CHAPTER Solutions Key

Geometric Reasoning

ARE YOU READY? PAGE 71

1.	В	2.	A
З.	F	4.	С
5.	D	6.	lin. pair
7.	vert. 🛦	8.	comp. 🕭
9.	natural, whole, integer, rational	10.	rational
11.	integer, rational	12.	rational
13.	rational	14.	whole, integer, rational
15.	Possible answer: B	16.	Possible answer: \overrightarrow{BD}
17.	Possible answer: \overrightarrow{CA}	18.	Possible answer: \overline{CD}
19.	Possible answer: plane	F	
20.	8 + x = 5	21.	6 <i>y</i> = −12
	-8 -8 -8 -8		$\frac{6y}{2} = \frac{-12}{2}$
	x = -3		y = -2
22.	9 = 6 <i>s</i>	23.	p - 7 = 9
	$\frac{9}{6} = \frac{6s}{6}$		+7 +7
	1.5 = s		p = 10
24.	$\frac{Z}{T} = 5$	25.	8.4 = -1.2r
	5 = 5(5)		$\frac{8.4}{1.2} = \frac{-1.2r}{1.2r}$
	$3(\frac{-}{5}) = 3(3)$		-1.2 -1.2 -7 = r
	Z = 25		

2-1 USING INDUCTIVE REASONING TO MAKE CONJECTURES, PAGES 74–79

CHECK IT OUT!

- **1.** 0.0004 **2.** odd
- 3. Female whales are longer than male whales.
- **4a.** Possible answer: $x = \frac{1}{2}$
- b. Possible answer:



c. Jupiter or Saturn

THINK AND DISCUSS

1. No; possible answer: a conjecture cannot be proven true just by giving examples, no matter how many.



EXERCISES

4.

6.1

1

GUIDED PRACTICE

1. Possible answer: A conjecture is based on observation and is not true until proven true in every case.

3. $\frac{4}{6}$

5. even

$$= 1 + 3 = 4 + 3 + 5 = 9 + 3 + 5 + 7 = 16$$

- Rule is n^2 .
- 7. The number of bacteria doubles every 20 min.

12. 42

- 8. Roosevelt was inaugurated at age 42.
- 9. The 3 pts. are collinear.
- **10.** Possible answer: x = -3

PRACTICE AND PROBLEM SOLVING

- 11. 5 р.м. 13.
- **14.** 2 = 1(2) 2 + 4 = 6 = 2(3) 2 + 4 + 6 = 12 = 3(4)Rule is n(n + 1).
- **15.** *n* 1
- **16.** About 5%(526) \approx 26 students will participate.
- **17.** Possible answer: y = -1
- **18.** Possible Answer: x = -1
- **19.** m∠1 = m∠2 = 90°
- **20.** Each term is the square of the previous term; $16^2 = 256, 256^2 = 65,536$
- **21.** Possible answer: each term is the previous term multiplied by $\frac{1}{2}$; $\frac{1}{16}$, $\frac{1}{32}$.
- **22.** The terms are multiples of 3 with alternating signs; -15, 18
- **23.** 2*n* + 1 **24.** T
- **25.** F; possible answer: n = 2
- 26. F; possible answer: 27. T



28. Amount increases by about \$50 per day. Therefore, about 300 + 2(50) = 400 is raised during the 6th day.

- **29.** $\frac{1}{11} = 0.\overline{09}, \frac{2}{11} = 0.\overline{18}, \frac{3}{11} = 0.\overline{27},...;$ fraction pattern is multiples of $\frac{1}{11}$, decimal pattern is repeating multiples of 0.09.
- **30.** 6 = 3 + 3; 8 = 3 + 5; 10 = 3 + 7 or 5 + 5; 12 = 5 + 7; 14 = 3 + 11 or 7 + 7
- **31.** 13 + 21 = 34; 21 + 34 = 55; 34 + 55 = 89; each term is the sum of the 2 previous terms.
- **32.** The middle number is the mean of the other 2 numbers.
- 33. 2n 1 is odd
- **34.** Feb. 19; possible answer: the weather or the whales' health
- **35.** Possible answer: Even numbers are divisible by 2, but odd numbers are not. So the conjecture is true for even numbers but not necessarily for all numbers.
- 36a. 8

37. C

TEST PREP

38. J

b. tenth day

For example, 2 - 4 is2 and 2 + 1 = 3 arenegative.both prime.

39. D

In 2010, 75 - 3(15) = 30 students predicted.

CHALLENGE AND EXTEND

40.	x	$x^2 + x + 11$
	1	13
	2	17
	3	23
	4	31
	5	41
	6	53
	7	67
	8	83

Possible answer: prime numbers. x = 10

41. Seats are up for election 6 years and 12 years later.6 is not divisible by 4, but 12 is; so seats are next up for election during a presidential election 12 years later.

42a.	Week	Sit-ups
	1	15
	2	35
	3	55
	4	75
	5	95
	6	115
	7	135
	8	155
	9	175
	10	195



c. Rob does 20(n-1) + 15 or 20n - 5 sit-ups during week *n*.





SPIRAL REVIEW

44. $y = 3x - 5$ 8 3(1) - 5 8 3 - 5 8 -2 X no	45. $\begin{array}{c c} y = 3x - 5 \\ \hline -11 & 3(-2) - 5 \\ -11 & -6 - 5 \\ -11 & -11 \checkmark \\ yes \end{array}$
46. $y = 3x - 5$ 4 3(3) - 5 4 9 - 5 $4 4 \checkmark$ yes	47. $\begin{array}{c c} y = 3x - 5 \\ \hline 0.5 & 3(-3.5) - 5 \\ 0.5 & -10.5 - 5 \\ 0.5 & -15.5 \ x \\ no \end{array}$
48. $A = s^{2} = x^{2}$ s = x P = 4s = 4x	49. $P = 2\ell + 2w$ = $2x + 2(4x - 3)$ = $10x - 6$
50. $P = 3s$ = 3(x + 2) = 3x + 6	51. $A = \pi r^{2} = 9\pi x^{2}$ $r^{2} = 9x^{2}$ $r = 3x$ $C = 2\pi r$ $= 2\pi (3x) = 6\pi x$
52. $(-1, -1) \rightarrow (-1, -1 +$	2) = (-1, 1)

- **52.** $(-1, -1) \rightarrow (-1, -1 + 2) = (-1, (0, 1) \rightarrow (0, 1 + 2) = (0, 3)$ $(4, 0) \rightarrow (4, 0 + 2) = (4, 2)$
- **53.** $(-1, -1) \rightarrow (-1 + 4, -1 1) = (3, -2)$ $(0, 1) \rightarrow (0 + 4, 1 - 1) = (4, 0)$ $(4, 0) \rightarrow (4 + 4, 0 - 1) = (8, -1)$

2-2 CONDITIONAL STATEMENTS, PAGES 81-87

CHECK IT OUT!

- Hypothesis: A number is divisible by 6. Conclusion: The number is divisible by 3.
- 2. If 2 🛦 are comp., then they are acute.
- 3. F; possible answer: 7
- Converse: If an animal has 4 paws, then it is a cat;
 F.

Inverse: If an animal is not a cat, then it does not have 4 paws; F.

Contrapositive: If an animal does not have 4 paws, then it is not a cat; T.

THINK AND DISCUSS

- **1.** T; F
- **3.** Yes; possible answer: "If x = 3, then 2x = 6" is true, and so is the conv. "If 2x = 6, then x = 3."

2. T



EXERCISES

GUIDED PRACTICE

- 1. converse 2. logically equivalent
- **3.** Hypothesis: A person is at least 16 years old. Conclusion: The person can drive a car.
- 4. Hypothesis: A figure is a rectangle. Conclusion: A figure is a parallelogram.
- **5.** Hypothesis: a b < a. Conclusion: *b* is a positive number.
- **6.** If a person is 18 years old, then that person is eligible to vote.

7. If
$$0 < a < b$$
, then $\left(\frac{a}{b}\right)^2 < \frac{a}{b}$.

- **8.** If something is a rotation, then it is a transformation.
- 9. T
- **10.** F; possible answer: x = 2 and y = -4
- 11. F; possible answer: April
- 12. Converse: If Brielle travels 10 mi in 20 min, then she drives at exactly 30 mi/h; F.
 Inverse: If Brielle does not drive at exactly 30 mi/h, then she does not travel 10 mi in 20 min; F.
 Contrapositive: If Brielle does not travel 10 mi in 20 min, then she does not drive at exactly 30 mi/h; T.

