $\dfrac{2}{x+4} + \dfrac{x}{x-3}$ **34.** $\dfrac{2x}{x^2-36} + \dfrac{x+4}{x+6}$ **35.** $\dfrac{2}{x^2-x-20} + \dfrac{3}{x^2+7x+12}$

36. $\dfrac{7x}{x^2-5x} + \dfrac{x^2}{x-5}$ **37.** $\dfrac{2x}{x-1} - \dfrac{9}{x-2}$ **38.** $\dfrac{2x+3}{3x+4} - \dfrac{x}{9x+12}$

39. $\dfrac{4x^2}{3x+4} - \dfrac{2}{2x-3}$ **40.** $\dfrac{6}{x^2+4x-32} - \dfrac{x-5}{x-4}$ **41.** $\dfrac{x+7}{x^2+13x+42} - \dfrac{10x}{x^2+8x+7}$

42. Environment The junior and senior classes of a high school are cleaning up a beach. Each class has pledged to clean 1600 m of shoreline. The junior class has 12 more students than the senior class.

a. Let s represent the number of students in the senior class. Write and simplify an expression in terms of s that represents the difference between the number of meters of shoreline each senior must clean and the number each junior must clean.

b. If there are 48 seniors, how many more meters of shoreline must each senior clean than the number each junior must clean? Round to the nearest tenth of a meter.

c. Multi-Step If it takes each student about 10 min to clean 15 m of shoreline, approximately how much sooner will the junior class finish than the senior class?

Simplify. Assume that all expressions are defined.

43. $\dfrac{\frac{4}{x+2}}{\frac{x+2}{6}}$ **44.** $\dfrac{\frac{2}{3x-4}}{5x+3}$ **45.** $\dfrac{\frac{1}{2x}+\frac{2}{3x}}{\frac{x-1}{x-3}}$

LINK Architecture

Andrea Palladio stated that his preferred room shapes were squares, circles, and rectangles with precise length-to-width ratios. Some of these shapes can be seen above in Il Redentore church that Palladio designed.

46. Architecture The Renaissance architect Andrea Palladio preferred that the length and width of rectangular rooms be limited to certain ratios. These ratios are listed in the table. Palladio also believed that the height of a room with vaulted ceilings should be the harmonic mean of the length and width.

a. The harmonic mean of two positive numbers a and b is equal to $\dfrac{2}{\frac{1}{a}+\frac{1}{b}}$. Simplify this expression.

b. Complete the table for a rectangular room with a width of 30 feet that meets Palladio's requirements for its length and height. If necessary, round to the nearest tenth.

c. What if...? A Palladian room has a length-to-width ratio of 4 : 3. If the length of this room is doubled, what effect should this change have on the room's width and height, according to Palladio's principles?

Rooms with a Width of 30 ft

Length-to-Width Ratio	Length (ft)	Height (ft)
2 : 1		
3 : 2		
4 : 3		
5 : 3		
$\sqrt{2}$: 1		

47. Critical Thinking Write two expressions whose sum is $\dfrac{x-3}{x+2}$.

48. **Write About It** The first step in adding rational expressions is to write them with a common denominator. This denominator does not necessarily need to be the least common denominator (LCD). Why is it often easier to use the LCD than it is to use other common denominators?

TEST PREP

49. Which best represents $\frac{3}{3x} + \frac{5}{9x}$?

Ⓐ $\frac{2}{3x}$ Ⓑ $\frac{7}{2x}$ Ⓒ $\frac{8}{9x}$ Ⓓ $\frac{14}{9x}$

50. Which of the following is equivalent to $\frac{5}{x+2} - \frac{8}{x+4}$?

Ⓕ $\frac{-3x+4}{x^2+8}$ Ⓖ $\frac{-5}{x+4}$ Ⓗ $\frac{-3x+4}{x^2+6x+8}$ Ⓙ $\frac{-3x+36}{x^2+6x+8}$

51. Which of the following is equivalent to $\dfrac{\frac{8}{7x}}{\frac{-4}{x+1}}$?

Ⓐ $-\frac{2x+2}{7x}$ Ⓑ $-\frac{32}{7x^2+7}$ Ⓒ $-\frac{2}{7x^2+7}$ Ⓓ $-\frac{7x}{2x+2}$

52. A three-day bicycle race has 3 stages of equal length. The table shows a rider's average speed in each of the stages. What is the rider's average speed for the entire race, rounded to the nearest tenth of a kilometer per hour?

Ⓕ 29.5 km/h Ⓗ 30.2 km/h

Ⓖ 29.7 km/h Ⓙ 30.7 km/h

Race Results

Stage	Speed (km/h)
1	35.5
2	31.1
3	25.6

CHALLENGE AND EXTEND

Simplify. Assume that all expressions are defined.

53. $\frac{x-1}{x+2} + \frac{4}{x^2-4} - \frac{6x}{x-2}$

54. $\frac{x^{-1}+y^{-1}}{x^{-1}-y^{-1}}$

55. $(x+2)^{-2} - (x^2-4)^{-1}$

56. $(x-y)^{-1} - (x+y)^{-1}$

57. What polynomial completes the equation $\frac{\blacksquare}{x^3+4x^2-5x} - \frac{x+4}{x^2-x} = \frac{5}{x+5}$?

SPIRAL REVIEW

Evaluate each expression for the given values of the variables. *(Lesson 1-4)*

58. $\frac{-x^2}{y^2-x^2}$ for $x = -2$ and $y = 3$

59. $\frac{m^2-mn}{n^2+10}$ for $m = -4$ and $n = 0$

Graph each logarithmic function. Find the asymptote. Then describe how the graph is transformed from the graph of its parent function. *(Lesson 7-7)*

60. $g(x) = 2\log(x-1)$

61. $h(x) = \log(x+4)$

Simplify. Identify any x-values for which the expression is undefined. *(Lesson 8-2)*

62. $\frac{2x^2+5x^3}{x}$

63. $\frac{x^2-2x-48}{x^2+10x+24}$

64. $\frac{x-2}{x^2-3x+2}$

8-4 Technology LAB

Explore Holes in Graphs

You can use a graphing calculator to explore the relationship between the graphs of rational functions and their simplified forms.

Use with Lesson 8-4

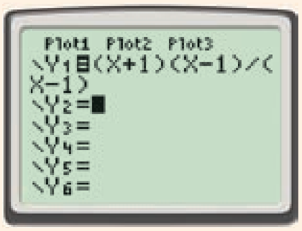
go.hrw.com
Lab Resources Online
KEYWORD: MB7 Lab8

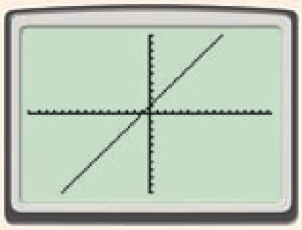
Activity

Use a graph and a table to identify holes in the graph of $f(x) = \dfrac{(x+1)(x-1)}{(x-1)}$.

1 Graph the function $f(x) = \frac{(x+1)(x-1)}{(x-1)}$ in the square window.

The graph appears to be identical to the graph of the function in simplified form, $f(x) = x + 1$.

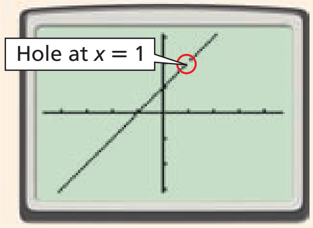




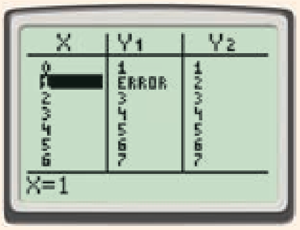
2 Change the window on your graph to the decimal window by pressing ZOOM and selecting **4:ZDecimal**.

Notice that there is a break, or *hole,* in the graph when $x = 1$ because the function is undefined at that x-value.

The hole appears only if you are in a friendly window that allows the calculator to evaluate the function exactly at that point.



3 Use a table to compare the function $f(x) = \frac{(x+1)(x-1)}{(x-1)}$ to the linear function $f(x) = x + 1$. The table suggests that the graphs are identical except when $x = 1$.



Try This

Use a graph and a table to identify the hole in the graph of each function.

1. $f(x) = \dfrac{(x-2)(x+3)}{x+3}$ **2.** $g(x) = \dfrac{(x+1)(x+3)}{x+1}$ **3.** $h(x) = \dfrac{x(x+2)}{x+2}$

4. Make a Conjecture Make a conjecture about where the holes in the graph of a rational function appear, based on the factors of the numerator and the denominator.

Use a graph and a table to identify the hole(s) in the graph of each function. Confirm your answer by factoring.

5. $f(x) = \dfrac{x^2 - 4x + 3}{x - 3}$ **6.** $g(x) = \dfrac{x^2 + x - 2}{x - 1}$ **7.** $h(x) = \dfrac{x^3 - x}{x^2 - 1}$

8-4 Rational Functions

Objectives

Graph rational functions.

Transform rational functions by changing parameters.

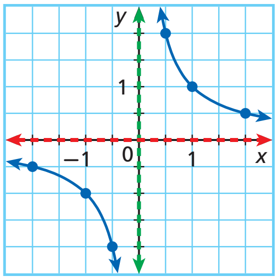
Vocabulary

rational function
discontinuous function
continuous function
hole (in a graph)

Why learn this?

Rational functions can be used to model the cost per person for group events, such as a band trip to a bowl game. (See Exercise 32.)

A **rational function** is a function whose rule can be written as a ratio of two polynomials. The parent rational function is $f(x) = \frac{1}{x}$. Its graph is a *hyperbola*, which has two separate branches. You will learn more about hyperbolas in Chapter 10.



Like logarithmic and exponential functions, rational functions may have asymptotes. The function $f(x) = \frac{1}{x}$ has a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$.

The rational function $f(x) = \frac{1}{x}$ can be transformed by using methods similar to those used to transform other types of functions.

Know it! Note

$$f(x) = \frac{a}{x - h} + k$$

$|a| \longrightarrow$ vertical stretch or compression factor
$a < 0 \longrightarrow$ reflection across the x-axis

$k \longrightarrow$ vertical translation down for $k < 0$; up for $k > 0$

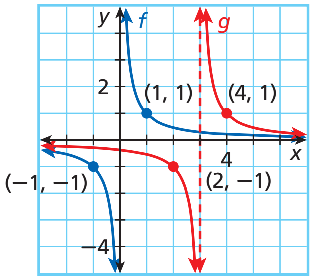
$h \longrightarrow$ horizontal translation left for $h < 0$; right for $h > 0$

EXAMPLE 1 **Transforming Rational Functions**

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

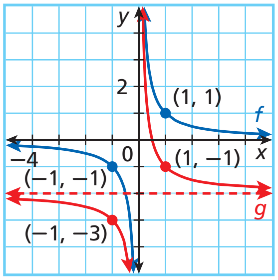
A $g(x) = \dfrac{1}{x - 3}$

Because $h = 3$, translate f 3 units right.



B $g(x) = \dfrac{1}{x} - 2$

Because $k = -2$, translate f 2 units down.



CHECK IT OUT! **Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.**

1a. $g(x) = \dfrac{1}{x+4}$ **1b.** $g(x) = \dfrac{1}{x} + 1$

The values of h and k affect the locations of the asymptotes, the domain, and the range of rational functions whose graphs are hyperbolas.

Know it! Note

Rational Functions

For a rational function of the form $f(x) = \dfrac{a}{x-h} + k$,

- the graph is a hyperbola.
- there is a vertical asymptote at the line $x = h$, and the domain is $\{x \mid x \neq h\}$.
- there is a horizontal asymptote at the line $y = k$, and the range is $\{y \mid y \neq k\}$.

EXAMPLE 2 Determining Properties of Hyperbolas

Identify the asymptotes, domain, and range of the function $g(x) = \dfrac{1}{x+2} + 4$.

$g(x) = \dfrac{1}{x-(-2)} + 4$ *$h = -2$, $k = 4$*

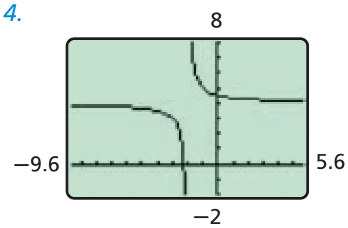
Vertical asymptote: $x = -2$ *The value of h is -2.*

Domain: $\{x \mid x \neq -2\}$

Horizontal asymptote: $y = 4$ *The value of k is 4.*

Range: $\{y \mid y \neq 4\}$

Check Graph the function on a graphing calculator. The graph suggests that the function has asymptotes at $x = -2$ and $y = 4$.



Caution!

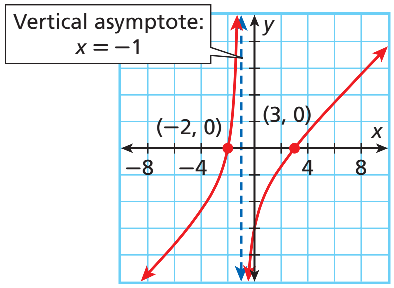
Graphing calculators may incorrectly connect two branches of the graph of a rational function with a nearly vertical segment that looks like an asymptote.

CHECK IT OUT! **2.** Identify the asymptotes, domain, and range of the function $g(x) = \frac{1}{x-3} - 5$.

A **discontinuous function** is a function whose graph has one or more gaps or breaks. The hyperbola graphed above and many other rational functions are discontinuous functions.

A **continuous function** is a function whose graph has no gaps or breaks. The functions you have studied before this, including linear, quadratic, polynomial, exponential, and logarithmic functions, are continuous functions.

The graphs of some rational functions are not hyperbolas. Consider the rational function $f(x) = \frac{(x-3)(x+2)}{x+1}$ and its graph. The numerator of this function is 0 when $x = 3$ or $x = -2$. Therefore, the function has x-intercepts at -2 and 3. The denominator of this function is 0 when $x = -1$. As a result, the graph of the function has a vertical asymptote at the line $x = -1$.



Zeros and Vertical Asymptotes — Rational Functions

If $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1, then the function f has

- zeros at each real value of x for which $p(x) = 0$.
- a vertical asymptote at each real value of x for which $q(x) = 0$.

EXAMPLE 3 Graphing Rational Functions with Vertical Asymptotes

Identify the zeros and vertical asymptotes of $f(x) = \frac{x^2 - 2x - 3}{x - 2}$. Then graph.

Step 1 Find the zeros and vertical asymptotes.

$f(x) = \frac{(x+1)(x-3)}{x-2}$ — *Factor the numerator.*

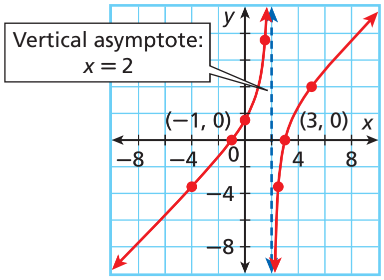
Zeros: -1 and 3 — *The numerator is 0 when $x = -1$ or $x = 3$.*

Vertical asymptote: $x = 2$ — *The denominator is 0 when $x = 2$.*

Step 2 Graph the function.

Plot the zeros and draw the asymptote. Then make a table of values to fill in missing points.

x	-4	-1	0	1.5	2.5	3	5
y	-3.5	0	1.5	7.5	-3.5	0	4



3. Identify the zeros and vertical asymptotes of $f(x) = \frac{x^2 + 7x + 6}{x + 3}$. Then graph.

> **Remember!** For a review of factoring and finding zeros of polynomial functions, see Lessons 6-4 and 6-5.

Some rational functions, including those whose graphs are hyperbolas, have a horizontal asymptote. The existence and location of a horizontal asymptote depends on the degrees of the polynomials that make up the rational function.

Note that the graph of a rational function can sometimes cross a horizontal asymptote. However, the graph will approach the asymptote when $|x|$ is large.

Horizontal Asymptotes — Rational Functions

Let $f(x) = \frac{p(x)}{q(x)}$, where p and q are polynomial functions in standard form with no common factors other than 1. The graph of f has at most one horizontal asymptote.

- If degree of $p >$ degree of q, there is no horizontal asymptote.
- If degree of $p <$ degree of q, the horizontal asymptote is the line $y = 0$.
- If degree of $p =$ degree of q, the horizontal asymptote is the line $y = \frac{\text{leading coefficient of } p}{\text{leading coefficient of } q}$.

EXAMPLE 4 Graphing Rational Functions with Vertical and Horizontal Asymptotes

Identify the zeros and asymptotes of each function. Then graph.

A $f(x) = \dfrac{x^2 + x - 6}{x}$

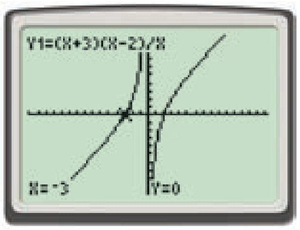
$f(x) = \dfrac{(x + 3)(x - 2)}{x}$ — *Factor the numerator.*

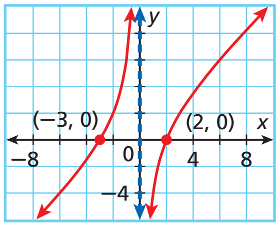
Zeros: -3 and 2 — *The numerator is 0 when $x = -3$ or $x = 2$.*

Vertical asymptote: $x = 0$ — *The denominator is 0 when $x = 0$.*

Horizontal asymptote: none — *Degree of p > degree of q*

Graph with a graphing calculator or by using a table of values.



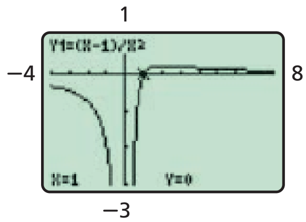


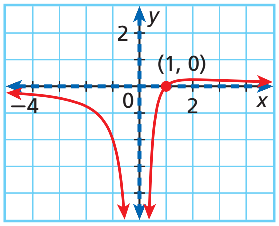
B $f(x) = \dfrac{x - 1}{x^2}$

Zero: 1 — *The numerator is 0 when $x = 1$.*

Vertical asymptote: $x = 0$ — *The denominator is 0 when $x = 0$.*

Horizontal asymptote: $y = 0$ — *Degree of p < degree of q*





Remember!

Recall from Lesson 6-1 that the leading coefficient of a polynomial is the coefficient of the first term when the polynomial is written in standard form.

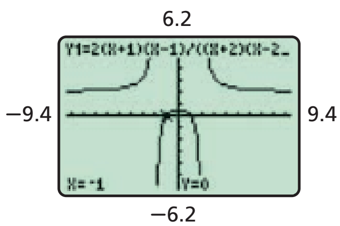
C $f(x) = \dfrac{2x^2 - 2}{x^2 - 4}$

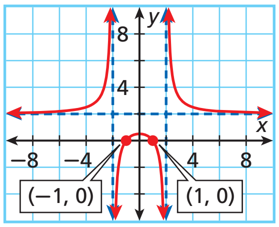
$f(x) = \dfrac{2(x + 1)(x - 1)}{(x + 2)(x - 2)}$ — *Factor the numerator and denominator.*

Zeros: -1 and 1 — *The numerator is 0 when $x = -1$ or $x = 1$.*

Vertical asymptotes: $x = -2$, $x = 2$ — *The denominator is 0 when $x = \pm 2$.*

Horizontal asymptote: $y = 2$ — *The horizontal asymptote is $y = \dfrac{\text{leading coefficient of } p}{\text{leading coefficient of } q} = \dfrac{2}{1} = 2$.*





CHECK IT OUT! **Identify the zeros and asymptotes of each function. Then graph.**

4a. $f(x) = \dfrac{x^2 + 2x - 15}{x - 1}$ **4b.** $f(x) = \dfrac{x - 2}{x^2 + x}$ **4c.** $f(x) = \dfrac{3x^2 + x}{x^2 - 9}$

In some cases, both the numerator and the denominator of a rational function will equal 0 for a particular value of x. As a result, the function will be undefined at this x-value. If this is the case, the graph of the function may have a *hole*. A **hole** is an omitted point in a graph.

Know it! Note

Holes in Graphs | **Rational Functions**

If a rational function has the same factor $x - b$ in both the numerator and the denominator, then there is a hole in the graph at the point where $x = b$, unless the line $x = b$ is a vertical asymptote.

EXAMPLE 5 Graphing Rational Functions with Holes

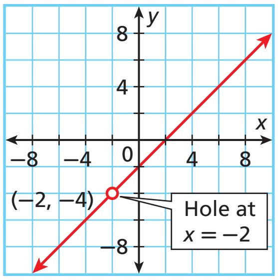
Identify holes in the graph of $f(x) = \dfrac{x^2 - 4}{x + 2}$. Then graph.

$$f(x) = \frac{(x-2)(x+2)}{(x+2)}$$ *Factor the numerator.*

There is a hole in the graph at $x = -2$. *The expression $x + 2$ is a factor of both the numerator and the denominator.*

For $x \neq -2$, $f(x) = \dfrac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}} = x - 2$ *Divide out common factors.*

The graph of f is the same as the graph of $y = x - 2$, except for the hole at $x = -2$. On the graph, indicate the hole with an open circle. The domain of f is $\{x \mid x \neq -2\}$.



CHECK IT OUT!

5. Identify holes in the graph of $f(x) = \dfrac{x^2 + x - 6}{x - 2}$. Then graph.

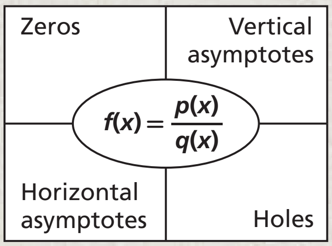
THINK AND DISCUSS

1. Explain how vertical asymptotes relate to the domain of a rational function.

2. Compare and contrast rational functions and polynomial functions.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the formula or method for identifying the characteristic of graphs of rational functions.



8-4 Exercises

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Homework Help Online
KEYWORD: MB7 8-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A function with a hole in its graph is __?__. (*continuous* or *discontinuous*)

SEE EXAMPLE 1 p. 592

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

2. $g(x) = \frac{1}{x} - 2$
3. $g(x) = \frac{1}{x + 5}$
4. $g(x) = \frac{1}{x - 1} + 4$

SEE EXAMPLE 2 p. 593

Identify the asymptotes, domain, and range of each function.

5. $f(x) = \frac{1}{x} - 1$
6. $f(x) = \frac{1}{x + 4} + 3$
7. $f(x) = \frac{2}{x - 2} - 8$

SEE EXAMPLE 3 p. 594

Identify the zeros and vertical asymptotes of each function. Then graph.

8. $f(x) = \frac{x^2 - x - 12}{x}$
9. $f(x) = \frac{x^2 - 5x}{x - 2}$
10. $f(x) = \frac{x^2}{x - 1}$

SEE EXAMPLE 4 p. 595

Identify the zeros and asymptotes of each function. Then graph.

11. $f(x) = \frac{x^2 + 3x + 2}{3 - x}$
12. $f(x) = \frac{x - 2}{x^2 + 6x}$
13. $f(x) = \frac{5x + 2}{x + 1}$

SEE EXAMPLE 5 p. 596

Identify holes in the graph of each function. Then graph.

