ARE YOU READY? PAGE 377

1. F
2. B
3. A
4. D
5. E

6. Use △ Sum Thm.
$$\begin{aligned} x° + 42° + 32° &= 180° \\ x° &= 180° - 42° - 32° \\ x° &= 106° \end{aligned}$$

7. Use △ Sum Thm.
$$\begin{aligned} x° + 53° + 90° &= 180° \\ x° &= 180° - 53° - 90° \\ x° &= 37° \end{aligned}$$

8. Use △ Sum Thm.
$$\begin{aligned} x° + x° + 32° &= 180° \\ 2x° &= 180° - 34° \\ 2x° &= 146° \\ x° &= 73° \end{aligned}$$

9. Use △ Sum Thm.
$$\begin{aligned} 2x° + x° + 57° &= 180° \\ 3x° &= 180° - 57° \\ 3x° &= 123° \\ x° &= 41° \end{aligned}$$

10. By Lin. Pair Thm.,
$m\angle 1 + 56 = 180$
$m\angle 1 = 124°$
By Vert. ∠s Thm.,
$m\angle 2 = 56°$
By Corr. ∠s Post.,
$m\angle 3 = m\angle 1 = 124°$
By Alt. Int. ∠s Thm.,
$m\angle 4 = 56°$

11. By Alt. Ext. ∠s Thm.,
$m\angle 2 = 101°$
By Lin. Pair Thm.,
$m\angle 1 + m\angle 2 = 180$
$m\angle 1 + 101 = 180$
$m\angle 1 = 79°$
Since $\ell \perp m$, $m \parallel n$
$\rightarrow \ell \perp n$,
$m\angle 3 = m\angle 4 = 90°$.

12. By Same-Side Int. ∠s Thm.,
$$\begin{aligned} 3x + 2x &= 180 \\ 5x &= 180 \\ x &= 36 \end{aligned}$$
By Lin. Pair Thm.,
$$\begin{aligned} m\angle 1 + 3(36) &= 180 \\ m\angle 1 + 108 &= 180 \\ m\angle 1 &= 72° \end{aligned}$$
By Corr. ∠s Post.,
$m\angle 2 = 3(36) = 108°$

13. 45°–45°–90° △
$$\begin{aligned} x &= \left(11\sqrt{2}\right)\sqrt{2} \\ &= 11(2) = 22 \end{aligned}$$

14. 30°-60°-90° △
$$\begin{aligned} 14 &= 2x \\ x &= 7 \end{aligned}$$

15. 45°-45°-90° △
$x = 3\sqrt{2}$

16. 30°-60°-90° △
$x = 2(8) = 16$

17. T (Lin. Pair Thm.); if 2 ∠s are supp., then they form a lin. pair; F (counterexample: any supp. but non-adj. pair of ∠s).

18. F (counterexample: a pair of ∠s with measure 30°); if 2 ∠s are rt ∠s, then they are ≅; T (Rt. ∠ ≅ Thm.).

19. F (counterexample: △ with side lengths 5, 6, 10); if a triangle is an acute triangle, then it is a scalene triangle; F (counterexample: any equilateral triangle).

6-1 PROPERTIES AND ATTRIBUTES OF POLYGONS, PAGES 382–388

CHECK IT OUT!

1a. not a polygon
b. polygon, nonagon
c. not a polygon

2a. regular, convex
b. irregular, concave

3a. Think: Use Polygon ∠ Sum Thm.
$(n - 2)180°$
$(15 - 2)180°$
$2340°$

b.
$$\begin{aligned} (10 - 2)180° &= 1440° \\ m\angle 1 + m\angle 2 + \ldots + m\angle 10 &= 1440° \\ 10m\angle 1 &= 1440° \\ m\angle 1 &= 144° \end{aligned}$$

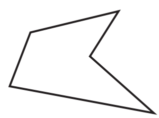
4a. Think: Use Polygon Ext. ∠ Sum Thm.
$$\begin{aligned} m\angle 1 + m\angle 2 + \ldots + m\angle 12 &= 360° \\ 12m\angle 1 &= 360° \\ m\angle 1 &= 30° \end{aligned}$$

b.
$$\begin{aligned} 4r + 7r + 5r + 8r &= 360 \\ 24r &= 360 \\ r &= 15 \end{aligned}$$

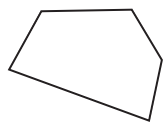
5. By Polygon Ext. ∠ Sum Thm., sum of ext. ∠ measures is 360°. Think: There are 8 ≅ ext. ∠s, so divide sum by 8.
$$m(\text{ext. } \angle) = \frac{360°}{8} = 45°$$

THINK AND DISCUSS

1. Possible answers:
Concave pentagon



Convex pentagon



A concave polygon seems to "cave in" or have a dent. A convex polygon does not have a dent.

2. Since polygon is not regular, you cannot assume that each of the ext. ∠s has the same measure.

3.

	Interior Angles	Exterior Angles
Sum of Angle Measures	$(n - 2)180°$	$360°$
One Angle Measure	$\frac{(n - 2)180°}{n}$	$\frac{360°}{n}$

EXERCISES

GUIDED PRACTICE

1. Possible answer: If a polygon is equil., all its sides are $\cong$, but all its $\measuredangle$s are not necessarily $\cong$. For a polygon to be regular, all its sides must be $\cong$, and all its $\measuredangle$s must be $\cong$.

2. polygon, decagon

3. not a polygon

4. polygon, quadrilateral

5. not a polygon

6. regular, convex

7. irregular, concave

8. irregular, convex

9. Think: Use Polygon $\angle$ Sum Thm.

$$(5 - 2)180° = 540°$$
$$3z + 4z + 5z + 3z + 5z = 540$$
$$20z = 540$$
$$z = 27$$
$$m\angle A = m\angle D = 3(27) = 81°$$
$$m\angle B = 4(27) = 108°$$
$$m\angle C = m\angle E = 5(27) = 135°$$

10. Think: Use Polygon $\angle$ Sum Thm.

$$(12 - 2)180° = 1800°$$
$$m\angle 1 + m\angle 2 + \ldots + m\angle 12 = 1800°$$
$$12m\angle 1 = 1800°$$
$$m\angle 1 = 150°$$

11. Think: Use Polygon $\angle$ Sum Thm.

$(n - 2)180°$
$(20 - 2)180°$
$3240°$

12. Think: Use Polygon Ext. $\angle$ Sum Thm.

$$4y + 2y + 6y + 4y = 360$$
$$16y = 360$$
$$y = 22.5$$

13. Think: Use Polygon Ext. $\angle$ Sum Thm.

$$m\angle 1 + m\angle 2 + \ldots + m\angle 5 = 360°$$
$$5m\angle 1 = 360°$$
$$m\angle 1 = 72°$$

14. pentagon

15. By Polygon $\angle$ Sum Thm., sum of $\angle$ measures is $(5 - 2)180 = 540°$. Think: $m\angle Q = m\angle S$ by def. of $\cong$ $\measuredangle$.

$$m\angle P + m\angle Q + m\angle R + m\angle S + m\angle T = 540$$
$$90 + m\angle S + 90 + m\angle S + 90 = 540$$
$$2m\angle S = 270$$
$$m\angle Q = m\angle S = 135°$$

PRACTICE AND PROBLEM SOLVING

16. polygon, hexagon

17. not a polygon

18. polygon, quadrilateral

19. irregular, concave

20. regular convex

21. irregular, convex

22. $2n + 6n + 2n + 5n = (4 - 2)180$

$$15n = 360$$
$$n = 24$$
$$m\angle R = m\angle T = 2(24) = 48°$$
$$m\angle S = 6(24) = 144°$$
$$m\angle V = 5(24) = 120°$$

23. $18m\angle = (18 - 2)180$

$$18m\angle = 2880$$
$$m\angle = 160°$$

24. $(7 - 2)180 = 900°$

25. $9m(\text{ext. } \angle) = 360$

$$m(\text{ext. } \angle) = 40°$$

26. $5a + 4a + 10a + 3a + 8a = 360$

$$30a = 360$$
$$a = 12$$

27. $6m\angle JKM = (6 - 2)180$

$$6m\angle JKM = 720$$
$$m\angle JKM = 120°$$

28. $6m\angle MKL = 360$

$$m\angle MKL = 60°$$

29. $x + x - 3 + 110 + 130 = (4 - 2)180$

$$2x + 237 = 360$$
$$2x = 123$$
$$x = 61.5$$

30. $2(90) + 2x + 2(x + 22) = (6 - 2)180$

$$4x + 224 = 720$$
$$4x = 496$$
$$x = 124$$

31. $5x = 360°$

$$x = 72°$$

32.

$$m\angle = m(\text{ext. } \angle)$$
$$n(m\angle) = n(m(\text{ext. } \angle))$$
$$(n - 2)180 = 360$$
$$180n = 720$$
$$n = 4$$

33.

$$m\angle = 4m(\text{ext. } \angle)$$
$$n(m\angle) = 4n(m(\text{ext. } \angle))$$
$$(n - 2)180 = 4(360)$$
$$180n = 1800$$
$$n = 10$$

34.

$$m(\text{ext. } \angle) = \frac{1}{8}m\angle$$
$$8n(m(\text{ext. } \angle)) = n(m\angle)$$
$$8(360) = (n - 2)180$$
$$3240 = 180n$$
$$n = 18$$

35. $(n - 2)180 = 540$

$$n - 2 = 3$$
$$n = 5$$

pentagon

36. $(n - 2)180 = 900$

$$n - 2 = 5$$
$$n = 7$$

heptagon

37. $(n - 2)180 = 1800$

$$n - 2 = 10$$
$$n = 12$$

dodecagon

38. $(n - 2)180 = 2520$

$$n - 2 = 14$$
$$n = 16$$

16-gon

39. $360 = n(120)$

$$n = 3$$
$$m\angle = 180 - 120 = 60°$$

40. $360 = n(72)$

$$n = 5$$
$$m\angle = 180 - 72 = 108°$$

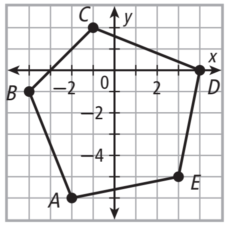
41. $360 = n(36)$

$$n = 10$$
$$m\angle = 180 - 36 = 144°$$

42. $360 = n(24)$

$$n = 15$$
$$m\angle = 180 - 24 = 156°$$

43. A; possible answer: this is not a plane figure, so it cannot be a polygon.

44. 

Check students' estimates; possible answer: pentagon is not equiangular; m$\angle A$ = 100°; m$\angle B$ = 113°; m$\angle C$ = 113°; m$\angle D$ = 101°; m$\angle E$ = 113°; yes, pentagon is not equiangular.

45a. heptagon **b.** (7 − 2)180 = 900°

c. $m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F + m\angle G = 900$

$95 + 125 + m\angle F + 130 + 130 + m\angle F + m\angle F = 900$

$3m\angle F + 480 = 900$

$3m\angle F = 420$

$m\angle F = 140°$

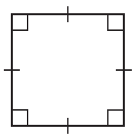
46. Let n be number of sides and s (= 7.5) be side length.

$P = ns$

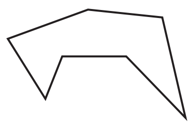
$45 = n(7.5)$

$n = 6$

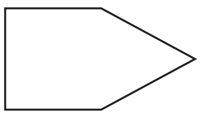
Polygon is a (regular) hexagon.

47. 

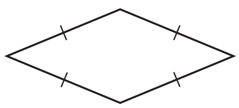
48. Possible answer:



49. Possible answer:



50. Possible answer:



51. The figure has 6 sides, so it is a hexagon. The 6 sides are ≅, so the hexagon is equilateral. The 6 ∠s are ≅, so the hexagon is equiangular. Since the hexagon is equilateral and equiangular, it is regular. No diagonal contains pts. in the interior, so it is convex.

52. As number of sides increases, isosc. △s formed by each side become thinner, and dists. from any pt. on base of each triangle to its apex approach same value. For a circle, each pt. is the same dist. from center. So polygon begins to resemble a circle.

TEST PREP

53. A

54. H

$(16 - 2)180 = 2520° \neq 2880°$

55. D

$49 + 107 + 2m\angle D + m\angle D = (4 - 2)180$

$3m\angle D = 204$

$m\angle D = 68°$

$m\angle C = 2(68) = 136°$

CHALLENGE AND EXTEND

56. ∠ measures are a, $a + 4$, …, $a + 16$, where a is a multiple of 4.

$a + a + 4 + \ldots + a + 16 = (5 - 2)180$

$5a + 40 = 540$

$5a = 500$

$a = 100$

∠ measures are 100°, 104°, 108°, 112°, and 116°.

57. $\overline{PQ} \cong \overline{ST}$, $\overline{QR} \cong \overline{RS}$, and $\angle Q \cong \angle S$. So by SAS, $\triangle PRQ \cong \triangle SRT$. By CPCTC, $\overline{PR} \cong \overline{RT}$, so $\triangle PRT$ is isosc. By Isosc. △ Thm., $\angle RTP \cong \angle RPT$, so $m\angle RTP = m\angle RPT = z°$.

By △ Sum Thm.,

$2z + y = 180$ (1)

By CPCTC and Isosc. △ Thm.,

$\angle PRQ \cong \angle SRT \cong \angle QPR \cong \angle RTS$

$m\angle PRQ = m\angle SRT = m\angle QPR = m\angle RTS = x°$

Since $PQRST$ is reg.,

$5m\angle QRS = (5 - 2)180$

$5(2x + y) = 540$

$2x + y = 108$ (2)

$5m\angle PTS = (5 - 2)180$

$5(y + z) = 540$

$y + z = 108$ (3)

Subtr. (3) from (1):

$z = 180 - 108 = 72°$

Subst: in (3):

$y + 72 = 108$

$y = 36°$

Subst. in (2):

$2x + 36 = 108$

$2x = 72$

$x = 36°$

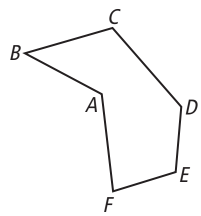
58. $\overline{KA} \parallel \overline{EF} \parallel \overline{LC}$. By Alt. Int. ∠s Thm., $\angle BLC \cong$ ext. $\angle A$ and $\angle CLD \cong$ ext. $\angle E$

$m\angle ALC = m\angle CLE = \frac{360}{10} = 36°$

$m\angle BLD = m\angle BLC + m\angle CLD = 72°$

59. Yes, if you allow for ∠ measures greater than 180°.

$m\angle A + m\angle B + m\angle C + m\angle D + m\angle E + m\angle F = 720°$



SPIRAL REVIEW

60. $x^2 + 3x - 10 = 0$

$(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

61. $x^2 - x - 12 = 0$

$(x - 4)(x + 3) = 0$

$x = 4$ or $x = -3$

62. $x^2 - 12x = -35$

$x^2 - 12x + 35 = 0$

$(x - 7)(x - 5) = 0$

$x = 7$ or $x = 5$

63. $x + 4 > 4$

$x > 0$

$4 + 4 > x$

$8 > x$

$0 < x < 8$

64. Check $x + 6 > 12$ and $6 + 12 > x$, since $6 < 12$.

$x + 6 > 12$

$x > 6$

$6 + 12 > x$

$18 > x$

$6 < x < 18$

65. Check $x + 3 > 7$ and $3 + 7 > x$, since $3 < 7$.

$x + 3 > 7$

$x > 4$

$3 + 7 > x$

$10 > x$

$4 < x < 10$

66. $c = 2a$
$= 2(6) = 12$

67. $c = 2a$
$10 = 2a$
$a = 5$
$b = a\sqrt{3} = 5\sqrt{3}$

6-2 PROPERTIES OF PARALLELOGRAMS, PAGES 391–397

CHECK IT OUT!