PRACTICE AND PROBLEM SOLVING

- **13.** Hypothesis: An animal is a tabby. Conclusion: The animal is a cat.
- 14. Hypothesis: Two lines intersect. Conclusion: Four angles are formed.
- **15.** Hypothesis: 8 oz of cereal cost \$2.99. Conclusion: 16 oz of cereal cost \$5.98.
- **16.** If a patient is ill, then you should monitor the patient's heart rate.
- 17. If the batter makes 3 strikes, then the batter is out.
- **18.** If segs. are \cong , then they have equal measures.

19. T

20. F; by Post. 1-1-5, if 2 planes intersect, they do so in exactly 1 line.

- 22. Converse: If an event is unlikely to occur, then the probability of the event is 0.1; F. Inverse: If the probability of an event is not 0.1, then the event is likely to occur; F. Contrapositive: If an event is likely to occur, then the probability of the event is not 0.1; T.
- 23. Converse: If the air temperature is 32°F or less, then freezing rain is falling; F.
 Inverse: If freezing rain is not falling, then the air temperature is greater than 32°F; F.
 Contrapositive: If the air temperature is greater than 32°F, then freezing rain is not falling; T

F

26.









- 34. If an animal is a dolphin, then it is a mammal.
- **35.** If a person is a Texan, then the person is an American.
- **36.** If x < -4, then x < -1.
- **37a.** Hypothesis: Only you can find it. Conclusion: Everything's got a moral.
 - **b.** If only you can find it, then everything's got a moral.

38. *x* = 5

21. T



- **41.** Possible answer: You did not go out in the sun.
- 42. If a mineral is calcite, then it has a hardness of 3; T.
- **43.** If a mineral has a hardness less than 5, then it is not apatite; T.
- **44.** If a mineral is not apatite, then it has a hardness of less than 5; F.
- 45. If a mineral is not apatite, then it is calcite; F.
- **46.** If a mineral has a hardness of 3, then it is not apatite; T.
- **47.** If a mineral is calcite, then it has a hardness less than 5; T.
- 48. Converse: If 2 ▲ have the same measure, then they are ≃; T.

Inverse: If 2 \measuredangle are not \cong , then they do not have the same measure; T.

Contrapositive: If $2 \leq 0$ not have the same measure, then they are not \cong ; T.



49. Possible answer: A conditional statement is false when the hypothesis is true and the conclusion is false. A conditional statement with a false hypothesis is always true because nothing has been guaranteed by the hypothesis.

TEST PREP

50.	С	51.	F
52.	D	53.	J

CHALLENGE AND EXTEND

- 54. No lines are pts. No pts. are lines.
- 55. Some students are adults. Some adults are students.



Possible answer: Figure A is not a rect., so it belongs outside the larger oval in the Venn diag. It cannot be inside the smaller oval, so it cannot be a square.

b. If a figure is not a rect., then it is not a square. By the contrapositve, since the figure is not a rect., it is not a square.

57. 3 true conditionals: $r \rightarrow q$, $q \rightarrow p$, and $r \rightarrow p$

SPIRAL REVIEW

58.
$$y = x + 3$$
 59. $y = 2x + 3$

60.
$$y = \frac{5}{2}x - 4$$
 61. T

1

63. T **64.** 13,131

65. $\frac{2}{81}$ **66.** $5x^5$

2-3 USING DEDUCTIVE REASONING TO VERIFY CONJECTURES, PAGES 88–93

CHECK IT OUT!

- 1. The myth rests on a false premise, that eelskin wallets are made from electric eels. The conclusion is a result of deductive reasoning.
- 2. Hypothesis: A student passes his classes. Conclusion: The student is eligible to play sports. The given statement "Ramon passed his classes" matches the hypothesis of the given conditional. By the Law of Detachment, Ramon is eligible to play sports. The conjecture is valid.
- **3.** Let *p*, *q*, and *r* represent the following: *p*: An animal is a mammal.
 - q: An animal has hair.

r: An animal is a dog.

You are given that $p \rightarrow q$ and $r \rightarrow p$. Since *p* is the conclusion of the 2nd conditional and the hypothesis of the 1st conditional, you can conclude that $r \rightarrow q$. The conjecture is valid by the Law of Syllogism.

4. Conclusion: Polygon P is not a quad.

THINK AND DISCUSS

- 1. Yes; the given information is false.
- 2. Possible answer: Using symbols instead of words forces you to look at the validity of the argument itself, without being distracted by the truth values of the individual statements.



EXERCISES

GUIDED PRACTICE

- 1. Possible answer: Inductive reasoning is based on a pattern of specific cases. Deductive reasoning is based on logical reasoning.
- **2.** The conclusion is based on logical reasoning. It is a result of deductive reasoning.
- **3.** The conclusion is based on logical reasoning. It is a result of deductive reasoning.
- 4. Hypothesis: You want to go on a field trip. Conclusion: You must have a signed permission slip. The given statement "Zola has a signed permission slip" matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Zola could have a permission slip for another reason. The conclusion is not valid.
- Hypothesis: The side lengths of a rect. are 3 ft and 4 ft. Conclusion: The rect.'s area is 12 ft². The given statement "A rect. has side lengths 3 ft and 4 ft" matches the hypothesis of the given conditional. By the Law of Detachment, the rect. has area 12 ft². The conjecture is valid.
- **6.** Let *p*, *q*, and *r* represent the following: *p*: You fly from Texas to California. *q*. You travel from the central to the Pacific time zone. *r*: You gain two hours. You are given that $p \rightarrow q$ and $q \rightarrow r$. Since *q* is the conclusion of the 1st conditional and the hypothesis of the 2nd conditional, you can conclude that $p \rightarrow r$. The conjecture is valid by the Law of Syllogism.
- 7. Let *p*, *q*, and *r* represent the following:
 - p: A figure is a square.
 - q: A figure is a rectangle.
 - r: A figure is a parallelogram.

You are given that $p \rightarrow q$ and $p \rightarrow r$. The Law of Syllogism cannot be used to draw a conclusion since p is the hypothesis of both conditionals. Even though the conjecture $r \rightarrow q$ is true, the logic used to draw the conclusion is not valid.

8. Conclusion: Alex's car might not start.

PRACTICE AND PROBLEM SOLVING

- **9.** The conclusion is based on mathematical calculation. So it is the result of deductive reasoning.
- **10.** Since the conclusion is based on a pattern of observation, it is a result of inductive reasoning.
- **11.** Hypothesis: One integer is odd and another integer is even.

Conclusion: The product of 2 integers is even. Statement "The product of 2 integers is 24" matches the conclusion of a true conditional. However, hypothesis is not necessarily true. For example, 6(4) = 24. Conclusion is not valid. 12. Let *p*, *q*, and *r* represent the following: *p*: An element is an alkali metal. *q*: An element reacts with water. *r*: An element is in the 1st column of the periodic table.
You are given that *p* → *q* and *r* → *p*. Since *p* is the

conclusion of the 2nd conditional and the hypothesis of the 1st conditional, you can conclude that $r \rightarrow q$. The conjecture is valid by the Law of Syllogism.

- **13.** Conclusion: Dakota gets better grades in Social Studies.
- **14.** If Cheetah-Net is 75 times as fast as dial-up, then dial-up is 75 times as slow as Cheetah-Net. Let *c* and *d* be the download times with Cheetah-Net and dial-up.

$$d = 75c$$

$$18 = 75c$$

$$\frac{18}{75} = \frac{75c}{75}$$

$$c = 0.24$$
 min or 14.4 s

15. valid **16.** invalid

17. valid **18.** invalid

- **19.** yes $(p \rightarrow q \text{ and } q \rightarrow r, \text{ so } p \rightarrow r)$; no (counterexample: x = 1); because the 1st conditional is false (same counterexample)
- 20. A: comp. ▲ are not necessarily adj., so they may not form a rt. ∠.
- 21. Possible answers: If Mary goes to the store, then I will go with her. Mary goes to the store. The conclusion "I will go with her" is valid by Law of Detachment. If Jon goes to the movies, then he will eat popcorn. If Jon eats popcorn, then he needs a drink. The conclusion "If Jon goes to the movies, then he will need a drink" is valid by Law of Syllogism.

22a. If a creature is a serpent, then it eats eggs.

 b. No; possible answer: the Pigeon did not correctly apply Law of Detachment; "Alice eats eggs" matches conclusion of conditional, not hypothesis.

TEST PREP

- 23. D
- **25.** 196 Hz

A G note is 3 octaves above low G. So its frequency is the frequency of low G, doubled 3 times. 2(2(2(24.50))) = 196 Hz

24. H

CHALLENGE AND EXTEND

- **26.** Either Andre is less than 35 years old, or he is not a natural-born citizen. Possible answer: Since there are 3 criteria and he meets 1, he must not meet 1 of the other 2.
- **27a.** If you live in San Diego then you live in the United States.
 - b. If you do not live in the United States, then you do not live in California. If you do not live in California, then you do not live in San Diego.