14. $f(x) = \frac{x^2 - 5x + 6}{x^2 - 4x + 3}$
15. $f(x) = \frac{x^2 - 4x + 4}{x - 2}$
16. $f(x) = \frac{4x + 20}{2x + 10}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–22	2
23–25	3
26–28	4
29–31	5

Extra Practice
Skills Practice p. S18
Application Practice p. S39

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformation and graph each function.

17. $g(x) = \frac{1}{x} - 5$
18. $g(x) = \frac{1}{x + 3}$
19. $g(x) = \frac{2}{x}$

Identify the asymptotes, domain, and range of each function.

20. $f(x) = \frac{1}{x + 6}$
21. $f(x) = \frac{4}{x} + 5$
22. $f(x) = \frac{3}{x - 4} - 1$

Identify the zeros and vertical asymptotes of each function. Then graph.

23. $f(x) = \frac{(x + 2)(x - 5)}{(x - 2)}$
24. $f(x) = \frac{(2 - x)(4 + x)}{(x - 1)}$
25. $h(x) = \frac{x^2 - 4}{x + 3}$

Identify the zeros and asymptotes of each function. Then graph.

26. $f(x) = \frac{x^2 - x - 2}{1 - x}$
27. $f(x) = \frac{x - 3}{x^2 - 4}$
28. $f(x) = \frac{2x^2 + x}{1 - x^2}$

Identify holes in the graph of each function. Then graph.

29. $f(x) = \frac{x^4}{x}$
30. $f(x) = \frac{-x^2 + x}{x - 1}$
31. $f(x) = \frac{x^2 - 14x + 49}{x - 7}$

32. **Band** Members of a high school band plan to play at a college bowl game. The trip will cost \$350 per band member plus a \$2000 deposit.
    a. Write a function to represent the total average cost of the trip per band member.
    b. Graph the function.
    c. **What if...?** Find the total average cost per person if 40 band members attend the bowl game.

Identify all zeros, asymptotes, and holes in the graph of each function.

33. $f(x) = \dfrac{x^2 - 2x - 3}{x^2 - 3x}$ **34.** $f(x) = \dfrac{x^3 - 1}{x - 1}$ **35.** $f(x) = \dfrac{6x - 5}{2 - 3x}$

36. $f(x) = \dfrac{x^2 + 6x + 8}{x^2}$ **37.** $f(x) = \dfrac{x}{x^2 - 9}$ **38.** $f(x) = \dfrac{x^2 - 9}{x^2 - 4}$

Write a rational function with the given characteristics.

39. zeros at -1 and 3 and vertical asymptote at $x = 0$

40. zero at 2, vertical asymptotes at $x = -2$ and $x = 0$, and horizontal asymptote at $y = 0$

41. zero at 2, vertical asymptote at $x = -1$, horizontal asymptote at $y = 1$, and hole at $x = -3$

LINK

Math History

The Agnesi curve is named for Maria Agnesi (1718–1799), a mathematician from Milan who wrote one of the earliest surviving mathematical works composed by a woman.

42. Math History The *Agnesi curve* is the graph of the function $y = \dfrac{a^3}{x^2 + a^2}$.

a. Graph the Agnesi curve for $a = 3$.

b. What are the domain and the range of the function?

c. Identify all asymptotes of the function.

43. Chemistry A chemist has 100 g of a 12% saline solution that she wants to strengthen to 25%. The percentage P of salt in the solution by mass can be modeled by $P(x) = \frac{100(12 + x)}{100 + x}$, where x is the number of grams of salt added.

a. Graph the function for $0 \le x \le 100$.

b. Use your graph to estimate how much salt the chemist must add to create a 25% solution.

44. Multi-Step The average cost per DVD purchased from a movie club is a function of the number of DVDs a member buys.

a. Graph the data in the table.

b. The function that describes the data in the table has the form $f(x) = \frac{40}{x} + k$, where k is a constant. What is the value of k?

c. What is the total cost of buying 15 DVDs from the club?

Number of DVDs	Average Cost ($)
1	55
2	35
4	25
5	23
10	19
20	17

45. ERROR ANALYSIS A student wrote the following description for the graph of $f(x) = \frac{(x - 1)(2x - 3)}{(x + 1)(x - 1)}$. Explain the error. Write a correct description.

The graph has vertical asymptotes at $x = 1$ and $x = -1$ and a horizontal asymptote at $y = 2$.

46. Critical Thinking Is it possible to have a rational function with no vertical asymptotes? Explain.

MULTI-STEP TEST PREP

47. This problem will prepare you for the Multi-Step Test Prep on page 608.

A race car driver makes a pit stop at the beginning of a lap. The time t in seconds that it takes the driver to complete the lap, including the pit stop, can be modeled by $t(r) = \frac{12r + 9000}{r}$, where r is the driver's average speed in miles per hour after the pit stop.

a. Graph the function.

b. What is the horizontal asymptote of the function, and what does it represent?

c. The driver's average speed after the pit stop is 200 mi/h. How long does it take the driver to complete the lap, including the pit stop?

48. **Critical Thinking** For what value(s) of x is $\frac{x^2 - 9}{x + 3} = x - 3$ a false statement? Explain.

49. **Write About It** Explain how to identify the domain of a rational function.

TEST PREP

50. The graph of which of the following rational functions has a hole?

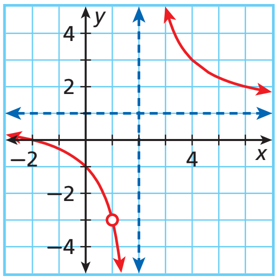
Ⓐ $f(x) = \frac{x^2 + 5x + 4}{x^2 + x - 12}$

Ⓑ $f(x) = \frac{x^2 - 2x + 1}{x^2 + 7x - 15}$

Ⓒ $f(x) = \frac{x^2 - 9}{x^2 - 2x - 7}$

Ⓓ $f(x) = \frac{x^2 + x - 30}{x^2 + 5x - 14}$

51. Which function is shown in the graph?



Ⓕ $f(x) = \frac{x^2 + x - 2}{x^2 - 3x + 2}$

Ⓖ $f(x) = \frac{x^2 + 3x + 2}{x^2 - x - 2}$

Ⓗ $f(x) = \frac{x^2 + x - 2}{x^2 + 3x + 2}$

Ⓙ $f(x) = \frac{x^2 - 3x + 2}{x^2 + x - 2}$

52. What is the horizontal asymptote of $f(x) = \frac{(2x + 4)(3x + 6)}{(x - 1)(x + 6)}$?

Ⓐ $y = -6$

Ⓑ $y = -2$

Ⓒ $y = 2$

Ⓓ $y = 6$

CHALLENGE AND EXTEND

Identify all zeros, asymptotes, and holes in the graph of each function. Then graph.

53. $f(x) = \frac{(x^2 - 3x + 2)(x - 3)}{(x - 1)(x^2 - 5x + 6)}$

54. $f(x) = \frac{(x^2 - 9)(3x + 2)}{(x^2 - 4)(x - 3)}$

55. Let $f(x) = \frac{1}{x^2 - 2x + c}$. Find c such that the graph of f has the given number of vertical asymptotes.

a. none **b.** one **c.** two

Write a rational function with the given characteristics.

56. no zeros, no vertical asymptotes, and a horizontal asymptote at $y = 1$

57. zero at 0, vertical asymptotes at $x = -3$ and $x = 3$, and holes at $x = -1$ and $x = 1$

SPIRAL REVIEW

58. **Sports** In the 1972–1973 school year, 817,073 females participated in high school sports in the United States. By the 2002–2003 school year, this number had increased to 2,856,358. To the nearest percent, what was the percent increase in the number of females participating in high school sports? *(Lesson 2-2)*

Solve. *(Lesson 7-5)*

59. $\log_3(5x - 2) = \log_3(2x + 8)$

60. $\log_2 x^2 = 4$

61. $\log_x \frac{1}{27} = 3$

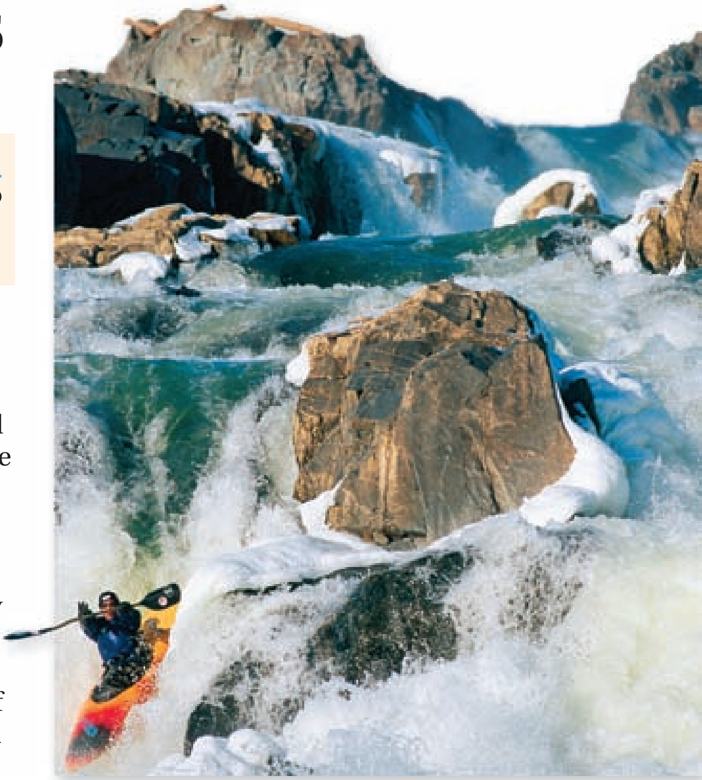
62. $\log_4 48 - \log_4 4x = 4$

Add or subtract. Identify any x-values for which the expression is undefined. *(Lesson 8-3)*

63. $\frac{5x - 7}{2x + 1} + \frac{3x - 6}{2x + 1}$

64. $\frac{x - 1}{x + 2} - \frac{x + 1}{x - 3}$

8-5 Solving Rational Equations and Inequalities



Objective
Solve rational equations and inequalities.

Vocabulary
rational equation
extraneous solution
rational inequality

Who uses this?
Kayakers can use rational equations to determine how fast a river is moving. (See Example 3.)

A **rational equation** is an equation that contains one or more rational expressions. The time t in hours that it takes to travel d miles can be determined by using the equation $t = \frac{d}{r}$, where r is the average rate of speed. This equation is a rational equation.

To solve a rational equation, start by multiplying each term of the equation by the least common denominator (LCD) of all of the expressions in the equation. This step eliminates the denominators of the rational expressions and results in an equation you can solve by using algebra.

EXAMPLE 1 Solving Rational Equations

Solve the equation $x + \frac{8}{x} = 6$.

$x(x) + \frac{8}{x}(x) = 6(x)$	*Multiply each term by the LCD, x.*
$x^2 + 8 = 6x$	*Simplify. Note that $x \neq 0$.*
$x^2 - 6x + 8 = 0$	*Write in standard form.*
$(x-2)(x-4) = 0$	*Factor.*
$x - 2 = 0$ or $x - 4 = 0$	*Apply the Zero Product Property.*
$x = 2$ or $x = 4$	*Solve for x.*

Check

$x + \frac{8}{x}$	6
$2 + \frac{8}{2}$	6
6	6 ✔

$x + \frac{8}{x}$	6
$4 + \frac{8}{4}$	6
6	6 ✔

Remember!
Factoring is not the only method of solving the quadratic equation that results in Example 1. You could also complete the square or use the Quadratic Formula.

CHECK IT OUT! **Solve each equation.**

1a. $\frac{10}{3} = \frac{4}{x} + 2$ **1b.** $\frac{6}{x} + \frac{5}{4} = -\frac{7}{4}$ **1c.** $x = \frac{6}{x} - 1$

An **extraneous solution** is a solution of an equation derived from an original equation that is not a solution of the original equation. When you solve a rational equation, it is possible to get extraneous solutions. These values should be eliminated from the solution set. Always check your solutions by substituting them into the original equation.

EXAMPLE 2 Extraneous Solutions

Solve each equation.

A $\dfrac{3x}{x-3}=\dfrac{2x+3}{x-3}$

$$\frac{3x}{x-3}(x-3)=\frac{2x+3}{x-3}(x-3)$$ *Multiply each term by the LCD, $x-3$.*

$$\frac{3x}{\cancel{x-3}}\cancel{(x-3)}=\frac{2x+3}{\cancel{x-3}}\cancel{(x-3)}$$ *Divide out common factors.*

$$3x=2x+3$$ *Simplify. Note that $x\neq 3$.*

$$x=3$$ *Solve for x.*

The solution $x=3$ is extraneous because it makes the denominators of the original equation equal to 0. Therefore, the equation has no solution.

Check Substitute 3 for x in the original equation.

$$\begin{array}{c|c} \dfrac{3(3)}{3-3} & \dfrac{2(3)+3}{3-3} \\ \hline \dfrac{9}{0} & \dfrac{9}{0}\ ✗ \end{array}$$ *Division by 0 is undefined.*

Remember!

A rational expression is undefined for any value of a variable that makes a denominator in the expression equal to 0.

B $\dfrac{2x-9}{x-7}+\dfrac{x}{2}=\dfrac{5}{x-7}$

$$\frac{2x-9}{x-7}\cdot 2(x-7)+\frac{x}{2}\cdot 2(x-7)=\frac{5}{x-7}\cdot 2(x-7)$$ *Multiply each term by the LCD, $2(x-7)$.*

$$\frac{2x-9}{\cancel{x-7}}\cdot 2\cancel{(x-7)}+\frac{x}{\cancel{2}}\cdot\cancel{2}(x-7)=\frac{5}{\cancel{x-7}}\cdot 2\cancel{(x-7)}$$ *Divide out common factors.*

$$2(2x-9)+x(x-7)=5(2)$$ *Simplify. Note that $x\neq 7$.*

$$4x-18+x^2-7x=10$$ *Use the Distributive Property.*

$$x^2-3x-28=0$$ *Write in standard form.*

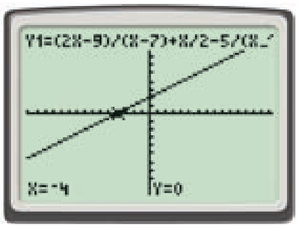
$$(x-7)(x+4)=0$$ *Factor.*

$$x-7=0 \text{ or } x+4=0$$ *Use the Zero Product Property.*

$$x=7 \text{ or } x=-4$$ *Solve for x.*

The solution $x=7$ is extraneous because it makes the denominators of the original equation equal to 0. The only solution is $x=-4$.

Check Write $\frac{2x-9}{x-7}+\frac{x}{2}=\frac{5}{x-7}$ as $\frac{2x-9}{x-7}+\frac{x}{2}-\frac{5}{x-7}=0$. Graph the left side of the equation as **Y1** and identify the values of x for which **Y1** $=0$.



The graph intersects the x-axis only when $x=-4$. Therefore, $x=-4$ is the only solution.

CHECK IT OUT! **Solve each equation.**

2a. $\dfrac{16}{x^2-16}=\dfrac{2}{x-4}$

2b. $\dfrac{1}{x-1}=\dfrac{x}{x-1}+\dfrac{x}{6}$

EXAMPLE 3

Problem-Solving Application

PROBLEM SOLVING

A kayaker spends an afternoon paddling on a river. She travels 3 mi upstream and 3 mi downstream in a total of 4 h. In still water, the kayaker can travel at an average speed of 2 mi/h. Based on this information, what is the average speed of the river's current? Is your answer reasonable?

1 Understand the Problem

The **answer** will be the average speed of the current.

List the **important information:**

- The kayaker spent 4 hours kayaking.
- She went 3 mi upstream and 3 mi downstream.
- Her average speed in still water is 2 mi/h.

2 Make a Plan

Let c represent the speed of the current. When the kayaker is going upstream, her speed is equal to her speed in still water minus c. When the kayaker is going downstream, her speed is equal to her speed in still water plus c.

	Distance (mi)	Average Speed (mi/h)	Time (h)
Up	3	$2 - c$	$\frac{3}{2-c}$
Down	3	$2 + c$	$\frac{3}{2+c}$

Helpful Hint

distance = rate × time
Therefore,
$\text{time} = \frac{\text{distance}}{\text{rate}}$.

$$\text{total time} = \text{time upstream} + \text{time downstream}$$

$$4 = \frac{3}{2-c} + \frac{3}{2+c}$$

3 Solve

$$4(2-c)(2+c) = \frac{3}{2-c}(2-c)(2+c) + \frac{3}{2+c}(2-c)(2+c) \quad \textit{The LCD is } (2-c)(2+c).$$

$$4(2-c)(2+c) = 3(2+c) + 3(2-c) \quad \textit{Simplify. Note that } c \neq \pm 2.$$

$$16 - 4c^2 = 6 + 3c + 6 - 3c \quad \textit{Use the Distributive Property.}$$

$$16 - 4c^2 = 12 \quad \textit{Combine like terms.}$$

$$-4c^2 = -4 \quad \textit{Solve for } c.$$

$$c = \pm 1$$

The speed of the current cannot be negative. Therefore, the average speed of the current is 1 mi/h.

4 Look Back

If the speed of the current is 1 mi/h, the kayaker's speed when going upstream is $2 - 1 = 1$ mi/h. It will take her 3 h to travel 3 mi upstream. Her speed when going downstream is $2 + 1 = 3$ mi/h. It will take her 1 hour to travel 3 mi downstream. The total trip will take 4 h, which is the given time.

CHECK IT OUT! **Use the information given above to answer the following.**

3. On a different river, the kayaker travels 2 mi upstream and 2 mi downstream in a total of 5 h. What is the average speed of the current of this river? Round to the nearest tenth.

EXAMPLE 4 Work Application

Jason can clean a large tank at an aquarium in about 6 hours. When Jason and Lacy work together, they can clean the tank in about 3.5 hours. About how long would it take Lacy to clean the tank if she works by herself?

Jason's rate: $\frac{1}{6}$ of the tank per hour

Lacy's rate: $\frac{1}{h}$ of the tank per hour, where h is the number of hours needed to clean the tank by herself

$$\text{Jason's rate} \times \text{hours worked} + \text{Lacy's rate} \times \text{hours worked} = \text{1 complete job}$$

$$\frac{1}{6}(3.5) + \frac{1}{h}(3.5) = 1$$

$$\frac{1}{6}(3.5)(6h) + \frac{1}{h}(3.5)(6h) = 1(6h) \qquad \textit{Multiply by the LCD, 6h.}$$

$$3.5h + 21 = 6h \qquad \textit{Simplify.}$$

$$21 = 2.5h \qquad \textit{Solve for h.}$$

$$8.4 = h$$

It will take Lacy about 8.4 hours, or 8 hours 24 minutes, to clean the tank when working by herself.

CHECK IT OUT!

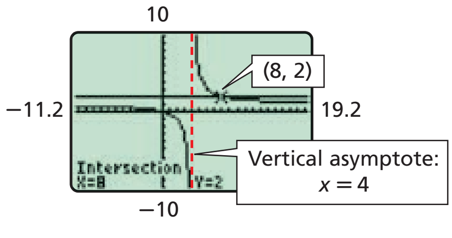
4. Julien can mulch a garden in 20 minutes. Together, Julien and Remy can mulch the same garden in 11 minutes. How long will it take Remy to mulch the garden when working alone?

A **rational inequality** is an inequality that contains one or more rational expressions. One way to solve rational inequalities is by using graphs and tables.

EXAMPLE 5 Using Graphs and Tables to Solve Rational Equations and Inequalities

Solve $\frac{x}{x-4} \le 2$ by using a graph and a table.

Use a graph. On a graphing calculator, let **Y1** $= \frac{x}{x-4}$ and **Y2** $= 2$.

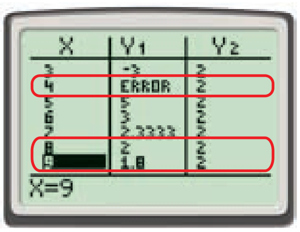


The graph of **Y1** is at or below the graph of **Y2** when $x < 4$ or when $x \ge 8$.

Remember!

The solution $x < 4$ or $x \ge 8$ can be written in set-builder notation as $\{x \mid x < 4 \cup x \ge 8\}$

Use a table. The table shows that **Y1** is undefined when $x = 4$ and that **Y1** $\le$ **Y2** when $x < 4$ or when $x \ge 8$.



The solution of the inequality is $x < 4$ or $x \ge 8$.

CHECK IT OUT!

Solve by using a graph and a table.

5a. $\frac{x}{x-3} \ge 4$

5b. $\frac{8}{x+1} = -2$

You can also solve rational inequalities algebraically. You start by multiplying each term by the least common denominator (LCD) of all the expressions in the inequality. However, you must consider two cases: the LCD is positive or the LCD is negative.

EXAMPLE 6

Solving Rational Inequalities Algebraically

Solve the inequality $\frac{8}{x+5} \le 4$ algebraically. Check your answer for reasonableness.

Case 1 LCD is **positive.**

Step 1 Solve for x.

$\frac{8}{x+5}(x+5) \le 4(x+5)$ *Multiply by the LCD.*

$8 \le 4x + 20$ *Simplify. Note that $x \ne -5$.*

$-12 \le 4x$ *Solve for x.*

$-3 \le x$

Step 2 Consider the sign of the LCD.

$x + 5 > 0$ *LCD is positive.*

$x > -5$ *Solve for x.*

For Case 1, the solution must satisfy $x \ge -3$ *and* $x > -5$, which simplifies to $x \ge -3$.

Case 2 LCD is **negative.**

Step 1 Solve for x.

$\frac{8}{x+5}(x+5) \ge 4(x+5)$ *Multiply by the LCD. Reverse the inequality.*

$8 \ge 4x + 20$ *Simplify. Note that $x \ne -5$.*

$-12 \ge 4x$ *Solve for x.*

$-3 \ge x$

Step 2 Consider the sign of the LCD.

$x + 5 < 0$ *LCD is negative.*

$x < -5$ *Solve for x.*

For Case 2, the solution must satisfy $x \le -3$ *and* $x < -5$, which simplifies to $x < -5$.

The solution set of the original inequality is the union of the solutions to both Case 1 and Case 2. The solution to the inequality $\frac{8}{x+5} \le 4$ is $x < -5$ or $x \ge -3$, or $\{x \mid x < -5 \cup x \ge -3\}$. The expression will be less than 4 when the denominator is negative or is very large, so the answer is reasonable.

> **Remember!**
> If you multiply or divide both sides of an inequality by a negative value, you must reverse the inequality symbol.