1a. $\overline{KN} \cong \overline{LM}$
$KN = LM = 28$ in.

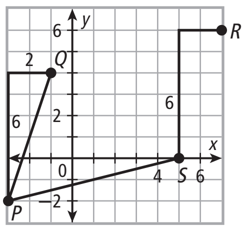
b. $\angle NML \cong \angle LKN$
$m\angle NML = m\angle LKN$
$= 74°$

c. O is mdpt. of $\overline{LN}$
$LO = \frac{1}{2}LN$
$= \frac{1}{2}(26) = 13$ in.

2a. $\overline{EJ} \cong \overline{JG}$
$EJ = JG$
$3w = w + 8$
$2w = 8$
$w = 4$
$JG = (4) + 8 = 12$

b. $\overline{FJ} \cong \overline{JH}$
$FJ = JH$
$4z - 9 = 2z$
$2z = 9$
$FH = 2JH$
$= 2(2z)$
$= 2(9) = 18$

3. **Step 1** Graph given pts.
Step 2 Find slope of $\overline{PQ}$ by counting units from P to Q.
Rise from -2 to 4 is 6.
Run from -3 to -1 is 2.
Step 3 Start at S and count same # of pts.
Rise of 6 from 0 is 6.
Run of 2 from 5 is 7.
Step 4 Use slope formula to verify that $\overline{QR} \parallel \overline{PS}$.
slope of $\overline{QR} = \frac{6-4}{7+1} = \frac{1}{4}$
slope of $\overline{PS} = \frac{0+2}{5+3} = \frac{1}{4}$
Coords. of vertex R are (7, 6).



4.

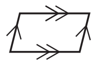
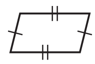
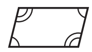
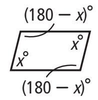
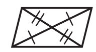
Statements	Reasons
1. $GHJN$ and $JKLM$ are ▱s.	1. Given
2. $\angle N$ and $\angle HJN$ are supp.; $\angle K$ and $\angle MJK$ are supp.	2. ▱ → cons. ∠s are supp.
3. $\angle HJN \cong \angle MJK$	3. Vert. ∠s Thm.
4. $\angle N \cong \angle K$	4. ≅ Supps. Thm.

THINK AND DISCUSS

1. Measure of opp. ∠ is 71°. Measure of each cons. ∠ is $180 - 71 = 109°$.

2. $XY = 21$, $WZ = 18$, *and* $YZ = 18$; possible answer: since $VWXY$ is a ▱, opp. sides are ≅, so $XY = VW = 21$. $\overline{WY}$ is a diag., and by Thm. 6-2-4, the other diag. bisects it, so $WZ = YZ = 36 \div 2 = 18$.

3.

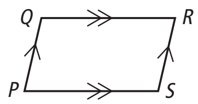
Properties of Parallelograms				
Opp. sides ∥:	Opp. sides ≅:	Opp. ∠s ≅:	Cons. ∠s supp.:	Diags. bisect each other:
				

EXERCISES

GUIDED PRACTICE

1. Only 1 pair of sides is ∥. By def., a ▱ has 2 pairs of ∥ sides.

2. Possible answer:



opp. sides: $\overline{PQ}$ and $\overline{RS}$, $\overline{QR}$ and $\overline{SP}$; opp. ∠s: $\angle P$ and $\angle R$, $\angle Q$ and $\angle S$

3. E is mdpt. of $\overline{BD}$
$BD = 2DE$
$= 2(18) = 36$

4. $\overline{CD} \cong \overline{AB}$
$CD = AB = 17.5$

5. E is mdpt. of $\overline{BD}$
$BE = DE = 18$

6. $\angle ABC$ and $\angle BCD$ are supp.
$m\angle ABC = 180 - m\angle BCD$
$= 180 - 110 = 70°$

7. $\angle ADC \cong \angle ABC$
$m\angle ADC = m\angle ABC$
$= 70°$

8. $\angle DAB \cong BCD$
$m\angle DAB = m\angle BCD$
$= 110°$

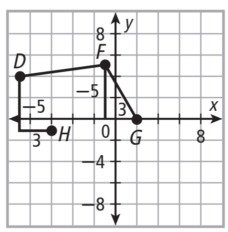
9. $\overline{JK} \cong \overline{LM}$
$JK = LM$
$7x = 3x + 14$
$4x = 14$
$x = 3.5$
$JK = 7(3.5) = 24.5$

10. $LM = 3(3.5) + 14$
$= 24.5$

11. $\angle L$ and $\angle M$ are supp.
$m\angle L + m\angle M = 180$
$2z - 3 + 5z - 6 = 180$
$7z = 189$
$z = 27$
$m\angle L = 2(27) - 3 = 51°$

12. $m\angle M = 5(27) - 6$
$= 129°$

13. **Step 1** Graph given pts.
Step 2 Find slope of $\overline{FG}$ by counting units from F to G.
Rise from 5 to 0 is -5.
Run from -1 to 2 is 3.
Step 3 Start at D and count same # of pts.
Rise of -5 from 4 is -1.
Run of 3 from -9 is -6.
Step 4 Use slope formula to verify that $\overline{DF} \parallel \overline{GH}$.
slope of $\overline{DF} = \frac{5-4}{-1+9} = \frac{1}{8}$
slope of $\overline{GH} = \frac{-1-0}{-6-2} = \frac{1}{8}$
Coords. of vertex H are $(-6, -1)$.



14.

Statements	Reasons
1. $PSTV$ is a $\square$; $\overline{PQ} \cong \overline{RQ}$.	1. Given
2. $\angle STV \cong \angle P$	2. $\square \rightarrow$ opp. $\angle$s $\cong$
3. $\angle P \cong \angle R$	3. Isoc. $\triangle$ Thm.
4. $\angle STV \cong \angle R$	4. Trans. Prop. of $\cong$

PRACTICE AND PROBLEM SOLVING

15. $JN = \frac{1}{2}JL$
$= \frac{1}{2}(165.8) = 82.9$

16. $LM = JK = 110$

17. $LN = JN = 82.9$

18. $m\angle JKL = m\angle JML$
$= 50°$

19. $m\angle KLM = 180 - m\angle JML$
$= 180 - 50 = 130°$

20. $m\angle MJK = m\angle KLM$
$= 130°$

21. $WV = VY$
$b + 8 = 5b$
$8 = 4b$
$b = 2$
$WV = (2) + 8 = 10$

22. $YW = 2WV$
$= 2(10) = 20$

23. $XV = ZV$
$3a - 7 = 2a$
$a = 7$
$XZ = 2ZV$
$= 2(2(7)) = 28$

24. $ZV = 2(7) = 14$

25. slope from P to R: rise $= 4$, run $= -6$
slope from V to T: rise of 4 from -1 is 3, run of -6 from 5 is -1
Coords. of T are $(-1, 3)$.

26.

Statements	Reasons
1. $ABCD$ and $AFGH$ are $\square$s.	1. Given
2. $\angle C \cong \angle A$, $\angle A \cong \angle G$	2. $\square \rightarrow$ opp. $\angle$s $\cong$
3. $\angle C \cong \angle G$	3. Trans. Prop. of $\cong$

27. $PQ = RS$ and $SP = QR$; given $PQ = QR$, all 4 side lengths are $=$.
$P = PQ + QR + RS + SP$
$84 = 4PQ$
$PQ = QR = RS = SP = 21$

28. $PQ = RS$, $SP = QR = 3RS$
$P = PQ + QR + RS + PS$
$84 = RS + 3RS + RS + 3RS$
$84 = 8RS$
$PQ = RS = 10.5$
$SP = QR = 3(10.5) = 31.5$

29. $PQ = RS = SP - 7$, $QR = SP$
$P = PQ + QR + RS + PS$
$84 = SP - 7 + SP + SP - 7 + SP$
$84 = 4SP - 14$
$98 = 4SP$
$QR = SP = 24.5$, $PQ = RS = 24.5 - 7 = 17.5$

30. $PQ = RS$, $QR = SP = RS^2$
$P = PQ + QR + RS + PS$
$84 = RS + RS^2 + RS + RS^2$
$84 = 2RS + 2RS^2$
$0 = RS^2 + RS - 42$
$0 = (RS + 7)(RS - 6)$
Since $RS > 0$, $PQ = RS = 6$, and $QR = SP = 6^2 = 36$.

31a. $\angle 3 \cong \angle 1$ (Corr. $\angle$s Post.)
$\angle 6 \cong \angle 1$ ($\square \rightarrow$ opp. $\angle$s $\cong$)
$\angle 8 \cong \angle 1$ ($\square \rightarrow$ opp. $\angle$s $\cong$)

b. $\angle 2$ is supp. to $\angle 1$ ($\square \rightarrow$ cons. $\angle$s supp.), $\angle 4$ is supp. to $\angle 1$ ($\square \rightarrow$ cons. $\angle$s supp.), $\angle 5$ is supp. to $\angle 1$ ($\square \rightarrow$ cons. $\angle$s supp.), and $\angle 7$ is supp. to $\angle 1$ (Subst.).

32. $\angle MPR \cong \angle RKM$ ($\square \rightarrow$ opp. $\angle$s $\cong$)

33. $\angle PRK \cong \angle KMP$ ($\square \rightarrow$ opp. $\angle$s $\cong$)

34. $\overline{MT} \cong \overline{RT}$ ($\square \rightarrow$ diags. bisect each other)

35. $\overline{PR} \cong \overline{KM}$ ($\square \rightarrow$ opp. sides $\cong$)

36. $\overline{MP} \parallel \overline{RK}$ (Def. of $\square$)

37. $\overline{MK} \parallel \overline{RP}$ (Def. of $\square$)

38. $\angle MPK \cong \angle RKP$ (Alt. Int. $\angle$s Thm.)

39. $\angle MTK \cong \angle RTP$ (Vert. $\angle$s Thm.)

40. $m\angle MKR + m\angle PRK = 180°$ ($\square \rightarrow$ cons $\angle$s supp.)

41. By props. of $\square$s,
$y = 61°$
$x + 61 = 18°$
$x = 119$
$z = x = 119°$

42. By Alt. Int. $\angle$s Thm.,
$x = 90°$
By props. of $\square$s,
$z = 53°$
By def. of comp. $\angle$s,
$y = 90 - 53 = 37°$

43. By Vert. $\angle$s Thm. and $\triangle$ Sum Thm.,
$31 + 125 + x = 180$
$x = 24°$
$y + (75 + 31) + 24 = 180$
$y = 50°$
By Alt. Int. $\angle$s Thm.,
$z = y = 50°$

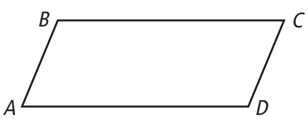
44a. $\overline{CD}$ **b.** $\angle 2$

c. $\angle 4$ **d.** opp. sides of a $\square$ are $\cong$

e. ASA **f.** CPCTC

g. bisect

45. Given: $ABCD$ is a $\square$,
Prove:
$\angle A$ and $\angle B$ are supp.
$\angle B$ and $\angle C$ are supp.
$\angle C$ and $\angle D$ are supp.
$\angle D$ and $\angle A$ are supp.



Statements	Reasons
1. $ABCD$ is a $\square$.	1. Given
2. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$	2. Def. of $\square$.
3. $\angle A$ and $\angle B$ are supp., $\angle B$ and $\angle C$ are supp., $\angle C$ and $\angle D$ are supp., $\angle D$ and $\angle A$ are supp.	3. Same-side Int. $\angle$s Thm.

46. $2x = y$
$4x = 2y$
$x = 2y - 9$
$3x = 9$
$x = 3$
$y = 2(3) = 6$

47. $y = x + 3$
$y + 7 = 3x$
$7 = 2x - 3$
$10 = 2x$
$x = 5$
$y = 5 + 3 = 8$

48a. $\angle B \cong \angle D$
$m\angle B = m\angle D$
$6x + 12 = 9x - 33$
$45 = 3x$
$x = 15$
$m\angle B = 6(15) + 12 = 102°$

b. $m\angle A = m\angle C = 180 - m\angle B$
$= 180 - 102 = 78°$
(▱ → cons. $\angle$s supp.)

49. Possible answer:

a. No; possible answer: Drawings show a counterexample, since all side pairs are ≅ but ▱s are ≇.

b. No; possible answer: For any given set of side lengths, a ▱ could have many different shapes.

50. Possible answer: a quad. is a 4-sided polygon. Since every ▱ is a polygon with 4 sides, every ▱ is a quad. A ▱ has 2 pairs of ∥ sides. Since sides of a quad. are not necessarily ∥, a quad. is not necessarily a ▱.

TEST PREP

51. A
$m\angle Q = m\angle S$
$3x + 25 = 5x - 5$
$30 = 2x$
$x = 15$

52. J

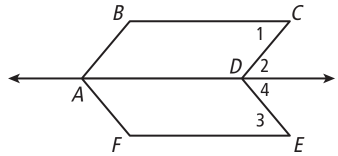
53. 26.4
$P = AB + BC + CD + DA$
$= CD + BC + CD + BC$
$= 2(5 + 8.2) = 26.4$

CHALLENGE AND EXTEND

54. Let given pts. be $A(0, 5)$, $B(4, 0)$, $C(8, 5)$, and possible 4th pts. be X, Y, Z.
$\overline{AC}$ is horiz., and $AC = |8 - 0| = 8$. So $\overline{XB}$ is horiz., and $X = (4 - 8, 0) = (-4, 0)$.
Similarly, $\overline{BY}$ is horiz., and $Y = (4 + 8, 0) = (12, 0)$.
From C to B is rise of 5 and run of 4; rise of 5 from A is $5 + 10 = 10$, run of 4 from A is $0 + 4 = 4$; so $Z = (4, 10)$.

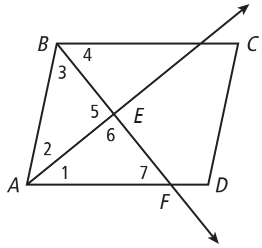
55. Let given pts. be $A(-2, 1)$, $B(3, -1)$, $C(-1, -4)$, and possible 4th pts. be X, Y, Z.
From C to B is rise of 3 and run of 4; rise of 3 from A is $1 + 3 = 4$, run of 4 from A is $-2 + 4 = 2$, so $X = (2, 4)$.
From B to C is rise of -3 and run of -4; rise of -3 from A is $1 - 3 = -2$, run of -4 from A is $-2 - 4 = -6$, so $Y = (-6, -2)$.
From A to B is rise of -2 and run of 5; rise of -2 from C is $-4 - 2 = -6$, run of 5 from C is $-1 + 5 = 4$; so $Z = (4, -6)$.

56.



Let $\angle 1 = x°$ and $\angle CDE = y°$. Draw $\overleftrightarrow{AD}$. $ABCD$ and $AFED$ are ▱s, so $\overline{BC} \parallel \overline{AD}$ and $\overline{FE} \parallel \overline{AD}$ by def. So $\angle 1 \cong \angle 2$ and $\angle 3 \cong \angle 4$ by the Alt. Int. $\angle$s Thm. Thus $m\angle 1 = m\angle 2$ and $m\angle 3 = m\angle 4$. Then $m\angle 1 + m\angle 3 = m\angle 2 + m\angle 4$ by the Add. Prop. of =. By the $\angle$ Add. Post., $m\angle 2 + m\angle 4 = m\angle CDE$. So $m\angle 1 + m\angle 3 = \angle CDE$. Since $ABCD$ and $AFED$ are ≅, with $\angle 1$ corr. to $\angle 3$, $m\angle 1 = m\angle 3$. So $m\angle 1 + m\angle 1 = m\angle CDE$ by subst. So $2m\angle 1 = m\angle CDE$, or $y = 2x$.

57.