- **c.** If you do not live in the United States, then you do not live in San Diego.
- d. They are contrapositives of each other.
- 28. If Cassie goes, at least 3 people will go, contradicting the hypothesis. If either Hanna or Amy goes, then Marc will go, so by Law of Syllogism, Dallas will also go: again, at least 3 people. Therefore neither Cassie, Amy, or Hanna will go. By elimination, Marc and Dallas will go.

SPIRAL REVIEW

29.
$$2(x + 5)$$

= $2x + 10$
30. $(4y + 6) - (3y - 5)$
= $4y + 6 - 3y + 5$
= $y + 11$

31. (3c + 4c) + 2(-7c + 7)= 7c - 14c + 14 = -7c + 14

32.
$$\left(\frac{1+4}{2}, \frac{2+5}{2}\right) = \left(\frac{5}{2}, \frac{7}{2}\right) = (2.5, 3.5)$$

33.
$$\left(\frac{-3+0}{2}, \frac{6+1}{2}\right) = \left(-\frac{3}{2}, \frac{7}{2}\right) = (-1.5, 3.5)$$

34.
$$\left(\frac{-2.5+2.5}{2}, \frac{9+(-3)}{2}\right) = \left(\frac{0}{2}, \frac{6}{2}\right) = (0, 3)$$

- **35.** Hypothesis: The fire alarm rings. Conclusion: Everyone should exit the building.
- **36.** Hypothesis: Two different lines intersect. Conclusion: Two lines intersect at exactly 1 pt.
- **37.** Hypothesis: $\overline{AB} \cong \overline{CD}$ Conclusion: AB = CD

2-4 BICONDITIONAL STATEMENTS AND DEFINITIONS, PAGES 96–101

CHECK IT OUT!

1a. Let *p* and *q* represent the following:

p: An ∠ is acute. *q*: An ∠'s measure is greater than 0° and less than 90° .

2 parts of biconditional $p \leftrightarrow q$ are $p \rightarrow q$ and $q \rightarrow p$. Conditional: If an \angle is acute, then its measure is greater than 0° and less than 90°.

Converse: If an \angle 's measure is greater than 0° and less than 90°, then the \angle is acute.

b. Let *x* and *y* represent the following:

x: Cho is a member.

y: Cho has paid the \$5 dues.

2 parts of biconditional $x \leftrightarrow y$ are $x \rightarrow y$ and $y \rightarrow x$. Conditional: If Cho is a member, then he has paid the \$5 dues.

Converse: If Cho has paid the \$5 dues, then he is a member.

2a. Converse: If it is Independence Day, then the date is July 4.

Biconditional: It is July 4th if and only if it is Independence Day.

- b. Converse: If pts. are collinear, then they lie on the same line.
 Biconditional: Pts. lie on the same line if and only if they are collinear.
- 3a. Conditional: If an ∠ is a rt. ∠, then its measure is 90°. (T)
 Converse: If an ∠'s measure is 90°, then it is a rt. ∠. (T)
 Since conditional and converse are true,

b. Conditional: $y = -5 \rightarrow y^2 = 25$; T Converse: $y^2 = 25 \rightarrow y = -5$; F If y = 5, then $y^2 = 25$, so converse is false. Therefore biconditional is false.

- **4a.** A figure is a quad. if and only if it is a 4-sided polygon.
- **b.** An \angle is a straight \angle if and only if its measure is 180°.

THINK AND DISCUSS

biconditional is true.

- **1.** Possible answer: Find truth values of conditional and converse that biconditional contains. If both are true, then biconditional is true.
- 2. A \triangle has 3 sides and 3 vertices. A quad. has 4 sides and 4 vertices.



EXERCISES

GUIDED PRACTICE

endpoint are collinear.

1. Possible answer: A biconditional statement contains the conditional and its converse. A conditional is not reversible, but a biconditional is.

the figure is a polygon.

2. Let *p* and *q* represent the following: *p*: Perry can paint the entire living room. *q*: Perry has enough paint.
2 parts of biconditional *p* ↔ *q* are *p* → *q* and *q* → *p*.
Conditional: If Perry can paint the entire living room, then he has enough paint.
Converse: If Perry has enough paint, then he can

Converse: If Perry has enough paint, then he can paint the entire living room.

- 3. Let *p* and *q* represent the following: *p*: Your medicine will be ready by 5 P.M. *q*: You drop your prescription off by 8 A.M.
 2 parts of biconditional *p* ↔ *q* are *p* → *q* and *q* → *p*.
 Conditional: If your medicine is ready by 5 P.M., then you dropped your prescription off by 8 A.M.
 Converse: If you drop your prescription off by 8 A.M., then your medicine will be ready by 5 P.M.
- 4. Converse: If a student is in the tenth grade, then the student is a sophomore.Biconditional: A student is a sophomore if and only if the student is in the tenth grade.
- Converse: If 2 segs. are ≅, then they have the same length.
 Biconditional: 2 segs. have the same length if and
 - only if they are \cong .
- **6.** Conditional: $xy = 0 \rightarrow x = 0$ or y = 0; T Converse: x = 0 or $y = 0 \rightarrow xy = 0$; T Since conditional and converse are true, biconditional is true.
- 7. Conditional: If a figure is a quad., then it is a polygon; T

Converse: If a figure is a polygon, then it is a quad.; F A \triangle is a polygon but not a quad., so converse is false. Therefore, biconditional is false.

- 8. 2 lines are || if and only if they are coplanar and never intersect.
- **9.** An animal is a hummingbird if and only if it is a tiny, brightly colored bird with narrow wings, a slender bill, and a long tongue.

PRACTICE AND PROBLEM SOLVING

- Conditional: If 3 pts. are coplanar, then they lie in the same plane.
 Converse: If 3 pts. lie in the same plane, then they are coplanar.
- Conditional: If a □ is a rect., then it has 4 rt. ▲. Converse: If a □ has 4 rt. ▲, then it is a rect.
- Conditional: If a lunar eclipse occurs, then Earth is between the Sun and the Moon.
 Converse: If Earth is between the Sun and the Moon, then a lunar eclipse occurs.
- Converse: If it is the weekend, then today is Saturday or Sunday.
 Biconditional: Today is Saturday or Sunday if and only if it is the weekend.
- 14. Converse: If Greg wins the race, then he has the fastest time.Biconditional: Greg has the fastest time if and only if he wins the race.
- **15.** Converse: If a \triangle is a rt. \triangle , then it contains a rt. \angle . Biconditional: A \triangle contains a rt. \angle if and only if it is a rt. \triangle
- **16.** Conditional is true because a swimmer is an athlete. Converse is false because an athlete might not be a swimmer. Therefore, biconditional is false. Possible counterexample: Felipe could be a runner.

- **17.** Conditional is true because if 2n is even, it is divisible by 2, so $\frac{2n}{2} = n$ is an integer. Converse is true because if *n* is an integer, then 2n is an integer divisible by 2, and so is even. Therefore, biconditional is true.
- **18.** A figure is a O if and only if it is the set of all pts. that are a fixed dist. from a given pt.
- **19.** A player is a catcher if and only if the player is positioned behind home plate and catches throws from the pitcher.
- **20.** no; possible answer: a = 3, b = -3
- **21.** yes (x = 5 is the solution of both equations)
- **22.** no; possible answer: y = -8
- **23.** no; possible answer: x = -2
- **24.** An equil. \triangle is a \triangle with 3 \cong sides.
- **25.** A square is a quad. with $4 \cong$ sides and 4 rt. \measuredangle .
- **26.** A cell is a white blood cell if and only if it defends the body against invading organisms by engulfing them or releasing antibodies.
- **27.** Possible answer: A bicycle is a vehicle that moves along the ground but is not an automobile.
- **28.** Possible answer: A computer is a machine that performs calculations but is not a calculator.
- **29.** Possible answer: Definition does not say that the rays have a common endpt.
- **30.** no **31.** no
 - **33.** 5 (since $2\pi(5) = 10\pi$)
- 34. quad.