CHECK IT OUT! **Solve each inequality algebraically.**

6a. $\frac{6}{x-2} \ge -4$ **6b.** $\frac{9}{x+3} < 6$

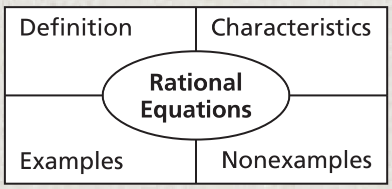
THINK AND DISCUSS

1. Explain why multiplying both sides of a rational equation by the LCD eliminates all of the denominators.

2. Explain why rational equations may have extraneous solutions.

3. Describe two methods for solving the inequality $\frac{12}{x} > 3$.

4. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the appropriate information related to rational equations.



Know it! Note

8-5 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 8-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How does a *rational expression* differ from a *rational equation*?

SEE EXAMPLE 1 p. 600

Solve each equation.

2. $\frac{1}{8} + \frac{2}{t} = \frac{17}{8t}$
3. $7 = \frac{1}{w} - 4$
4. $\frac{1}{r-5} = \frac{7}{2r}$
5. $\frac{1}{x} = \frac{x}{6} - \frac{5}{6}$
6. $m + \frac{12}{m} = 7$
7. $k + \frac{1}{k} = 2$

SEE EXAMPLE 2 p. 601

8. $\frac{-2x}{x+2} + \frac{x}{3} = \frac{4}{x+2}$
9. $\frac{x}{x-3} + \frac{x}{2} = \frac{6x}{2x-6}$
10. $\frac{3}{x(x+1)} - 1 = \frac{3}{x^2+x}$

SEE EXAMPLE 3 p. 602

11. **Transportation** A river barge travels at an average of 8 mi/h in still water. The barge travels 60 mi up the Mississippi River and 60 mi down the river in a total of 16.5 h. What is the average speed of the current in this section of the Mississippi River? Round to the nearest tenth. Is your answer reasonable?

SEE EXAMPLE 4 p. 603

12. **Work** Each month Leo must make copies of a budget report. When he uses both the large and the small copier, the job takes 30 min. If the small copier is broken, the job takes him 50 min. How long will the job take if the large copier is broken?

SEE EXAMPLE 5 p. 603

Solve by using a graph and a table.

13. $\frac{x-5}{x} > 2$
14. $\frac{3}{x+6} = 3$
15. $\frac{x+3}{2x} < 2$

SEE EXAMPLE 6 p. 604

Solve each inequality algebraically.

16. $\frac{4}{x+1} < 4$
17. $\frac{12}{x-4} \leq 3$
18. $\frac{10}{x+8} > 2$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
19–24	1
25–27	2
28	3
29	4
30–32	5
33–35	6

Extra

Skills Practice p. S19
Application Practice p. S39

Solve each equation.

19. $4 + \frac{1}{x} = \frac{10}{2x}$
20. $\frac{5}{4} = \frac{n-3}{n-4}$
21. $\frac{1}{a-7} = 3$
22. $\frac{1}{x} - \frac{3}{4} = \frac{x}{4}$
23. $\frac{14}{z} = 9 - z$
24. $x + \frac{4}{x} = 4$
25. $\frac{4x}{x-3} + \frac{x}{2} = \frac{12}{x-3}$
26. $\frac{3x}{x+1} = \frac{2x-1}{x+1}$
27. $\frac{2}{x(x-1)} = 1 + \frac{2}{x-1}$
28. **Multi-Step** A passenger jet travels from Los Angeles to Mumbai, India, in 22 h. The return flight takes 17 h. The difference in flight times is caused by winds over the Pacific Ocean that blow primarily from west to east. If the jet's average speed in still air is 550 mi/h, what is the average speed of the wind during the round-trip flight? Round to the nearest mile per hour. Is your answer reasonable?
29. **Art** A glassblower can produce a set of simple glasses in about 2 h. When the glassblower works with an apprentice, the job takes about 1.5 h. How long would it take the apprentice to make a set of glasses when working alone?

Solve by using a graph and a table.

30. $\frac{1}{x} > 1$
31. $\frac{x+1}{x+2} = 2$
32. $\frac{x}{x-5} \leq 0$

LINK

Ice Skating

The temperature of the ice in many indoor skating rinks is kept between 24°F and 28°F. Ice that is colder than these temperatures may chip too easily, and ice that is warmer may result in too much friction as skate blades pass over it.

Solve each inequality algebraically.

33. $\frac{1}{3x} < 2$ **34.** $\frac{9}{x-4} \geq -6$ **35.** $\frac{9}{x+10} > 3$

36. Ice Skating A new skating rink will be approximately rectangular in shape and will have an area of no more than 17,000 square feet.

a. Write an inequality expressing the possible perimeter P of the skating rink in feet in terms of its width w.

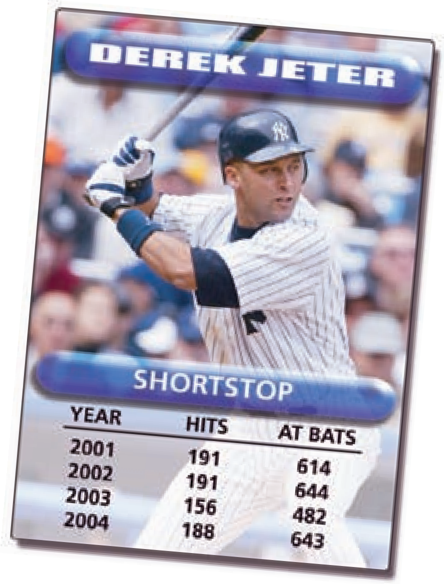
b. Is 400 feet a reasonable value for the perimeter? Explain.

37. Baseball The baseball card shows statistics for a professional player during four seasons.

a. A player's batting average is equal to his number of hits divided by his number of at bats. For which year listed on the card did Derek Jeter have the greatest batting average?

b. Write and solve an equation to find how many additional consecutive hits h Jeter would have needed to raise his batting average in 2004 to that of his average in 2001.

c. What if...? How many additional hits in a row would Jeter have needed to raise his batting average in 2003 to .500? Check your answer for reasonableness.



Solve each equation or inequality.

38. $\frac{15n}{n-3} = \frac{5}{n-3} - 8$ **39.** $\frac{z}{z+1} = \frac{z}{z-4}$ **40.** $\frac{4}{x} + 6 = \frac{1}{x^2}$

41. $\frac{8}{x} - \frac{3}{x} = \frac{6}{x-1}$ **42.** $\frac{2(x+4)}{x-4} = \frac{3x}{x-4}$ **43.** $\frac{1}{a-1} + \frac{4}{a+1} = \frac{7}{a^2-1}$

44. $\frac{6}{r} \geq \frac{5}{2}$ **45.** $\frac{8}{x+1} > 4$ **46.** $x \geq \frac{4}{x}$

Use a graphing calculator to solve each rational equation. Round your answers to the nearest hundredth.

47. $\frac{1}{x^2} = 5$ **48.** $\frac{1}{x^2} = x^2 - 1$ **49.** $\frac{1}{x-1} = x - 1$

50. Critical Thinking The reciprocal of a number plus $\frac{7}{2}$ equals 2. Find the number.

MULTI-STEP TEST PREP

51. This problem will prepare you for the Multi-Step Test Prep on page 608.

The average speed for the winner of the 2002 Indy 500 was 25 mi/h greater than the average speed for the 2001 winner. In addition, the 2002 winner completed the 500 mi race 32 min faster than the 2001 winner.

a. Let s represent the average speed of the 2001 winner in miles per hour. Write expressions in terms of s for the time in hours that it took the 2001 and 2002 winners to complete the race.

b. Write a rational equation that can be used to determine s. Solve your equation to find the average speed of the 2001 winner to the nearest mile per hour.

52. **Critical Thinking** An equation has the form $\frac{a}{x} + \frac{x}{b} = c$, where a, b, and c are constants and $b \neq 0$. How many values of x could make this equation true?

53. **Write About It** Describe the steps needed to solve the rational equation $\frac{3x}{5} = \frac{3}{x} - 6$.

TEST PREP

54. What value of x makes the equation $\frac{1}{x} + \frac{3}{x+3} = \frac{6}{x}$ true?

(A) $-\frac{15}{2}$ (B) $-\frac{12}{5}$ (C) $-\frac{3}{2}$ (D) $-\frac{6}{7}$

55. How many solutions does the equation $\frac{x+2}{x-4} - \frac{1}{x} = \frac{4}{x^2 - 4x}$ have?

(F) 0 (G) 1 (H) 2 (J) 3

56. If $x \neq -2$, which is equivalent to $\frac{4x}{x-2} = 6 + \frac{10}{x-2}$?

(A) $\frac{4x}{x-2} = \frac{16}{x-2}$

(B) $4x = 6(x-2) + 10$

(C) $4x = 6 + 10$

(D) $\frac{4}{-2} = 6 + \frac{5}{x-1}$

57. **Short Response** Water flowing through both a small pipe and a large pipe can fill a water tank in 7 h. Water flowing through the small pipe alone can fill the tank in 15 h.

a. Write an equation that can be used to find the number of hours it would take to fill the tank using only the large pipe.

b. How many hours would it take to fill the tank using only the large pipe? Show your work, or explain how you determined your answer.

CHALLENGE AND EXTEND

Solve each equation or inequality.

58. $\frac{4x}{x^2 + x - 6} = \frac{7x}{x^2 - 5x - 24}$

59. $\frac{1 - 4x^{-1} + 3x^{-2}}{1 - 9x^{-2}} = \frac{x-1}{x+3}$

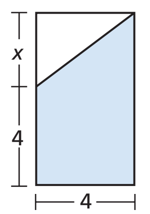
60. $\frac{3x}{x+2} - \frac{2}{x+4} \geq 7$

61. $\frac{6}{x-3} > \frac{x}{4} + 5$

62. Marcus and Will are painting a barn. Marcus paints about twice as fast as Will. On the first day, they have worked for 6 h and completed about $\frac{1}{3}$ of the job when Will gets injured. If Marcus has to complete the rest of the job by himself, about how many additional hours will it take him?

SPIRAL REVIEW

63. Write and simplify an expression in terms of x that represents the area of the shaded portion of the rectangle. *(Previous course)*



Simplify by rationalizing each denominator. *(Lesson 1-3)*

64. $\frac{-3\sqrt{3}}{\sqrt{8}}$

65. $\frac{5}{4\sqrt{7}}$

Given: y varies inversely as x. Write and graph each inverse variation function. *(Lesson 8-1)*

66. $y = -2$ when $x = 5$

67. $y = 2$ when $x = \frac{3}{2}$

CHAPTER 8

SECTION 8A

MULTI-STEP TEST PREP

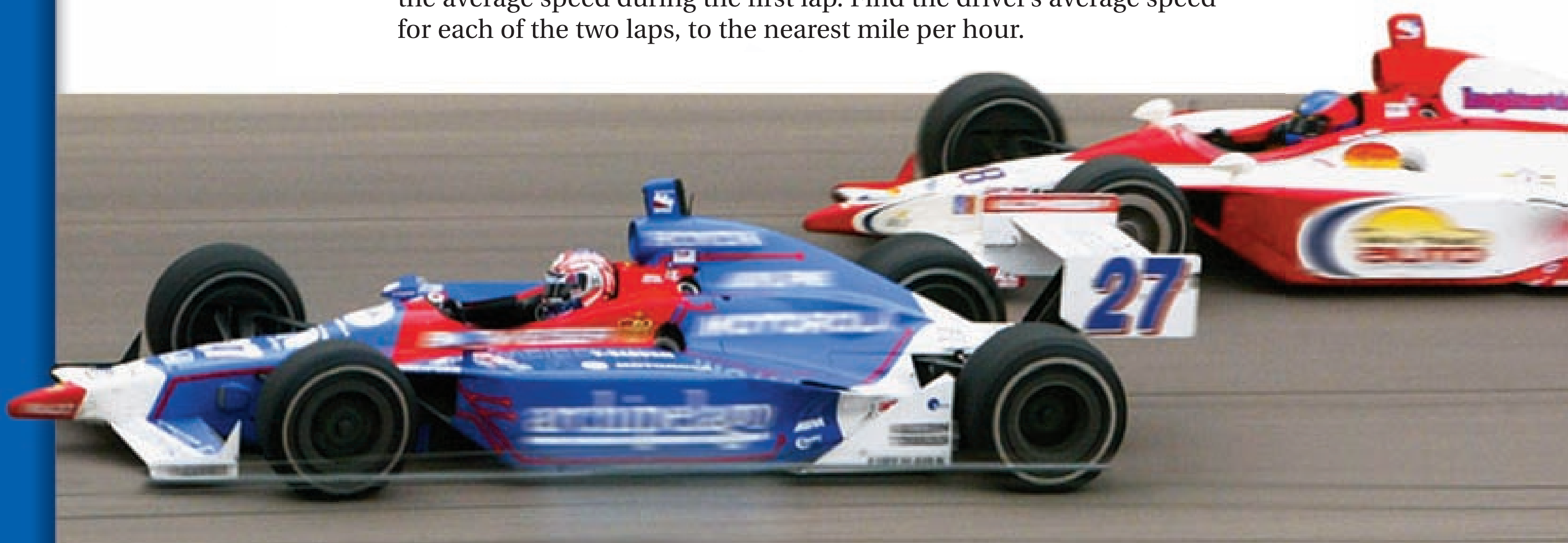
Rational Functions

Math in the Fast Lane The Indianapolis 500 is one of the most exciting events in sports. Each spring, 33 drivers compete in the 500 mi race, sometimes hitting speeds of more than 220 mi/h.

Winners of the Indianapolis 500

Year	Winner	Winning Time
1911	Roy Harroun	6 h 42 min
1990	Arie Luyendyk	2 h 41 min 18 s
2004	Buddy Rice	3 h 15 s

1. Write a rational function that can be used to model the race, where the independent variable represents the average speed in miles per hour and the dependent variable represents the time in hours it takes to complete the race. What type of variation function is it?
2. To the nearest mile per hour, how much faster was the average winning speed in 2004 than in 1911?
3. In 1990, Arie Luyendyk set the record for the fastest Indy 500 average speed, about 186 mi/h. The time in hours to finish the race based on Arie Luyendyk's record can be modeled by the function $t = \frac{500}{186 + s}$, where s is the speed above 186 in miles per hour. Graph the function, and evaluate it for $s = 10$. What does this value of the function represent?
4. During the race, a driver completes one lap with an average speed of 200 mi/h and then completes the following lap at an average speed of 210 mi/h. What is the driver's average speed for the two laps, to the nearest tenth of a mile per hour?
5. Each lap in the Indy 500 is 2.5 mi. A driver completes two laps in 1.5 min. The average speed during the second lap is 8 mi/h faster than the average speed during the first lap. Find the driver's average speed for each of the two laps, to the nearest mile per hour.



READY TO GO ON?

Quiz for Lessons 8-1 Through 8-5

8-1 Variation Functions

1. The mass m in kilograms of a bronze statue varies directly as its volume V in cubic centimeters. If a statue made from 1000 cm^3 of bronze has a mass of 8.7 kg, what is the mass of a statue made from 4500 cm^3 of bronze?
2. The time t in hours needed to clean the rides at an amusement park varies inversely with the number of workers n. If 6 workers can clean the rides in 6 hours, how many hours will it take 10 workers to clean the rides?

8-2 Multiplying and Dividing Rational Expressions

Simplify. Identify any x-values for which the expression is undefined.

3. $\dfrac{5x^3}{10x^2 + 5x}$
4. $\dfrac{x^2 - 2x - 3}{x^2 + 5x + 4}$
5. $\dfrac{-x + 6}{x^2 - 3x - 18}$

Multiply or divide. Assume that all expressions are defined.

6. $\dfrac{x + 3}{x + 2} \cdot \dfrac{2x - 4}{x^2 - 9}$
7. $\dfrac{9x^6 y}{27x^2 y^5} \div \dfrac{x}{6y^2}$
8. $\dfrac{2x^3 - 18x}{x^2 - 2x - 8} \div \dfrac{x^2 + x - 12}{x^2 - 16}$

8-3 Adding and Subtracting Rational Expressions

Add or subtract. Identify any x-values for which the expression is undefined.

9. $\dfrac{3x + 2}{x - 2} - \dfrac{x + 5}{x - 2}$
10. $\dfrac{x^2 - x}{x^2 - 25} + \dfrac{3}{x + 5}$
11. $\dfrac{x}{x - 3} - \dfrac{1}{x + 3}$
12. A plane's average speed when flying from one city to another is 550 mi/h and is 430 mi/h on the return flight. To the nearest mile per hour, what is the plane's average speed for the entire trip?

8-4 Rational Functions

Using the graph of $f(x) = \frac{1}{x}$ as a guide, describe the transformations and graph each function.

13. $g(x) = \dfrac{1}{x - 4}$
14. $g(x) = \dfrac{1}{x + 1} + 2$

Identify the zeros and asymptotes of each function. Then graph.

15. $f(x) = \dfrac{x^2 - 16}{x - 3}$
16. $f(x) = \dfrac{2x}{x^2 - 4}$

8-5 Solving Rational Equations and Inequalities

Solve each equation.

17. $y - \dfrac{10}{y} = 3$
18. $\dfrac{x}{x - 8} = \dfrac{24 - 2x}{x - 8}$
19. $\dfrac{-3x}{3} - \dfrac{x + 15}{x + 9} = 1$
20. A restaurant has two pastry ovens. When both ovens are used, it takes about 3 hours to bake the bread needed for one day. When only the large oven is used, it takes about 4 hours to bake the bread for one day. Approximately how long would it take to bake the bread for one day if only the small oven were used?

8-6 Radical Expressions and Rational Exponents

Objectives
Rewrite radical expressions by using rational exponents.
Simplify and evaluate radical expressions and expressions containing rational exponents.

Vocabulary
index
rational exponent

Who uses this?
Guitar makers use radical expressions to ensure that the strings produce the correct notes. (See Example 6.)

You are probably familiar with finding the square and square root of a number. These two operations are inverses of each other. Similarly, there are roots that correspond to larger powers.

5 and −5 are **square** roots of 25 because $5^2 = 25$ and $(-5)^2 = 25$.
2 is the **cube** root of 8 because $2^3 = 8$.
2 and −2 are **fourth** roots of 16 because $2^4 = 16$ and $(-2)^4 = 16$.
a is the nth root of b if $a^n = b$.

The nth root of a real number a can be written as the radical expression $\sqrt[n]{a}$, where n is the **index** (plural: *indices*) of the radical and a is the radicand. When a number has more than one real root, the radical sign indicates only the principal, or positive, root.

Reading Math
When a radical sign shows no index, it represents a square root.

Numbers and Types of Real Roots

Case	Roots	Example
Odd index	1 real root	The real 3rd root of 8 is 2.
Even index; positive radicand	2 real roots	The real 4th roots of 16 are ±2.
Even index; negative radicand	0 real roots	−16 has no real 4th roots.
Radicand of 0	1 root of 0	The 3rd root of 0 is 0.

EXAMPLE 1 Finding Real Roots

Find all real roots.

A **fourth roots of 81**
A positive number has two real fourth roots. Because $3^4 = 81$ and $(-3)^4 = 81$, the roots are 3 and −3.

B **cube roots of −125**
A negative number has one real cube root. Because $(-5)^3 = -125$, the root is −5.

C **sixth roots of −729**
A negative number has no real sixth roots.

CHECK IT OUT! **Find all real roots.**
1a. fourth roots of −256 **1b.** sixth roots of 1 **1c.** cube roots of 125

The properties of square roots in Lesson 1-3 also apply to nth roots.

Know it! Note

Properties of nth Roots

For $a > 0$ and $b > 0$,

WORDS	NUMBERS	ALGEBRA
Product Property of Roots		
The nth root of a product is equal to the product of the nth roots.	$\sqrt[3]{16} = \sqrt[3]{8} \cdot \sqrt[3]{2} = 2\sqrt[3]{2}$	$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$
Quotient Property of Roots		
The nth root of a quotient is equal to the quotient of the nth roots.	$\sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4}$	$\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$

EXAMPLE 2 **Simplifying Radical Expressions**

Simplify each expression. Assume that all variables are positive.

A $\sqrt[3]{27x^6}$

$\sqrt[3]{3^3 \cdot x^3 \cdot x^3}$ *Factor into perfect cubes.*

$\sqrt[3]{3^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^3}$ *Product Property*

$3 \cdot x \cdot x$ *Simplify.*

$3x^2$

B $\sqrt[3]{\frac{x^3}{7}}$

$\frac{\sqrt[3]{x^3}}{\sqrt[3]{7}}$ *Quotient Property*

$\frac{x}{\sqrt[3]{7}}$ *Simplify the numerator.*

$\frac{x}{\sqrt[3]{7}} \cdot \frac{\sqrt[3]{7}}{\sqrt[3]{7}} \cdot \frac{\sqrt[3]{7}}{\sqrt[3]{7}}$ *Rationalize the denominator.*

$\frac{x\sqrt[3]{7^2}}{\sqrt[3]{7^3}}$ *Product Property*

$\frac{x\sqrt[3]{49}}{7}$ *Simplify.*

Remember!

When an expression contains a radical in the denominator, you must rationalize the denominator. To do so, rewrite the expression so that the denominator contains no radicals.

CHECK IT OUT! **Simplify each expression. Assume that all variables are positive.**

2a. $\sqrt[4]{16x^4}$ **2b.** $\sqrt[4]{\frac{x^8}{3}}$ **2c.** $\sqrt[3]{x^7} \cdot \sqrt[3]{x^2}$

A **rational exponent** is an exponent that can be expressed as $\frac{m}{n}$, where m and n are integers and $n \neq 0$. Radical expressions can be written by using rational exponents.

Know it! Note

Rational Exponents

For any natural number n and integer m,

WORDS	NUMBERS	ALGEBRA
The exponent $\frac{1}{n}$ indicates the nth root.	$16^{\frac{1}{4}} = \sqrt[4]{16} = 2$	$a^{\frac{1}{n}} = \sqrt[n]{a}$
The exponent $\frac{m}{n}$ indicates the nth root raised to the mth power.	$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$	$a^{\frac{m}{n}} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

EXAMPLE 3 Writing Expressions in Radical Form

Write the expression $(-125)^{\frac{2}{3}}$ in radical form, and simplify.

Writing Math

The denominator of a rational exponent becomes the index of the radical.

Method 1 Evaluate the root first.		**Method 2** Evaluate the power first.	
$\left(\sqrt[3]{-125}\right)^2$	*Write with a radical.*	$\sqrt[3]{(-125)^2}$	*Write with a radical.*
$(-5)^2$	*Evaluate the root.*	$\sqrt[3]{15{,}625}$	*Evaluate the power.*
25	*Evaluate the power.*	25	*Evaluate the root.*

CHECK IT OUT! Write each expression in radical form, and simplify.

3a. $64^{\frac{1}{3}}$ **3b.** $4^{\frac{5}{2}}$ **3c.** $625^{\frac{3}{4}}$

EXAMPLE 4 Writing Expressions by Using Rational Exponents

Write each expression by using rational exponents.