Given: $ABCD$ is a ▱. $\overrightarrow{AE}$ *bisects* $\angle DAB$.
$\overrightarrow{BE}$ *bisects* $\angle CBA$.

Prove: $\overrightarrow{AE} \perp \overrightarrow{BE}$

Statements	Reasons
1. $ABCD$ is a ▱. $\overrightarrow{AE}$ bisects $\angle DAB$. $\overrightarrow{BE}$ bisects $\angle CBA$.	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Def. of $\angle$ bisector
3. $\overline{BC} \parallel \overline{AD}$	3. Def. of ▱
4. $\angle 4 \cong \angle 7$	4. Alt. Int. $\angle$s Thm.
5. $\angle 3 \cong \angle 7$	5. Trans. Prop. of ≅
6. $\overline{AE} \cong \overline{AE}$	6. Reflex. Prop. of ≅
7. $\triangle ABE \cong \triangle AFE$	7. AAS
8. $\angle 5 \cong \angle 6$	8. CPCTC
9. $\angle 5$ and $\angle 6$ are supp.	9. Lin. Pair Thm.
10. $\angle 5$ and $\angle 6$ are rt. $\angle$s.	10. ≅ $\angle$s supp. → rt. $\angle$s
11. $\overrightarrow{AE} \perp \overrightarrow{BE}$	11. Def. of ⊥

SPIRAL REVIEW

58. negative correlation

59. no correlation

60. alt. int. $\angle$s

61. alt. ext. $\angle$s

62. same-side int. $\angle$s

63. corr. $\angle$s

64. $(n)120 = (n - 2)180$
$360 = 60n$
$n = 6$ sides
$6m(\text{ext. } \angle) = 360$
$m(\text{ext. } \angle) = 60°$

65. $(n)135 = (n - 2)180$
$360 = 45n$
$n = 8$ sides
$8m(\text{ext. } \angle) = 360$
$m(\text{ext. } \angle) = 45°$

66. $(n)156 = (n - 2)180$
$360 = 24n$
$n = 15$ sides
$15m(\text{ext. } \angle) = 360$
$m(\text{ext. } \angle) = 24°$

6-3 CONDITIONS FOR PARALLELOGRAMS, PAGES 398–405

CHECK IT OUT!

1. Think: Show that $\overline{PQ}$ and $\overline{RS}$. are $\parallel$ and $\cong$.
$a = 2.4 \rightarrow PQ = 7(2.4) = 16.8$, $RS = 2(2.4) + 12 = 16.8$, so $PQ \cong RS$
$b = 9 \rightarrow m\angle Q = 10(9) - 16 = 74°$, $m\angle R = 9(9) + 25 = 106°$
$m\angle Q + m\angle R = 180°$, so $\angle Q$ and $\angle R$ are supp.
By Conv. of Alt. Int. $\angle$s Thm., $\overline{PQ} \parallel \overline{RS}$. So $PQRS$ is a ▱ by Thm. 6-3-1 since 1 pair of opposite sides is $\parallel$ and $\cong$.

2a. Yes; possible answer: the diag. of the quad. forms 2 $\triangle$s with 2 $\cong$ pairs of $\angle$s. By 3rd $\angle$s Thm., 3rd pair of $\angle$s in $\triangle$s are $\cong$. So both pairs of opp. $\angle$s of the quad. are $\cong$. By Thm. 6-3-3, quad. is a ▱.

b. No; 2 pairs of cons. sides are $\cong$, but none of the sets of conditions for a ▱ are met.

3. Possible answers: Find slopes of both pairs of opp. sides.
slope of $\overline{KL} = \frac{7-0}{-5+3} = -\frac{7}{2}$
slope of $\overline{MN} = \frac{-2-5}{5-3} = -\frac{7}{2}$
slope of $\overline{LM} = \frac{5-7}{3+5} = -\frac{1}{4}$
slope of $\overline{KN} = \frac{-2-0}{5+3} = -\frac{1}{4}$
Since both pairs of opp. sides are $\parallel$, $KLMN$ is a ▱ by definition.

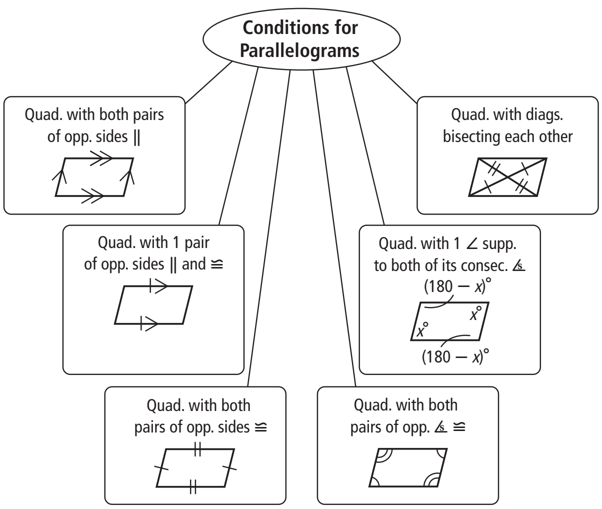
4. Possible answer: by Thm. 6-3-2, $ABRS$ is a ▱. Since $\overline{AB}$ is vert. and $\overline{RS} \parallel \overline{AB}$, $\overline{RS}$ is vert., so $\angle$ of binoculars stays the same.

THINK AND DISCUSS

1. Possible answer: Conclusion of each thm. is "The quad. is a ▱."

2. Possible answer: In Lesson 6-2, "A quad. is a ▱," is the hypothesis of each thm., rather than the conclusion.

3.



EXERCISES

GUIDED PRACTICE

1. **Step 1** Find $\overline{EJ}$ and $\overline{JG}$.
$EJ = t + 12$ | $JG = 3t$
$EJ = 6 + 12 = 18$ | $JG = 3(6) = 18$
Step 2 Find $\overline{FJ}$ and $\overline{JH}$.
$FJ = 2s$ | $JG = s + 5$
$FJ = 2(5) = 10$ | $JG = 5 + 5 = 10$
Since $EJ = JG$ and $FJ = JH$, $EFGH$ is a ▱ by Thm. 6-3-5 since its diagonals bisect each other.

2. $\angle L = 5m + 36$
$\angle L = 5(14) + 36 = 106°$
$\angle P = 6n - 1$
$\angle P = 6(12.5) - 1 = 74°$
$\angle Q = 4m + 50$
$\angle Q = 4(14) + 50 = 106°$
Since $106° + 74° = 180°$, $\angle P$ is supp. to both $\angle L$ and $\angle Q$. $KLPQ$ is a ▱ by Thm. 6-3-4 since an angle is supp. to both its cons $\angle$s.

3. Yes; both pairs of opp. $\angle$s of the quad. are $\cong$. By Thm. 6-3-3, the quad. is a ▱.

4. No; 1 pair of opp. sides of quad. are $\cong$. 1 diag. is bisected by other diag. None of the conditions for a ▱ are met.

5. Yes; possible answer: a pair of alt. int. $\angle$s are $\cong$, so 1 pair of opp. sides are $\parallel$. The same pair of opp. sides are $\cong$. By Thm. 6-3-1, quad. is a ▱.

6. Possible answer: Find slopes of both pairs of opp. sides.
slope of $\overline{WX} = \frac{3+2}{-3+5} = \frac{5}{2}$
slope of $\overline{YZ} = \frac{0-5}{1-3} = \frac{5}{2}$
slope of $\overline{XY} = \frac{5-3}{3+3} = \frac{1}{3}$
slope $\overline{WZ} = \frac{0+2}{1+5} = \frac{1}{3}$
Since both pairs of opp. sides are $\parallel$, $WXYZ$ is a ▱ by definition.

7. Possible answer: Find slopes of both pairs of opp. sides.
slope of $\overline{RS} = \frac{-1+5}{-2+1} = -4$
slope of $\overline{TU} = \frac{-5+1}{5-4} = -4$
slope of $\overline{ST} = \frac{-1+1}{4+2} = 0$
slope of $\overline{RU} = \frac{-5+5}{5+1} = 0$
Since both pairs of opp. sides are $\parallel$, $RSTU$ is a ▱ by def.

8. Since $\overline{AD} \parallel \overline{BC}$ and $\overline{AD} \cong \overline{BC}$, $ABCD$ is a ▱ by Thm. 6-3-1, so $\overline{AB} \parallel \overline{CD}$ by def. of ▱.

PRACTICE AND PROBLEM SOLVING

9. $BC = 3(3.2) + 7 = 16.6$, $GH = 8(3.2) - 9 = 16.6$
$BH = 3(7) + 7 = 28$, $CG = 6(7) - 14 = 28$
$BCGH$ is a ▱ by Thm. 6-3-2.

10. $UV = 10(19.5) - 6 = 189$, $TW = 8(19.5) + 33 = 189$
$m\angle V = 2(22) + 41 = 85°$, $m\angle W = 7(22) - 59 = 95°$
$\overline{UV} \cong \overline{TW}$; $\angle V$ and $\angle W$ are supp., so by Conv. of Same-Side Int. ∠s Thm., $\overline{UV} \parallel \overline{TW}$. $TUVW$ is a ▱ by Thm. 6-3-1.

11. Yes; both pairs of opp. sides are ≅, since all sides are ≅, so quad is a ▱ by Thm. 6-3-2.

12. Yes; by ∠ Add. Post., 1 pair of opp. ∠s are ≅, and by 3rd ∠s Thm., 2nd pair of opp. ∠s are ≅. So quad is a ▱ by Thm. 6-3-3.

13. No; by looking at the angles, we can be sure that one pair of sides are ∥. This is not enough.

14. slope of $\overline{JK} = \frac{7}{-2} = -\frac{7}{2}$; slope of $\overline{LM} = \frac{-7}{2} = -\frac{7}{2}$
slope of $\overline{KL} = \frac{-1}{5} = -\frac{1}{5}$; slope of $\overline{JM} = \frac{-1}{5} = -\frac{1}{5}$
both pairs of opp. sides have the same slope, so $\overline{JK} \parallel \overline{LM}$ and $\overline{KL} \parallel \overline{MJ}$; $JKLM$ is a ▱ by definition.

15. slope of $\overline{PQ} = \frac{5}{3}$; slope of $\overline{RS} = \frac{-5}{-3} = \frac{5}{3}$
slope of $\overline{QR} = \frac{-6}{6} = -1$; slope of $\overline{PS} = \frac{-6}{6} = -1$.
So, $\overline{PQ} \parallel \overline{RS}$ and $\overline{PS} \parallel \overline{QR}$.
$PQRS$ is a ▱ by def.

16. Possible answer: The brackets are always the same length, so it is always true that $AB = CD$. The bolts are always the same dist. apart, so it is always true that $BC = DA$. By Thm. 6-3-2, $ABCD$ is always a ▱. The side $\overline{AD}$ stays horiz. no matter how you move the tray. Since $\overline{BC} \parallel \overline{AD}$, $\overline{BC}$ stays horiz. Since $\overline{BC}$ holds the tray in position, the tray will stay horiz. no matter how it is moved.

17. No; the given ∠ measures only indicate that 1 ∠ of the quad. is supp. to 1 of its cons. ∠s. By Thm. 6-3-4, you must know that 1 ∠ is supp. to both of its cons. ∠s in order to conclude that quad. is a ▱.

18. No; you are only given the measures of the 4 ∠s formed by the diags. None of the sets of conditions for a ▱ are met.

19. Yes; diags. of the quad. bisect each other. By Thm. 6-3-5, the quad. is a ▱.

20. Think: Opp. sides must be ≅.

$2a + 6 = 3a - 10$; $16 = a$

$6b - 3 = 5(16) + 1$; $6b = 84$; $b = 14$

21. Think: Middle ∠ must be supp. to cons. ∠s.

$4a - 8 + 8a - 10 = 180$
$12a = 198$
$a = 16.5$
$4(16.5) - 8 + 5b + 6 = 180$
$5b = 116$
$b = 23.2$

22. Think: Diags. must bisect each other.

$5b - 7 = 3b + 6$; $2b = 13$; $b = 6.5$

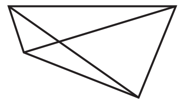
$2a = 3(6.5) - 5$; $2a = 14.5$; $a = 7.25$

23. Think: 1 pair of opp. sides must be ≅ and ∥. For conditions of Conv. of Alt. Int. ∠s Thm., given ∠s must be ≅.

$3a + 1.8 = 4a - 6.6$; $8.4 = a$

$1.4b = b + 8$; $0.4b = 8$; $b = 20$

24. Possible answer:
If the diags. of a quad. are ≅, you cannot necessarily conclude that the quad. is a ▱.



25. Possible answer: The red and green △s are isosc. rt. △s, so the measure of each acute ∠ of the △s is 45°. Each of the smaller ∠s of the yellow stripe is comp. to 1 of the acute ∠s of the rt. △s, so the measure of each of the smaller ∠s of the yellow stripe is $90° - 45° = 45°$. Each of the larger ∠s of the yellow stripe is supp. to 1 of the acute ∠s of the rt. △s, so the measure of each of the larger ∠s of the yellow stripe is $180° - 45° = 135°$. So the yellow stripe is quad. in which both pairs of opp. ∠s are ≅. By Thm. 6-3-3, the shape of the yellow stripe is a ▱.

26a. Reflex. Prop. of ≅ **b.** $\triangle BCD$
c. SSS **d.** $\angle 3$
e. $\angle 2$ **f.** Conv. of Alt. Int. ∠s Thm.
g. def. of ▱

27a. $\angle Q$ **b.** $\angle S$
c. $\overline{SP}$ **d.** $\overline{RS}$
e. ▱

28. **Given:** $ABCD$ is a ▱, E is the mdpt. of $\overline{AB}$, and F is the mdpt. of $\overline{CD}$.
Prove: $AEFD$ and $EBCF$ are ▱s.
Proof: Since $ABCD$ is a ▱, $\overline{AB} \parallel \overline{CD}$, so $\overline{AE} \parallel \overline{DF}$ and $\overline{EB} \parallel \overline{FC}$. Since opp. sides of a ▱ are ≅, $\overline{AB} \cong \overline{CD}$. It is given that E is the mdpt. of $\overline{AB}$, and F is the mdpt. of $\overline{CD}$. Because these two segs. are ≅, it follows that $\overline{AE} \cong \overline{EB} \cong \overline{DF} \cong \overline{FC}$. Since $\overline{AE} \parallel \overline{DF}$ and $\overline{AE} \cong \overline{DF}$, $AEFD$ is a ▱. Similarly, $EBCF$ is a ▱.

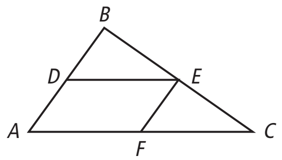
29.

Statements	Reasons
1. $\angle E \cong \angle G$, $\angle F \cong \angle H$	1. Given
2. $m\angle E = m\angle G$, $m\angle F = m\angle H$	2. Def. of ≅ ∠s
3. $m\angle E + m\angle F + m\angle G + m\angle H = 360°$	3. Polygon Sum Thm.
4. $m\angle E + m\angle F + m\angle E + m\angle F = 360°$, $m\angle E + m\angle H + m\angle E + m\angle H = 360°$	4. Subst.
5. $2m\angle E + 2m\angle F = 360°$, $2m\angle E + 2m\angle H = 360°$	5. Distrib. Prop. of =
6. $m\angle E + m\angle F = 180°$, $m\angle E + m\angle H = 180°$	6. Div. Prop. of =
7. $\angle E$ is supp. to $\angle F$ and $\angle H$.	7. Def. of supp. ∠s
8. $\overline{EF} \parallel \overline{GH}$, $\overline{FG} \parallel \overline{HE}$	8. Conv. of Same-Side Int. ∠s Thm.
9. $EFGH$ is a ▱.	9. Def. of ▱

30.