32. no

- **35.** A statement is a biconditional if and only if it can be written in the form "*p* if and only if *q*." Conditional: If a statement is a biconditional, then it can be written in the form "*p* if and only if *q*." Converse: If a statement can be written in the form "*p* if and only if *q*," if and only if *q*," then it is a biconditional. Since conditional and converse are true, biconditional is true.
- 36. Possible answer: If you write the def. as a biconditional, "A ray is an ∠ bisector iff it divides the ∠ into 2 ≅ ▲," then you can use it either forward or backward. If you know the ray is an ∠ bisector, then you can conclude that the 2 ▲ formed are ≅. If you know that 2 adj. ▲ formed by a ray are ≅, then you can conclude that the ray is an ∠ bisector.
- 37a. If I say it, then I mean it. If I mean it, then I say it.
 - **b.** Possible answer: The biconditional Alice implies is "I say it if and only if I mean it." This biconditional is not true. People often mean things without saying them or say things they don't mean.

TEST PREP

38. A

 $m \angle S \neq 80^\circ$ but $\angle S$ is acute.

39. G

40. B Converse is also true. **41.** Conditional: If you get a traffic ticket, then you are speeding.

Converse: If you are speeding, then you will get a traffic ticket.

The biconditional is false because both statements are false. It is possible to get a traffic ticket for running a red light while not speeding. Also, it is possible to speed without getting a ticket.

CHALLENGE AND EXTEND

- **42.** The two ovals within Venn diagram will exactly overlap each other. If one condition is met then the other is necessarily met, which is true of the conditions in a good def.
- **43a.** If an ∠ does not measure 105°, then the ∠ is not obtuse.
 - b. If an ∠ is not obtuse, then it does not measure 105°.
 - c. It is the contrapositive of the original.
 - **d.** F; the inverse is false, so the biconditonal formed is false.
- **44.** T; It is given that conditional is true. Converse "If D is in int. of $\angle ABC$, then $m \angle ABD + m \angle DBC = m \angle ABC$ " is true by \angle Add. Post. Since conditional and its converse are true, biconditional is true.
- **45.** Possible answer: n = 2 (since *n* is not divisible by 4, but $n^2 = 4$ is even.)

SPIRAL REVIEW

- **46.** The graph is shifted 5 units up and is wider than graph of parent function.
- **47.** The graph is reflected across *x*-axis and shifted 1 unit down, and is narrower than graph of parent function.
- **48.** y = (x + 2)(x 2)= $x^2 - 4$

The graph is shifted 4 units down.

49. T

51. *S* **52.** F; poss. answer: *n* = 0

50. Y

53. F; poss. answer: *x* = 2 **54.** T

READY TO GO ON? PAGE 103

1. 31	2. January
3. –1	^{4.} #

- 5. Possible answer: A male lion weighs about 412.4 lb.
- 6. negative 7. Possible answer: 6
- Hypothesis: An ∠'s measure is 107°. Conclusion: An ∠ is obtuse.
- 9. If a number is a whole number, then it is an integer.
- 10. If a figure is a square, then it is a rect.
- **11.** If a figure is a square, then its diags. are \cong .
- **12.** F; possible answer: an \angle that measures 60°

13. T

14. Converse: If a number is divisible by 4, then it is even; T Inverse: If a number is not even, then it is not divisible by 4; T

Contrapositive: If a number is not divisible by 4, then it is not even; F

- 15. Hypothesis: Sue finishes her science project. Conclusion: Sue can go to the movie. The given statement "Sue goes to the movie" matches the conclusion of a true conditional. But this does not mean the hypothesis is true. Sue could have gone to the movie on another night. The conclusion is not valid.
- 16. Let *p*, *q*, and *r* represent the following. *p*: 1 ∠ of a △ is 90°. *q*: A △ is a rt. △. *r*: A △'s acute ∠ measures are comp.
 You are given that *p* → *q* and *q* → *r*. Since *q* is the conclusion of the 1st conditional and the hypothesis

of the 2nd conditional, you can conclude that $p \rightarrow r$, or that if $1 \angle of a \bigtriangleup is 90^\circ$, then its acute \angle measures are comp.

17. Converse: If the sum of 2 ∠ measures is 180°, then the are supp.
Biconditional: 2 are supp. if and only if the sum of their measures is 180°.

18. T

2-5 ALGEBRAIC PROOF, PAGES 104-109

CHECK IT OUT!

1.
$$\frac{1}{2}t = -7$$
 Given equation
 $2\left(\frac{1}{2}t\right) = 2(-7)$ Mult. Prop. of =
 $t = -14$ Simplify.

2. 1 Understand the Problem Answer will be temperature in °C. Important information:

•
$$C = \frac{5}{9}(F - 32)$$
 • $F = 86^{\circ}F$

2 Make a Plan

Subst. given information into formula and solve. **3 Solve**

$C = \frac{5}{9}(F - 32)$	Given equation
$=\frac{5}{9}(86-32)$	Subst.
$=\frac{5}{9}(54)$	Simplify.
= 30°C	Simplify.

4 Look Back

Check answer by substituting it back into original formula. $_{\mbox{\tiny E}}$

$$C = \frac{3}{9}(F - 32)$$

30 $\stackrel{?}{=} \frac{5}{9}(86 - 32)$
9(30) $\stackrel{?}{=} 5(54)$
270 = 270 ✓

3.	\angle Add. Post. Subst. Simplify. Subtr. Prop. of = Add. Prop. of =	
4a.	Sym. Prop. of $=$	b. Reflex. Prop. of =

c. Trans. Prop. of = **d.** Sym. Prop. of \cong

THINK AND DISCUSS

- **1.** Mult. Prop. of =
- **2.** Use a \cong symbol for geometric figures. Use an = sign for numbers.
- 3.

Property	Equality	Congruence
Reflexive	1 = 1	$\overline{AB} \cong \overline{AB}$
Symmetric	If $x = y$, then $y = x$.	If $\overline{AB} \cong \overline{BC}$, then $\overline{BC} \cong \overline{AB}$.
Transitive	If $x = 0$ and $0 = y$, then $x = y$.	If $\overline{AB} \cong \overline{BC}$ and $\overline{BC} \cong \overline{CD}$, then $\overline{AB} \cong \overline{CD}$.

EXERCISES

GUIDED PRACTICE

1. Possible answer: A proof is an argument that uses logic, definitions, and previously proven statements to show that a statement is always true.

3. $t - 3.2 = -8.3$ Given equation + 3.2 $+ 3.2$ Add. Prop. of = t = -5.1 Simplify. 4. $2p - 30 = -4p + 6$ Given equation +4p $+4p$ Add. Prop. of = 6p - 30 = 6 Simplify. + 30 $+ 30$ Add. Prop. of = 6p = 36 Simplify. $\frac{6p}{6} = \frac{36}{6}$ Div. Prop. of = p = 6 Simplify. 5. $\frac{x + 3}{-2} = 8$ Given equation $(-2)\left(\frac{x + 3}{-2}\right) = -2(8)$ Mult. Prop. of = x + 3 = -16 Simplify. $\frac{-3}{x} = \frac{-3}{-19}$ Subtr. Prop. of = Simplify. 6. $\frac{1}{2}n = \frac{3}{4}$ Given equation $2\left(\frac{1}{2}n\right) = 2\left(\frac{3}{4}\right)$ Mult. Prop. of = $n = \frac{3}{2}$ Simplify.	2.	y + 1 = 5 -1 y = 4	Given equation Subtr. Prop. of = Simplify.
4. $2p - 30 = -4p + 6$ +4p $+4p6p - 30 = 66p - 30 = 66p - 30 = 66p = 366p = 366p = 366p = 36p = 65. \frac{x + 3}{-2} = 8(-2)\left(\frac{x + 3}{-2}\right) = -2(8)x + 3 = -16\frac{-3}{x} = -196. \frac{1}{2}n = \frac{3}{4}2\left(\frac{1}{2}n\right) = 2\left(\frac{3}{4}\right)n = \frac{3}{2}Siven equationMult. Prop. of a simplify.Given equation2\left(\frac{1}{2}n\right) = 2\left(\frac{3}{4}\right)Mult. Prop. of a simplify.Mult. Prop. of a simplify.Subtr. Prop. of a simplify.Simplify.$	3.	$\frac{t - 3.2 = -8.3}{+ 3.2}$ $\frac{+ 3.2}{t = -5.1}$	Given equation Add. Prop. of = Simplify.
5. $\frac{x+3}{-2} = 8$ Given equation $(-2)\left(\frac{x+3}{-2}\right) = -2(8)$ Mult. Prop. of a x+3 = -16Simplify. $\frac{-3}{-3} = -3$ Subtr. Prop. of Simplify. 6. $\frac{1}{2}n = \frac{3}{4}$ Given equation $2\left(\frac{1}{2}n\right) = 2\left(\frac{3}{4}\right)$ Mult. Prop. of a $n = \frac{3}{2}$ Simplify.	4.	2p - 30 = -4p + 4p + 4p + 4p + 4p - 6p - 30 = 6 - 6 - 36 - 6p = 36 - 6p = 36 - 6 - 6 - 6 - 6 - 6 - 6 - 6 - 6 - 6	+ 6 Given equation Add. Prop. of = Simplify. Add. Prop. of = Simplify. Div. Prop. of = Simplify.
6. $\frac{1}{2}n = \frac{3}{4}$ Given equation $2\left(\frac{1}{2}n\right) = 2\left(\frac{3}{4}\right)$ Mult. Prop. of $\frac{3}{2}$ Simplify.	5.	$\frac{x+3}{-2} = 8$ $(-2)\left(\frac{x+3}{-2}\right) = -2$ $x+3 = -2$ $\frac{-3}{x} = -2$	Given equation $2(8) \qquad Mult. Prop. of = 16 \qquad Simplify.$ $3 \qquad Subtr. Prop. of = 19 \qquad Simplify.$
2	6.	$\frac{\frac{1}{2}n = \frac{3}{4}}{2\left(\frac{1}{2}n\right)} = 2\left(\frac{1}{2}n\right) = \frac{3}{2}$	Given equation $\left(\frac{3}{4}\right)$ Mult. Prop. of = Simplify.