A $\sqrt[4]{7^3}$

$7^{\frac{3}{4}}$ *$\sqrt[n]{a^m} = a^{\frac{m}{n}}$*

B $\sqrt[3]{11^6}$

$11^{\frac{6}{3}}$ *$\sqrt[n]{a^m} = a^{\frac{m}{n}}$*

$11^2 = 121$ *Simplify.*

CHECK IT OUT! Write each expression by using rational exponents.

4a. $\left(\sqrt[4]{81}\right)^3$ **4b.** $\sqrt[3]{10^9}$ **4c.** $\sqrt[4]{5^2}$

Rational exponents have the same properties as integer exponents. (See Lesson 1-5.)

Know it! Note

Properties of Rational Exponents

For all nonzero real numbers a and b and rational numbers m and n,

WORDS	NUMBERS	ALGEBRA
Product of Powers Property To multiply powers with the same base, add the exponents.	$12^{\frac{1}{2}} \cdot 12^{\frac{3}{2}} = 12^{\frac{1}{2}+\frac{3}{2}} = 12^2 = 144$	$a^m \cdot a^n = a^{m+n}$
Quotient of Powers Property To divide powers with the same base, subtract the exponents.	$\frac{125^{\frac{2}{3}}}{125^{\frac{1}{3}}} = 125^{\frac{2}{3}-\frac{1}{3}} = 125^{\frac{1}{3}} = 5$	$\frac{a^m}{a^n} = a^{m-n}$
Power of a Power Property To raise one power to another, multiply the exponents.	$\left(8^{\frac{2}{3}}\right)^3 = 8^{\frac{2}{3}\cdot 3} = 8^2 = 64$	$(a^m)^n = a^{m \cdot n}$
Power of a Product Property To find the power of a product, distribute the exponent.	$(16 \cdot 25)^{\frac{1}{2}} = 16^{\frac{1}{2}} \cdot 25^{\frac{1}{2}} = 4 \cdot 5 = 20$	$(ab)^m = a^m b^m$
Power of a Quotient Property To find the power of a quotient, distribute the exponent.	$\left(\frac{16}{81}\right)^{\frac{1}{4}} = \frac{16^{\frac{1}{4}}}{81^{\frac{1}{4}}} = \frac{2}{3}$	$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$

EXAMPLE 5 Simplifying Expressions with Rational Exponents

Simplify each expression.

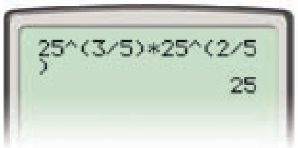
A $25^{\frac{3}{5}} \cdot 25^{\frac{2}{5}}$

$25^{\frac{3}{5}+\frac{2}{5}}$ *Product of Powers*

25^1 *Simplify.*

25 *Evaluate the power.*

Check Enter the expression in a graphing calculator.



B $\dfrac{8^{\frac{1}{3}}}{8^{\frac{2}{3}}}$

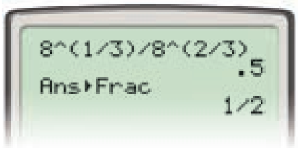
$8^{\frac{1}{3}-\frac{2}{3}}$ *Quotient of Powers*

$8^{-\frac{1}{3}}$ *Simplify.*

$\dfrac{1}{8^{\frac{1}{3}}}$ *Negative Exponent Property*

$\dfrac{1}{2}$ *Evaluate the power.*

Check Enter the expression in a graphing calculator.



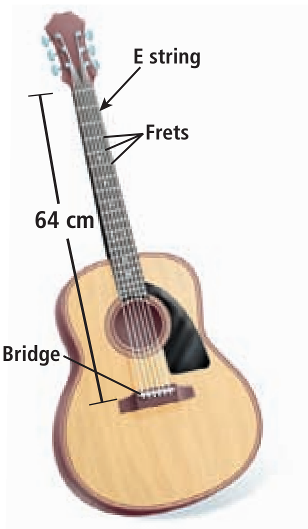
CHECK IT OUT! Simplify each expression.

5a. $36^{\frac{3}{8}} \cdot 36^{\frac{1}{8}}$ **5b.** $(-8)^{-\frac{1}{3}}$ **5c.** $\dfrac{5^{\frac{9}{4}}}{5^{\frac{1}{4}}}$

EXAMPLE 6 Music Application

Frets are small metal bars positioned across the neck of a guitar so that the guitar can produce the notes of a specific scale.

To find the distance a fret should be placed from the bridge, multiply the length of the string by $2^{-\frac{n}{12}}$, where n is the number of notes higher than the string's root note. Where should a fret be placed to produce a G note on the E string (3 notes higher)?



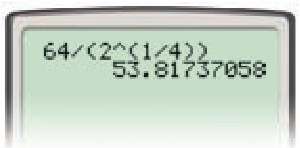
$$64\left(2^{-\frac{n}{12}}\right) = 64\left(2^{-\frac{3}{12}}\right)$$ *Use 64 cm for the length of the string, and substitute 3 for n.*

$$= 64\left(2^{-\frac{1}{4}}\right)$$ *Simplify.*

$$= 64\left(\frac{1}{2^{\frac{1}{4}}}\right)$$ *Negative Exponent Property*

$$= \frac{64}{2^{\frac{1}{4}}}$$ *Simplify.*

$$\approx 53.82$$ *Use a calculator.*



The fret should be placed about 53.82 cm from the bridge.

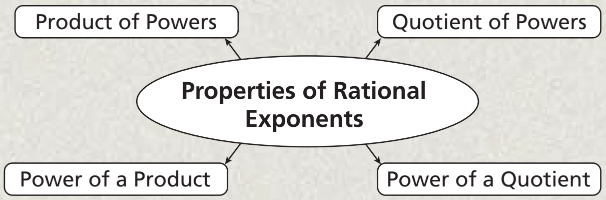
CHECK IT OUT! **6.** Where should a fret be placed to produce the E note that is one octave higher on the E string (12 notes higher)?

THINK AND DISCUSS

Know it! Note

1. Explain why $\sqrt[n]{a^n}$ is equal to a for all natural numbers a and n.

2. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, give a numeric and an algebraic example of the given property of rational exponents.



8-6 Exercises

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GUIDED PRACTICE

1. **Vocabulary** The *index* of the expression $\sqrt[3]{4^2}$ is __?__ . (2, 3, or 4)

SEE EXAMPLE 1 p. 610

Find all real roots.

2. cube roots of 27
3. fourth roots of 625
4. cube roots of 0

SEE EXAMPLE 2 p. 611

Simplify each expression. Assume that all variables are positive.

5. $\sqrt[3]{8x^3}$
6. $\sqrt[4]{\dfrac{32}{x^4}}$
7. $\sqrt[3]{\dfrac{125x^6}{6}}$
8. $\sqrt{50x^3}$
9. $\sqrt[4]{x^8} \cdot \sqrt[3]{x^4}$
10. $\sqrt[3]{\dfrac{x^5}{4}}$
11. $\dfrac{\sqrt{40x^4}}{\sqrt[3]{-x^3}}$
12. $\sqrt[4]{\dfrac{x^{12}y^4}{3}}$

SEE EXAMPLE 3 p. 612

Write each expression in radical form, and simplify.

13. $36^{\frac{3}{2}}$
14. $32^{\frac{3}{5}}$
15. $(-27)^{\frac{1}{3}}$
16. $8^{\frac{2}{3}}$

SEE EXAMPLE 4 p. 612

Write each expression by using rational exponents.

17. $\sqrt[5]{9^{10}}$
18. $\sqrt{8^3}$
19. $\left(\sqrt[6]{5}\right)^3$
20. $\left(\sqrt[3]{27}\right)^2$

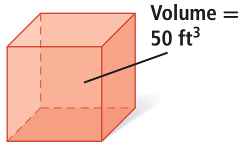
SEE EXAMPLE 5 p. 613

Simplify each expression.

21. $13^{\frac{1}{2}} \cdot 13^{\frac{3}{2}}$
22. $\dfrac{9^{\frac{4}{3}}}{9^{\frac{2}{3}}}$
23. $\left(64^{\frac{1}{2}}\right)^{\frac{1}{3}}$
24. $\left(\dfrac{8}{27}\right)^{\frac{1}{3}}$
25. $25^{-\frac{1}{2}}$
26. $7^{\frac{1}{4}} \cdot 7^{-\frac{3}{4}}$
27. $(-125)^{-\frac{1}{3}}$
28. $\left(6^{\frac{1}{2}}\right)^6$

SEE EXAMPLE 6 p. 613

29. **Geometry** The side length of a cube can be determined by finding the cube root of the volume. What is the side length to the nearest *inch* of the cube shown?



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
30–32	1
33–40	2
41–44	3
45–48	4
49–56	5
57	6

Extra Practice

Skills Practice p. S19

Application Practice p. S39

Find all real roots.

30. cube roots of -64 **31.** fifth roots of 32 **32.** fourth roots of -16

Simplify each expression. Assume that all variables are positive.

33. $\sqrt[3]{9x} \cdot \sqrt[3]{3x^2}$ **34.** $\sqrt[4]{324x^8}$ **35.** $\sqrt[3]{\dfrac{x^6}{250}}$ **36.** $\sqrt{\dfrac{x^5}{45}}$

37. $\sqrt[3]{56x^9}$ **38.** $\dfrac{\sqrt[4]{x^{10}}}{\sqrt[4]{x^4}}$ **39.** $\sqrt[5]{x^7} \cdot \sqrt[5]{x^6}$ **40.** $\sqrt[3]{-54x^9y^3}$

Write each expression in radical form, and simplify.

41. $64^{\frac{1}{2}}$ **42.** $216^{\frac{2}{3}}$ **43.** $(-1000)^{\frac{4}{3}}$ **44.** $6^{\frac{3}{2}}$

Write each expression by using rational exponents.

45. $\sqrt[3]{14^3}$ **46.** $\left(\sqrt[5]{-8}\right)^4$ **47.** $\left(\sqrt[4]{144}\right)^2$ **48.** $\sqrt{48^3}$

Simplify each expression.

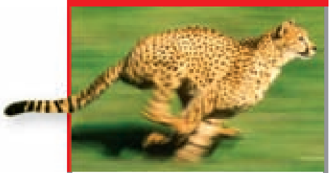
49. $(8 \cdot 64)^{\frac{2}{3}}$ **50.** $144^{-\frac{1}{2}}$ **51.** $\left(\dfrac{2^3}{27}\right)^{\frac{1}{3}}$ **52.** $2^{\frac{1}{2}} \cdot 2^{\frac{1}{4}}$

53. $\left(\dfrac{49}{81}\right)^{-\frac{1}{2}}$ **54.** $\dfrac{12^{\frac{1}{4}}}{12^{\frac{3}{4}}}$ **55.** $\left(5^{\frac{1}{3}}\right)^{\frac{1}{3}}$ **56.** $\left(\dfrac{27}{27^{\frac{1}{3}}}\right)^{\frac{1}{2}}$

57. Banking The initial amount deposited in a savings account is \$1000. The amount a in dollars in the account after t years can be represented by the function $a(t) = 1000\left(2^{\frac{t}{24}}\right)$. To the nearest dollar, what will the amount in the account be after 6 years?

LINK

Biology



Suppose that a cheetah, a lion, a house cat, and the fastest human runner competed in a 100 m race. At top speed, the cheetah would finish in 3.20 s, the lion in 4.47 s, the house cat in 7.46 s, and the human in 9.77 s.

58. Biology The formula $P = 73.3\sqrt[4]{m^3}$, known as Kleiber's law, relates the metabolism rate P of an organism in Calories per day and the body mass m of the organism in kilograms. The table shows the typical body mass of several members of the cat family.

Typical Body Mass

Animal	Mass (kg)
House cat	4.5
Cheetah	55.0
Lion	170.0

a. What is the metabolism rate of a cheetah to the nearest Calorie per day?

b. Multi-Step Approximately how many more Calories of food does a lion need to consume each day than a house cat does?

59. Medicine Iodine-131 is a radioactive material used to treat certain types of cancer. Iodine-131 has a half-life of 8 days, which means that it takes 8 days for half of an initial sample to decay. The percent of radioactive material that remains after t days can be determined from the expression $100\left(\frac{1}{2}\right)^{\frac{t}{h}}$, where h is the half-life in days.

a. What percent of a sample of iodine-131 will remain after 20 days?

b. What if...? Another form of radioactive iodine used in cancer treatment is iodine-125. Iodine-125 has a half-life of 59 days. A hospital has 20 g of iodine-125 and 20 g of iodine-131 left over from treating patients. How much more iodine-125 than iodine-131 will remain after a period of 30 days?

60. Meteorology The formula $W = 35.74 + 0.6215T - 35.75V^{\frac{4}{25}} + 0.4275TV^{\frac{4}{25}}$ relates the wind chill temperature W to the air temperature T in degrees Fahrenheit and the wind speed V in miles per hour. Use a calculator to find the wind chill to the nearest degree when the air temperature is 40°F and the wind speed is 35 mi/h.

MULTI-STEP TEST PREP

61. This problem will prepare you for the Multi-Step Test Prep on page 636.

For a pendulum with a length L in meters, the expression $2\pi\sqrt{\frac{L}{g}}$ models the time in seconds for the pendulum to complete one back-and-forth swing. In this expression, g is the acceleration due to gravity, 9.8 m/s^2.

a. Simplify the expression by rationalizing the denominator.

b. To the nearest tenth of a second, how long does it take a pendulum with a length of 0.35 m to complete one back-and-forth swing?

Write each expression by using rational exponents. Assume that all variables are positive.

62. $\sqrt[4]{20x^3}$ **63.** $\sqrt{(5x)^7}$ **64.** $\left(\sqrt[5]{-9}\sqrt[3]{x}\right)^4$ **65.** $\left(\sqrt[4]{11x^8}\right)^6$

Simplify each expression, and write it by using a radical. Assume that all variables are positive.

66. $\left(-8x^{12}\right)^{\frac{2}{3}}$ **67.** $5^{\frac{7}{4}}x^{\frac{3}{4}}$ **68.** $\left(-12x^{15}\right)^{\frac{3}{5}}$

69. $\left(a^2b^4\right)^{\frac{1}{3}}$ **70.** $\left(\frac{a^4}{b}\right)^{\frac{1}{4}}$ **71.** $a^{\frac{3}{4}}\left(4b^6\right)^{\frac{1}{4}}$

72. Botany Duckweed is a quickly growing plant that floats on the surface of lakes and ponds. The initial mass of a population of duckweed plants is 100 kg. The mass of the population doubles every 60 h and can be represented by the function $m(t) = 100 \cdot 2^{\frac{t}{60}}$, where t is time in hours. To the nearest kilogram, what is the mass of the plants after 24 h?

Explain whether each statement is sometimes, always, or never true for nonzero values of the variable.

73. $\sqrt[3]{x^6} = x^2$ **74.** $(x)^{\frac{1}{3}} = (-x)^{\frac{1}{3}}$ **75.** $-\sqrt[4]{x^8} = x^{-2}$ **76.** $-\sqrt[3]{x} < 0$

Estimation **Identify the pair of consecutive integers that each value is between. Then use a calculator to check your answer.**

77. $\sqrt[3]{18}$ **78.** $\sqrt[4]{200}$ **79.** $\sqrt[3]{-80}$

80. Physics Air pressure decreases with altitude according to the formula $P = 14.7(10)^{-0.000014a}$, where P is the air pressure in pounds per square inch and a is the altitude in feet above sea level.

a. Use the formula to estimate the air pressure in Denver, Colorado, which is 5280 ft above sea level.

b. Use the formula to estimate the air pressure at the top of Mount Everest, which is 29,028 ft above sea level.

81. ERROR ANALYSIS Below are two methods of simplifying an expression. Which is incorrect? Explain the error.

A

$625^{\frac{1}{3}} \div 625^{\frac{4}{3}}$

$625^{\frac{1}{3} \div \frac{4}{3}}$

$625^{\frac{1}{4}}$

5

B

$625^{\frac{1}{3}} \div 625^{\frac{4}{3}}$

$625^{\frac{1}{3} - \frac{4}{3}}$

625^{-1}

$\frac{1}{625}$

82. How many different positive integer values of n result in a whole number nth root of 64? What are these values?

83. **Critical Thinking** Describe two ways to find the sixth root of 10 on a calculator.

84. **Write About It** Explain whether the expression $x^{2.4}$ contains a rational exponent.

TEST PREP

85. Which of the following represents a real number?

Ⓐ $6^{-\frac{4}{3}}$ Ⓑ $(-9)^{\frac{3}{2}}$ Ⓒ $\sqrt[4]{-11}$ Ⓓ $\left(\sqrt[4]{-14}\right)^3$

86. The surface area S of a sphere with volume V is $S = (4\pi)^{\frac{1}{3}}(3V)^{\frac{2}{3}}$. What effect does increasing the volume of a sphere by a factor of 8 have on its surface area?

Ⓕ The surface area doubles.

Ⓖ The surface area triples.

Ⓗ The surface area increases by a factor of 4.

Ⓙ The surface area increases by a factor of 8.

87. If $a = x^6$, what is $\sqrt[4]{a}$?

Ⓐ $(\sqrt{x})^3$ Ⓑ x^2 Ⓒ $x^2\sqrt{x}$ Ⓓ $x^{\frac{2}{3}}$

88. Which expression is equivalent to $\sqrt[3]{\dfrac{56a^6}{7}}$?

Ⓕ $2a^2$ Ⓖ $8a^2$ Ⓗ $2a^3$ Ⓙ $8a^3$

CHALLENGE AND EXTEND

89. Write an expression by using rational exponents for the square root of the square root of the square root of 20.

90. Simplify the expression $2^{\frac{1}{3}} \cdot 4^{\frac{1}{6}} \cdot 8^{\frac{1}{9}}$.

91. **Critical Thinking** For what real values of a is $\sqrt[3]{a}$ greater than a?

92. Any nonzero real number has three cube roots, only one of which is real. Show that the cube roots of 1 are 1, $\frac{-1 + i\sqrt{3}}{2}$, and $\frac{-1 - i\sqrt{3}}{2}$.

SPIRAL REVIEW

Add or subtract, if possible. *(Lesson 4-1)*

$A = \begin{bmatrix} -2 & -1 & 0 \\ 3 & 2 & 6 \end{bmatrix}$ $B = \begin{bmatrix} 1 & -1 \\ 0 & -2 \end{bmatrix}$ $C = \begin{bmatrix} 5 & 7 \\ 0 & -4 \end{bmatrix}$ $D = \begin{bmatrix} 9 & 8 & -2 \\ 6 & 3 & -1 \end{bmatrix}$

93. $A + D$ 94. $B - C$ 95. $B + C$

Use the description to write each quadratic function in vertex form. *(Lesson 5-1)*

96. The parent function $f(x) = x^2$ is vertically compressed by a factor of 3 and then translated 1 unit right to create g.

97. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 3 units up to create h.

Identify the zeros and the asymptotes of each function. Then graph. *(Lesson 8-4)*

98. $f(x) = \dfrac{x^2 - 4}{x + 5}$ 99. $f(x) = \dfrac{x + 3}{x^2 + 6x + 5}$ 100. $f(x) = \dfrac{4x - 3}{x + 6}$

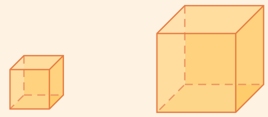
Connecting Algebra to **Geometry**

Area and Volume Relationships

When you change the linear dimensions of a solid figure, its surface area and volume may change in different ways.

Recall that when you multiply the side length of a cube by a constant a, the surface area increases by a factor of a^2 and the volume increases by a factor of a^3, as shown.

When you want to change the surface area or volume of a figure but maintain the same linear proportions, you can use the reverse process. If the surface area increases by a factor of a, the linear dimensions increase by a factor of $\sqrt{a}$. If the volume increases by a factor of a, the linear dimensions increase by a factor of $\sqrt[3]{a}$.

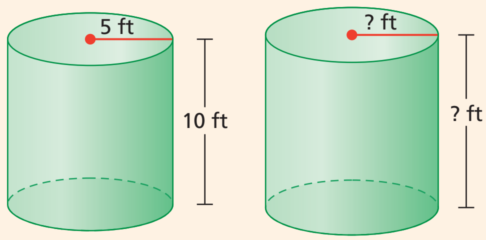


Side Length	$s = 1$ cm	$s = \mathbf{2}(1) = 2$ cm
Surface Area	$A = 6$ cm^2	$A = \mathbf{2}^2(6) = 24$ cm^2
Volume	$V = 1$ cm^3	$V = \mathbf{2}^3(1) = 8$ cm^3

Example

A cylindrical water storage tank has a radius of 5 ft and a height of 10 ft. A new tank similar to the first is constructed with 20% more capacity. What are the radius and height of the new tank?

The capacity of the larger tank is 120% of the smaller tank. So, the volume is increased by a factor of 1.2.



Step 1 Find the scale factor for the linear dimensions.

$\sqrt[3]{1.2} \approx 1.0627$ *Take the cube root.*

Step 2 Find the new dimensions.

$1.0627(5) \approx 5.31$

$1.0627(10) \approx 10.63$

Multiply the original dimensions by the scale factor.

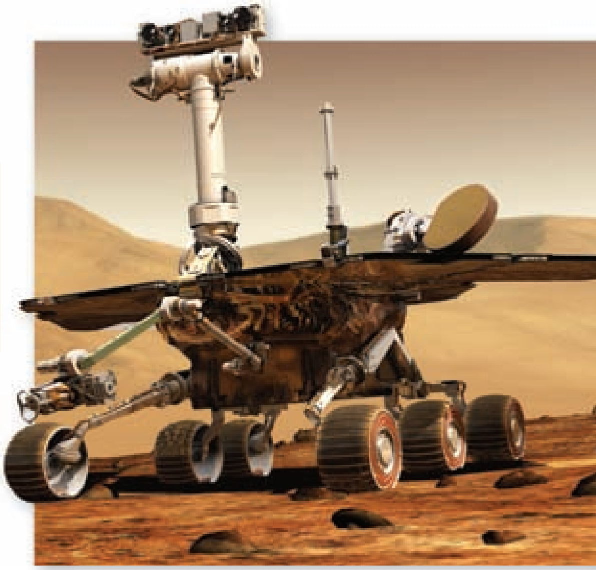
The radius is about 5.31 ft, and the height is about 10.63 ft.

Try This

Solve each problem. If necessary, round your answers to the nearest thousandth.

1. Marsha wants to double the surface area of a circular pond. How should she change the radius? the diameter?
2. The volume of a sphere is increased by a factor of 100. The new radius is 30 cm. What was the radius of the original sphere?
3. The surface area of a cube is decreased from 150 cm^2 to 96 cm^2. By what factor has the volume changed?
4. A store owner wants to create giant ice-cream cones that contain 3 times the volume of a traditional cone. How should he change the radius and height of the traditional cone?