Statements	Reasons
1. $\overline{JL}$ and $\overline{KM}$ bisect each other.	1. Given
2. $\overline{JN} \cong \overline{LN}$, $\overline{KN} \cong \overline{MN}$	2. Def. of bisect
3. $\angle JNK \cong \angle LNM$, $\angle KNL \cong \angle MNJ$	3. Vert. $\angle$s Thm.
4. $\triangle JNK \cong \triangle LNM$, $\triangle KNL \cong \triangle MNJ$	4. SAS
5. $\angle JKN \cong \angle LMN$, $\angle KLN \cong \angle MJN$	5. CPCTC
6. $\overline{JK} \parallel \overline{LM}$, $\overline{KL} \parallel \overline{MJ}$	6. Conv. of Alt Int. $\angle$s Thm.
7. *JKLM* is a ▱.	7. Def. of ▱

31. Possible answer:

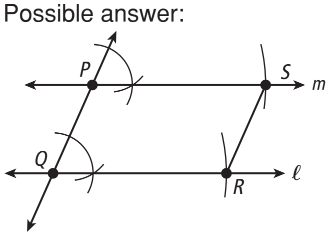


Given: $\overline{DE}$ and $\overline{EF}$ are midsegments of $\triangle ABC$.
Prove: *ADEF* is a ▱.

Statements	Reasons
1. $\overline{DE}$ and $\overline{EF}$ are midsegs. of $\triangle ABC$.	1. Given
2. $\overline{DE} \parallel \overline{FA}$, $\overline{AD} \parallel \overline{EF}$	2. $\triangle$ Midseg. Thm.
3. *ADEF* is a ▱.	3. Def. of ▱

32. Possible answer: A quad. is a ▱ if and only if both pairs of opp. sides are ≅. A quad. is a ▱ if and only if both pairs of opp. $\angle$s are ≅. A quad. is a ▱ if and only if its diags. bisect each other.

33. Possible answer:



Draw line ℓ. Draw P, not on ℓ. Draw a line through P that intersects ℓ at Q. Construct $m \parallel$ to ℓ through P. Place the compass point at Q and mark off a seg. on ℓ. Label the second endpoint of this seg as R. Using the same compass setting, place the compass point at P and mark off a ≅ seg. on m. Label the second endpoint of this seg. S. Draw $\overline{RS}$. Since $\overline{PS} \parallel \overline{QR}$ and $\overline{PS} \cong \overline{QR}$, *PSRQ* is a ▱ by Thm. 6-3-1.

34a. No; none of sets of conditions for a ▱ are met.

b. Yes; since $\angle S$ and $\angle R$ are supp., $\overline{PS} \parallel \overline{QR}$. Thus *PQRS* is a ▱ by Thm. 6-3-1.

c. Yes; draw $\overline{PR}$. $\angle QPR \cong \angle SRP$ (Alt. Int. $\angle$s Thm.) and $\overline{PR} \cong \overline{PR}$ (Reflex Prop. of ≅). So $\triangle QPR \cong \triangle SRP$ (AAS), and $\overline{PQ} \cong \overline{SR}$ (CPCTC). Since $\overline{PQ} \parallel \overline{SR}$ and $\overline{PQ} \cong \overline{SR}$, *PQRS* is a ▱ by Thm. 6-3-1.

TEST PREP

35. B
By Conv. of Alt. Int. $\angle$s Thm., $\overline{WX} \parallel \overline{YZ}$; need $\overline{WX} \cong \overline{YZ}$ to meet conditions of Thm. 6-3-1.

36. G
Slope of $\overline{AB}$: rise of 4 and run of 2, or rise of -4 and run of -2; rise of ± 4 from C is $1 \pm 4 = 5$ or -3, run of ± 2 from C is $6 \pm 2 = 8$ or 4. So D could be at (8, 5) or (4, −3).

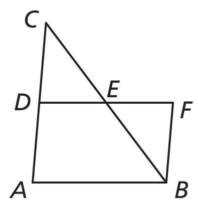
37. No; possible answer: slope of $\overline{RS} = \frac{3}{4}$, slope of $\overline{TV} = 1$; $\overline{RS}$ and $\overline{TV}$ do not have same slope, so $\overline{RS} \nparallel \overline{TV}$; $\overline{RS}$ and $\overline{TV}$ are opp. sides of *RSTV*; by def., both pairs of opp. sides of a ▱ are ∥, so *RSTV* is not a ▱.

CHALLENGE AND EXTEND

38. The top and bottom of each step form a small ▱ with the back of the stairs and the base of the railing. The vertices of each ▱ have joints that allow the pieces to move. But the lengths of the sides of ▱s stay the same. Since they start out as ▱s with opp. sides that are ≅, and the lengths do not change, they remain ▱s. Therefore the top and bottom of each step, and thus also the upper platform, remain ∥ to ground regardless of the position of the staircase.

39. Let intersection and vertices be $P(-2, 1.5)$, $A(-7, 2)$, $B(2, 6.5)$, $C(x, y)$, and $D(u, v)$. P is mdpt. of $\overline{AC}$ and $\overline{BD}$.
$(-2, 1.5) = \left(\frac{-7 + x}{2}, \frac{2 + y}{2}\right)$
$x = 3$, $y = 1$; $C = (3, 1)$
$(-2, 1.5) = \left(\frac{2 + u}{2}, \frac{6.5 + v}{2}\right)$
$u = -6$, $v = -3.5$; $D = (-6, -3.5)$

40. Possible answer:



Draw F collinear with D and E such that $\overline{DE} \cong \overline{EF}$. Since E is the mdpt. of $\overline{BC}$, $\overline{CE} \cong \overline{EB}$. By the Vert. $\angle$s Thm., $\angle CED \cong \angle BEF$. Thus $\triangle CED \cong \triangle BEF$ by SAS. By CPCTC, $\overline{CD} \cong \overline{FB}$. Since D is the mdpt. of $\overline{AC}$, $\overline{CD} \cong \overline{AD}$. So by the Trans. Prop. of ≅, $\overline{AD} \cong \overline{FB}$. Also by CPCTC, $\angle CDE \cong \angle BFE$. By Conv. of Alt. Int. $\angle$s Thm., $\overline{AC} \parallel \overline{FB}$. Thus *DFBA* is a ▱ since 1 pair of opp. sides are ∥ and ≅. Since *DFBA* is a ▱, $\overline{DE} \parallel \overline{AB}$ by definition. Since opp. sides of a ▱ are ≅, $\overline{AB} \cong \overline{DF}$ and $AB = DF$ by the def. of ≅ segs. Since $\overline{DE} \cong \overline{EF}$, E is the mdpt. of $\overline{DF}$, and $DE = \frac{1}{2}DF$. By subst., $DE = \frac{1}{2}AB$.

SPIRAL REVIEW

41.

x	−5	−2	0	0.5
y	−38	−17	−3	0.5

42.

x	-5	-2	0	0.5
y	-1.5	0	1	1.25

43.

x	-5	-2	0	0.5
y	77	14	2	2.75

44. It is given that $\overline{BC} \cong \overline{DA}$ and that $\angle DBC \cong \angle BDA$. By Reflex. Prop. of $\cong$, $\overline{DB} \cong \overline{DB}$. Therefore $\triangle ABD \cong \triangle CDB$ by SAS.

45. Possible answer: It is given that $\overline{TW} \cong \overline{VW}$. Because $\angle UWV$ is a rt. $\angle$ and is supp. to $\angle UWT$, $\angle UWT$ is also a rt. $\angle$. Thus $\angle UWV \cong \angle UWT$. By the Reflex. Prop. of $\cong$, $\overline{UW} \cong \overline{UW}$. Therefore $\triangle TUW \cong \triangle VUW$ by SAS.

46. $KN = NM$
$x + 6 = 2x - 2$
$8 = x$
$NM = 2(8) - 2 = 14$

47. $JK = LM$
$16z - 4 = 8z + 4$
$8z = 8$
$z = 1$
$LM = 8(1) + 4 = 12$

48. $JN = NL$
$4y = 3y + 2$
$y = 2$
$JL = 2JN$
$= 2(4y) = 8y$
$= 8(2) = 16$

49. $JK = LM = 12$

6A READY TO GO ON? PAGE 407

1. polygon; octagon

2. not a polygon

3. not a polygon

4. polygon; pentagon

5. $(n - 2)180°$
$(16 - 2)180°$
$2520°$

6. $(n)\text{m}\angle = (n - 2)180$
$6\text{m}\angle = 720$
$\text{m}\angle = 120°$

7. $14z + 8z + 7z + 11z = 360$
$40z = 360$
$z = 9$
Ext. $\angle$ measures are $14(9) = 126°$, $8(9) = 72°$, $7(9) = 63°$, and $11(9) = 99°$.

8. $10\text{m(ext. }\angle) = 360$
$\text{m(ext. }\angle) = 36°$

9. N is mdpt. of $\overline{KM}$.
$KM = 2KN$
$= 2(13.5) = 27$ cm

10. $\overline{KJ} \cong \overline{LM}$
$KJ = LM = 17$ cm

11. $MN = KN = 13.5$ cm

12. $\angle JKL$ and $\angle KJM$ are supp.
$\text{m}\angle JKL + \text{m}\angle KJM = 180$
$\text{m}\angle JKL + 102 = 180$
$\text{m}\angle JKL = 78°$

13. $\angle JML \cong \angle JKL$
$\text{m}\angle JML = \text{m}\angle JKL$
$= 78°$

14. $\angle KLM \cong \angle KJM$
$\text{m}\angle KLM = \text{m}\angle KJM$
$= 102°$

15. slope from B to C: rise of -5 and run of 1
rise of -5 from A is $1 - 5 = -4$; run of 1 from A is $-3 + 1 = -2$; $D = (-2, -4)$

16. $WX = YZ$
$6b - 7 = 10b - 19$
$12 = 4b$
$b = 3$
$WX = 6(3) - 7 = 11$

17. $YZ = WX = 11$

18. $\angle X$ and $\angle W$ are supp.
$\text{m}\angle X + \text{m}\angle Y = 180$
$5a - 39 + 3a + 27 = 180$
$8a = 192$
$a = 24$
$\text{m}\angle X = 5(24) - 39 = 81°$

19. $\text{m}\angle W = 3(24) + 27 = 99°$

20. $x = 6 \rightarrow RS = 7(6) + 6 = 48$, $TV = 9(6) - 6 = 48$
$y = 4.5 \rightarrow RV = 8(4.5) - 8 = 28$, $ST = 6(4.5) + 1 = 28$
$RS \cong TV$, $ST \cong RV \rightarrow RSTV$ is a ▱ (Thm. 6-3-2)

21. $m = 12 \rightarrow \text{m}\angle G = 2(12) + 31 = 55°$,
$\text{m}\angle J = 7(12) - 29 = 55°$
$n = 9.5 \rightarrow \text{m}\angle K = 12(9.5) + 11 = 125°$
$\angle K$ supp. to $\angle G$, and $\angle J \rightarrow GHJK$ is a ▱ (Thm. 6-3-4).

22. Yes; both pairs of opp. sides are $\parallel$, so quad. is a ▱ by definition.

23. No; one pair of opposites $\angle$s of the quad. are $\cong$. None of the sets of conditions for a ▱ are met.

24. No; the diagonals are divided into two segments at their point of intersection, and each segment of one diagonal is $\cong$ to a segment of the other diagonal. None of the sets of conditions for a ▱ are met.

25. Slope of $\overline{CD} = \frac{4}{5}$; slope of $\overline{EF} = \frac{-4}{-5} = \frac{4}{5}$
slope of $\overline{DE} = \frac{-2}{6} = -\frac{1}{3}$; slope of $\overline{FC} = \frac{-2}{6} = -\frac{1}{3}$
Both pairs of opp. sides are $\parallel$, so quad. is a ▱ by definition.

6-4 PROPERTIES OF SPECIAL PARALLELOGRAMS, PAGES 408–415

CHECK IT OUT!

1a. Think: rect. $\rightarrow$ ▱ $\rightarrow$ opp. sides $\cong$
$\overline{HJ} \cong \overline{GK}$
$HJ = GK = 48$ in.

b. Think: rect. $\rightarrow$ diags. $\cong$
$\overline{HK} \cong \overline{GJ}$
$HK = GJ$
$= 2JL$
$= 2(30.8) = 61.6$ in.

2a. $CG = GF$
$5a = 3a + 17$
$2a = 17$
$a = 8.5$
$CD = CG$
$= 5(8.5) = 42.5$

b. Think: rhombus $\rightarrow$ ▱ $\rightarrow$ cons. $\angle$s supp.
$\text{m}\angle GCD + \text{m}\angle CDF = 180$
$b + 3 + 6b - 40 = 180$
$7b = 217$
$b = 31$
Think: Use Thm. 6-4-5.
$\text{m}\angle GCH = \frac{1}{2}\text{m}\angle GCD$
$= \frac{1}{2}(31 + 3) = 17°$

3. **Step 1** Show that $\overline{SV}$ and $\overline{TW}$ are $\cong$.

$SV = \sqrt{11^2 + 1^2} = \sqrt{122}$

$TW = \sqrt{1^2 + 11^2} = \sqrt{122}$

Since $SV = TW$, $\overline{SV} \cong \overline{TW}$.

Step 2 Show that $\overline{SV}$ and $\overline{TW}$ are $\perp$.

slope of $\overline{SV} = \frac{1}{11}$; slope of $\overline{TW} = \frac{-11}{1} = -11$

Since $\left(\frac{1}{11}\right)(-11) = -1$, $\overline{SV}$ and $\overline{TW}$ are $\perp$.

Step 3 Show that $\overline{SV}$ and $\overline{TW}$ bisect each other.

mdpt. of $\overline{SV} = \left(\frac{-5+6}{2}, \frac{-4-3}{2}\right) = \left(\frac{1}{2}, -\frac{7}{2}\right)$

mdpt. of $\overline{TW} = \left(\frac{0+1}{2}, \frac{2-9}{2}\right) = \left(\frac{1}{2}, -\frac{7}{2}\right)$

Since $\overline{SV}$ and $\overline{TW}$ have same mdpt., they bisect each other.

Diags. are $\cong$ $\perp$ bisectors of each other.

4. Possible answer:

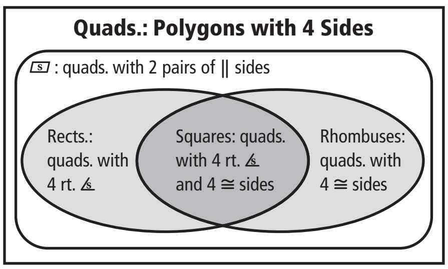
Statements	Reasons
1. $PQTS$ is a rhombus.	1. Given
2. $\overline{PT}$ bisects $\angle QPS$.	2. Thm. 6-4-5
3. $\angle QPR \cong \angle SPR$	3. Def. of $\angle$ bisector
4. $\overline{PQ} \cong \overline{PS}$	4. Def. of rhombus
5. $\overline{PR} \cong \overline{PR}$	5. Reflex. Prop. of $\cong$
6. $\triangle QPR \cong \triangle SPR$	6. SAS
7. $\overline{RQ} \cong \overline{RS}$	7. CPCTC

THINK AND DISCUSS

1. Thm. 6-4-2; possible answer: when the thm. is written as a conditional statement, it is easier to identify the hypothesis and the conclusion.

2. Same properties: 2 pairs of $\parallel$ sides, opp. sides $\cong$, opp. $\angle$s $\cong$, cons. $\angle$s supp., diags. bisect each other; Special properties: 4 $\cong$ sides, $\perp$ diags., each diag. bisects a pair of opp. $\angle$s

3.