7. $0 = 2(r - 3) + 4$	Given equation
0 = 2r - 2	Distribute.
+2 +2	Add. Prop. of $=$
2 = 2r	Simplify.
$\frac{2}{2} = \frac{2r}{2}$	Div. Prop. of $=$
1 = <i>r</i>	Simplify.
<i>r</i> = 1	Sym. Prop. of =

8. 1 Understand the Problem

Answer will be amount of fat in g.

Important information: • C = 9f + 90

2 Make a Plan

Subst. given information into formula and solve. 3 Solve

• C = 102 calories

C = 9f + 90Given equation 102 = 9f + 90Subst. <u>-90</u> <u>-90</u> Subtr. Prop. of = 12 = 9fSimplify. $\frac{12}{2} = \frac{9f}{2}$ Div. Prop. of = 9 9 $\frac{4}{2} = f$ Simplify. 3 Cereal contains $\frac{4}{3}$ g of fat.

4 Look Back

Check answer by substituting it back into original formula.

$$C = 9f + 90$$

$$102 \stackrel{?}{=} 9\left(\frac{4}{3}\right) + 90$$

$$90 \stackrel{?}{=} 12 + 90$$

$$102 = 102 \checkmark$$

9. 1 Understand the Problem Answer will be number of movie rentals.

Important information:

```
• C = $5.75 + $0.89m
                         • C = $11.98
```

2 Make a Plan

Subst. given information into formula and solve. 3 Solve

C = 5.75 + 0.89m $11.98 = 5.75 + 0.89m$ $-5.75 - 5.75$ $6.23 = 0.89m$ $6.23 - 5.75$	 Given equation Subst. Subtr. Prop. of = Simplify.
$\frac{1}{0.89} = m$ $7 = m$	Div. Prop. of $=$ Simplify.

Elias rented 7 movies.

Div. Prop. of =

4 Look Back Check answer by substituting it back into original formula. C = 5.75 + 0.89m $11.98 \stackrel{?}{=} 5.75 + 0.89(7)$ 11.98 ² 5.75 + 6.23 11.98 = 11.98 🗸 11. Seg. Add. Post. **10.** Def. of \cong segs. Subst. Subst. Subtr. Prop. of = Subtr. Prop. of = Subtr. Prop. of = Add. Prop. of =

Div. Prop. of =

12. Reflex. Prop. of \cong 13. Trans. Prop. of $=$	$\left(\frac{1+x}{y+1}\right) = (3,5)$
14. Sym. Prop. of = 15. Trans. Prop. of \cong	
PRACTICE AND PROBLEM SOLVING	$\frac{1+x}{2} = 3$ Midpt. formula
16. $5x - 3 = 4(x + 2)$ Given equation $5x - 3 = 4x + 8$ Distrib. Prop. $x - 3 = 8$ Subtr. Prop. of = $x = 11$ Add. Prop. of =	$1 + x = 6$ $x = 5$ Mult. Prop. of = $\frac{y + 1}{2} = 5$ Midpt. formula $y + 1 = 10$ Mult. Prop. of =
17. $1.6 = 2n$ Given equation 0.5 = n Div. Prop. of =	y = 9 Subtr. Prop. of = 34. $C = 35 + 21h + 1.1p$ Given equation
18. $\frac{z}{3} - 2 = -10$ Given equation $\frac{z}{3} = -8$ Add. Prop. of = z = -24 Mult. Prop. of =	169.5 = 35 + 21(3) + 1.1pSubst. $169.5 = 98 + 1.1p$ Simplify. $71.5 = 1.1p$ Subtr. Prop. of = $65 = p$ Div. Prop. of =Cost of parts was \$65.
19. $-(h+3) = 72$ Given equation $-h-3 = 72$ Distrib. Prop. $-h = 75$ Add. Prop. of = $h = -75$ Mult. Prop. of =	35a. $C = 92.5 + 79.96 + 983$ Given equation + 10,820x 1,733.65 = 92.5 + 79.96 + 983 Subst. + 10,820x
20. $9y + 17 = -19$ Given equation 9y = -36 Subtr. Prop. of = y = -4 Div. Prop. of =	1,733.65 = 1,155.46 + 10,820x Simplify. 578.19 = 10,820x Subtr. Prop. of = $0.0534 \approx x$ Div. Prop. of = Average cost of gas per mile is $\approx 0.05
21. $\frac{1}{2}(p-16) = 13$ Given equation p-16 = 26 Mult. Prop. of = p = 42 Add. Prop. of =	b. 1 gal costs \approx 1 gal $\cdot \frac{32 \text{ mi}}{1 \text{ gal}} \cdot \frac{\$0.05}{1 \text{ mi}} \approx \1.71
22. $T = 0.03c + 0.05b$ Given equation $147 = 0.03c + 0.05(150)$ Subst. $147 = 0.03c + 7.5$ Simplify. $139.5 = 0.03c$ Subtr. Prop. of = $4,650 = c$ Div. Prop. of = $4,650$ bottles were collected. Given equation	36. Given <i>PR</i> , you know from Reliex. Prop. of = that $PR = PR$. By def. of \cong segs., $\overline{PR} \cong \overline{PR}$. Given that $\overline{PR} \cong \overline{ST}$, you know from def. of \cong segs. that $PR = ST$. By Sym. Prop. of =, $ST = PR$. By def. of \cong segs., $\overline{ST} \cong \overline{PR}$. Given that $\overline{AB} \cong \overline{CD}$ and $\overline{CD} \cong \overline{EF}$, you know from def. of \cong segs. that $AB = CD$ and $CD = EF$. By Trans. Prop. of = $AB = FE$. Therefore $\overline{AB} \cong \overline{FE}$ by
23. Z Add. Post.24. Z Add. Post.Subst.Subst.Simplify.Distrib. Prop.Subtr. Prop. of =Simplify.Add. Prop. of =Subtr. Prop. of =Div. Prop. of =Div. Prop. of =	def. of \cong segs. 37a. $x + 15 \le 63$ Given inequal. $x \le 48$ Subtr. Prop. of Inequal. b. $-2x > 36$ Given inequal.
25. Sym. Prop. of \cong 26. Reflex. Prop. of $=$	x < -18 Div. Prop. of Inequal.
27. Trans. Prop. of = 28. Reflex. Prop. of \cong 29. Estimate: 2(3x - 1) = 94 3x - 1 = 47 3x = 48	 38. Possible answer: The conclusion of a deductive proof has been proven true in all cases, but a conjecture is based on observation and is not proven to be true. TEST PREP
x = 16	39. B 40. H
Equation: 2(2.1x = 0.87) = 0.4.26 Given equation	41. D
3.1x - 0.87 = 47.18 Div. Prop. of = 3.1x = 48.05 Add. Prop. of =	42. 90° $m \angle 1 + m \angle 2 + m \angle 3 = 180^{\circ}$
x = 15.5 Div. Prop. of =	$4m\angle 1 = 180^{\circ}$
Possible answer: The exact solution rounds to the estimate.	$m \angle 1 = 45^{\circ}$
30. $3x - 1$ 31. $\angle A \cong \angle T$	$m\angle 3 = 2m\angle 1$
32. $\overline{NP} \cong \overline{BC}$	= 2(40) = 90
-	

CHALLENGE AND EXTEND

43.	PR = PA + RA	Seg. Add. Post.
	PA = QB, QB = RA	Given
	PA = RA	Trans. Prop. of =
	PR = PA + PA	Subst.
	<i>P</i> A = 18	Given
	<i>PR</i> = 18 + 18	Subst.
	<i>PR</i> = 36 in.	Simplify.