8-7 Radical Functions



Objectives

Graph radical functions and inequalities.

Transform radical functions by changing parameters.

Vocabulary

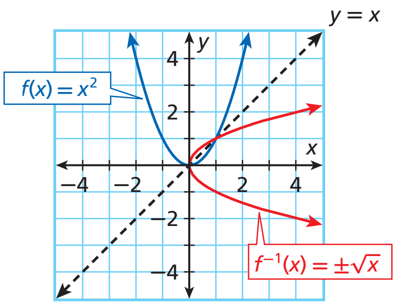
radical function

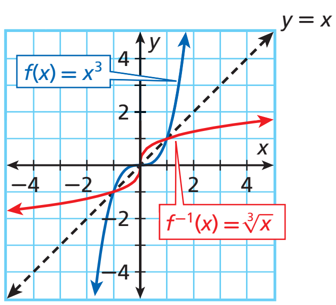
square-root function

Who uses this?

Aerospace engineers use transformations of radical functions to adjust for gravitational changes on other planets. (See Example 5.)

Recall that exponential and logarithmic functions are inverse functions. Quadratic and cubic functions have inverses as well. The graphs below show the inverses of the quadratic parent function and the cubic parent function.





Notice that the inverse of $f(x) = x^2$ is not a function because it fails the vertical line test. However, if we limit the domain of $f(x) = x^2$ to $x \geq 0$, its inverse is the function $f^{-1}(x) = \sqrt{x}$.

A **radical function** is a function whose rule is a radical expression. A **square-root function** is a radical function involving $\sqrt{x}$. The square-root parent function is $f(x) = \sqrt{x}$. The cube-root parent function is $f(x) = \sqrt[3]{x}$.

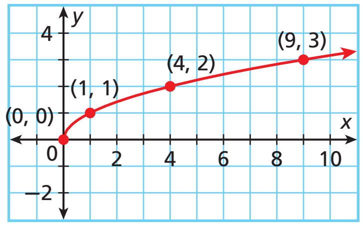
EXAMPLE 1 Graphing Radical Functions

Graph the function, and identify its domain and range.

A $f(x) = \sqrt{x}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Because the square root of a negative number is imaginary, choose only nonnegative values for x.

x	$f(x) = \sqrt{x}$	$(x, f(x))$
0	$f(0) = \sqrt{0} = 0$	(0, 0)
1	$f(1) = \sqrt{1} = 1$	(1, 1)
4	$f(4) = \sqrt{4} = 2$	(4, 2)
9	$f(9) = \sqrt{9} = 3$	(9, 3)



The domain is $\{x \mid x \geq 0\}$, and the range is $\{y \mid y \geq 0\}$.

Helpful Hint

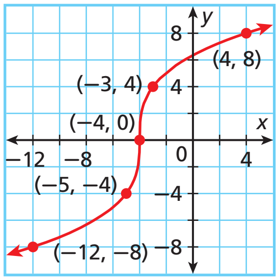
When using a table to graph square-root functions, choose x-values that make the radicands perfect squares.

Graph the function, and identify its domain and range.

B $f(x) = 4\sqrt[3]{x+4}$

Make a table of values. Plot enough ordered pairs to see the shape of the curve. Choose both negative and positive values for x.

x	$4\sqrt[3]{x+4}$	$(x, f(x))$
-12	$4\sqrt[3]{-12+4} = 4\sqrt[3]{-8} = -8$	$(-12, -8)$
-5	$4\sqrt[3]{-5+4} = 4\sqrt[3]{-1} = -4$	$(-5, -4)$
-4	$4\sqrt[3]{-4+4} = 4\sqrt[3]{0} = 0$	$(-4, 0)$
-3	$4\sqrt[3]{-3+4} = 4\sqrt[3]{1} = 4$	$(-3, 4)$
4	$4\sqrt[3]{4+4} = 4\sqrt[3]{8} = 8$	$(4, 8)$

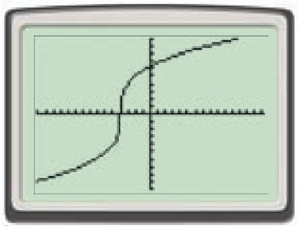


Helpful Hint

When using a table to graph cube-root functions, choose x-values that make the radicands perfect cubes.

The domain is the set of all real numbers. The range is also the set of all real numbers.

Check Graph the function on a graphing calculator.



The graphs appear to be identical.

CHECK IT OUT! **Graph each function, and identify its domain and range.**

1a. $f(x) = \sqrt[3]{x}$ **1b.** $f(x) = \sqrt{x+1}$

The graphs of radical functions can be transformed by using methods similar to those used to transform linear, quadratic, polynomial, and exponential functions. This lesson will focus on transformations of square-root functions.

Know it! Note

Transformations of the Square-Root Parent Function $f(x) = \sqrt{x}$

Transformation	$f(x)$ Notation	Examples
Vertical translation	$f(x) + k$	$y = \sqrt{x} + 3$ 3 units up $y = \sqrt{x} - 4$ 4 units down
Horizontal translation	$f(x - h)$	$y = \sqrt{x-2}$ 2 units right $y = \sqrt{x+1}$ 1 unit left
Vertical stretch/compression	$af(x)$	$y = 6\sqrt{x}$ vertical stretch by 6 $y = \frac{1}{2}\sqrt{x}$ vertical compression by $\frac{1}{2}$
Horizontal stretch/compression	$f\left(\frac{1}{b}x\right)$	$y = \sqrt{\frac{1}{5}x}$ horizontal stretch by 5 $y = \sqrt{3x}$ horizontal compression by $\frac{1}{3}$
Reflection	$-f(x)$ $f(-x)$	$y = -\sqrt{x}$ across x-axis $y = \sqrt{-x}$ across y-axis

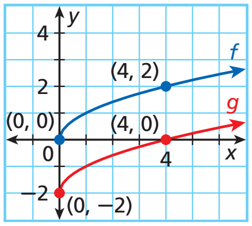
EXAMPLE 2 Transforming Square-Root Functions

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

A $g(x) = \sqrt{x} - 2$

$g(x) = f(x) - 2$

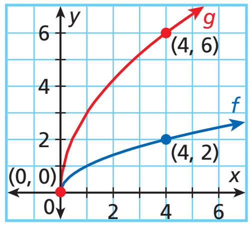
Translate f 2 units down.



B $g(x) = 3\sqrt{x}$

$g(x) = 3 \cdot f(x)$

Stretch f vertically by a factor of 3.



CHECK IT OUT! Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

2a. $g(x) = \sqrt{x} + 1$

2b. $g(x) = \frac{1}{2}\sqrt{x}$

Transformations of square-root functions are summarized below.

Know it! Note

$$f(x) = a\sqrt{\frac{1}{b}(x - h)} + k$$

$|a| \longrightarrow$ vertical stretch or compression factor
$a < 0 \longrightarrow$ reflection across the x-axis

$h \longrightarrow$ horizontal translation

$|b| \longrightarrow$ horizontal stretch or compression factor
$b < 0 \longrightarrow$ reflection across the y-axis

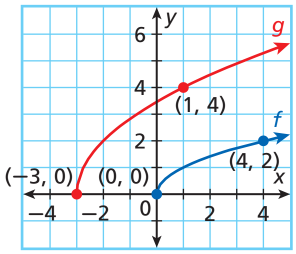
$k \longrightarrow$ vertical translation

EXAMPLE 3 Applying Multiple Transformations

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

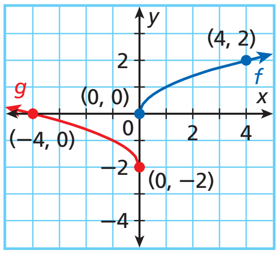
A $g(x) = 2\sqrt{x + 3}$

Stretch f vertically by a factor of 2, and translate it 3 units left.



B $g(x) = \sqrt{-x} - 2$

Reflect f across the y-axis, and translate it 2 units down.



CHECK IT OUT! Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

3a. $g(x) = \sqrt{-x} + 3$

3b. $g(x) = -3\sqrt{x} - 1$

EXAMPLE 4 Writing Transformed Square-Root Functions

Use the description to write the square-root function g.

The parent function $f(x) = \sqrt{x}$ is stretched horizontally by a factor of 2, reflected across the y-axis, and translated 3 units left.

Step 1 Identify how each transformation affects the function.

$$\left.\begin{array}{l}\text{Horizontal stretch by a factor of 2: } |b| = 2 \\ \text{Reflection across the } y\text{-axis: } b \text{ is negative}\end{array}\right\} b = -2$$

Translation 3 units left: $h = -3$

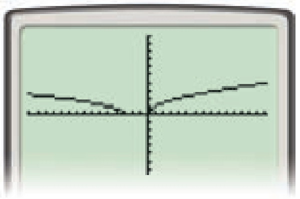
Step 2 Write the transformed function.

$$g(x) = \sqrt{\frac{1}{b}(x - h)}$$

$$g(x) = \sqrt{\frac{1}{-2}[x-(-3)]} \qquad \textit{Substitute } -2 \textit{ for } b \textit{ and } -3 \textit{ for } h.$$

$$g(x) = \sqrt{-\frac{1}{2}(x + 3)} \qquad \textit{Simplify.}$$

Check Graph both functions on a graphing calculator. The graph of g indicates the given transformations of f.



CHECK IT OUT! **Use the description to write the square-root function g.**

4. The parent function $f(x) = \sqrt{x}$ is reflected across the x-axis, stretched vertically by a factor of 2, and translated 1 unit up.

EXAMPLE 5 *Space Exploration Application*

Special airbags are used to protect scientific equipment when a rover lands on the surface of Mars. On Earth, the function $f(x) = \sqrt{64x}$ approximates an object's downward velocity in feet per second as the object hits the ground after bouncing x ft in height.

The corresponding function for Mars is compressed vertically by a factor of about $\frac{3}{5}$. Write the corresponding function g for Mars, and use it to estimate how fast a rover will hit Mars's surface after a bounce of 45 ft in height.

Step 1 To compress f vertically by a factor of $\frac{3}{5}$, multiply f by $\frac{3}{5}$.

$$g(x) = \frac{3}{5}f(x) = \frac{3}{5}\sqrt{64x}$$

Step 2 Find the value of g for a bounce height of 45 ft.

$$g(45) = \frac{3}{5}\sqrt{64(45)} \approx 32 \qquad \textit{Substitute 45 for x and simplify.}$$

The rover will hit Mars's surface with a downward velocity of about 32 ft/s at the end of the bounce.

CHECK IT OUT! **Use the information on the previous page to answer the following.**

5. The downward velocity function for the Moon is a horizontal stretch of f by a factor of about $\frac{25}{4}$. Write the velocity function h for the Moon, and use it to estimate the downward velocity of a landing craft at the end of a bounce 50 ft in height.

In addition to graphing radical functions, you can also graph radical inequalities. Use the same procedure you used for graphing linear and quadratic inequalities.

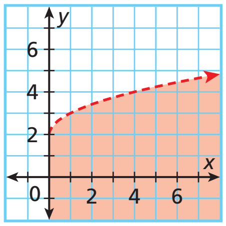
EXAMPLE 6 Graphing Radical Inequalities

Graph the inequality $y < \sqrt{x} + 2$.

Step 1 Use the related equation $y = \sqrt{x} + 2$ to make a table of values.

x	0	1	4	9
y	2	3	4	5

Step 2 Use the table to graph the boundary curve. The inequality sign is $<$, so use a dashed curve and shade the area below it.



Because the value of x cannot be negative, do not shade left of the y-axis.

Check Choose a point in the solution region, such as $(1, 0)$, and test it in the inequality.

$$y < \sqrt{x} + 2$$
$$0 \overset{?}{<} \sqrt{1} + 2$$
$$0 < 3 \checkmark$$

CHECK IT OUT! **Graph each inequality.**

6a. $y > \sqrt{x + 4}$

6b. $y \geq \sqrt[3]{x - 3}$

THINK AND DISCUSS

1. Explain whether radical functions have asymptotes.

2. Explain how to determine the domain of the function $f(x) = \sqrt{2x + 2}$.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example of the transformation of the square-root function $f(x) = \sqrt{x}$.

Transformation	Equation	Graph
Vertical translation		
Horizontal translation		
Reflection		
Vertical stretch		

8-7 Exercises

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Homework Help Online
KEYWORD: MB7 8-7
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Explain why $f(x) = \sqrt{3x} + 4$ is a radical function.

SEE EXAMPLE 1 p. 619

Graph each function, and identify its domain and range.

2. $f(x) = \sqrt{x+6}$
3. $f(x) = \sqrt{x} - 1$
4. $f(x) = 2\sqrt{x-3}$
5. $f(x) = 3\sqrt[3]{x}$
6. $f(x) = \sqrt[3]{x} + 2$
7. $f(x) = \sqrt[3]{x-2}$

SEE EXAMPLE 2 p. 621

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

8. $g(x) = \sqrt{x} - 7$
9. $h(x) = 3\sqrt{x}$
10. $j(x) = \sqrt{x-5}$

SEE EXAMPLE 3 p. 621

11. $g(x) = \frac{1}{2}\sqrt{x} - 1$
12. $h(x) = \sqrt{\frac{1}{3}(x+4)}$
13. $j(x) = \sqrt{-(x-3)}$
14. $g(x) = -2\sqrt{x} - 4$
15. $h(x) = \sqrt{-2(x+2)}$
16. $j(x) = 3\sqrt{x+3} + 3$

SEE EXAMPLE 4 p. 622

Use the description to write the square-root function g.

17. The parent function $f(x) = \sqrt{x}$ is stretched vertically by a factor of 4 and then translated 5 units left and 2 units down.
18. The parent function $f(x) = \sqrt{x}$ is reflected across the y-axis, then compressed horizontally by a factor of $\frac{1}{2}$, and finally translated 7 units right.

SEE EXAMPLE 5 p. 622

19. **Space Exploration** On Earth, the function $f(x) = \frac{6}{5}\sqrt{x}$ approximates the distance in miles to the horizon observed by a person whose eye level is x feet above the ground. The graph of the corresponding function for Mars is a horizontal stretch of f by a factor of about $\frac{9}{5}$. Write the corresponding function g for Mars, and use it to estimate the distance to the horizon for an astronaut whose eyes are 6 ft above Mars's surface.

SEE EXAMPLE 6 p. 623

Graph each inequality.

20. $y \geq \sqrt{x}$
21. $y \leq \sqrt{x-4}$
22. $y < \sqrt{x} - 3$
23. $y > \sqrt[3]{x}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
24–29	1
30–32	2
33–38	3
39–41	4
42	5
43–46	6

Extra Practice

Skills Practice p. S19
Application Practice p. S39

Graph each function, and identify its domain and range.

24. $f(x) = \sqrt{x-2}$
25. $f(x) = -3\sqrt{x}$
26. $f(x) = 2\sqrt{x+1} - 3$
27. $f(x) = \sqrt[3]{x+1}$
28. $f(x) = \sqrt[3]{x} - 4$
29. $f(x) = -2\sqrt[3]{x-3}$

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

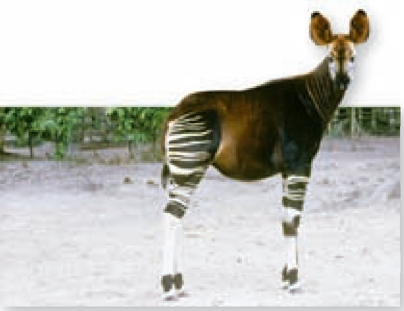
30. $g(x) = \sqrt{x} + 2$
31. $h(x) = \sqrt{x-4}$
32. $j(x) = 0.5\sqrt{x}$
33. $g(x) = \sqrt{3(x+5)}$
34. $h(x) = \frac{1}{4}\sqrt{-x}$
35. $j(x) = \sqrt{x+4} - 1$
36. $g(x) = -4\sqrt{x} + 1$
37. $h(x) = 3\sqrt{-x} + 2$
38. $j(x) = \frac{1}{3}\sqrt{-(x+2)}$

Use the description to write the square-root function g.

39. The parent function $f(x) = \sqrt{x}$ is compressed vertically by a factor of $\frac{1}{3}$ and then translated 3 units left.
40. The parent function $f(x) = \sqrt{x}$ is reflected across the y-axis, stretched horizontally by a factor of 6, and then translated 2 units right.

41. The parent function $f(x) = \sqrt{x}$ is reflected across the x-axis and then translated 1 unit left and 4 units down.

42. **Manufacturing** A company manufactures cans for pet food. The function $f(x) = \sqrt{\frac{x}{40}}$ models the radius in centimeters of a can holding x cm^3 of dog food. The graph of the corresponding function for cans of cat food is a horizontal compression of f by a factor of about $\frac{3}{5}$. Write the corresponding function g for cans of cat food, and use it to estimate the radius of a can holding 216 cm^3 of cat food.



Okapi

Graph each inequality.

43. $y < \sqrt{x+5}$ **44.** $y \geq \sqrt{x-1}$ **45.** $y > \sqrt[3]{x} + 2$ **46.** $y \leq \sqrt[3]{x+3}$

47. **Biology** The function $h(m) = 241m^{-\frac{1}{4}}$ can be used to approximate an animal's resting heart rate h in beats per minute, given its mass m in kilograms.

a. A common shrew is one of the world's smallest mammals. What is the resting heart rate of a common shrew with a mass of 0.01 kg?

b. An okapi is an African animal related to the giraffe. What is the resting heart rate of an okapi with a mass of 300 kg?

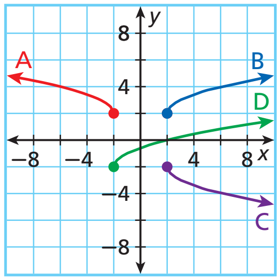
Describe how $f(x) = \sqrt{x}$ was transformed to produce each function.

48. $g(x) = 6\sqrt{x+1}$ **49.** $h(x) = 3\sqrt{x-1} - 9$ **50.** $j(x) = -\sqrt{x-3} - 7$

Match each function to its graph.

51. $f(x) = \sqrt{x+2} - 2$ **52.** $g(x) = \sqrt{x-2} + 2$

53. $h(x) = \sqrt{-(x+2)} + 2$ **54.** $j(x) = -\sqrt{x-2} - 2$



55. **Aviation** Pilots use the function $D(A) = 3.56\sqrt{A}$ to approximate the distance D in kilometers to the horizon from an altitude A in meters.

a. What is the approximate distance to the horizon observed by a pilot flying at an altitude of 11,000 m?

b. **What if...?** How will the approximate distance to the horizon appear to change if the pilot descends by 4000 m?

56. **Earth Science** The speed in miles per hour of a tsunami can be modeled by the function $s(d) = 3.86\sqrt{d}$, where d is the average depth in feet of the water over which the tsunami travels. Graph this function. Use the graph to predict the speed of a tsunami over water with a depth of 1500 feet.

57. **Astrophysics** New stars can form inside an interstellar cloud of gas when a cloud fragment, called a clump, has a mass M that is greater than what is known as the Jean's mass. The Jean's mass M_J is given by $M_J = 100\sqrt{\frac{(T+273)^3}{n}}$, where T is the temperature of the gas in degrees Celsius and n is the density of the gas in molecules per cubic centimeter. An astronomer discovers a gas clump with $M = 137$, $T = -263$, and $n = 1000$. Will the clump form a star? Justify your answer.

58. **Multi-Step** The formula $v = \sqrt{4909gR}$ approximates the velocity in miles per hour necessary to escape the gravity of a planet with acceleration due to gravity g in ft/s^2 and radius R in miles. On Earth, which has a radius of 3960 mi, the acceleration due to gravity is 32 ft/s^2. On the Moon, which has a radius of 1080 mi, the acceleration due to gravity is about $\frac{1}{6}$ that on Earth. How much faster would a vehicle need to be traveling to escape Earth's gravity than to escape the Moon's gravity?

MULTI-STEP TEST PREP

59. This problem will prepare you for the Multi-Step Test Prep on page 636.

For a pendulum with length x in meters, the function $T(x) = 2\pi\sqrt{\frac{x}{9.8}}$ gives the period of the pendulum in seconds. The period of a pendulum is the time it takes the pendulum to complete one back-and-forth swing.

a. Graph the function.

b. Describe the graph of T as a transformation of $f(x) = \sqrt{x}$.

c. By what factor must the length of a pendulum be increased to double its period?

Tell whether each statement is sometimes, always, or never true.

60. For $n > 0$, the value of $\sqrt{n}$ is greater than the value of $\sqrt[3]{n}$.

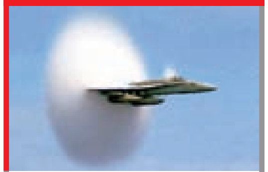
61. The domain of a radical function is all real numbers.

62. The range of $f(x) = a\sqrt[3]{x - h}$, where a and h are nonzero real numbers, is all real numbers.

63. The range of $f(x) = a\sqrt{x} + k$, where a and k are nonzero real numbers, is all real numbers.

LINK

Physics



A sonic boom is a shock wave produced by an aircraft flying at or above the speed of sound in air. Occasionally, an unusual cone-shaped cloud forms around a plane when the plane's speed is near that of sound, as shown above.

Graph each inequality, and tell whether the point $(1, 2)$ is a solution.

64. $f(x) \geq \sqrt{x - 3}$

65. $f(x) \leq \sqrt{x + 5}$

66. $f(x) > \sqrt[3]{x} - 3$

67. **Physics** The speed s of sound in air in meters per second is given by the function $s = \sqrt{k(T + 273.15)}$, where T is the air temperature in degrees Celsius and k is a positive constant. The table shows the speed of sound in air at a pressure of 1 atmosphere.

Temperature (°C)	Speed of Sound in Air (m/s)
0	331
10	337
20	343
30	348
40	354

a. Graph the data in the table.

b. Use your graph to predict the speed of sound in air at 25°C.

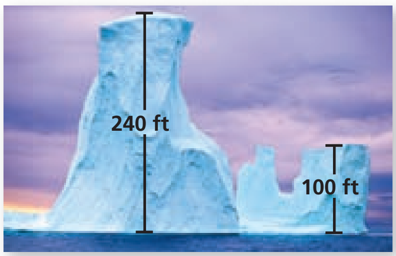
c. Based on the function above, at what temperature would the speed of sound in air be 0 m/s? Explain.

68. **Medicine** A pharmaceutical company samples the raw materials it receives before they are used in the manufacture of drugs. For inactive ingredients, the company uses the function $s(x) = \sqrt{x} + 1$ to determine the number of samples s that should be taken from a shipment of x containers.

a. Describe the graph of s as a transformation of $f(x) = \sqrt{x}$. Then graph the function.

b. How many samples should be taken from a shipment of 45 containers of an inactive ingredient?

69. **Multi-Step** The time t in seconds required for an object to fall from a certain height can be modeled by the function $t = \frac{\sqrt{h}}{4}$, where h is the initial height of the object in feet. To the nearest tenth of a second, how much longer will it take for a piece of an iceberg to fall to the ocean from a height of 240 ft than from a height of 100 ft?