EXERCISES

GUIDED PRACTICE

1. rhombus; rectangle; square

2. rect. ▱ → diags. bisect each other

$TQ = \frac{1}{2}QS$

$= \frac{1}{2}(380) = 190$ ft

3. $\overline{PQ} \cong \overline{RS}$

$PQ = RS = 160$ ft

4. T is mdpt. of $\overline{QS}$.

$ST = TQ = 190$ ft

5. rect. → diags. are $\cong$

$\overline{PR} \cong \overline{QS}$

$PR = QS = 380$ ft

6.
$$\begin{aligned} BC &= CD \\ 4x + 15 &= 7x + 2 \\ 13 &= 3x \\ x &= 4\tfrac{1}{3} \end{aligned}$$

$$\begin{aligned} AB &= BC \\ &= 4\left(4\tfrac{1}{3}\right) + 15 = 32\tfrac{1}{3} \end{aligned}$$

7. $\overline{AC} \perp \overline{BD}$
$$\begin{aligned} m\angle AFB &= 90 \\ 12y &= 90 \\ y &= 7.5 \\ m\angle ABC + m\angle BCD &= 180 \\ m\angle ABC + 2m\angle FCD &= 180 \\ m\angle ABC + 2(4(7.5) - 1) &= 180 \\ m\angle ABC + 58 &= 180 \\ m\angle ABC &= 122° \end{aligned}$$

8. **Step 1** Show that $\overline{JL}$ and $\overline{KM}$ are $\cong$.

$JL = \sqrt{5^2 + 7^2} = \sqrt{74}$

$KM = \sqrt{7^2 + 5^2} = \sqrt{74}$

Since $JL = KM$, $\overline{JL} \cong \overline{KM}$.

Step 2 Show that $\overline{JL}$ and $\overline{KM}$ are $\perp$.

slope of $\overline{JL} = \frac{7}{5}$; slope of $\overline{KM} = \frac{-5}{7} = -\frac{5}{7}$

Since $\left(\frac{7}{5}\right)\left(-\frac{5}{7}\right) = -1$, $\overline{JL}$ and $\overline{KM}$ are $\perp$.

Step 3 Show that $\overline{JL}$ and $\overline{KM}$ bisect each other.

mdpt. of $\overline{JL} = \left(\frac{-3+2}{2}, \frac{-5+2}{2}\right) = \left(-\frac{1}{2}, -\frac{3}{2}\right)$

mdpt. of $\overline{KM} = \left(\frac{-4+3}{2}, \frac{1-4}{2}\right) = \left(-\frac{1}{2}, -\frac{3}{2}\right)$

Since $\overline{JL}$ and $\overline{KM}$ have same mdpt., they bisect each other.

Diags. are $\cong$ $\perp$ bisectors of each other.

9. Possible answer:

Statements	Reasons
1. $RECT$ is a rect.; $\overline{RX} \cong \overline{TY}$	1. Given
2. $\overline{XY} \cong \overline{XY}$	2. Reflex. Prop. of $\cong$
3. $RX = TY$, $XY = XY$	3. Def. of $\cong$ segs.
4. $RX + XY = TY + XY$	4. Add. Prop. of =
5. $RX + XY = RY$, $TY + XY = TX$	5. Seg. Add. Post.
6. $RY = TX$	6. Subst.
7. $\overline{RY} \cong \overline{TX}$	7. Def. of $\cong$ segs.
8. $\angle R$ and $\angle T$ are rt. $\angle$s.	8. Def. of rect.
9. $\angle R \cong \angle T$	9. Rt. $\angle$ $\cong$ Thm.
10. $RECT$ is a ▱.	10. Rect. → ▱
11. $\overline{RE} \cong \overline{CT}$	11. ▱ → opp. sides $\cong$
12. $\triangle REY \cong \triangle TCX$	12. SAS

PRACTICE AND PROBLEM SOLVING

10. $JL = 2JP$

$= 2(14.5) = 29$ in.

11. $KL = JM = 25$ in.

12. $KM = JL = 29$ in.

13. $MP = \frac{1}{2}KM$

$= \frac{1}{2}(29) = 14\frac{1}{2}$ in.

14. $WX = XY$
$9a - 18 = 3a + 15$
$6a = 33$
$a = 5.5$
$VW = WX$
$= 9(5.5) - 18$
$= 31.5$

15. $m\angle XZW = 90$
$10b - 5 = 90$
$10b = 95$
$b = 9.5$
$m\angle VWX + m\angle WVY = 180$
$m\angle VWX + 4(9.5) + 10 = 180$
$m\angle VWX = 132°$
$m\angle WYX = \frac{1}{2}m\angle VYX$
$= \frac{1}{2}m\angle VWX$
$= \frac{1}{2}(132) = 66°$

16. **Step 1** Show that $\overline{PR}$ and $\overline{QS}$ are $\cong$.
$PR = \sqrt{11^2 + 5^2} = \sqrt{146}$
$QS = \sqrt{5^2 + 11^2} = \sqrt{146}$
Since $PR = QS$, $\overline{PR} \cong \overline{QS}$.
Step 2 Show that $\overline{PR}$ and $\overline{QS}$ are $\perp$.
slope of $\overline{PR} = \frac{-5}{11} = -\frac{5}{11}$; slope of $\overline{QS} = \frac{-11}{-5} = \frac{11}{5}$
Since $\left(-\frac{5}{11}\right)\left(\frac{11}{5}\right) = -1$, $\overline{PR}$ and $\overline{QS}$ are $\perp$.
Step 3 Show that $\overline{PR}$ and $\overline{QS}$ bisect each other.
mdpt. of $\overline{PR} = \left(\frac{-4 + 7}{2}, \frac{0 - 5}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$
mdpt. of $\overline{QS} = \left(\frac{4 - 1}{2}, \frac{3 - 8}{2}\right) = \left(\frac{3}{2}, -\frac{5}{2}\right)$
Since $\overline{PR}$ and $\overline{QS}$ have same mdpt., they bisect each other.
Diags. are $\cong$ $\perp$ bisectors of each other.

17. Possible answer:

Statements	Reasons
1. *RHMB* is a rhombus. $\overline{HB}$ is a diag. of *RHMB*.	1. Given
2. $\overline{MH} \cong \overline{RH}$	2. Def. of rhombus
3. $\overline{HB}$ bisects $\angle RHM$.	3. Rhombus → each diag. bisects opp. $\angle$s
4. $\angle MHX \cong \angle RHX$	4. Def. of $\angle$ bisector
5. $\overline{HX} \cong \overline{HX}$	5. Reflex. Prop. of $\cong$
6. $\triangle MHX \cong \triangle RHX$	6. SAS
7. $\angle HMX \cong \angle HRX$	7. CPCTC

18. $m\angle 1 = 90 - 61 = 29°$ (comp. $\angle$s)
$m\angle 2 = 61°$ (Alt. Int. $\angle$s Thm.)
$m\angle 3 = 90°$ (def. of rect.)
$m\angle 4 = m\angle 1 = 29°$ (Alt. Int. $\angle$s Thm.)
$m\angle 5 = 90°$ (def. of rect.)

19. $m\angle 1 = 90 - 36 = 54°$ (comp. $\angle$s)
$m\angle 2 = 36°$ (diags. $\cong$ → $\triangle$s $\cong$ by SSS, $\angle$s $\cong$ by CPCTC)
$m\angle 3 = 90 - m\angle 2 = 54°$ (comp. $\angle$s)
$m\angle 4 = 180 - (m\angle 2 + 36) = 108°$ ($\triangle$ Sum Thm., Alt. Int. $\angle$s Thm.)
$m\angle 5 = 180 - m\angle 4 = 72°$ (supp. $\angle$s)

20. $m\angle 1 = 90°$ (rect. is a rhombus, Thm. 6-4-4)
$m\angle 2 = m\angle 3$, $m\angle 2 + m\angle 3 = 90°$
→ $m\angle 2 = m\angle 3 = 45°$ (Thm 6-4-5, comp. $\angle$s)
$m\angle 4 = 45°$ (same reasoning as $\angle 2$, $\angle 3$)
$m\angle 5 = m\angle 3 = 45°$ (rect. → ▱, Alt. Int. $\angle$s Thm.)

21. $m\angle 2 = 27°$ (Isosc. $\triangle$ Thm.)
$m\angle 1 = 180 - (27 + 27) = 126°$ ($\triangle$ Sum Thm.)
$m\angle 3 = m\angle 2 = 27°$ (Thm. 6-4-5)
$m\angle 4 = m\angle 1 = 126°$ (rhombus → ▱ → opp. $\angle$s $\cong$)
$m\angle 5 = 27°$ (Thm. 6-4-5)

22. $m\angle 1 = m\angle 2$, $m\angle 1 + m\angle 2 + 70 = 180$
→ $m\angle 1 = m\angle 2 = 55°$ (Isosc. $\triangle$ Thm.)
$m\angle 3 = m\angle 2 = 55°$ (Thm. 6-4-5)
$m\angle 4 = 70°$ (rhombus → ▱ → opp. $\angle$s $\cong$)
$m\angle 5 = m\angle 1 = 55°$ (Thm. 6-4-5)

23. $m\angle 1 = 90 - 26 = 64°$ (Thm. 6-4-4, comp. $\angle$s)
$m\angle 2 = m\angle 1 = 64°$ (Thm. 6-4-5)
$m\angle 3 = 26°$ (rhombus → ▱, Alt. Int. $\angle$s Thm.)
$m\angle 4 = 90°$ (Thm. 6-4-4)
$m\angle 5 = m\angle 2 = 64°$ (rhombus → ▱, Alt. Int. $\angle$s Thm.)

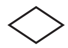
24. always (Thm. 6-4-1) **25.** sometimes

26. sometimes **27.** sometimes

28. always (all 4 sides $\cong$) **29.** always (has 4 sides)

30. always (4 rt. $\angle$s) **31.** sometimes

32. No; possible answer: a rhombus with int $\angle$s that measure 70°, 110°, 70°, and 110° is equliateral, but it is not equiangular. A rect. with side lengths 5, 7, 5, and 7 is equiangular, but it is not equilateral.

33a. 1.  polygon
2.  polygon
3.  polygon
4.  polygon
5.  Not a polygon

b. 1. triangle; reg. 2. quad.; reg.
3. hexagon; reg. 4. quad.; irreg.

c. Shape 2 appears to be a square.
Shape 4 appears to be a rhombus.

d. Assume polygon is reg.
$6m\angle = (6 - 2)180 = 720$
$m\angle = 120°$

34. You cannot use Thm. 6-2-1 to justify the final statement because you do not know that *JKLM* is a ▱. That is what is being proven. Instead, Thm. 6-3-2 states that if both pairs of opp. sides of a quad. are $\cong$, then the quad. is a ▱. So *JKLM* is a ▱ by Thm. 6-3-2.

35a. rect. → ▱ **b.** $\overline{HG}$

c. reflex. Prop. of $\cong$ **d.** def. of rect.

e. $\angle GHE$ **f.** SAS

g. CPCTC

36a. slope of $\overline{AB} = \frac{-2}{2} = -1$; slope of $\overline{CD} = \frac{2}{-2} = -1$

slope of $\overline{BC} = \frac{-5}{-5} = 1$; slope of $\overline{AD} = \frac{-5}{-5} = 1$

b. Rect.; adj. sides are $\perp$.

c. By Thm. 6-4-2, the diags. of a rect. are $\cong$.

37. Possible answer:

Statements	Reasons
1. *VWXY* is a rhombus.	1. Given
2. $\overline{WX} \cong \overline{YX}$	2. Def. of rhombus
3. *VWXY* is a ▱.	3. Rhombus → ▱
4. $\overline{WZ} \cong \overline{YZ}$	4. ▱ → diags. bisect each other
5. $\overline{XZ} \cong \overline{XZ}$	5. Reflex. Prop. of $\cong$
6. $\triangle WZX \cong \triangle YZX$	6. SSS
7. $\angle WZX \cong \angle YZX$	7. CPCTC
8. $\angle WZX$ and $\angle YZX$ are supp.	8. Lin. Pair Thm.
9. $\angle WZX$ and $\angle YZX$ are rt. $\angle$s.	9. $\cong$ $\angle$s supp. → rt. $\angle$s
10. $m\angle WZX = m\angle YZX = 90°$	10. Def. of rt. $\angle$
11. $\overline{VX} \perp \overline{WY}$	11. Def. of $\perp$

38. Possible answer: It is given that *ABCD* is a rect. By def. of a rect., $\angle A$, $\angle B$, $\angle C$, and $\angle D$ are rt. $\angle$s. So $\angle A \cong \angle C$ and $\angle B \cong \angle D$ because all rt. $\angle$s are $\cong$. By Thm. 6-3-3, *ABCD* is a ▱.

39. Possible answer:

Statements	Reasons
1. *ABCD* is a rhombus.	1. Given
2. *ABCD* is a ▱.	2. Rhombus → ▱
3. $\angle B \cong \angle D$, $\angle A \cong \angle C$	3. ▱ → opp. $\angle$s $\cong$
4. $\overline{AB} \cong \overline{BC} \cong \overline{CD} \cong \overline{DA}$	4. Def. of rhombus
5. *E*, *F*, *G* and *H* are the mdpts. of sides.	5. Given
6. $\overline{EB} \cong \overline{BF} \cong \overline{HD} \cong \overline{DG}$, $\overline{EA} \cong \overline{AH} \cong \overline{FC} \cong \overline{CG}$	6. Def. of mdpt.
7. $\triangle BEF \cong \triangle DGH$, $\triangle AEH \cong \triangle CGF$	7. SAS
8. $\overline{EF} \cong \overline{GH}$, $\overline{EH} \cong \overline{GF}$	8. CPCTC
9. EFGH is a ▱.	9. Quad. with opp. sides $\cong$ → ▱

40. $5 = 2w$

$w = 2.5$ cm

$\ell = w\sqrt{3} = 2.5\sqrt{3}$ cm

$P = 2\ell + 2w$

$= 2(2.5) + 2\left(2.5\sqrt{3}\right)$

$= 5 + 5\sqrt{3}$ cm

≈ 13.66 cm

$A = \ell w$

$= \left(2.5\sqrt{3}\right)(2.5)$

$= 6.25\sqrt{3}$ cm

≈ 10.83 cm^2

41. $s = 7\sqrt{2}$ in.

$P = 4s$

$= 28\sqrt{2}$ in.

≈ 39.60 in.

$A = s^2$

$= \left(7\sqrt{2}\right)^2 = 98$ in.^{2}

42. $s = \sqrt{3^2 + 4^2} = 5$ cm

$P = 4s = 20$ cm

$A = 4\left(\frac{1}{2}(3)(4)\right) = 24$ cm^2

43a. By def., a square is a quad. with 4 $\cong$ sides. So it is true that both pairs of opp. sides are $\cong$. Therefore, a square is a ▱ by Thm. 6-3-2.

b. By def., a square is a quad. with 4 rt. $\angle$s and 4 $\cong$ sides. So a square is a rect., because by def., a rect. is a quad. with 4 rt. $\angle$s.

c. By def., a square is a quad. with 4 rt. $\angle$s and 4 $\cong$ sides. So a square is a rhombus, because by def., a rhombus is a quad. with 4 $\cong$ sides.

44. (1) Both pairs of opp. sides are $\parallel$.
Both pairs of opp. sides are $\cong$.
Both pairs of opp. $\angle$s are $\cong$.
All pairs of cons. $\angle$s are supp. Its diags. bisect each other.

(2) Its diags. are $\cong$.

(3) Its diags. are $\perp$. Each diag. bisects a pair of opp. $\angle$s.