44. Possible answer: You cannot add geometric figures.

45. 7 - 3x > 19 Given -3x > 12 Subtr. Prop. of Inequal. x < -4 Div. Prop. of Inequal.

SPIRAL REVIEW, PAGE 109

- 46. the interest rate the account earns
- 47. Check students' contructions.
- 48. Check students' contructions.
- **49.** deductive reasoning **50.** inductive reasoning

2-6 GEOMETRIC PROOF, PAGES 110-116

CHECK IT OUT!

- 1. 1. Given
 - 2. Def. of mdpt.
 - 3. Given
 - 4. Trans. Prop. of \cong

2a. $\angle 1$ and $\angle 2$ are supp., and $\angle 2$ and $\angle 3$ are supp.

- **b.** $m \angle 1 + m \angle 2 = m \angle 2 + m \angle 3$
- **c.** Subtr. Prop. of = $\mathbf{d}. \angle 1 \cong \angle 3$

3.	Statements	Reasons
	1. $\angle 1$ and $\angle 2$ are comp.,	1. Given
	2. $m \angle 1 + m \angle 2 = 90^{\circ}$, $m \angle 2 + m \angle 2 = 90^{\circ}$	2. Def. of comp. 🔬
	$m \angle 2 + m \angle 3 = 90$ 3. $m \angle 1 + m \angle 2 = m \angle 2 + m \angle 3$	3. Subst.
	4. m∠2 = m∠2	4. Reflex. Prop. of =
	5. m∠1 = m∠3	5. Subtr. Prop.
	6. ∠1 ≅ ∠3	6. Def. of \cong $\&$

THINK AND DISCUSS

- 1. the last step
- 2. Possible answer: so another person can follow your proof and you can verify that your logical reasoning is correct.
- 3. postulate; theorem; definition; property

4.	1. Write the)	2. Draw a	1	3. State the)	4. State the		5. Plan your)
	conjecture		diagram to		given		conclusion		argument	
	to be proven.	Ι.	represent the	Ι.	information	Ι.	of the		and prove	
		Γ	hypothesis	ľ	and mark it	Γ	conjecture in	Γ	the	
			of the		on the		terms of the		conjecture.	
			conjecture.	ļ	diagram.		diagram.			

EXERCISES

GUIDED PRACTICE

- 1. statements; reasons 2. theorem
- 3. 1. Given
 - 2. Subst.
 - Simplify.
 - 4. Add. Prop. of =
 - 5. Simplify.
- 6. Def. of supp. 🖄
- **4a.** Def. of ≅ <u>∕</u>s
- **b.** $\angle 1$ and $\angle 2$ are supp.
- c. Subst.
- **d.** $\angle 1$ and $\angle 3$ are supp.

5.	Statements	Reasons
	1. X is the mdpt. of \overline{AY} ,	1. Given
	Y is the mdpt. of XB.	
	2. $\underline{AX} \cong \underline{XY}, XY \cong YB$	2. Def. of mdpt.
	3. $\overline{AX} \cong \overline{YB}$	3. Trans. Prop. of \cong

PRACTICE AND PROBLEM SOLVING

 Given Def. of ∠ bisector Def. of ≅ ▲ Given Subst. ∠ Add. Post. Subst. 	 7a. m∠1 + m∠2 = 180°, m∠3 + m∠4 = 180° b. Subst. c. m∠1 = m∠4 d. Def. of ≅ ▲
6. ∠ Add. Post. 7. Subst. 8. Simplify. 9. Def. of rt. ∠	d. Def. of ≌ <u>&</u>

8a. Def. of rt. ∠
c. m∠2 = m∠3

- **b.** $m \angle 1 + m \angle 2 = m \angle BAC$
- d. Subst.
- **e.** $\angle 1$ and $\angle 3$ are comp.

9.	Statements	Reasons
	1. $\overline{BE} \cong \overline{CE}, \overline{DE} \cong \overline{AE}$	1. Given
	2. <i>BE</i> = <i>CE</i> , <i>DE</i> = <i>AE</i>	 Def. of
	3. $AE + BE = AB$,	3. Seg. Add. Post.
	CE + DE = CD	
	4. $DE + CE = AB$	4. Subst.
	5. $AB = CD$	5. Subst.
	6. $\overline{AB} \cong \overline{CD}$	6. Def. of \cong segs.

10.	Statements	Reasons		
	1. ∠1 and ∠3 are comp., ∠2	1. Given		
	and ∠4 are comp.			
	2. m∠1 + m∠3 = 90°,	2. Def. of comp.		
	m∠2 + m∠4 = 90°	<u>/s</u>		
	$3. m \angle 1 + m \angle 3 = m \angle 2 + m \angle 4$	3. Subst.		
	4. ∠ 3 ≅ ∠4	4. Given		
	5. m∠ 3 = m∠4	5. Def. of ≅		
	6. m∠ 1 = m∠2	6. Subtr. Prop.		
		of =		
	7. ∠1 ≅ ∠2	7. Def. of \cong $\&$		
11. 1	$11. m/1 + 48^\circ = 180^\circ$ 12 . $m/2 + 63^\circ = 90^\circ$			

 $m \angle 1 = 132^{\circ}$

m∠3 = 59°

13. $m \angle 3 + 31^\circ = 90^\circ$

 $m\angle 2 = 27^{\circ}$

14. \cong Supps. Thm.

 Possible answer: because the ac an be supp. or comp. to the same ∠ or to 2 ≅ a.

16. sometimes 17. s	ometimes
-----------------------------------	----------

18. sometimes **19.** never **20.** 4n + 5 + 8n - 5 = 180

12n = 180n = 15

- **21.** 9x 6 = 8.5x + 2
9x = 8.5x + 8**22.** 4z + 3z + 6 = 90
7z + 6 = 90
7z + 6 = 90
7z = 84
z = 16
- 23. Possible answer: A thm. and a post. are both true statements of geometric facts. They are different because a post. is assumed to be true, while a thm. must be proven to be true.
- **24a.** Given: *Y* is mdpt. of \overline{AC} . *X* is the mdpt. of \overline{AB} . Prove: $XY = \frac{1}{2}BC$
 - **b.** Given: $\angle C$ is a rt. \angle . Prove: $\angle A$ and $\angle B$ are comp.
 - **c.** Given: $\angle C$ is a rt. \angle . Prove: $(AB)^2 = (AC)^2 + (BC)^2$

TEST PREP

25. C 27. D

CHAI	IFNGE		FYTEND
UNAL	LLNGL	AND	LAILNU

28.	Statements	Reasons
	1. m∠ <i>LAN</i> = 30°	1. Given
	2. m∠1 + m∠2 = m∠LAN	2. ∠ Add. Post.
	3. m∠1 + m∠2 = 30°	3. Subst.
	4. m∠1 = 15°	4. Given
	5. 15° + m∠2 = 30°	5. Subst.
	6. m∠2 = 15°	6. Subtr. Prop. of =
	7. m∠1 = m∠2	7. Trans. Prop. of $=$
	8. <u>∠1</u> ≅ ∠2	8. Def. of ≅
	9. AM bisects ∠LAN.	9. Def. of ∠ bisector

26. G

29. Step 1 Find a.

2a + 3.5 = 2.5a - 52a + 8.5 = 2.5a8.5 = 0.5*a* 17 = a**Step 2** Find \angle measures. $2a + 3.5 = 2(17) + 3.5 = 37.5^{\circ}$ $3a + 1.5 = 3(17) + 1.5 = 52.5^{\circ}$ $2.5a - 5 = 2.5(17) - 5 = 37.5^{\circ}$ 30. Step 1 Find x. $4x^2 - 6 + (-2x^2 + 19x) = 180$ $2x^2 + 19x - 6 = 180$ $2x^2 + 19x - 186 = 0$ (x-6)(2x+31) = 0x = 6 (since $-2x^2 + 19x > 0$) **Step 2** Find \angle measures. $4x^2 - 6 = 4(6)^2 - 6 = 138^\circ$ $-2x^{2} + 19x = -2(6)^{2} + 19(6) = 42^{\circ}$

SPIRAL REVIEW



2-7 FLOWCHART AND PARAGRAPH PROOFS, PAGES 118-125

CHECK IT OUT!