70. **Critical Thinking** Explain why a vertical compression of a square-root function by a factor of $\frac{1}{2}$ is equivalent to a horizontal stretch of a square-root function by a factor of 4.

71. **Critical Thinking** Why does the square-root function have a limited domain but the cube-root function does not?

72. **Write About It** Describe how a horizontal translation and a vertical translation of the function $f(x) = \sqrt{x}$ each affects the function's domain and range.

TEST PREP

73. What is the domain of the function $f(x) = \sqrt{x - 9}$?

Ⓐ All real numbers
Ⓑ $x \geq -9$
Ⓒ $x \geq 0$
Ⓓ $x \geq 9$

74. Which situation could best be modeled by a cube-root function?

Ⓕ The volume of a cube as a function of its edge length
Ⓖ The diameter of a circle as a function of its area
Ⓗ The edge length of a cube as a function of its surface area
Ⓙ The radius of a sphere as a function of its volume

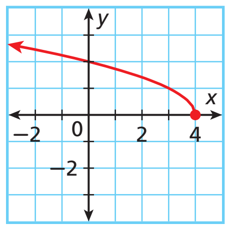
75. The function g is a translation 2 units left and 5 units up of $f(x) = \sqrt{x}$. Which of the following represents g?

Ⓐ $g(x) = \sqrt{x + 2} + 5$
Ⓑ $g(x) = 2\sqrt{x} + 5$
Ⓒ $g(x) = \sqrt{x + 5} + 2$
Ⓓ $g(x) = 5\sqrt{x - 2}$

76. Which function has a range of $\{y \mid y \leq -2\}$?

Ⓕ $f(x) = \sqrt{x} - 2$
Ⓖ $f(x) = \sqrt{x - 2}$
Ⓗ $f(x) = \sqrt{-x} - 2$
Ⓙ $f(x) = -\sqrt{x} - 2$

77. **Short Response** Describe how the graph of $f(x) = \sqrt{x}$ was transformed to produce the graph shown. Then write the equation of the graphed function.



CHALLENGE AND EXTEND

78. The function $f(x) = \sqrt{x}$ is transformed solely by using translations and reflections to produce g. The domain of g is all real numbers greater than or equal to 3, and the range is all real numbers less than or equal to 2. What is the equation that represents g?

79. Write the equation of a square-root function whose graph has its endpoint at $(-3, 4)$ and passes through the point $(2, 2)$.

SPIRAL REVIEW

Solve and graph. *(Lesson 2-1)*

80. $-4x + 5 < -7$

81. $12 \geq 4(x - 5)$

82. $2(x + 1) \geq x - 2$

Use substitution to solve each system of equations. *(Lesson 3-2)*

83. $\begin{cases} y = 2x - 10 \\ 2x + y = 14 \end{cases}$

84. $\begin{cases} y = 3x - 2 \\ 3x = 2y \end{cases}$

85. $\begin{cases} -8x + y = 36 \\ y = x - 4 \end{cases}$

Solve each equation. *(Lesson 8-5)*

86. $\frac{7}{x} + x = \frac{16}{3}$

87. $4 + \frac{2}{x} = \frac{9}{2}$

88. $\frac{-5x^2}{x + 5} = \frac{3x - 2}{x + 5}$

8-8 Solving Radical Equations and Inequalities

Objective
Solve radical equations and inequalities.

Vocabulary
radical equation
radical inequality

Who uses this?
Police officers can use radical equations to determine whether a car is speeding. (See Example 6.)

A **radical equation** contains a variable within a radical. Recall that you can solve quadratic equations by taking the square root of both sides. Similarly, radical equations can be solved by raising both sides to a power.

Know it! Note

Solving Radical Equations

Steps	Example
1. Isolate the radical.	$\sqrt[3]{x} - 2 = 0$ $\sqrt[3]{x} = 2$
2. Raise both sides of the equation to the power equal to the index of the radical.	$\left(\sqrt[3]{x}\right)^3 = (2)^3$
3. Simplify and solve.	$x = 8$

EXAMPLE 1 Solving Equations Containing One Radical

Solve each equation.

A $2\sqrt{x+1} = 14$

$\frac{2\sqrt{x+1}}{2} = \frac{14}{2}$ *Divide by 2.*

$\sqrt{x+1} = 7$ *Simplify.*

$\left(\sqrt{x+1}\right)^2 = 7^2$ *Square both sides.*

$x + 1 = 49$ *Simplify.*

$x = 48$ *Solve for x.*

Check

$2\sqrt{x+1}$	$= 14$
$2\sqrt{48+1}$	14
$2\sqrt{49}$	14
$2(7)$	14 ✔

B $5\sqrt[3]{4x+3} = 15$

$\frac{5\sqrt[3]{4x+3}}{5} = \frac{15}{5}$ *Divide by 5.*

$\sqrt[3]{4x+3} = 3$ *Simplify.*

$\left(\sqrt[3]{4x+3}\right)^3 = 3^3$ *Cube both sides.*

$4x + 3 = 27$ *Simplify.*

$4x = 24$ *Solve for x.*

$x = 6$

Check

$5\sqrt[3]{4x+3}$	$= 15$
$5\sqrt[3]{4(6)+3}$	15
$5\sqrt[3]{27}$	15
$5(3)$	15 ✔

Remember!
For a square root, the index of the radical is 2.
$\sqrt{x+1} = \sqrt[2]{x+1}$

CHECK IT OUT! **Solve each equation.**

1a. $4 + \sqrt{x-1} = 5$ **1b.** $\sqrt[3]{3x-4} = 2$ **1c.** $6\sqrt{x+10} = 42$

EXAMPLE 2 Solving Equations Containing Two Radicals

Solve $\sqrt{35x} = 5\sqrt{x+2}$.

$$\left(\sqrt{35x}\right)^2 = \left(5\sqrt{x+2}\right)^2 \qquad \textit{Square both sides.}$$

$$35x = 25(x+2) \qquad \textit{Simplify.}$$

$$35x = 25x + 50 \qquad \textit{Distribute 25.}$$

$$10x = 50 \qquad \textit{Solve for x.}$$

$$x = 5$$

Check

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{35x} = 5\sqrt{x+2}} \\
\hline
\sqrt{35 \cdot 5} & 5\sqrt{5+2} \\
5\sqrt{7} & 5\sqrt{7}\ \checkmark
\end{array}$$

CHECK IT OUT! **Solve each equation.**

2a. $\sqrt{8x+6} = 3\sqrt{x}$ **2b.** $\sqrt[3]{x+6} = 2\sqrt[3]{x-1}$

Raising each side of an equation to an even power may introduce extraneous solutions.

EXAMPLE 3 Solving Equations with Extraneous Solutions

Solve $\sqrt{x+18} = x - 2$.

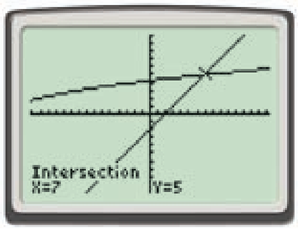
Helpful Hint

You can also use the intersect feature on a graphing calculator to find the point where the two curves intersect.

Method 1 Use a graphing calculator. Let **Y1** $= \sqrt{x+18}$ and **Y2** $= x - 2$.

The graphs intersect in only one point, so there is exactly one solution.

The solution is $x = 7$.



Method 2 Use algebra to solve the equation.

Step 1 Solve for x.

$$\sqrt{x+18} = x - 2$$

$$\left(\sqrt{x+18}\right)^2 = (x-2)^2 \qquad \textit{Square both sides.}$$

$$x + 18 = x^2 - 4x + 4 \qquad \textit{Simplify.}$$

$$0 = x^2 - 5x - 14 \qquad \textit{Write in standard form.}$$

$$0 = (x+2)(x-7) \qquad \textit{Factor.}$$

$$x + 2 = 0 \text{ or } x - 7 = 0 \qquad \textit{Solve for x.}$$

$$x = -2 \text{ or } x = 7$$

Step 2 Use substitution to check for extraneous solutions.

$$\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{x+18} = x - 2} \\
\hline
\sqrt{-2+18} & -2-2 \\
\sqrt{16} & -4 \\
4 & -4\ ✘
\end{array}
\qquad
\begin{array}{r|l}
\multicolumn{2}{c}{\sqrt{x+18} = x - 2} \\
\hline
\sqrt{7+18} & 7-2 \\
\sqrt{25} & 5 \\
5 & 5\ \checkmark
\end{array}$$

Because $x = -2$ is extraneous, the only solution is $x = 7$.

CHECK IT OUT! **Solve each equation.**

3a. $\sqrt{2x+14} = x + 3$ **3b.** $\sqrt{-9x+28} = -x + 4$

You can use similar methods to solve equations containing rational exponents. You raise both sides of the equation to the reciprocal of the exponent. You can also rewrite any expressions with rational exponents in radical form and solve as you would other radical equations.

EXAMPLE 4 Solving Equations with Rational Exponents

Solve each equation.

A $(3x - 1)^{\frac{1}{5}} = 2$

$$\begin{aligned} \sqrt[5]{3x-1} &= 2 && \textit{Write in radical form.} \\ \left(\sqrt[5]{3x-1}\right)^5 &= 2^5 && \textit{Raise both sides to the fifth power.} \\ 3x - 1 &= 32 && \textit{Simplify.} \\ 3x &= 33 && \textit{Solve for } x. \\ x &= 11 \end{aligned}$$

Remember!

To find a power of a power, multiply the exponents.

$\left[(x + 12)^{\frac{1}{2}}\right]^2$

$(x + 12)^{\frac{1}{2} \cdot 2}$

$x + 12$

B $x = (x + 12)^{\frac{1}{2}}$

Step 1 Solve for x.

$$\begin{aligned} x^2 &= \left[(x+12)^{\frac{1}{2}}\right]^2 && \textit{Raise both sides to the reciprocal power.} \\ x^2 &= x + 12 && \textit{Simplify.} \\ x^2 - x - 12 &= 0 && \textit{Write in standard form.} \\ (x+3)(x-4) &= 0 && \textit{Factor.} \\ x + 3 = 0 \text{ or } x - 4 &= 0 && \textit{Solve for } x. \\ x = -3 \text{ or } x &= 4 \end{aligned}$$

Step 2 Use substitution to check for extraneous solutions.

$$\begin{array}{c|l} \multicolumn{2}{c}{x = (x + 12)^{\frac{1}{2}}} \\ \hline -3 & (-3 + 12)^{\frac{1}{2}} \\ -3 & 9^{\frac{1}{2}} \\ -3 & 3 \text{ ✗} \end{array} \qquad \begin{array}{c|l} \multicolumn{2}{c}{x = (x + 12)^{\frac{1}{2}}} \\ \hline 4 & (4 + 12)^{\frac{1}{2}} \\ 4 & 16^{\frac{1}{2}} \\ 4 & 4 \text{ ✓} \end{array}$$

The only solution is $x = 4$.

CHECK IT OUT! **Solve each equation.**

4a. $(x + 5)^{\frac{1}{3}} = 3$ **4b.** $(2x + 15)^{\frac{1}{2}} = x$ **4c.** $3(x + 6)^{\frac{1}{2}} = 9$

A **radical inequality** is an inequality that contains a variable within a radical. You can solve radical inequalities by graphing or by using algebra.

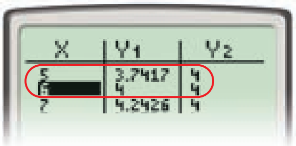
EXAMPLE 5 Solving Radical Inequalities

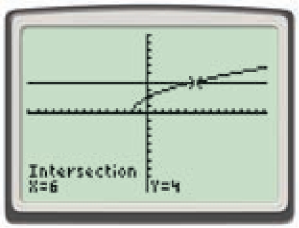
Solve $\sqrt{2x + 4} \le 4$.

Method 1 Use a graph and a table.

On a graphing calculator, let **Y1** $= \sqrt{2x + 4}$ and **Y2** $= 4$. The graph of **Y1** is at or below the graph of **Y2** for values of x between -2 and 6. Notice that **Y1** is undefined when $x < -2$.

The solution is $-2 \le x \le 6$.





Method 2 Use algebra to solve the inequality.

Step 1 Solve for x.

$$\sqrt{2x+4} \le 4$$

$$\left(\sqrt{2x+4}\right)^2 \le (4)^2 \quad \text{Square both sides.}$$

$$2x + 4 \le 16 \quad \text{Simplify.}$$

$$2x \le 12 \quad \text{Solve for } x.$$

$$x \le 6$$

Step 2 Consider the radicand.

$$2x + 4 \ge 0 \quad \text{The radicand cannot be negative.}$$

$$2x \ge -4 \quad \text{Solve for } x.$$

$$x \ge -2$$

The solution of $\sqrt{2x+4} \le 4$ is $x \ge -2$ and $x \le 6$, or $-2 \le x \le 6$.

Remember!

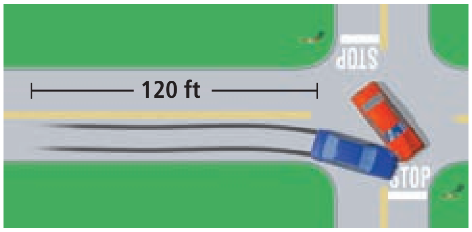
A radical expression with an even index and a negative radicand has no real roots.

CHECK IT OUT! **Solve each inequality.**

5a. $\sqrt{x-3} + 2 \le 5$ **5b.** $\sqrt[3]{x+2} \ge 1$

EXAMPLE 6 *Automobile Application*

The speed s in miles per hour that a car is traveling when it goes into a skid can be estimated by using the formula $s = \sqrt{30fd}$, where f is the coefficient of friction and d is the length of the skid marks in feet.



After an accident, a driver claims to have been traveling the speed limit of 45 mi/h. The coefficient of friction under accident conditions was 0.7. Is the driver telling the truth about his speed? Explain.

Use the formula to determine the greatest possible length of the driver's skid marks if he were traveling 45 mi/h.

$$s = \sqrt{30fd}$$

$$45 = \sqrt{30(0.7)d} \quad \text{Substitute 45 for } s \text{ and 0.7 for } f.$$

$$45 = \sqrt{21d} \quad \text{Simplify.}$$

$$(45)^2 = \left(\sqrt{21d}\right)^2 \quad \text{Square both sides.}$$

$$2025 = 21d \quad \text{Simplify.}$$

$$96 \approx d \quad \text{Solve for } d.$$

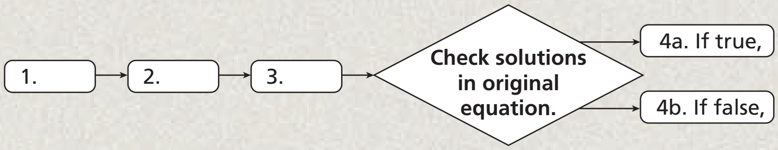
If the driver were traveling 45 mi/h, the skid marks would measure about 96 ft. Because the skid marks actually measure 120 ft, the driver must have been driving faster than 45 mi/h.

CHECK IT OUT! **6.** A car skids to a stop on a street with a speed limit of 30 mi/h. The skid marks measure 35 ft, and the coefficient of friction was 0.7. Was the car speeding? Explain.

THINK AND DISCUSS

1. Describe two methods that can be used to solve $\sqrt{x+2} = 6$.
2. Explain the relationship between solving a quadratic equation of the form $x^2 = a$ and a square-root equation of the form $\sqrt{x} = b$, where a and b are real numbers.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write a step needed to solve a radical equation with extraneous solutions.

Know it! Note



8-8 Exercises

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Homework Help Online
KEYWORD: MB7 8-8
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Is $4x + \sqrt{9} = 5$ a *radical equation*? Explain.

SEE EXAMPLE 1 p. 628

Solve each equation.

2. $\sqrt{x-9} = 5$
3. $\sqrt{3x} = 6$
4. $\sqrt[3]{x-2} = 2$

SEE EXAMPLE 2 p. 629

5. $\sqrt{3x-1} = \sqrt{2x+4}$
6. $2\sqrt{x} = \sqrt{x+9}$
7. $\sqrt[5]{x+4} = \sqrt[5]{3x-2}$
8. $2\sqrt[3]{x} = \sqrt[3]{x+7}$
9. $\sqrt{x+6} - \sqrt{2x-4} = 0$
10. $4\sqrt{x+1} = 3\sqrt{x+2}$

SEE EXAMPLE 3 p. 629

11. $\sqrt{x+56} = x$
12. $\sqrt{x+18} = x-2$
13. $\sqrt{3x-11} = x-3$
14. $\sqrt{x+6} - x = 4$
15. $\sqrt{-x-1} = x+1$
16. $\sqrt{15x+10} = 2x+3$

SEE EXAMPLE 4 p. 630

17. $(x-5)^{\frac{1}{2}} = 3$
18. $(2x+1)^{\frac{1}{3}} = 2$
19. $(4x+5)^{\frac{1}{2}} = x$
20. $2(x-50)^{\frac{1}{3}} = -10$
21. $2(x+1)^{\frac{1}{2}} = 1$
22. $(45-9x)^{\frac{1}{2}} = x-5$

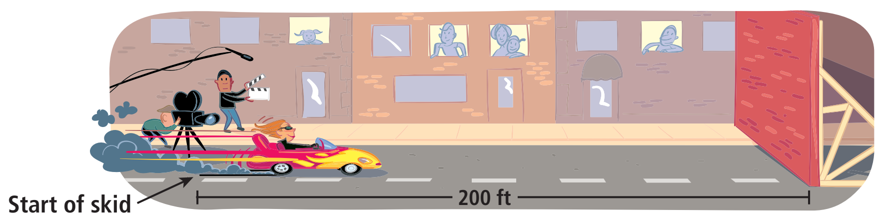
SEE EXAMPLE 5 p. 630

Solve each inequality.

23. $\sqrt{x+5} - 1 \le 4$
24. $\sqrt{2x} + 6 \le 10$
25. $\sqrt{2x+5} < 5$

SEE EXAMPLE 6 p. 631

26. **Stunts** The formula $s = \sqrt{21d}$ relates a stunt car's speed s in miles per hour at the beginning of a skid to the length d of the skid in feet. A stunt driver must skid her car to a stop just in front of a wall. When the driver starts her skid, she is traveling at 64 mi/h. When the driver comes to a stop, how many feet will be between her car and the wall? Round to the nearest foot.



PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
27–32	1
33–35	2
36–38	3
39–41	4
42–44	5
45	6

Extra Practice

Skills Practice p. S19

Application Practice p. S39

Solve each equation.

27. $\sqrt{x-12}=9$

28. $\sqrt[3]{2x+1}-3=0$

29. $5\sqrt{x+7}=25$

30. $\sqrt[4]{2x+6}=2$

31. $3=\frac{1}{4}\sqrt{3x+30}$

32. $-3=2\sqrt{x-7}-7$

33. $\sqrt{4x+12}=\sqrt{6x}$

34. $5\sqrt{x-1}=\sqrt{x+1}$

35. $\sqrt[3]{4x}=\sqrt[3]{x+7}$

36. $x+3=\sqrt{x+5}$

37. $\sqrt{3x+13}+3=2x$

38. $\sqrt{x+8}-x=-4$

39. $(x-9)^{\frac{1}{2}}=4$

40. $(5x+1)^{\frac{1}{4}}=4$

41. $(3x+28)^{\frac{1}{2}}=x$

Solve each inequality.

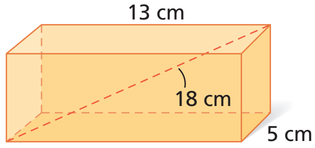
42. $\sqrt{3x+3}\le 6$

43. $\sqrt{x-3}\le 4$

44. $\sqrt{8x+1}\ge 7$

45. **Construction** The diameter d in inches of a rope needed to lift a weight of w tons is given by the formula $d=\frac{\sqrt{15w}}{\pi}$. How much weight can be lifted with a rope with a diameter of 1.5 in.?

46. **Geometry** The length of a diagonal d of a rectangular prism is given by $d=\sqrt{\ell^2+w^2+h^2}$, where ℓ is the length, w is the width, and h is the height.



a. What is the height of the prism shown? Round to the nearest tenth.

b. **What if...?** Suppose that the length, width, and height of the prism are doubled. What effect will this change have on the length of the diagonal?

Solve each equation for the indicated variable.

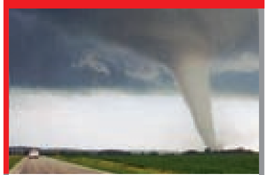
47. $r=\sqrt{\frac{A}{\pi}}$ for A

48. $r=\sqrt[3]{\frac{3V}{4\pi}}$ for V

49. $v=\sqrt{\frac{2E}{m}}$ for E

LINK

Tornadoes



The Fujita Tornado Scale goes up to category F12, even though scientists expect that Earth's most powerful tornadoes will reach wind speeds of no higher than those of category F5.

50. **Tornadoes** The Fujita Tornado Scale is used to estimate the wind velocity of a tornado based on the damage that the tornado causes. The equation $V=k(F+2)^{\frac{3}{2}}$ can be used to determine a tornado's minimum wind velocity V in miles per hour, where k is a constant and F is the tornado's category number on the Fujita Scale.

Fujita Tornado Scale

Damage Level	Category	Minimum Wind Velocity (mi/h)
Moderate	F1	73
Significant	F2	113
Severe	F3	158
Devastating	F4	207
Incredible	F5	261

a. Based on the information in the table, what is the value of the constant k?

b. What would be the minimum wind velocity of an F6 tornado?

c. Winds on Neptune can reach velocities of more than 600 mi/h. Use the equation given above to determine the Fujita category of this wind velocity.

51. **Amusement Parks** For a spinning amusement park ride, the velocity v in meters per second of a car moving around a curve with a radius r meters is given by $v=\sqrt{ar}$, where a is the car's acceleration in m/s^2.

a. For safety reasons, a ride has a maximum acceleration of 39.2 m/s^2. If the cars on the ride have a velocity of 14 m/s, what is the smallest radius that any curve on the ride may have?

b. What is the acceleration of a car moving at 8 m/s around a curve with a radius of 2.5 m?

MULTI-STEP TEST PREP

52. This problem will prepare you for the Multi-Step Test Prep on page 636.

The time T in seconds for a pendulum to complete one back-and-forth swing is given by $T = 2\pi\sqrt{\frac{L}{9.8}}$, where L is the length of the pendulum in meters.

a. Find the length of a pendulum that completes one back-and-forth swing in 2.2 s. Round to the nearest hundredth of a meter.

b. A clockmaker needs a pendulum that will complete 120 back-and-forth swings in one minute. To the nearest hundredth of a meter, how long should the pendulum be?