TEST PREP

45. D

By Thm. 6-4-5, $\angle LKM \cong \angle JKM$. $\overline{JK} \cong \overline{JM}$, so $\triangle JKM$ is isosc.; by Isosc. $\triangle$ Thm., $\angle JMK \cong \angle JKM$. So

$m\angle J + m\angle JMK + m\angle JKM = 180$

$m\angle J + x + x = 180$

$m\angle J = (180 - 2x)°$

46. The perimeter of $\triangle RST$ is 7.2 cm.

Possible answer: Opp. sides of a rect. are $\cong$, so $RS = QT = 2.4$ and $ST = QR = 1.8$. Diags. of a rect. bisect each other, so $QS = 2QP = 2(1.5) = 3$. The diags. of a rect. are $\cong$, so $TR = QS = 3$. Therefore the perimeter of $\triangle RST$ is $2.4 + 1.8 + 3 = 7.2$.

47. H

Cons. sides need not be $\cong$.

CHALLENGE AND EXTEND

48. Think: By Alt. Int. $\angle$s Thm. and Thm. 6-4-4, given $\angle$s.

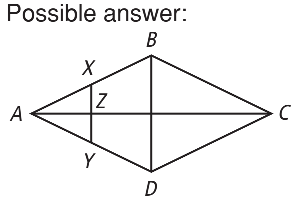
$3x^2 - 15 + x^2 + x = 90$

$4x^2 + x - 105 = 0$

$(4x + 21)(x - 5) = 0$

$x = 5$ or -5.25

49. Possible answer:

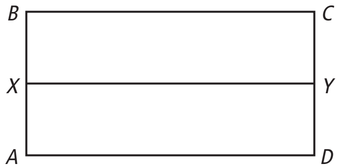


Given: *ABCD* is a rhombus. *X* is mdpt. of $\overline{AB}$. *Y* is mdpt. of $\overline{AD}$.

Prove: $\overline{XY} \parallel \overline{BD}$; $\overline{XY} \perp \overline{AC}$

Proof: Since *X* is the mdpt. of $\overline{AB}$ and *Y* is the mdpt. of $\overline{AD}$, $\overline{XY}$ is a midseg. of $\triangle ABD$ by def. By the $\triangle$ Midsegment Thm., $\overline{XY} \parallel \overline{BD}$. By Thm 6-4-4, since *ABCD* is a rhombus then its diags. are $\perp$. So $\overline{AC} \perp \overline{BD}$. Since also $\overline{BD} \parallel \overline{XY}$, it follows by the $\perp$ Transv. Thm. that $\overline{AC} \perp \overline{XY}$.

50. Possible answer: The midseg. of a rect. is a seg. whose endpoints are mdpts. of opp. sides of the rect.



Given: $ABCD$ is a rect.
X is mdpt. of $\overline{AB}$.
Y is mdpt. of $\overline{CD}$.

Prove: $AXYD \cong BXYC$

Proof: A rect. is a ▱, so $ABCD$ is a ▱. Since opp. sides of a ▱ are ≅, $\overline{AB} \cong \overline{CD}$ and $\overline{AD} \cong \overline{BC}$. Since X is the mdpt. of $\overline{AB}$, $\overline{AX} \cong \overline{XB}$. Since Y is the mdpt. of $\overline{CD}$, $\overline{DY} \cong \overline{YC}$. But because $\overline{AB} \cong \overline{CD}$, you can conclude that $\overline{AX} \cong \overline{XB} \cong \overline{DY} \cong \overline{YC}$. Opp. sides of a ▱ are ∥ by def., so $\overline{AX} \parallel \overline{DY}$. Since also $\overline{AX} \cong \overline{DY}$, $AXYD$ is a ▱ by Thm. 6-3-1. But since $ABCD$ is a rect. ∠A is a rt. ∠. So ▱$AXYD$ contains a rt. ∠ and is therefore a rect. By similar reasoning, you can conclude that $BXYC$ is a rect. Since $\overline{XY} \cong \overline{XY}$ by the Reflex. Prop. of ≅, all corr. sides are ≅. Also, all rt. ∠s are ≅, so all corr. ∠s are ≅. Therefore $AXYD \cong BXYC$ by def. of ≅.

51. 11 1-by-1s, 8 1-by-2s, 5 1-by-3s, 2 1-by-4s, 1 1-by-5, 6 2-by-1s, 4 2-by-2s, 2 2-by-3s, 3 3-by-1s, 2 3-by-2s, 1 3-by-3
45 rects.

SPIRAL REVIEW

52. change $= 20 - 1.1c$
$= 20 - 1.1(2 + 1.8(5)) = 7.9$
change is \$7.90

53. T ($a = (-3)b \rightarrow a = 3(-b)$)

54. F; possible answer: suppose a ⊙ has a diam. of 4 cm and an area of 4π cm^2. If diam. is doubled to 8 cm, area of ⊙ changes to 16π cm^2. New area is 4 times as large as original area.

55. No; none of the conditions for a ▱ are met.

56. Yes; 135° ∠ is supp. to both of its cons. ∠s, so by Thm. 6-3-4, quad. is a ▱.

CONSTRUCTION, PAGE 415

Check students' constructions.

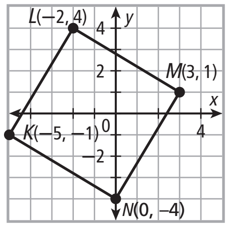
6-5 CONDITIONS FOR SPECIAL PARALLELOGRAMS, PAGES 418–425

CHECK IT OUT!

1. Both pairs of opp. sides of $WXYZ$ are ≅, so $WXYZ$ is a ▱. The contractor can use the carpenter's square to see if one ∠ of $WXYZ$ is a rt. ∠. If so, then by Thm. 6-5-1, the frame is a rect.

2. Not valid; by Thm. 6-5-1, if one ∠ of a ▱ is a rt. ∠, then the ▱ is a rect. To apply this thm., you need to know that $ABCD$ is a ▱.

3a. **Step 1** Graph ▱$KLMN$.



Step 2 Determine if $KLMN$ is a rect.

$KM = \sqrt{8^2 + 2^2}$
$= \sqrt{68} = 2\sqrt{17}$
$LN = \sqrt{2^2 + 8^2}$
$= \sqrt{68} = 2\sqrt{17}$

Since $KM = LN$, diags. are ≅. $KLMN$ is a rect.

Step 3 Determine if $KLMN$ is a rhombus.

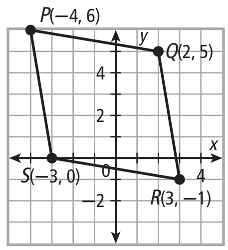
slope of $KM = \frac{2}{8} = \frac{1}{4}$

slope of $LN = \frac{-8}{2} = -4$

Since $\left(\frac{1}{4}\right)(-4) = -1$, $\overline{KM} \perp \overline{LN}$. $KLMN$ is a rhombus.

Since $KLMN$ is a rect. and a rhombus, $KLMN$ is a square.

b. **Step 1** Graph ▱$PQRS$.



Step 2 Determine if $PQRS$ is a rect.

$PR = \sqrt{7^2 + 7^2}$
$= \sqrt{98} = 7\sqrt{2}$
$QS = \sqrt{5^2 + 5^2}$
$= \sqrt{50} = 5\sqrt{2}$

Since $PR \neq QS$, $PQRS$ is not a rect.
Thus $PQRS$ is not a square.

Step 3 Determine if $PQRS$ is a rhombus.

slope of $\overline{PR} = \frac{-7}{7} = -1$

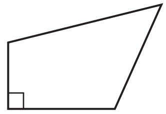
slope of $\overline{QS} = \frac{-5}{-5} = 1$

Since $(-1)(1) = -1$, $\overline{PR} \perp \overline{QS}$. $PQRS$ is a rhombus.

THINK AND DISCUSS

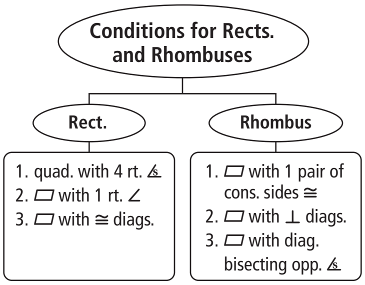
1. rect.; rhombus; square

2. Possible answer:



3. If a quad. is a rect., then it is a ▱. If a ▱ has one rt. ∠, then it is a rect. Thus these defs. are equivalent.

4.



EXERCISES

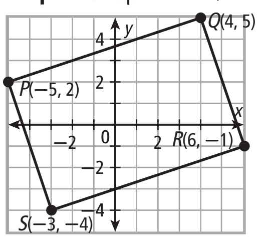
GUIDED PRACTICE

1. Possible answer: If *WXYZ* is both a rhombus and a rect., then it is a square. All 4 sides of *WXYZ* are ≅. So *WXYZ* is a rhombus and therefore a ▱. If the diags. of a ▱ are ≅, then by Thm. 6-5-2, ▱ is a rect. So the club members can measure the diags., and if they are equal, *WXYZ* is both a rhombus and a rect., and therefore it is a square.

2. Not valid; by Thm. 6-5-2, if the diags. of a ▱ are ≅, then the ▱ is a rect. To apply this thm., you need to know that *ABCD* is a ▱.

3. Valid ($\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD} \rightarrow$ ▱, $\overline{AB} \perp \overline{BC}$ $\rightarrow \angle B$ a rt. $\angle \rightarrow$ rect.)

4. Step 1 Graph ▱*PQRS*.



Step 2 Determine if *PQRS* is a rect.

$PR = \sqrt{11^2 + 3^2} = \sqrt{130}$

$QS = \sqrt{7^2 + 9^2} = \sqrt{130}$

Since $PR = QS$, diags. are ≅. *PQRS* is a rect.

Step 3 Determine if *PQRS* is a rhombus.

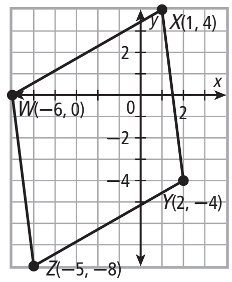
slope of $\overline{PR} = \frac{-3}{11} = -\frac{3}{11}$

slope of $\overline{QS} = \frac{-9}{-7} = \frac{9}{7}$

Since $\left(-\frac{3}{11}\right)\left(\frac{9}{7}\right) \neq -1$, *PQRS* is not a rhombus.

Thus *PQRS* is not a square.

5. Step 1 Graph ▱*WXYZ*.



Step 2 Determine if *WXYZ* is a rect.

$WY = \sqrt{8^2 + 4^2}$
$= \sqrt{80} = 4\sqrt{5}$

$XZ = \sqrt{6^2 + 12^2}$
$= \sqrt{180} = 6\sqrt{5}$

Since $WY \neq XZ$, *WXYZ* is not a rect.

Thus *WXYZ* is not a square.

Step 3 Determine if *WXYZ* is a rhombus.

slope of $\overline{WY} = \frac{-4}{8} = -\frac{1}{2}$

slope of $\overline{XZ} = \frac{-12}{-6} = 2$

Since $\left(-\frac{1}{2}\text{v}\right)(2) = -1$, $\overline{WY} \perp \overline{XZ}$. *WXYZ* is a rhombus.

PRACTICE AND PROBLEM SOLVING

6. Both pairs of opp. sides of PQRS are ≅, so *PQRS* is a ▱. Since $PZ = QZ$ and $RZ = SZ$, it follows that $PR = QS$ by the Seg. Add. Post. Thus $\overline{PR} \cong \overline{QS}$. So the diags. of ▱*PQRS* are ≅. The frame is a rect. by Thm. 6-5-2.

7. valid (by Thms. 6-3-5 and 6-5-4)

8. Not valid; by Thm. 6-5-5, if one diag. of a ▱ bisects a pair of opp. ∠s, then the ▱ is a rhombus. To apply this thm., you need to know that *EFGH* is a ▱.

9. Step 1 Determine if *ABCD* is a rect.

$AC = \sqrt{14^2 + 2^2} = \sqrt{200} = 10\sqrt{2}$

$BD = \sqrt{2^2 + 14^2} = 10\sqrt{2}$

$AC = BD$, so diags. are ≅. *ABCD* is a rect.

Step 2 Determine if *ABCD* is a rhombus.

slope of $\overline{AC} = \frac{-2}{14} = -\frac{1}{7}$

slope of $\overline{BD} = \frac{-14}{-2} = 7$

Since $\left(-\frac{1}{7}\right)(7) = -1$, $AC \perp BD$. *ABCD* is a rhombus.

Since *ABCD* is a rect. and a rhombus, *ABCD* is a square.

10. Step 1 Determine if *JKLM* is a rect.

$JL = \sqrt{12^2 + 4^2} = 4\sqrt{10}$

$KM = \sqrt{2^2 + 6^2} = 2\sqrt{10}$

$JL \neq KM$, so *JKLM* is not a rect., and therefore not a square.

Step 2 Determine if *JKLM* is a rhombus.

slope of $\overline{JL} = \frac{4}{12} = \frac{1}{3}$

slope of $\overline{KM} = \frac{-6}{2} = -3$

Since $\left(\frac{1}{3}\right)(-3) = -1$, $JL \perp KM$. *JKLM* is a rhombus.

11. diags. bisect each other → ▱; one ∠ a rt. ∠ → rect.

12. diags. bisect each other → ▱

13. diags. bisect each other → ▱; diags. ≅ → rect.; diags. ⊥ → rhombus; rect., rhombus → square

14. both pairs of opp. sides ≅ → ▱

15. both pairs of opp. sides ≅ → ▱; one ∠ a rt. ∠ → rect.; all 4 sides ≅ → rhombus; rect., rhombus → square

16. ASA → two △s are ≅ → all 4 sides ≅ → ▱, rhombus

17. B; possible answer: it is given that *ABCD* is a ▱. $\overline{AC}$ and $\overline{BD}$ are its diags. By Thm. 6-5-2, if diags. of a ▱ are ≅, you can conclude that the ▱ is a rect. There is not enough information to conclude that *ABCD* is a square.

18. $\overline{JL}$ and $\overline{KM}$ bisect each other.

19. $\overline{PR} \cong \overline{QS}$

20. $\overline{AB}$ is horiz. and $\overline{AC}$ is vert. So $\overline{BD}$ is vert. and $\overline{CD}$ is horiz. *D* has same *x*-coord. as *B* and same *y*-coord. as *C*, so $D = (-5, 4)$.

21. $AB = BC = \sqrt{50}$; so *D* is opp. *B*.
A is rise of 5 and run of −5 from *B*;
so *D* is rise of 5 and run of −5 from *C*.
Therefore $D = (7 - 5, 1 + 5) = (2, 6)$.

22. $AB = BC = 4\sqrt{2}$; so *D* is opp. *B*.
A is rise of 4, run of −4 from *B*;
so *D* is rise of 4, run of −4 from *C*.
Therefore $D = (0 - 4, -6 + 4) = (-4, -2)$.

23. $AB = BC = 5$; so *D* is opp. *B*.
A is rise of −4, run of 3 from *B*;
so *D* is rise of −4, run of 3 from *C*.
Therefore $D = (-5 + 3, 2 - 4) = (-2, -2)$.

24. Given ∠ must be rt. ∠.
$5x - 3 = 90$
$5x = 93$
$x = 18.6$

25. All 4 sides must be ≅.
$14 - x = 2x + 5$
$9 = 3x$
$x = 3$

26. Diags. must be ⊥.
$13x + 5.5 = 90$
$13x = 84.5$
$x = 6.5$

27. Rhombus; since diags. bisect each other the quad. is a ▱. Since the diags. are ⊥., the quad. is a rhombus.

28a. ▱ → opp. sides ≅ **b.** $\overline{EH} \cong \overline{EH}$

c. SSS **d.** ∠*GHE*

e. CPCTC **f.** ▱ → cons. △s supp.

g. ∠*FEH*, ∠*GHE* are rt.△s **h.** ▱ with 1 rt. ∠ → rect.