1.	Statements	Reasons
	1. <i>RS</i> = <i>UV</i> , <i>ST</i> = <i>TU</i>	1. Given
	2. RS + ST = TU + UV	2. Add. Prop. of =
	3. RS + ST = RT,	3. Seg. Add. Post.
	TU + UV = TV	
	4. $RT = TV$	4. Subst.
	5. $\overline{RT} \cong \overline{TV}$	5. Def. of \cong segs.



3.	Statements	Reasons
	1. ∠ <i>WXY</i> is a rt. ∠.	1. Given
	2. m $\angle WXY = 90^{\circ}$	2. Def. of rt. ∠
	3. $m \angle 2 + m \angle 3 = m \angle WXY$	3. ∠ Add. Post.
	4. m∠2 + m∠3 = 90°	4. Subst.
	5. ∠1 ≅ ∠3	5. Given
	6. m∠1 = m∠3	6. Def. of ≅ ∡
	7. m∠2 + m∠1 = 90°	7. Subst.
	8. ∠1 and ∠2 are comp.	8. Def. of comp. 🖄

4. It is given that $\angle 1 \cong \angle 4$. By Vert. \measuredangle Thm., $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By Trans. Prop. of \cong (twice), $\angle 2 \cong \angle 4$, and $\angle 2 \cong \angle 3$.

THINK AND DISCUSS

- Possible answer: There may be more than one thm. that you can apply to a proof, and the steps in a proof may sometimes be written in a different order.
- 2. Answers will vary.



EXERCISES

GUIDED PRACTICE

1. flowchart 2. paragraph

3.	Statements	Reasons
	1. ∠1 ≃ ∠2	1. Given
	2. ∠1 and ∠2 are supp.	2. Lin. Pair Thm.
	3. ∠1 and ∠2 are rt. ≰	3. ≅ ≰ supp. → rt. ≰
4.	$\begin{array}{c} \swarrow 2 \text{ and } \swarrow 4 \\ \text{are supp.} \\ \text{Lin. Pair Thm.} \end{array} \qquad \qquad$	ihm.

5.	Statements	Reasons
	1. ∠2 ≃ ∠4	1. Given
	2. ∠1 ≅ ∠2, ∠3 ≅ ∠4	2. Vert. 🛦 Thm.
	3. ∠1 ≅ ∠4	3. Trans. Prop. of \cong
	4. ∠1 ≅ ∠3	4. Trans. Prop. of \cong

6. It is given that BD bisects ∠ABC, so ∠1 ≅ ∠2 by def. of ∠ bis. By Vert. & Thm., ∠1 ≅ ∠4 and ∠2 ≅ ∠3. By Trans. Prop. of ≅, ∠4 ≅ ∠2, and thus ∠4 ≅ ∠3. Therefore BG bisects ∠FBH by def. of ∠ bis.

PRACTICE AND PROBLEM SOLVING

7.	Statements	Reasons
	1. <i>B</i> is mdpt. of AC.	1. Given
	2. $\overline{AB} \cong \overline{BC}$	2. Def. of mdpt.
	3. $AB = BC$	3. Def. of \cong segs.
	4. AD + DB = AB,	4. Seg. Add. Post.
	BE + EC = BC	
	5. $AD + DB = BE + EC$	5. Subst.
	6. $AD = EC$	6. Given
	7. <i>DB</i> = <i>BE</i>	7. Subtr. Prop. of =



9.	Statements	Reasons
	1. ∠1 ≅ ∠4	1. Given
	2. ∠1 ≅ ∠2	2. Vert. \land Thm.
	3. ∠4 ≅ ∠2	3. Trans. Prop. of \cong
	4. m∠4 = m∠2	4. Def. of ≅
	5. ∠3 and ∠4 are supp.	5. Lin. Pair Thm.
	6. m∠3 + m∠4 = 180°	6. Def. of supp. 🔬
	7. m∠3 + m∠2 = 180°	7. Subst.
	8. ∠2 and ∠3 are supp.	8. Def. of supp. 🔬

- **10.** Since $\angle 1$ and $\angle 2$ are comp., $m\angle 1 + m\angle 2 = 90^{\circ}$. $\angle 1 \cong \angle 3$ by Vert. \measuredangle Thm. Thus $m\angle 1 = m\angle 3$. By subst., $m\angle 2 + m\angle 3 = 90^{\circ}$, so $\angle 2$ and $\angle 3$ are comp.
- 11. 13 cm; by conv. of the Common Segs. Thm.
- **12.** 90°; \cong \pounds supp. \rightarrow rt. \pounds
- 13. 37°; By Vert. 🛦 Thm.
- **14.** By the Common Segs. **15.** By the Vert. ▲ Thm., Thm., 2x + 4 = 5x - 2 6 = 3x **11** y = 121y = 11
- **16.** By the Vert. \measuredangle Thm., 2x + 40 = 5x + 16 24 = 3xx = 8

x = 2

17. A; diagram is marked with Prove information instead of Given information.



- 19. Possible answer: Both & adj. to given rt. ∠ must be rt.
 & because they form lin. pairs with the given ∠. The fourth ∠ is a vert. ∠ of given ∠, so it, too, is a rt. ∠.
 Since all 4 & are rt. &, they are all ≅ by Rt. & ≅ Thm.
- 20. Answers will vary.

TEST PREP

21. C 22. H ∠5 and ∠8 are vert. ▲. m∠2 = 90 + 38 = 128°
23. D

CHALLENGE AND EXTEND



$= m \angle BOC + m \angle COD$	
5. m $\angle BOC = m \angle BOC$	5. Reflex.
	Prop. of $=$
6. m∠ AOB = m∠ COD	6. Subtr. Prop
	of =
7. ∠AOB \cong ∠COD	7. Def. of \cong \measuredangle

26. It is given that $\angle 2$ and $\angle 5$ are rt. \measuredangle . By Rt. $\angle \cong$ Thm., $\angle 2 \cong \angle 5$. By def. of $\cong \measuredangle$, $m\angle 2 = m\angle 5$. It is also given that $m\angle 1 + m\angle 2 + m\angle 3 = m\angle 4 + m\angle 5$ $+ m\angle 6$. By Subtr. Prop. of =, $m\angle 1 + m\angle 3 = m\angle 4$ $+ m\angle 6$. $\angle 3 \cong \angle 6$ by Vert. \measuredangle Thm. By def. of $\cong \measuredangle$, $m\angle 3 = m\angle 6$. By Subtr. Prop. of =, $m\angle 1 = m\angle 4$. So by def. of $\cong \measuredangle$, $\angle 1 \cong \angle 4$.

27. Step 1 Find x and y.

By Vert. & Thm., By def. of supp. ∡, 3x + 1 = 6y + x - 6 3x + 1 + 2x + 2y + 1 = 1802x = 6y - 75x + 2y = 178x = 3y - 3.55(3y - 3.5) + 2y = 17815y - 17.5 + 2y = 17817y = 195.5y = 11.5x = 3y - 3.5 = 3(11.5) - 3.5 = 31Step 2 Find ∠ measures. $3x + 1 = 3(31) + 1 = 94^{\circ}$ $2x + 2y + 1 = 2(31) + 2(11.5) + 1 = 86^{\circ}$ $6y + x - 6 = 6(11.5) + 31 - 6 = 94^{\circ}$ $m(4th \angle) = m(2nd \angle) = 86^{\circ}$

SPIRAL REVIEW 28. y = 2x + 14**29.** 7x - y = -33-6x + 18 = 2x + 143x + y = -710x = -4018 = 8x + 144 = 8xx = -41 3x + y = -7= x2 3(-4) + y = -7v = 2x + 14-12 + y = -7*y* = 5 = 2+ 14 = 15(-4, 5) $\left(\frac{1}{2}\right)$, 15 **30.** 2(-x + 3y = 10)+2x + y = 8-2x + 6y = 20+2x + y = 87y = 28y = 4-x + 3y = 10-x + 3(4) = 10

31-34. Check students' drawings.

-x = -2

x = 2

-x + 12 = 10

- 35. Converse: If a positive integer is a composite number, then it has more than 2 factors. Biconditional: A positive integer has more than 2 factors if and only if it is a composite number.
- 36. Converse: If a quad. has exactly 1 pair of || sides, then it is a trapezoid.Biconditional: A quad. is a trapezoid if and only if it has exactly 1 pair of || sides.