53. **Art** Gabriel plans to cover a circular area on a mural with yellow paint.

a. Write a radical inequality that can be used to determine the possible radius r of the circle given that Gabriel has enough paint to cover at most A ft^2.

b. If Gabriel can cover up to 80 ft^2, is 20 a reasonable value of r? Explain.

54. **/// ERROR ANALYSIS ///** Below are two solutions to the equation $2\sqrt{3x + 3} = 12$. Which is incorrect? Explain the error.

A

$$2\sqrt{3x+3} = 12$$
$$\sqrt{3x+3} = 6$$
$$\left(\sqrt{3x+3}\right)^2 = 6^2$$
$$3x + 3 = 36$$
$$x = 11$$

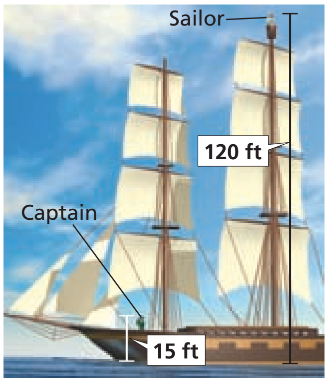
B

$$2\sqrt{3x+3} = 12$$
$$2\left(\sqrt{3x+3}\right)^2 = 12^2$$
$$2(3x+3) = 144$$
$$6x + 6 = 144$$
$$x = 23$$

Graphing Calculator **Use a graphing calculator to solve each equation. Graph each side of the equation on the same screen, and find the point(s) of intersection.**

55. $1.6x - 4 = 1.4\sqrt{x + 8.7}$ 56. $3(x + 7.4)^{\frac{2}{3}} = 8.8$ 57. $\sqrt[3]{x^2 + 4.2} = 2.7x - 4.2$

58. **Multi-Step** On a clear day, the approximate distance d in miles that a person can see is given by $d = 1.2116\sqrt{h}$, where h is the person's height in feet above the ocean.



a. To the nearest tenth of a mile, how far can the captain on the clipper ship see?

b. How much farther, to the nearest tenth of a mile, will the sailor be able to see than will the captain?

c. A pirate ship is approaching the clipper ship at a relative speed of 10 mi/h. Approximately how many minutes sooner will the sailor be able to see the pirate ship than will the captain?

59. **Chemistry** The formula $s = \sqrt[3]{\frac{m}{\rho}}$ relates the side length s of a metal cube to its mass m and its density ρ. The density of gold is 19.30 g/cm^3, and the density of lead is 11.34 g/cm^3. How much greater is the mass of a cube of gold than the mass of a cube of lead if both cubes have a side length of 5 cm?

60. **Critical Thinking** Without solving the equation, how can you tell that $\sqrt{5x + 17} + 5 = 2$ has no real solutions?

61. **Write About It** Describe how solving a radical equation is similar to solving a rational equation.

TEST PREP

62. Solve $\sqrt[3]{2x + 4} = 3$.

Ⓐ -0.5 Ⓑ -1.5 Ⓒ 2.5 Ⓓ 11.5

63. How many solutions does $x - 1 = \sqrt{5x - 9}$ have?

Ⓕ 0 Ⓖ 1 Ⓗ 2 Ⓙ 3

64. The surface area S of a cone is given by the formula $S = \pi\sqrt{r^2 + h^2}$, where r is the radius of the base and h is the height. What is the approximate height of a cone with a surface area of 40 square inches and a base radius of 8 inches?

Ⓐ 5 inches Ⓒ 15 inches
Ⓑ 10 inches Ⓓ 20 inches

65. The equation $V = \left(\frac{A}{6}\right)^{\frac{3}{2}}$ relates the volume V of a cube to its surface area A. Which of the following is equivalent to this equation?

Ⓕ $A = 6V^{\frac{2}{3}}$ Ⓖ $A = (6V)^{\frac{2}{3}}$ Ⓗ $A = 36V^{\frac{1}{3}}$ Ⓙ $A = (216V)^{\frac{1}{2}}$

66. **Gridded Response** What value of x makes $(2x - 3)^{\frac{1}{4}} = 3$ a true statement?

CHALLENGE AND EXTEND

Indicate whether each of the following statements is sometimes, always, or never true. Equations of the form $\sqrt{x + a} = b$ have at least one real solution when

67. Both a and b are positive.

68. Both a and b are negative.

69. a is negative and b is positive.

70. a is positive and b is negative.

Solve each equation.

71. $\sqrt{x} = \frac{9}{\sqrt{x}}$

72. $\sqrt{\sqrt{x + 2}} = 4$

73. $\sqrt{x^2 - 64} = x - 4$

74. **Biology** The surface area S of a human body in square meters can be approximated by $S = \sqrt{\frac{hm}{36}}$, where h is height in meters and m is mass in kilograms. Between the ages of 4 and 17, an athlete's height increased by 75% and mass increased by 350%. By approximately what percent did the surface area of the athlete's skin increase?

SPIRAL REVIEW

75. **Entertainment** The cost of driving through a safari park is \$2.00 per person and \$10.00 per car. For each car, the total cost C can be modeled by the function $C(n) = 2.00n + 10.00$, where n is the number of people. *(Lesson 2-6)*

  a. The manager of the park announces a half-price discount on the charge per car. Write the new cost function $D(n)$. Assume that the charge per person does not change.

  b. Graph $C(n)$ and $D(n)$ in the same coordinate plane.

  c. Describe the transformation that has been applied.

Use inverse operations to write the inverse of each function. *(Lesson 7-2)*

76. $f(x) = \frac{x}{2} + 4$

77. $f(x) = -3x - 1$

78. $f(x) = \frac{x - 2}{7}$

Simplify each expression. Assume that all variables are positive. *(Lesson 8-6)*

79. $\sqrt[3]{64x^9}$

80. $\sqrt[4]{\frac{x^8}{81}}$

81. $\sqrt[3]{\frac{18x^2}{x^4}}$

CHAPTER 8

MULTI-STEP TEST PREP

SECTION 8B

Radical Functions

Tick Tock A pendulum clock keeps time by using weights, gears, and a pendulum. The length of the pendulum determines how fast it swings, and the speed of the pendulum determines how fast the hands of the clock advance.

A clockmaker is building a replica of an antique pendulum clock for a museum display and finds that it is running too slowly. As he attempts to fix the clock, he tries pendulums of different lengths. He records the pendulum length and period data shown in the table. The period of a pendulum is the time it takes for the pendulum to complete one back-and-forth swing.

Pendulum Swings

Length (cm)	Period (s)
10	0.6
20	0.9
30	1.1
40	1.3
50	1.4
60	1.6

1. Create a scatter plot of the data, using pendulum length as the independent variable and period as the dependent variable.
2. Experiment with a graphing calculator to find a function rule that models the data in the scatter plot. Describe your model as a transformation of $f(x) = \sqrt{x}$.
3. What is a reasonable domain for this situation? Explain.
4. Use your model to determine the period of a pendulum that has a length of 16 cm. Round to the nearest tenth of a second.
5. From his observations, the clockmaker concludes that the pendulum needs to have a period of 1 s. To the nearest centimeter, how long should the pendulum be?
6. The function $y = 2\pi\sqrt{\frac{x}{9.8}}$ gives the period y of a pendulum in seconds in terms of the pendulum's length x in meters. Graph this function with the data from the table and explain whether the function is a reasonable model for the data.

READY TO GO ON?

CHAPTER 8

SECTION 8B

Quiz for Lessons 8-6 Through 8-8

8-6 Radical Expressions and Rational Exponents

Simplify each expression. Assume that all variables are positive.

1. $\sqrt{32x^3}$ **2.** $\sqrt[3]{8y^{12}z^6}$ **3.** $\sqrt[4]{\frac{a^4}{9}}$

Write each expression in radical form, and simplify.

4. $4^{\frac{3}{2}}$ **5.** $16^{\frac{5}{4}}$ **6.** $(-27)^{\frac{2}{3}}$

Write each expression by using rational exponents.

7. $\sqrt[4]{8^3}$ **8.** $\left(\sqrt[5]{243}\right)^2$ **9.** $\left(\sqrt[3]{-1000}\right)^2$

10. In an experiment involving fruit flies, the initial population is 112. The growth of the population can be modeled by the function $n(t) = 112 \cdot 2^{\frac{t}{50}}$, where n is the number of fruit flies and t is the time in hours. Based on this model, what is the population of fruit flies after 1 week?

8-7 Radical Functions

Graph each function, and identify its domain and range.

11. $f(x) = -\sqrt{x} + 4$ **12.** $f(x) = \sqrt[3]{x+1}$

13. Water is draining from a tank connected to two pipes. The speed f in feet per second at which water drains through the first pipe can be modeled by $f(x) = \sqrt{64(x-2)}$, where x is the depth of the water in the tank in feet. The graph of the corresponding function for the second pipe is a translation of f 4 units right. Write the corresponding function g, and use it to estimate the speed at which water drains through the second pipe when the depth of the water is 10 ft.

14. Use the description to write the square-root function g. The parent function $f(x) = \sqrt{x}$ is reflected across the x-axis and then translated 2 units right and 3 units down.

Graph each inequality.

15. $y > \sqrt{x} + 4$ **16.** $y \le \sqrt{x-2}$

8-8 Solving Radical Equations and Inequalities

Solve each equation.

17. $-2\sqrt[3]{5x-5} = -10$ **18.** $\sqrt{x+4} = x - 8$ **19.** $3\sqrt[3]{x-2} = \sqrt[3]{6x}$

20. The formula $d = \sqrt[3]{\frac{4w}{0.02847}}$ relates the average diameter d of a cultured pearl in millimeters to its weight w in carats. To the nearest tenth of a carat, what is the weight of a cultured pearl with an average diameter of 7 mm?

Solve each inequality.

21. $\sqrt{x+5} < 4$ **22.** $\sqrt[3]{2x} \ge -2$ **23.** $\sqrt{x-6} - 10 \le 4$

CHAPTER 8 Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. A(n) ___?___ is a function whose rule is a ratio of two polynomials.

2. A(n) ___?___ is a relationship that can be written in the form $y = kx$, where k is the ___?___.

8-1 Variation Functions *(pp. 569–576)*

EXAMPLES

■ **The cost in dollars of apples a varies directly as the number of pounds p, and $a = 3.12$ when $p = 2.4$. Find p when $a = 1.04$.**

$\frac{a_1}{p_1} = \frac{a_2}{p_2}$ *Use a proportion.*

$\frac{3.12}{2.4} = \frac{1.04}{p_2}$ *Substitute.*

$3.12p = 2.4(1.04)$ *Find the cross products.*

$p = 0.8$ *Solve for p.*

Apples that cost \$1.04 have a weight of 0.8 lb.

■ **The base b of a parallelogram with fixed area varies inversely as the height h, and $b = 12$ cm when $h = 8$ cm. Find b when $h = 3$ cm.**

$b = \frac{k}{h}$ *b varies inversely with h.*

$12 = \frac{k}{8}$ *Substitute.*

$k = 96$ *Solve for k.*

$b = \frac{96}{h}$ *Substitute 96 for k.*

$b = \frac{96}{3}$ *Substitute 3 for h.*

$b = 32$ *Solve for b.*

The base is 32 cm when the height is 3 cm.

EXERCISES

Given: y varies directly as x. Write and graph each direct variation function.

3. $y = 2$ when $x = 6$

4. $y = 4$ when $x = 1$

5. The number of tiles n needed to cover a floor varies directly as the area a of the floor, and $n = 180$ when $a = 20$ ft^2. Find n when $a = 34$ ft^2.

6. The simple interest I earned over a particular period of time varies jointly as the principal P and rate r, and $I = \$264$ when $P = \$1100$ and $r = 0.12$. Find P when $I = \$360$ and $r = 0.09$.

Given: y varies inversely as x. Write and graph each inverse variation function.

7. $y = 3$ when $x = 2$

8. $y = 4$ when $x = 1$

9. For a fixed voltage, the current I flowing in a wire varies inversely as the resistance R of the wire. If the current is 8 amperes when the resistance is 15 ohms, what will the resistance be when the current is 5 amperes?

10. Determine whether the data set represents a direct variation, an inverse variation, or neither.

x	2	5	10
y	25	10	5

8-2 Multiplying and Dividing Rational Expressions *(pp. 577–582)*

EXAMPLES

- **Simplify $\frac{4-x}{x^2-x-20}$. Identify any x-values for which the expression is undefined.**

$$\frac{-1\cancel{(x+4)}}{(x-5)\cancel{(x+4)}} = \frac{-1}{x-5}$$

Factor. Then divide out common factors.

Undefined at $x = 5$ and $x = -4$

- **Divide. Assume that all expressions are defined.**

$$\frac{x^2-9}{x+2} \div \frac{x+3}{x^2+7x+10}$$

$$\frac{x^2-9}{x+2} \cdot \frac{x^2+7x+10}{x+3}$$

Rewrite as multiplication.

$$\frac{(x-3)\cancel{(x+3)}}{\cancel{x+2}} \cdot \frac{\cancel{(x+2)}(x+5)}{\cancel{x+3}} = (x-3)(x+5)$$

EXERCISES

Simplify. Identify any x-values for which the expression is undefined.

11. $\frac{24x^{14}}{9x^{16}}$ **12.** $\frac{6x^3}{3x+12}$ **13.** $\frac{x^2+x-12}{x^2+5x+4}$

Multiply. Assume that all expressions are defined.

14. $\frac{x+5}{3x+1} \cdot \frac{9x+3}{x^2-25}$ **15.** $\frac{x}{x-4} \cdot \frac{-x+2}{x^2+x-6}$

16. $\frac{x^2+2x-3}{x^2-x-2} \cdot \frac{x-2}{x+3}$ **17.** $\frac{9x^2-1}{x^2-9} \cdot \frac{x+3}{3x+1}$

Divide. Assume that all expressions are defined.

18. $\frac{x^3y}{4xy^4} \div \frac{x}{8y^2}$ **19.** $\frac{x^2+2x-15}{x-2} \div \frac{x^2-9}{2x-4}$

20. $\frac{3x-21}{3x} \div \frac{x^2-49}{x^2+7x}$ **21.** $\frac{x^2+4x+3}{x^2+2x-8} \div \frac{3x+3}{x-2}$

8-3 Adding and Subtracting Rational Expressions *(pp. 583–590)*

EXAMPLES

- **Add. Identify any x-values for which the expression is undefined.**

$$\frac{6x-3}{x^2-x-12} + \frac{x}{x+3}$$

$$\frac{6x-3}{(x-4)(x+3)} + \frac{x}{x+3}\left(\frac{x-4}{x-4}\right)$$

$$\frac{6x-3+x(x-4)}{(x-4)(x+3)}$$

Add the numerators.

$$\frac{x^2+2x-3}{(x-4)(x+3)}$$

Simplify the numerator.

$$\frac{\cancel{(x+3)}(x-1)}{(x-4)\cancel{(x+3)}} = \frac{x-1}{x-4}$$

Factor the numerator.

Undefined at $x = 4$ and $x = -3$

- **Simplify. Assume that all expressions are defined.**

$$\frac{\frac{x+2}{6x}}{\frac{x}{x-4}} = \frac{\frac{x+2}{\cancel{6x}}\cancel{(6x)}(x-4)}{\frac{x}{\cancel{x-4}}(6x)\cancel{(x-4)}}$$

The LCD is $(6x)(x-4)$.

$$\frac{(x+2)(x-4)}{x(6x)} = \frac{(x+2)(x-4)}{6x^2}$$

EXERCISES

Add. Identify any x-values for which the expression is undefined.

22. $\frac{4}{x^2+4} + \frac{x^2+8}{x^2+4}$ **23.** $\frac{1}{x+3} + \frac{1}{x-3}$

24. $\frac{x}{x^2-4} + \frac{1}{x-2}$ **25.** $\frac{2x-3}{3x+7} + \frac{6}{4x-1}$

Find the least common multiple for each pair.

26. $x^2 - 9$ and $x^2 - 6x + 9$

27. $x^2 + 2x - 35$ and $x^2 + 9x + 14$

Subtract. Identify any x-values for which the expression is undefined.

28. $\frac{2x}{x+4} - \frac{3}{x+4}$ **29.** $\frac{x}{x+5} - \frac{5}{x-5}$

30. $\frac{1}{x^2-x-6} - \frac{x}{x+2}$ **31.** $\frac{2x}{2x+1} - \frac{7}{3x-1}$

Simplify. Assume that all expressions are defined.

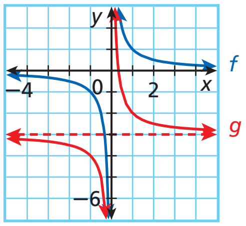
32. $\frac{\frac{x-6}{5}}{\frac{x+2}{8}}$ **33.** $\frac{\frac{x+3}{3x}}{\frac{x^2-9}{6x-9}}$ **34.** $\frac{\frac{x}{4}-\frac{1}{x}}{\frac{x+2}{x-2}}$

35. A jet's average speed is 520 mi/h when flying from Dallas to Chicago and 580 mi/h on the return trip. What is the jet's average speed for the entire trip?

8-4 Rational Functions *(pp. 592–599)*

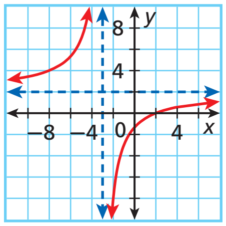
EXAMPLES

- **Using the graph of $f(x)=\frac{1}{x}$ as a guide, describe the transformation and graph $g(x)=\frac{1}{x}-3$.**



Because $k=-3$, translate f down 3 units.

- **Identify the zeros and asymptotes of $f(x)=\frac{2x-4}{x+3}$. Then graph.**



Zero: 2
Vertical asymptote: $x=-3$
Horizontal asymptote: $y=2$

EXERCISES

Using the graph of $f(x)=\frac{1}{x}$ as a guide, describe the transformation and graph each function.

36. $g(x)=\dfrac{1}{x-4}$ **37.** $g(x)=\dfrac{1}{x-2}+3$

Identify the asymptotes, domain, and range of each function.

38. $f(x)=\dfrac{2}{x-1}-3$ **39.** $f(x)=\dfrac{3}{x+2}+1$

Identify the zeros and asymptotes of each function. Then graph.

40. $f(x)=\dfrac{x^2-3x}{x+4}$ **41.** $f(x)=\dfrac{x-3}{x^2+6x+5}$

42. $f(x)=\dfrac{2x-4}{x+3}$ **43.** $f(x)=\dfrac{x^2-9}{x-2}$

44. Identify holes in the graph of $f(x)=\frac{x^2-3x-18}{x+3}$. Then graph.

8-5 Solving Rational Equations and Inequalities *(pp. 600–607)*

EXAMPLE

- **Solve the equation $\frac{30}{x+1}+x=10$.**

$$\frac{30}{x+1}(x+1)+x(x+1)=10(x+1)$$

$30+x^2+x=10x+10$ *Simplify. $x\neq-1$*

$x^2-9x+20=0$ *Write in standard form.*

$(x-4)(x-5)=0$ *Factor.*

$x=4$ or $x=5$ *Solve for x.*

EXERCISES

Solve each equation.

45. $x-\dfrac{6}{x}=1$ **46.** $\dfrac{4x}{x-5}=\dfrac{3x+5}{x-5}$

47. $\dfrac{3x}{x+2}=\dfrac{2x+2}{x+2}$ **48.** $\dfrac{x}{x+4}+\dfrac{x}{2}=\dfrac{2x}{2x+8}$

Solve each inequality.

49. $\dfrac{x+4}{x}>-2$ **50.** $\dfrac{2}{x-3}<4$

8-6 Radical Expressions and Rational Exponents *(pp. 610–617)*

EXAMPLES

Simplify each expression. Assume that all variables are positive.

- $\sqrt[3]{-8x^9}=\sqrt[3]{(-2^3)}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{x^3}\cdot\sqrt[3]{x^3}=-2x^3$

- $\sqrt[4]{8x^6}\cdot\sqrt[4]{2x^2}=\sqrt[4]{16x^8}=\sqrt[4]{2^4}\cdot\sqrt[4]{x^4}\cdot\sqrt[4]{x^4}=2x^2$

- **Write the expression $\left(\sqrt{16}\right)^3$ by using rational exponents.**

$16^{\frac{3}{2}}$ $\left(\sqrt[n]{a}\right)^m=a^{\frac{m}{n}}$

EXERCISES

Simplify each expression. Assume that all variables are positive.

51. $\sqrt[3]{27x^6}$ **52.** $\sqrt[4]{81x^{12}}$ **53.** $\sqrt[3]{\dfrac{8x^3}{3}}$

Write each expression by using rational exponents.

54. $\left(\sqrt[3]{-27}\right)^2$ **55.** $\sqrt[4]{16^3}$ **56.** $\left(\sqrt{9}\right)^3$

Simplify each expression.

57. $17^{\frac{1}{3}}\cdot17^{\frac{2}{3}}$ **58.** $\left(9^4\right)^{\frac{1}{2}}$ **59.** $\left(\dfrac{1}{16}\right)^{\frac{1}{4}}$

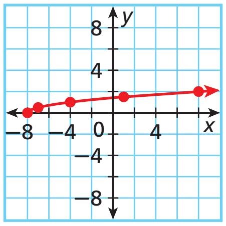
8-7 Radical Functions *(pp. 619–627)*

EXAMPLE

- **Graph $f(x) = \frac{\sqrt{x+8}}{2}$, and identify its domain and range.**

Make a table of values. Then graph.

x	y
−8	0
−7	0.5
−4	1
1	1.5
8	2



D: $\{x \mid x \geq -8\}$; R: $\{y \mid y \geq 0\}$

EXERCISES

Graph each function, and identify its domain and range.

60. $f(x) = \sqrt{x} + 5$

61. $f(x) = -4\sqrt[3]{x}$

Using the graph of $f(x) = \sqrt{x}$ as a guide, describe the transformation and graph each function.

62. $g(x) = -\sqrt{x} + 1$

63. $h(x) = \sqrt{4x}$

64. $j(x) = \sqrt{-(x-8)}$

65. $k(x) = -\frac{1}{2}\sqrt{x} + 1$

66. Use the description to write the square-root function g. The parent function $f(x) = \sqrt{x}$ is stretched vertically by a factor of 3 and translated 4 units left.

Graph each inequality.

67. $y < \sqrt{x}$

68. $y < \sqrt[3]{x+4}$

8-8 Solving Radical Equations and Inequalities *(pp. 628–635)*

EXAMPLES

Solve each equation.

- $4\sqrt[3]{x-4} = 12$

$\sqrt[3]{x-4} = 3$ *Divide by 4.*

$\left(\sqrt[3]{x-4}\right)^3 = 3^3$ *Cube both sides.*

$x - 4 = 27$ *Simplify.*

$x = 31$ *Solve for x.*

- $\sqrt{x+15} = x - 5$

$\left(\sqrt{x+15}\right)^2 = (x-5)^2$ *Square both sides.*

$x + 15 = x^2 - 10x + 25$

$x^2 - 11x + 10 = 0$ *Write in standard form.*

$(x-10)(x-1) = 0$ *Factor.*

$x = 10$ or $x = 1$ *Solve for x.*

Use substitution to check for extraneous solutions.