29a. slope of $\overline{AB} = -\frac{1}{3}$; slope of $\overline{CD} = \frac{1}{-3} = -\frac{1}{3}$
slope of $\overline{BC} = \frac{-3}{1} = -3$; slope of $\overline{AD} = \frac{-3}{1} = -3$

b. slope of $\overline{AC} = \frac{-4}{4} = -1$; slope of $\overline{BD} = \frac{-2}{-2} = 1$; the slopes are negative reciprocals of each other, so $\overline{AC} \perp \overline{BD}$.

c. *ABCD* is a rhombus, since it is a ▱ and its diags. are ⊥ (Thm. 6-5-4).

30a. $\overline{RT}$ **b.** $\overline{QT}$

c. ⊥ lines **d.** Rt. ∠ ≅ Thm.

e. SAS **f.** $\overline{QR}$

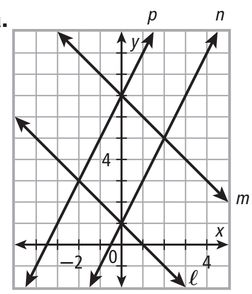
g. rhombus

31. Possible answer:

Statements	Reasons
1. *ABCD* is a ▱. ∠*A* is a rt. ∠.	1. Given
2. m∠*A* = 90°	2. Def. of rt. ∠
3. ∠*A* and ∠*B* are supp.	3. ▱ → cons. △s supp.
4. m∠*A* + m∠*B* = 180°	4. Def. of supp. △s
5. 90° + m∠*B* = 180°	5. Subst.
6. m∠*B* = 90°	6. Subtr. Prop. of =
7. ∠*C* ≅ ∠*A*, ∠*D* ≅ ∠*B*	7. ▱ → opp. △s ≅
8. m∠*C* = m∠*A*, m∠*D* = m∠*B*	8. Def. of ≅ △s
9. m∠*C* = 90°, m∠*D* = 90°	9. Trans. Prop. of =
10. ∠*B*, ∠*C*, and ∠*D* are rt. △s.	10. Def. of rt. ∠
11. *ABCD* is a rect.	11. Def. of rect.

32. Possible answer: It is given that $\overline{JK} \cong \overline{KL}$. Since opp. sides of a ▱ are ≅, $\overline{JK} \cong \overline{LM}$ and $\overline{KL} \cong \overline{MJ}$. By Trans. Prop. of ≅, $\overline{JK} \cong \overline{MJ}$. So $\overline{JK}$ is ≅ to each of the other 3 sides of *JKLM*. Therefore *JKLM* is a rhombus by definition.

33a.



b. Slopes of sides are 2, −1, 2, −1, so quad. is ▱ but not. rect. Side lengths are $2\sqrt{5}$ and $2\sqrt{2}$, so quad. is not rhombus.

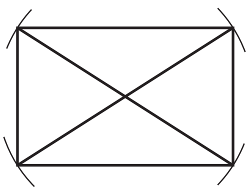
c. Lines change to *n*: $y = x + 1$ and *p*: $y = x + 7$; new slopes are 1, −1, 1, −1; new side lengths are all $3\sqrt{2}$. So the quad. becomes a square.

34. Possible answer:

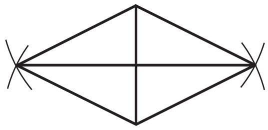
Statements	Reasons
1. *FHJN* and *GLMF* are ▱s. $\overline{FG} \cong \overline{FN}$	1. Given
2. $\overline{FH} \parallel \overline{NJ}$, $\overline{GL} \parallel \overline{FM}$	2. Def. of ▱
3. *FGKN* is a ▱.	3. Def. of ▱
4. *FGKN* is a rhombus.	4. ▱ with 1 pair cons. sides ≅ → rhombus

35. A ▱ is a rect. if and only if its diags. are ≅; a ▱ is a rhombus if and only if its diags. are ⊥; no; possible answer: Thms. 6-4-5 and 6-5-5 are not converses. The conclusion of the conditional in Thm. 6-4-5 refers to both diags. of a ▱. The hypothesis of the conditional in Thm 6-5-5 refers to only one diag. of a ▱.

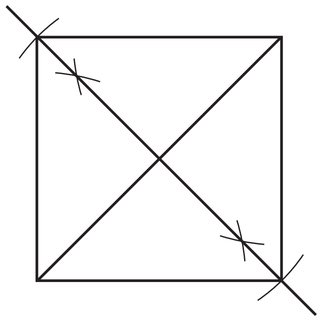
36. Possible answer: Draw 2 pairs of arcs from same center on same compass setting. Draw 2 lines, through center and each pair of arcs. 2 lines bisect each other and are $\cong$, so they are diags. of a rect. Draw sides to complete rect.



37. Possible answer: Draw seg. for 1st diag. Construct $\perp$ bisector. Seg. between 2 pairs of arcs in construction is bisected by 1st seg., so segs. are diags. of a rhombus.



38. Possible answer: Draw seg. for 1st diag. Construct $\perp$ bisector. Set compass to half length of diags., and construct 2nd diag. Diags. are $\perp$ bisectors of each other and are $\cong$, so they are diags. of a square.



TEST PREP

39. A
(condition for Thm. 6-5-2)

40. G
Slope of $\overline{WX}$ = slope of $\overline{YZ} = 1$; slope of $\overline{WZ}$ = slope of $\overline{XY} = -1$; $WX = 4\sqrt{2}$; $XY = 7\sqrt{2}$. So *WXYZ* is a rect. but not a square.

41a. Think: Use Vert. $\angle$s Thm.

$$m\angle KNL = m\angle JNM$$
$$15x = 13x + 12$$
$$2x = 12$$
$$x = 6$$

b. Yes; $m\angle JKN = 6(6) = 36°$ and $m\angle LMN = 5(6) + 6 = 36°$, so by Conv. of Alt. Int. $\angle$s Thm., $\overline{JK} \parallel \overline{LM}$. Since $\overline{KL} \parallel \overline{JM}$, *JKLM* is a ▱ by def.

c. No; by subst., Lin. Pair Thm., and Rt. $\angle \cong$ Thm., all 4 $\angle$s at *N* are rt. $\angle$s. Since *JKLM* is a ▱, *N* is mdpt. of both diags. By SAS, $\triangle KNL \cong \triangle KNJ$, so by CPCTC, $\angle LKN \cong \angle JKN$. Therefore $m\angle JKL = 2m\angle JKN = 2(36) = 72° \neq 90°$.

d. Yes; from part c., diags of ▱*JKLM* are $\perp$. So by Thm. 6-5-4, *JKLM* is a rhombus.

CHALLENGE AND EXTEND

42. Possible answer:

Statements	Reasons
1. $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$, $\overline{BE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$	1. Given
2. $m\angle ABC = 90°$, $m\angle DEF = 90°$, $m\angle BEF = 90°$	2. Def. of $\perp$
3. $\angle ABC$, $\angle DEF$, and $\angle BEF$ are rt. $\angle$s.	3. Def. of rt. $\angle$
4. $\triangle ABC$ and $\triangle DEF$ are rt. $\triangle$s.	4. Def. of rt. $\triangle$
5. $\triangle ABC \cong \triangle DEF$	5. HL
6. $\overline{BC} \cong \overline{EF}$	6. CPCTC
7. *EBCF* is a ▱.	7. Thm. 6-3-1
8. *EBCF* is a rect.	8. Thm. 6-5-1

43a. Possible answer: If a quad. is a rect., then it has four rt. $\angle$s. If a quad. is a rhombus, then it has four $\cong$ sides. By def., a quad. with four rt. $\angle$s and four $\cong$ sides is a square. Therefore statement is true.

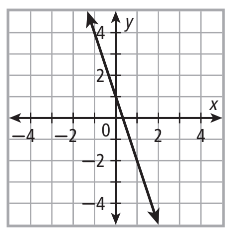
b. No; possible answer: if a quad. is a rect., then it is a ▱. By Thm. 6-5-3, if 1 pair of cons. sides of a ▱ are $\cong$, then the ▱ is a rhombus. So if 1 pair of cons. sides of a rect. are $\cong$, it is a rhombus. If a quad. is a rect. and a rhombus, then it is a square.

c. No; possible answer: if a quad. is a rhombus, then it is a ▱. By Thm. 6-5-1, if 1 $\angle$ of a ▱ is a rt. then the ▱ is a rect. So if 1 $\angle$ of a rhombus is a rt. $\angle$, it is a rect. If a quad. is a rhombus and a rect., then it is a square.

44. Diags. of the ▱ are $\perp$, so it is a rhombus.

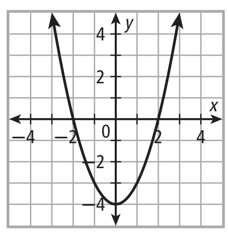
SPIRAL REVIEW

45.

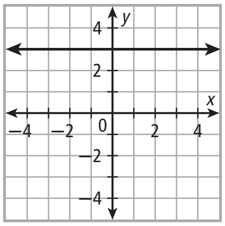


linear

46.



nonlinear

47. 

linear

48. $c = \sqrt{8^2 + 10^2}$
$= \sqrt{164} = 2\sqrt{41}$
$P = 20 + 2\sqrt{41} + 2\sqrt{41}$
$= 20 + 4\sqrt{41} \approx 45.6$

49. $c = \sqrt{(12 - 6)^2 + (9 - 4)^2} = \sqrt{61}$
$P = 6 + 9 + 12 + 4 + \sqrt{61}$
$= 31 + \sqrt{61} \approx 38.8$

50. Cons. $\angle$s are supp. So
$8x + 10 + 10x + 44 = 180$
$18x = 126$
$x = 7$

51. Opp. sides are $\cong$. So
$2y + 7 = 7z + 1$
$5y = 9z + 2$
Think: Eliminate z.
$18y + 63 = 63z + 9$
$35y = 63z + 14$
$17y - 63 = 5$
$17y = 68$
$y = 4$

52. Subst. for y to find z.
$5(4) = 9z + 2$
$18 = 9z$
$z = 2$

6-6 PROPERTIES OF KITES AND TRAPEZOIDS, PAGES 427–435

CHECK IT OUT!

1. Perimeter of kite doubles:
$P = 20\sqrt{17} + 8\sqrt{185} \approx 191.3$ in.
Daryl needs approx. 191.3 in. of binding.
$\frac{191.3}{72} \approx 2.7$ packages of binding
In order to have enough, Daryl must buy 3 packages of binding.

2a. Think: $\triangle PQR$ is isosc., so $\angle QPT \cong \angle QRT$.
$m\angle PQR + m\angle QPT + m\angle QRT = 180$
$78 + 2m\angle QRT = 180$
$2m\angle QRT = 102$
$m\angle QRT = 51°$

b. Think: Kite → 1 pair opp. $\angle$s $\cong$.
$m\angle QPS = m\angle QRS$
$= m\angle QRT + m\angle TRS$
$= 51 + 59 = 110°$

c. Think: Use Quad. $\angle$ Sum Thm.
$m\angle PSR + m\angle QRS + m\angle PQR + m\angle QPS = 360$
$m\angle PSR + 110 + 78 + 110 = 360$
$m\angle PSR = 62°$

3a. Think: Use Same-Side Int. $\angle$s Thm., isosc. trap. → base $\angle$s $\cong$.
$m\angle G + m\angle H = 180$
$m\angle G + 49 = 180$
$m\angle G = 131°$
$\angle F \cong \angle G$
$m\angle F = m\angle G = 131°$

b. Think: Isosc. trap. → diags. $\cong$.
$\overline{KM} \cong \overline{JL}$
$KM = JL$
$= JN + NL$
$= 10.6 + 14.8 = 25.4$

4. Think: 1 pair base $\angle$s $\cong$ → trap. isosc.
$\angle Q \cong \angle S$
$m\angle Q = m\angle S$
$2x^2 + 19 = 4x^2 - 13$
$32 = 2x^2$
$16 = x^2$
$x = 4$ or -4

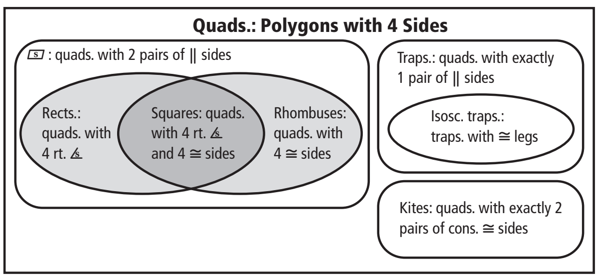
5. Think: Use Trap. Midseg. Thm.
$XY = \frac{1}{2}(EH + FG)$
$16.5 = \frac{1}{2}(EH + 25)$
$33 = EH + 25$
$8 = EH$

THINK AND DISCUSS

1. No; possible answer: if the legs are ||, then the trap. has two pairs of || sides. But by def., a trap. has exactly one pair of || sides, so the figure would be a ▱.

2. Possible answer: Similarities: The endpts. of both are the mdpts. of two sides. Both are || to another side. Differences: A △ has three midsegs., while a trap. has just one. To find the length of a midseg. of a △., you find half the measure of just one side; to find the length of a midseg. of a trap., you must average the lengths of two sides.

3.



EXERCISES

GUIDED PRACTICE

1. bases: $\overline{RS}$ and $\overline{PV}$; legs: $\overline{PR}$ and $\overline{VS}$; midseg.: $\overline{QT}$

2. Possible answer: In a ▱, two pairs of opp. sides are $\cong$. In a kite, exactly two distinct pairs of cons. sides are $\cong$.

3. 1 Understand the Problem

Answer has 2 parts:

- Total amount of lead needed
- Number of suncatchers that can be sealed

2 Make a Plan

Diags. of a kite are $\perp$, so 4 $\triangle$s are rt. $\triangle$s. Use Pyth. Thm. and props. of kites to find unknown side lengths. Add these lengths to find perimeter of kite.

3 Solve

$JK = \sqrt{JH^2 + KH^2}$

$= \sqrt{2.75^2 + 2.75^2} = \sqrt{15.125}$

$KL = JK = \sqrt{15.125}$

$JM = \sqrt{JH^2 + MH^2}$

$= \sqrt{2.75^2 + 5.5^2} = \sqrt{37.8125}$

$LM = JM = \sqrt{37.8125}$

perimeter of $JKLM = 2\sqrt{15.125} + 2\sqrt{37.8125}$

≈ 20.1 in.

20.1 in. of lead is needed to seal edges.

One 3-ft length of lead contains 36 in.

$\frac{2(36)}{20.1} \approx 3.6$

3 sun catchers can be sealed.

4 Look Back

To estimate perimeter, change side lengths into decimals and round. $\sqrt{15.125} \approx 4$, and $\sqrt{37.8125} \approx 6$. Perimeter of sun catcher is approx. $2(4) + 2(6) = 20$. So 20.1 in. is a reasonable answer.