READY TO GO ON? PAGE 127

1. $m - 8 = 13$ Giv m = 21 Ad	ven equation ld. Prop. of =
2. $4y - 1 = 27$ Gi 4y = 28 Ac y = 7 Di	ven equation dd. Prop. of = v. Prop. of =
3. $-\frac{x}{3} = 2$ Given -x = 6 Mult. x = -6 Div. F	n equation Prop. of = Prop. of =
4. Sym. Prop. of =	5. Reflex. Prop. of \cong
6. Trans. Prop. of \cong	7. Trans. Prop. of $=$
8a. Given (given information)	 b. ∠1 and ∠3 are supp. (deduce from line 1)
 c. Reflex. Prop. of ≅ (reason why ∠3 ≅ . 	d. ∠1 \cong ∠4 (apply \cong Supps. Thm. to lines 2 and 3)



11. It is given that $\angle 1 \cong \angle 3$. By Vert. \measuredangle Thm., $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$. By Trans. Prop. of \cong , $\angle 2 \cong \angle 3$ and thus $\angle 2 \cong \angle 4$.

STUDY GUIDE: REVIEW, PAGES 130-133

- 1. theorem
- deductive reasoning
 conjecture

and

3. counterexample

LESSON 2-1

5. The rightmost \triangle is duplicated, rotated 180°, and

shifted right. The next two items are Z



- 6. Each term is $\frac{1}{6}$ greater than previous one. The next two terms are $\frac{5}{6}$ and 1.
- 7. The white section is halved. If the white section is a rect. but not a square, it is halved horiz. and the upper portion is colored yellow. If the white section is a square, it is halved vert. and the left portion is colored yellow. The next 2 items are and



8. odd

- 9. positive
- 10. F; 0 is a whole number but not a natural number
- **11.** T **12.** T
- 13. F; during a leap year, there are 29 days in February.
- 14. Check students' constructions. Possible answer: The 3 ∠ bisector of a △ intersect at the int. of △.

LESSON 2-2

- 15. If it is Monday, then it is a weekday.
- **16.** If something is a lichen, then it is a fungus.
- **17.** T
- **18.** F; possible answer: $\sqrt{2}$ and $\sqrt{2}$
- **19.** Converse: If $m \angle X = 90^\circ$, then $\angle X$ is a rt. \angle ; T Inverse: If $\angle X$ is not a rt. \angle , then $m \angle X \neq 90^\circ$; T Contrapositive: If $m \angle X \neq 90^\circ$, then $\angle X$ is not a rt. \angle ; T
- **20.** Converse: If x = 2, then x is a whole number; T Inverse: If x is not a whole number, then $x \neq 2$; T Contrapositive: If $x \neq 2$, then x is not a whole number; F

LESSON 2-3

- **21.** Let *p*, *q*, *r*, and *s* be the following; *p*: The team practices, beginning at 8 A.M. on weekdays and at 12 noon on Saturday. *q*: Sue swims, beginning at 8 A.M. on weekdays and at 12 noon on Saturday. *r*: The pool opens at 8 A.M. on weekdays and at 12 noon on Saturday. Using symbols, given information is $p \rightarrow q, r \rightarrow p$, and *r*. By Law of Detachment, *p* is valid, so conjecture is not valid.
- **22.** Using symbols, given information is $p \rightarrow q, r \rightarrow p$, and *r*. By Law of Syllogism, $r \rightarrow q$, and by Law of Detachment, *q* is valid; so conjecture is valid.
- **23.** Using symbols, given information is $p \rightarrow q, r \rightarrow p$, and *r*. By Law of Detachment, *p* is valid, so conjecture is not valid.
- **24.** Let *p* be hypothesis: Cost of Sara's call is \$2.57 Let *q* be conclusion: Sara's call lasted 7 min. 2.57 = 2.15 + 0.07x

$$0.42 = 0.07x$$

x + 1 = 7

So $p \rightarrow q$. Since statement "Cost of Sara's call is \$2.57." matches hypothesis, can conclude that Sara's call lasted 7 min.

- **25.** Let *p* be hypothesis: Paolo's call lasts 10 min. Let *q* be conclusion: Cost of Paolo's call is \$2.78. 2.15 + 0.07(10 - 1) = 2.78So $p \rightarrow q$. Since statement "Paolo's call lasts 10 min." matches hypothesis, can conclude that cost of Paolo's call is \$2.78.
- **26.** No conclusion; the number and lengths of calls are unknown.

LESSON 2-4

27. yes	28. no; <i>x</i> = 2
29. no; seg. with endpts.	(3, 7) and $(-5, -1)$
30. yes	
31. comp.	32. positive
33. greater than 50 mi/h	34. 4 <i>s</i>

LESSON 2-5

35.	$\frac{m}{-5} + 3 = -4.5$	Given equation
	$\frac{m}{5} = -7.5$	Subtr. Prop. of =
	m = 37.5	Mult. Prop. of =
36.	-47 = 3x - 59 $12 = 3x$ $4 = x$	Given equation Add. Prop. of = Div. Prop. of =
37.	Reflex. Prop. of =	38. Sym. Prop. of \cong
39.	Trans. Prop. of =	40. figure ABCD
41.	m∠5 = m∠2	42. $\overline{CD} \cong \overline{EF}$
43.	l = Prt 4200 = P(0.06) 4200 = P(0.24) \$17,500 = P	(4) Given equation Subst. Simplify. Div. Prop. of =

LESSON 2-6

44.	1. Given	45a. Given
	2. Def. of comp. 🔬	b . $TU = UV$
	3. Given	
	4. Def. of ≅	c. $SU + UV = SV$
	5. Subst.	d. Subst.
	6. Def. of comp. 🖄	
46.	Think: Use def. of	47. Think: Use def. of
	supp. 🔬.	comp. 🗟.
	z - 2 + 2 + 7z = 180	3x + 2x + 5 = 90
	8 <i>z</i> = 180	5x = 85
	z = 22.5	x = 17

LESSON 2-7



- 49. It is given that ∠ADE and ∠DAE are comp. and ∠ADE and ∠BAC are comp. By ≅ Comps. Thm., ∠DAE ≅ ∠BAC. By Reflex. Prop. of ≅, ∠CAE ≅ ∠CAE. By Common & Thm., ∠DAC ≅ ∠BAE.
- **50.** *w* = 45; by Vert. <u>&</u> Thm.

51. x = 45; since $\cong 45$ supp. \rightarrow rt. As

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3. even

1.

Possible answer: ∠1 and ∠2 are comp. but not adj.

2.5



- 5. Hypothesis: It rains. Conclusion: The show is cancelled.
- 6. If 2 lines are ||, then they do not intersect.



- 9. Converse: If you live in Kentucky, then you live in the United States; T
- Inverse: If you do not live in the United States, then you do not live in Kentucky; T
- 11. Contrapositive: If you do not live in Kentucky, then you do not live in the United States; F
- 12. Let *p*, *q*, and *r* be the following: *p*: it is colder than 50°F. *q*: Tom wears a sweater. *r*: it is 46°F today.
 You are given that *p* → *q* and that *r* is true. You also know that *r* → *p*, By Law of Detachment, *q* is true, so conjecture is valid.
- 13. Let *p*, *q*, and *r* be the following:*p*: A figure is a square.*q*: A figure is a quad.
 - *r*: A figure is a polygon.

You are given that $p \rightarrow q$ and $q \rightarrow r$. By Law of Syllogism, $p \rightarrow r$. The statement "Figure ABCD is a square" matches p, so by Law of Detachment, r is true for figure *ABCD*. Therefore, figure *ABCD* is a polygon.

- Conditional: If Chad works on Saturday, then he gets paid overtime. Converse: If Chad gets paid overtime, he will work on Saturday.
- 15. F; A, B, and C with B not between A and C

16. 8 - 5s = 1Given equation-5s = -7Subtr. Prop. of =s = 1.4Div. Prop. of =

17. 0.4t + 3 = 1.6 Given equation

 0.4t = -1.4 Subtr. Prop. of =

 t = -3.5 Div. Prop. of =

 18. 38 = -3w + 2 Given equation

 36 = -3w Subtr. Prop. of =

 -12 = w Div. Prop. of =

- **19.** Trans. Prop. of = **20.** Reflex. Prop. of =
- **21.** Trans. Prop. of \cong **22.** Sym. Prop. of \cong

23.	Statements	Reasons
	1. ∠AFB \cong ∠EFD	1. Given
	2. ∠EFD \cong ∠BFC	2. Vert. \land Thm.
	3. ∠AFB \cong ∠BFC	3. Trans. Prop. of \cong
	4. <i>FB</i> bisects ∠AFC.	 Def. of ∠ bisector

24. It is given that $\angle AFB \cong \angle EFD$. By Vert. \measuredangle Thm., $\angle EFD \cong \angle BFC$. Therefore, $\angle AFB \cong \angle BFC$ by Trans. Prop. of \cong . So \overrightarrow{FB} bisects $\angle AFC$. by Def. of \angle bisector.