$\sqrt{x+15}$	$= x - 5$
$\sqrt{10+15}$	$10 - 5$
5	5 ✔

$\sqrt{x+15}$	$= x - 5$
$\sqrt{1+15}$	$1 - 5$
4	−4 ✘

The solution $x = 1$ is extraneous. The only solution is $x = 10$.

EXERCISES

Solve each equation.

69. $\sqrt{x+6} - 7 = -2$

70. $\frac{\sqrt[3]{2x-2}}{6} = 1$

71. $\sqrt{10x} = 3\sqrt{x+1}$

72. $2\sqrt[5]{x} = \sqrt[5]{64}$

73. $\sqrt{6x-12} = x - 2$

74. $\sqrt{x+1} = x - 5$

75. $(4x+7)^{\frac{1}{2}} = 3$

76. $(x-4)^{\frac{1}{4}} = 3$

77. $x = (2x+35)^{\frac{1}{2}}$

78. $(x+3)^{\frac{1}{3}} = -6$

Solve each inequality.

79. $\sqrt{x-4} \leq 3$

80. $\sqrt{2x+7} - 6 > -1$

81. $\sqrt{3x} - 4 < 2$

82. $\sqrt[3]{x-1} > -2$

83. The time T in seconds required for a pendulum to complete one back-and-forth swing can be determined from the formula $T = 2\pi\sqrt{\frac{L}{9.8}}$, where L is the length of the pendulum in meters. Estimate the length of a pendulum that completes one back-and-forth swing in 2.5 s.

84. A tetrahedron is a triangular pyramid with four congruent faces. The side length s in meters of a tetrahedron is given by the formula $s = \left(6V\sqrt{2}\right)^{\frac{1}{3}}$, where V is the volume of the tetrahedron in cubic meters. What is the volume of a tetrahedron with a side length of 8 m? Round to the nearest tenth.

CHAPTER 8

CHAPTER TEST

1. The monthly minimum payment p due on a certain credit card with a fixed rate varies directly as the balance b, and $p = \$19.80$ when $b = \$1100$. Find p when $b = \$3000$.
2. The time t that it takes Hannah to bike to school varies inversely as her average speed s. If she can bike to school in 25 min when her average speed is 6 mi/h, what would her average speed need to be to get to school in 20 min?
3. Simplify $\frac{x^2 - x - 6}{x^2 - 4x + 3}$. Identify any x-values for which the expression is undefined.

Multiply or divide. Assume that all expressions are defined.

4. $\frac{x - 9}{2x - 10} \cdot \frac{x - 5}{x^2 - 81}$
5. $\frac{3x^3 - 9x^2}{x^2 - 16} \div \frac{2x - 6}{x^2 - 8x + 16}$

Add or subtract. Identify any x-values for which the expression is undefined.

6. $\frac{5}{x - 5} + \frac{x}{2x - 10}$
7. $\frac{5x}{x - 7} - \frac{9x - 6}{x + 3}$
8. Lorraine averaged 62 words per minute when typing the first 3 pages of a 6-page report. Her average typing speed for the last 3 pages was 45 words per minute. To the nearest word per minute, what was Lorraine's average typing speed for the entire report?
9. Identify the zeros and asymptotes of $f(x) = \frac{3x + 3}{x + 2}$. Then graph.

Solve each equation.

10. $2 + \frac{3}{x - 1} = 10$
11. $\frac{x}{x - 1} + \frac{x}{3} = \frac{5}{x - 1}$
12. Beth can tile a floor in about 6 h. When Beth and Mike work together, they can tile a floor in about 2.4 h. About how long would it take Mike to tile a floor if he works by himself?

Simplify each expression. Assume that all variables are positive.

13. $\sqrt[3]{-32x^6}$
14. $8^{-\frac{2}{3}}$
15. $\frac{27^{\frac{2}{3}}}{27^{\frac{1}{3}}}$
16. Write the expression $\sqrt[5]{x^2}$ by using a rational exponent.
17. Graph the function $f(x) = \sqrt{x + 2} - 4$ and identify its domain and range.
18. Graph the inequality $y \le \sqrt{x} - 2$.

Solve each equation.

19. $\sqrt{x + 7} = 5$
20. $\sqrt{2x + 1} = \sqrt{x + 9}$
21. $(3x + 1)^{\frac{1}{3}} = -2$
22. The formula $s = \sqrt{\frac{A}{4.828}}$ can be used to approximate the side length s of a regular octagon with area A. A stop sign is shaped like a regular octagon with a side length of 12.4 in. To the nearest square inch, what is the area of the stop sign?
23. Solve the inequality $\sqrt{2x + 1} > 3$.

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT

There is a set of criteria that your calculator must meet in order for it to be allowed in the testing facility when you take the SAT. For example, calculators that make noise or have QWERTY keypads are not allowed. For complete guidelines, check www.collegeboard.com.

HOT TIP! **If you do not already have a graphing calculator, consider purchasing or borrowing one because it may give you an advantage when solving some problems on the SAT. Be sure to spend time getting used to any new calculator before test day.**

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

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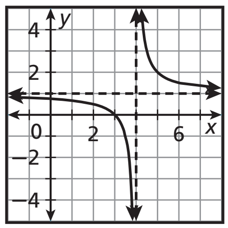
1. Which of the following functions is graphed below?



(A) $f(x) = (x+3)(x-4)$

(B) $f(x) = (x-3)(x+4)$

(C) $f(x) = \dfrac{x-3}{x-4}$

(D) $f(x) = \dfrac{x-3}{x+4}$

(E) $f(x) = \dfrac{x+3}{x+4}$

---

2. If each of the following expressions is defined, which is equivalent to $x-1$?

(A) $\dfrac{(x+1)(x-1)}{x-1}$

(B) $\dfrac{(x-1)(x+2)}{x+1} \cdot \dfrac{x+1}{x+2}$

(C) $\dfrac{(x+1)(x+2)}{x-2} \div \dfrac{x+2}{x-2}$

(D) $\dfrac{x+1}{x+2} + \dfrac{x-1}{x+2}$

(E) $\dfrac{2x-2}{x-2} - \dfrac{x-1}{x-2}$

---

3. The cube root of the square of a real number n is 16. What is the value of n?

(A) $\dfrac{4}{3}$

(B) $\dfrac{8}{3}$

(C) 4

(D) 12

(E) 64

---

4. If y varies inversely as the square of x and $y = 1$ when $x = 2$, what is the value of y when $x = -4$?

(A) -2

(B) $-\dfrac{1}{2}$

(C) $\dfrac{1}{4}$

(D) 4

(E) 16

---

5. If $\sqrt[3]{12x+28} = 4$, what is the value of x^3?

(A) -8

(B) 3

(C) 12

(D) 27

(E) 64

CHAPTER 8

TEST TACKLER

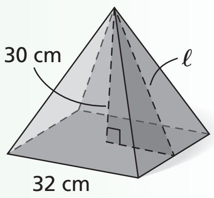
Standardized Test Strategies

Any Question Type: Use a Diagram

Diagrams are often useful when you are solving problems. For some problems, a diagram is provided for you and you must correctly interpret it. In other situations, you can sketch your own diagram to help you visualize a problem.

EXAMPLE 1

Short Response The height h of a square pyramid can be determined from the equation $h = \sqrt{\ell^2 - \left(\frac{s}{2}\right)^2}$, where ℓ is the slant height of the pyramid and s is the side length of the square base. What is the slant height ℓ of the square pyramid shown? Show your work.



To solve this problem, you must use information from the diagram.

$$30 = \sqrt{\ell^2 - \left(\frac{32}{2}\right)^2}$$ *Substitute 30 for h and 32 for s.*

$$30 = \sqrt{\ell^2 - 16^2}$$ *Simplify.*

$$900 = \ell^2 - 256$$ *Square both sides.*

$$1156 = \ell^2$$ *Add 256 to both sides.*

$$\pm 34 = \ell$$ *Solve for ℓ.*

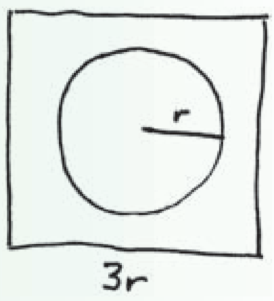
The slant height of the pyramid is 34 centimeters.

EXAMPLE 2

Multiple Choice A circular fountain with radius r feet is built into a square base with a side length of $3r$ feet. What is the probability that a penny hitting the square base at random will land in the circular fountain?

Ⓐ $\frac{\pi}{9}$ Ⓑ $\frac{\pi}{3}$ Ⓒ $\frac{1}{3}$ Ⓓ $\frac{1}{9}$

A diagram would be helpful with this problem. Sketch a square with side length 3r to represent the square base. Then sketch a circle inside it with radius r to represent the circular fountain.



$$\frac{\pi r^2}{(3r)^2}$$ *The probability that the penny will land in the fountain is the ratio of the area of the fountain to the area of the base.*

$$\frac{\pi r^2}{9r^2} = \frac{\pi}{9}$$ *Simplify.*

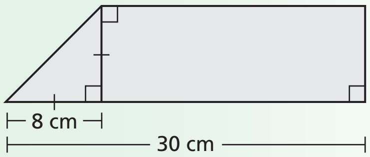
The correct answer is A.

HOT TIP! If you sketch your own diagram to help you solve a problem, be sure to label it with any measurements you are given.

Read each test item and answer the questions that follow.

Item A

Multiple Choice What is the area of this composite figure?



Ⓐ 176 cm^2 Ⓒ 240 cm^2

Ⓑ 208 cm^2 Ⓓ 272 cm^2

1. How can you use the information given in the diagram to determine the length of the rectangle?
2. Explain how you can use the diagram to determine the rectangle's width.
3. What is the area of the triangle? What is the area of the rectangle?

Item B

Multiple Choice A square courtyard has a perimeter of 200 meters. What is the approximate length of a sidewalk that lies along one of the courtyard's diagonals?

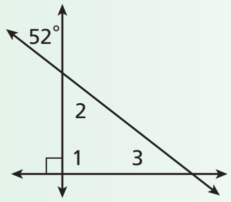
Ⓕ 50 meters Ⓗ 71 meters

Ⓖ 57 meters Ⓙ 87 meters

4. Sketch a diagram that can help you visualize the situation.
5. How can you use the information given in the problem to label each side of the courtyard in your diagram with its length?
6. Into what shapes does the diagonal sidewalk divide the courtyard?
7. What equation can you use to determine the length of the sidewalk?

Item C

Short Response What are the measures of the three numbered angles of this triangle? Explain how you determined your answer.



8. Based on the information in the diagram, what type of angle is ∠1? What is its measure?
9. What is the relationship between the 52° angle and ∠2? What is the measure of ∠2?
10. How can you use the measures of ∠1 and ∠2 to determine the measure of ∠3?

Item D

Extended Response A rectangular pool is surrounded on all four sides by a tiled lounging area. The length of the pool is 5 feet greater than the width. The width of the lounging area is 10 feet greater than twice the width of the pool. The length of the lounging area is 5 times the width of the pool.

a. Write a rational expression that represents the ratio of the area of the pool to the entire area of the pool and lounging area.

b. Determine the value of the ratio if the width of the pool is 30 feet.

11. Sketch a diagram that can help you visualize the situation.
12. Explain how you determined the labels for the dimensions of your diagram.
13. Is it necessary to draw your diagram to scale? Why or why not?
14. What expression represents the area of the pool? What expression represents the entire area of the pool and lounging area?

CHAPTER 8

STANDARDIZED TEST PREP

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KEYWORD: MB7 TestPrep

CUMULATIVE ASSESSMENT, CHAPTERS 1–8

Multiple Choice

1. Given: y varies jointly as x and z, and $y = 16$ when $x = \frac{1}{2}$ and $z = 8$. What equation represents the joint variation function?

Ⓐ $y = \frac{4x}{z}$

Ⓑ $y = 4x$

Ⓒ $y = \frac{1}{4}xz$

Ⓓ $y = 4xz$

2. What is the solution of the equation $\sqrt{3x + 2} = 3\sqrt{2x - 2}$?

Ⓕ $x = \frac{4}{15}$

Ⓖ $x = \frac{8}{15}$

Ⓗ $x = \frac{4}{3}$

Ⓙ $x = \frac{8}{3}$

3. Which is equivalent to $(3 - 5i)(2 + i)$?

Ⓐ 11

Ⓑ $11 - 7i$

Ⓒ $11 + 7i$

Ⓓ $1 - 7i$

4. Which is equivalent to $\frac{4x^2y^3}{5xy^2} \div \frac{2y}{10xy}$?

Ⓕ $\frac{4y}{25}$

Ⓗ $4x^2y$

Ⓖ $\frac{4x^2}{y}$

Ⓙ $4x^2y^5$

5. What is the slope of the line $3y = 2x + 9$?

Ⓐ $\frac{2}{3}$

Ⓒ 3

Ⓑ $\frac{3}{2}$

Ⓓ 9

6. Which expression can be simplified to a rational number?

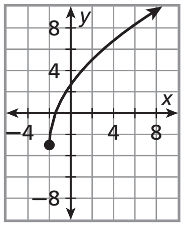
Ⓕ $\sqrt{1} + \sqrt{8}$

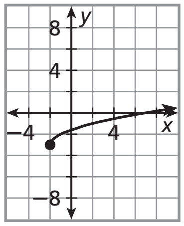
Ⓗ $\left(\sqrt{15}\right)^2$

Ⓖ $\sqrt{10} \cdot \sqrt{25}$

Ⓙ $\sqrt{\frac{20}{4}}$

7. Which is the graph of the function $f(x) = 4\sqrt{x + 2} - 3$?

Ⓐ


Ⓒ


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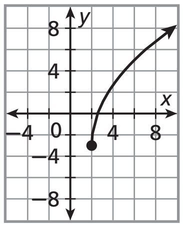

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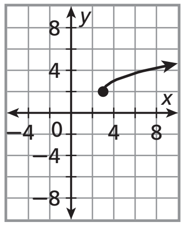


8. At track practice, Jamie ran 0.5 mile farther than twice the distance Rochelle ran. If x represents the distance in miles that Rochelle ran, which expression represents the distance that Jamie ran?

Ⓕ $0.5(2x)$

Ⓗ $2(x + 0.5)$

Ⓖ $0.5x + 2$

Ⓙ $2x + 0.5$

9. Which equation best describes the relationship between x and y shown in the table?

x	1	3	6	10	15
y	−1	5	14	26	41

Ⓐ $y = -3x + 2$

Ⓑ $y = -2x + 1$

Ⓒ $y = 2x - 3$

Ⓓ $y = 3x - 4$

10. A triangle with vertices at $(1, 4)$, $(-2, 3)$, and $(5, 0)$ is translated 2 units right and 3 units down. Which are the coordinates of a vertex of the image?

Ⓕ $(-5, 5)$

Ⓗ $(0, 0)$

Ⓖ $(-1, 1)$

Ⓙ $(3, 3)$

HOT TIP! If you have extra time at the end of a test, go back and check your answers. Remember that you can always check the solution to an equation by substituting your answer to see if it makes the equation true.

11. What is the standard form of the expression $(2x^2 - x + 4) - (3x^3 + x^2 - 2x)$?

Ⓐ $-3x^3 - x^2 - 3x + 4$

Ⓑ $-3x^3 + x^2 + x + 4$

Ⓒ $x^2 + x + 4 - 3x^3$

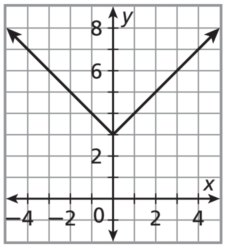
Ⓓ $3x^3 + 3x^2 + 3x + 4$

12. At what point does the graph of $f(x) = \dfrac{2x^2 - x - 3}{x + 1}$ have a hole?

Ⓕ $(-1, -5)$ Ⓗ $(1.5, 0)$

Ⓖ $(-1, 0)$ Ⓙ $(1.5, 2.5)$

13. What function is graphed below?



Ⓐ $f(x) = |3x|$ Ⓒ $f(x) = 3|x|$

Ⓑ $f(x) = |x + 3|$ Ⓓ $f(x) = |x| + 3$

Gridded Response

14. What value of x makes the equation true?

$\frac{7}{4} = \frac{3}{x} + 1$

15. What value completes the square for the expression below?

$x^2 - 3x + \blacksquare$

16. Simplify the expression.

$\left(\sqrt[3]{-8}\right)^2$

17. What value of x makes the equation true?

$\log_5(x + 8) = 2$

18. Simplify the expression.

$\dfrac{5x}{x + \frac{1}{4}} - \dfrac{20x}{4x + 1}$

Short Response

19. The function $K = \frac{5}{9}(F - 32) + 273$ expresses temperature in kelvins K as a function of temperature in degrees Fahrenheit F.

a. Find the inverse of the function.

b. What does the inverse represent?

c. Use the inverse to find the temperature in degrees Fahrenheit that is equivalent to 300 kelvins.

20. The WNBA Most Valuable Player award is given to the player with the greatest number of total points, which are tallied based on the number of first-, second-, and third-place votes that the player receives. The table shows the number of votes for the top three nominees in 2004. Find the number of points awarded for each vote.

Player	First-Place Votes	Second-Place Votes	Third-Place Votes	Total Points
L. Leslie	33	0	19	425
L. Jackson	15	18	15	351
D. Taurasi	0	13	7	126

21. The graph of $f(x) = \frac{1}{2}x^2 + c$ is a parabola with its vertex at $(0, 3)$.

a. What is the value of c? Explain how you determined this value.

b. Graph the function f.

Extended Response

22. The information in the table describes a polynomial function.

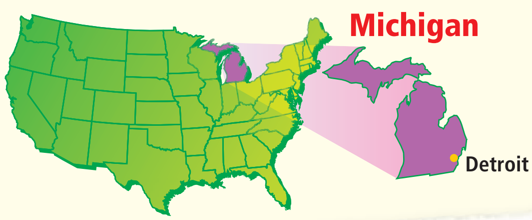
Leading Coefficient	1
Degree	3
Zeros	$-1, 2, 4$
Local Minimum	≈ -4.1
Local Maximum	≈ 8.2
***y*-intercept**	8

a. Describe the end behavior of the graph of the function. Justify your answer.

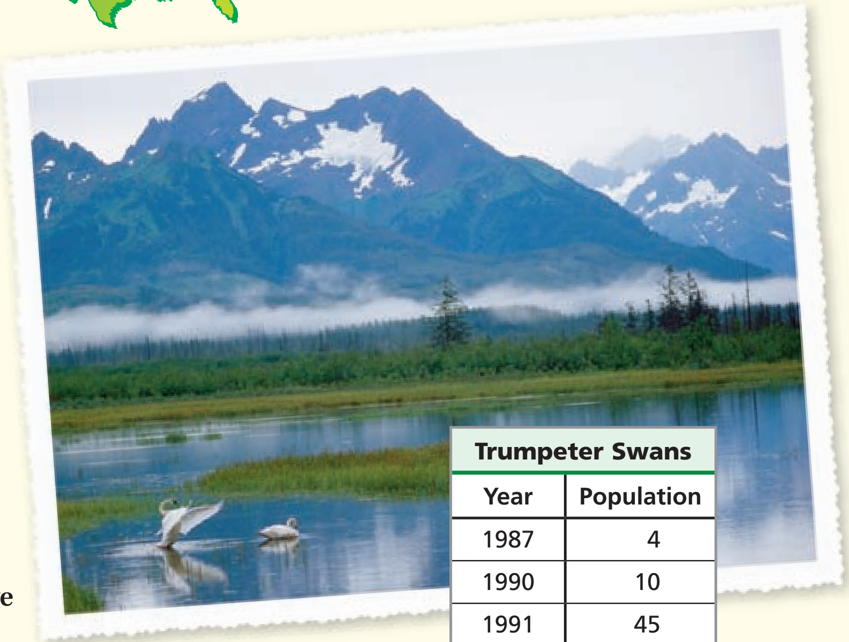
b. How many turning points does the graph of the function have? Justify your answer.

c. Sketch a graph of the function.

Real-World CONNECTIONS



The Return of the Trumpeter Swan



The majestic trumpeter swan was once abundant throughout Michigan, but by 1900, the species had been hunted almost to extinction. Since 1985, the Detroit Zoo has been working with Michigan State University to reintroduce the species to Michigan's wetlands. The program has been a great success—the population of trumpeter swans continues to grow every year.

Trumpeter Swans	
Year	Population
1987	4
1990	10
1991	45
1996	140
1999	285
2000	400
2003	565

Choose one or more strategies to solve each problem. For 1–3, use the table.

1. The table shows the growth of the swan population in Michigan. Use an exponential model to predict the population in 2012.
2. In 2000, there were about 100 trumpeter swans in southwest Michigan, 50 swans in eastern Michigan, and 191 swans in Seney National Wildlife Refuge. If this population distribution continues, about how many swans will be in each region in 2012?
3. In what year do you predict that the total population of trumpeter swans in Michigan will exceed 6000? Justify your answer.
4. A cygnet is a young swan. In 1997, there were 60 trumpeter swan cygnets in Michigan. In each of the next 2 years, their population increased by 30% compared with the year before. If this rate of increase continues, in what year will the population of cygnets exceed 1500?

The Motor City

In 1903, Henry Ford opened a small car company in Detroit that employed 10 people. Within a decade, Detroit had become the heart of America's automotive industry, earning it the nickname the "Motor City." Today, Detroit remains an important center for automotive research.

Choose one or more strategies to solve each problem. For 1–3, use the table.

1. Automotive engineers use the equation $s = \sqrt{30fd}$ to study the relationship between a vehicle's speed s in miles per hour and its stopping distance d in feet once the brakes have been applied. In this equation, f is the coefficient of friction, which depends in part on the condition of the road.

a. Determine the coefficient of friction to the nearest tenth for dry pavement.

b. Predict the stopping distance to the nearest foot for a vehicle moving at 65 mi/h.

Stopping Distances on Dry Pavement

Speed (mi/h)	Distance (ft)
10	4.2
20	16.8
30	37.8
40	67.2

2. For a vehicle on wet pavement, the coefficient of friction is 0.4. How does driving on wet pavement affect the stopping distance for a given speed?

3. Engineers want to design brakes that will reduce stopping distances by 10%. How would this change the equation relating speed and stopping distance?

4. The equation $v_{\text{max}} = \sqrt{14.88fr}$ gives the maximum velocity in miles per hour that a vehicle can safely travel around a curve that has a radius of r feet. If the velocity is greater than v_{max}, the tires will slip. Engineers find that under snowy conditions, $v_{\text{max}} = 15$ mi/h for a freeway off-ramp that has a radius of 50 ft. To the nearest tenth, what is the coefficient of friction for the off-ramp in these conditions?