4. Think: Kite → diags. $\perp$ → $\angle VZY$ and $\angle VYZ$ are comp.

$m\angle VZY + m\angle VYZ = 90$

$m\angle VZY + 49 = 90$

$m\angle VZY = 41°$

5. Think: $\triangle XYZ$ is isosc., so $\angle VXY \cong \angle VZY$.

$m\angle VXW + m\angle VXY = m\angle WXY$

$m\angle VXW + 41 = 104$

$\angle VXW = 63°$

6. Think: Use Quad. $\angle$ Sum Thm., kite → 1 pair opp. $\angle$s $\cong$.

$m\angle XWZ + m\angle WXY + m\angle XYZ + m\angle YZW = 360$

$m\angle XWZ + 104 + 2m\angle VYZ + 104 = 360$

$m\angle XWZ + 104 + 2(49) + 104 = 360$

$m\angle XWZ = 54°$

7. Think: Use Same-Side Int. $\angle$s Thm., isosc. trap. → base $\angle$s $\cong$.

$m\angle C + m\angle D = 180$

$74 + m\angle D = 180$

$m\angle D = 106°$

$\angle A \cong \angle D$

$m\angle A = m\angle D$

$= 106°$

8. Think: Isosc. trap. → diags. $\cong$.

$\overline{RT} \cong \overline{SV}$

$RT = SV$

$RW + TW = SV$

$17.7 + TW = 23.3$

$TW = 5.6$

9. Think: 1 pair base $\angle$s $\cong$ → trap. isosc.

$\angle E \cong \angle H$

$m\angle E = m\angle H$

$12z^2 = 7z^2 + 20$

$5z^2 = 20$

$z^2 = 4$

$z = 2$ or -2

10. Think: Diags. $\cong$ → trap. isosc.

$\overline{MQ} \cong \overline{LP}$

$MQ = LP$

$7y - 6 = 4y + 11$

$3y = 17$

$y = \frac{17}{3} = 5\frac{2}{3}$

11. Think: Use Trap. Midseg. Thm.

$XY = \frac{1}{2}(PS + QR)$

$22 = \frac{1}{2}(30 + QR)$

$44 = 30 + QR$

$14 = QR$

12. Think: Use Trap. Midseg. Thm.

$AZ = \frac{1}{2}(DF + JK)$

$= \frac{1}{2}(11.9 + 7.1)$

$= \frac{1}{2}(19) = 9.5$

PRACTICE AND PROBLEM SOLVING

13. 1 Understand the Problem

Answer has 2 parts:

- Amount of iron needed to outline 1 kite
- Amount of iron needed for 1 complete section

2 Make a Plan

Diags. of a kite are $\perp$., so 4 $\triangle$s are rt. $\triangle$s. Use Pyth. Thm. and props. of kites to find unknown side lengths. Add these lengths to find perimeter of kite.

3 Solve

shorter side length $= \sqrt{7^2 + 7^2} = 7\sqrt{2}$

longer side length $= \sqrt{7^2 + 17^2} = 13\sqrt{2}$

perimeter $= 2\left(7\sqrt{2}\right) + 2\left(13\sqrt{2}\right)$

$= 40\sqrt{2} \approx 56.6$ in.

56.6 in. of iron is needed for 1 kite.

For 1 complete section, need iron for 4 kites and 1 square of side length $2(7 + 17) = 48$ in.

Amount of iron $= 4\,(40\sqrt{2}) + 4(48) \approx 418.3$ in.

4 Look Back

To estimate perimeter, change side lengths into decimals and round. $7\sqrt{2} \approx 10$, and $13\sqrt{2} \approx 18$. Perimeter of one kite is approx. $2(10) + 2(18) = 56$. So 56.6 in. is a reasonable answer.

14. Think: Kite → diags. $\perp$.

$m\angle XDA + m\angle DAX = 90$

$m\angle XDA + 32 = 90$

$m\angle XDA = 58°$

15. Think: Kite → 1 pair opp. $\angle$s $\cong$.

$m\angle ABC = m\angle ADC$

$= m\angle XDA + m\angle XDC$

$= 58 + 64 = 122°$

16. Think: Use Quad. $\angle$ Sum Thm.

$m\angle BCD + m\angle ADC + m\angle BAD + m\angle ABC = 360$

$m\angle BCD + 122 + 2m\angle DAX + 122 = 360$

$m\angle BCD + 122 + 2(32) + 122 = 360$

$m\angle BCD = 52°$

17. Think: Use Same-Side Int. $\angle$s Thm., Thm. 6-6-3.

$m\angle L + m\angle M = 180$

$m\angle L + 118 = 180$

$m\angle L = 62°$

$\angle Q \cong \angle L$

$m\angle Q = m\angle L = 62°$

18. Think: Use Thm. 6-6-5.

$\overline{RJ} \cong \overline{SK}$

$RJ = SK$

$= SZ + KZ$

$= 62.6 + 34 = 96.6$

19. Think: 1 pair base $\angle$s $\cong$ → trap. isosc.

$\angle X \cong \angle W$

$m\angle X = m\angle W$

$a^2 + 15 = 2a^2 - 65$

$80 = a^2$

$a = \pm\sqrt{80}$

$= \pm 4\sqrt{5}$

20. Think: Diags. ≅ → trap. isosc.

$\overline{GJ} \cong \overline{FH}$
$GJ = FH$
$4x - 1 = 9x - 15$
$14 = 5x$
$x = 2.8$

21. Think: Use Trap. Midseg. Thm.

$PQ = \frac{1}{2}(AV + BT)$
$= \frac{1}{2}(4.2 + 3) = 3.6$

22. Think: Use Trap. Midseg. Thm.

$MN = \frac{1}{2}(JT + KR)$
$52.5 = \frac{1}{2}(32.5 + KR)$
$105 = 32.5 + KR$
$72.5 = KR$

23. Sometimes (opp. ∠s supp. only when trap. is isosc.)

24. Sometimes (kites with non-≅ opp. ∠s of 30° and 150° or 20° and 150°)

25. Never (this →s both opp. pairs of ∠s are ≅, so quad. would be a ▱, not a kite)

26. △ formed by dashed line is 30°-60°-90°. So shorter leg measures a, where

$6 = 2a$
$3 = a$
4th edge measures $20 - 2a = 20 - 2(3) = 14$ ft
$P = 20 + 6 + 14 + 6 = 46$ ft
$C = 1.29P$
$\approx 1.3(46) \approx \$60$
about $60.

27. Think: Use Same-Side Int. ∠s Thm.

$m\angle 1 + 98 = 180$ $\quad$ $m\angle 2 + 52 = 180$
$m\angle 1 = 82°$ $\quad$ $m\angle 2 = 128°$

28. Think: Use Thm 6-6-2, Quad. ∠ Sum Thm.

$m\angle 1 = 116°$
$m\angle 1 + m\angle 2 + 116 + 82 = 360$
$116 + m\angle 2 + 116 + 82 = 360$
$m\angle 2 + 314 = 360$
$m\angle 2 = 46°$

29. Think: All 4 △s are rt. △s; left pair of △s is ≅.

$m\angle 1 + 39 = 90$ $\quad$ $m\angle 2 + 74 = 90$
$m\angle 1 = 51°$ $\quad$ $m\angle 2 = 16°$

30. Think: Top left and bottom right ∠s of isosc. trap. are supp.; also use Alt. Int. ∠s Thm.

$(m\angle 2 + 34) + (72 + 34) = 180$
$m\angle 2 + 140 = 180$
$m\angle 2 = 40°$

Think: By ≅ △s, lower part of top right ∠ measures 40°. By Ext. ∠ Thm.

$m\angle 1 = 72 + 40 = 112°$

31. Think: Use Thm. 6-6-2, Quad. ∠ Sum Thm.

$3x = 48$
$x = 16$
$m\angle 1 + 3(16) + 9(16) + 48 = 360$
$m\angle 1 + 240 = 360$
$m\angle 1 = 120°$

32. Think: Use Same-Side Int. ∠s Thm.

$40z + 5 + 10z = 180$
$50z = 175$
$z = 3.5$
$18(3.5) + m\angle 1 = 180$
$63 + m\angle 1 = 180$
$m\angle 1 = 117°$

33a. $EF = \sqrt{(3 + 1)^2 + (4 - 3)^2} = \sqrt{17}$

$FG = \sqrt{(2 - 3)^2 + (0 - 4)^2} = \sqrt{17}$
$GH = \sqrt{(-3 - 2)^2 + (-2 - 0)^2} = \sqrt{29}$
$EH = \sqrt{(-3 + 1)^2 + (-2 - 3)^2} = \sqrt{29}$
$\overline{EF} \cong \overline{FG} \ncong \overline{GH} \cong \overline{EH}$ → $EFGH$ is a kite.

b. $m\angle E = m\angle G = 126°$

34. $12t = \frac{1}{2}(16t + 10)$

$24t = 16t + 10$
$8t = 10$
$t = 1.25$
length of midseg. $= 12(1.25) = 15$

35. $n + 6 = \frac{1}{2}(n + 3 + 3n - 5)$

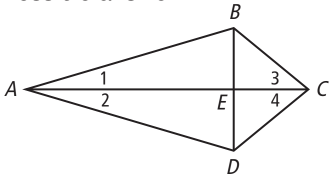
$2n + 12 = 4n - 2$
$14 = 2n$
$7 = n$
length of midseg. $= (7) + 6 = 13$

36. $4c = \frac{1}{2}(c^2 + 6 + c^2 + 2)$

$4c = c^2 + 4$
$c^2 - 4c + 4 = 0$
$(c - 2)^2 = 0$
$c = 2$
length of midseg. $= 4(2) = 8$

37. $m\angle PAQ + m\angle AQB +$
$m\angle PBQ + m\angle APB = 360$
$m\angle PAQ + 72 +$
$m\angle PAQ + 72 = 360$
$m\angle PAQ + 72 = 180$
$m\angle PAQ = 108°$
$m\angle OAQ + m\angle AQB +$
$m\angle OBQ + m\angle AOB = 360$
$m\angle OAQ + 72 +$
$m\angle OAQ + 28 = 360$
$2m\angle OAQ + 100 = 360$
$2m\angle OAQ = 260$
$m\angle OAQ = 130°$
$m\angle OBQ = m\angle OBP + m\angle PBQ$
$m\angle OAQ = m\angle OBP + m\angle PAQ$
$130 = m\angle OBP + 108$
$22° = m\angle OBP$

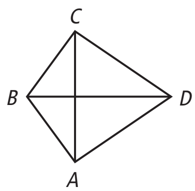
38. Possible answer:



Given: *ABCD* is a kite with $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$.
Prove: $\overline{AC}$ bisects $\angle DAB$ and $\angle DCB$. $\overline{AB}$ bisects $\overline{BD}$.

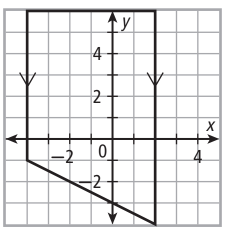
Statements	Reasons
1. $\overline{AB} \cong \overline{AD}$ and $\overline{CB} \cong \overline{CD}$.	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflex. Prop. of $\cong$
3. $\triangle ABC \cong \triangle ADC$	3. SSS
4. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	4. CPCTC
5. $\overline{AC}$ bisects $\angle DAB$ and $\angle DCB$.	5. Def. of $\angle$ bisector
6. $\overline{AE} \cong \overline{AE}$	6. Reflex. Prop. of $\cong$
7. $\triangle ABE \cong \triangle ADE$	7. SAS
8. $\overline{BE} \cong \overline{DE}$	8. CPCTC
9. $\overline{AC}$ bisects $\overline{BD}$.	9. Def. of seg. bisector

39. Possible answer:



Given: *ABCD* is a kite with $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$.
Prove: $\overline{BD} \perp \overline{AC}$
It is given that $\overline{AB} \cong \overline{CB}$ and $\overline{AD} \cong \overline{CD}$. This means that *B* and *D* are equidistant from *A* and from *C*. By the Conv. of the $\perp$ Bisector Thm., if a pt. is equidist. from the endpts. of a seg., then it is on the $\perp$ bisector of the seg. Through any two pts. there is exactly one line, so the line containing *B* and *D* must be the $\perp$ bisector of $\overline{AC}$. Therefore $\overline{BD} \perp \overline{AC}$.

40.

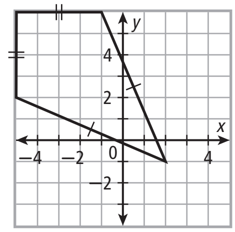


$\overline{AB}$ and $\overline{CD}$ are vert.; $\overline{BC}$ is horiz.;

slope of $\overline{DA} = \frac{3}{-6} = -\frac{1}{2}$

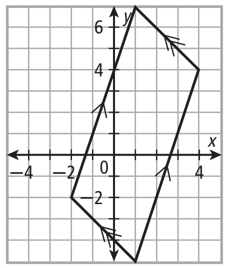
Exactly 2 sides are $\parallel$, so quad. is a trap.

41.



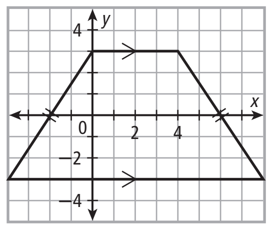
$AB = BC = 4$; $CD = \sqrt{3^2 + 7^2} = \sqrt{58}$;
$DA = \sqrt{7^2 + 3^2} = \sqrt{58}$
Exactly 2 pairs of cons. sides are $\cong$, so quad is a kite.

42.



Diags. have equations $y = x$ and $x = 1$; they intersect at (1, 1), and bisect each other, but are not $\perp$. Therefore quad is a ▱.

43.



$\overline{AD}$ and $\overline{BC}$ are horiz.; $AB = \sqrt{4^2 + 6^2} = 2\sqrt{13}$,
$CD = \sqrt{4^2 + 6^2} = 2\sqrt{13}$
$\overline{AD} \parallel \overline{BC}$ and $\overline{AB} \cong \overline{CD}$; so quad. is an isosc. trap.

44. Extend $\overline{BA}$ and $\overline{CD}$ to meet in center of window, *X*.
$\angle AXC = 360 \div 8 = 45°$; by Isosc. $\triangle$ Thm.,
$\angle XBC \cong \angle XCB$, so
$m\angle XBC = \frac{1}{2}(180 - m\angle AXC) = 67.5°$
Since $\overline{BC} \parallel \overline{AD}$, by Corr. $\angle$s Post.,
$m\angle B = m\angle C = 67.5°$
by Lin. Pair Thm.,
$m\angle A = m\angle D = 180 - 67.5° = 112.5°$

45. Possible answer: Common props.: exactly one pair of $\parallel$ sides; two pairs of cons. $\angle$s supp.; length of midseg. is the average of the lengths of the bases; special props. of isosc. trap.: $\cong$ legs; two pairs of $\cong$ base $\angle$s; $\cong$ diags.

46a. $\left(\dfrac{0+2a}{2}, \dfrac{0+2b}{2}\right) = (a, b)$

b. $\left(\dfrac{c+c+2d}{2}, \dfrac{2b+0}{2}\right) = (c+d, b)$

c. slope of $\overline{QR} = \dfrac{2b-2b}{c-2a} = 0$

slope of $\overline{PS} = \dfrac{0-0}{c+2d-0} = 0$

slope of $\overline{MN} = \dfrac{b-b}{c+d-a} = 0$

All 3 segs. are ∥.

d. $QR = c - 2a$; $PS = c + 2d - 0 = c + 2d$;
$MN = c + d - a$; $c + d - a = \frac{1}{2}(c - 2a + c + 2d)$,
so $MN = \frac{1}{2}(PS + QR)$

TEST PREP

47. B

48. H

$\frac{1}{2}(6 + 26) = 16$

49. 18

By △ Midseg. Thm., $DE = \frac{1}{2}(24) = 12$ in.

midseg. length $= \frac{1}{2}(12 + 24) = 18$ in.

CHALLENGE AND EXTEND

50. Possible answer:

Statements	Reasons
1. *WXYZ* is a trap. with $\overline{XZ} \cong \overline{YW}$.	1. Given
2. Draw $\overline{XU}$ through *X* so that $\overline{XU} \perp \overline{WZ}$. Draw $\overline{YV}$ through *Y* so that $\overline{YV} \perp \overline{WZ}$.	2. There is exactly 1 line through a pt. not on a line that is ⊥ to that line.
3. $m\angle XUZ = 90°$, $m\angle YVW = 90°$	3. Def. of ⊥ lines
4. ∠*XUZ* and ∠*YVW* are rt. ∠s.	4. Def. of rt. ∠
5. △*XUZ* and △*YVW* are rt. △s.	5. Def. of rt. △
6. $\overline{XU} \parallel \overline{YV}$	6. 2 lines ⊥ to 3rd line → 2 lines ∥
7. $\overline{XY} \parallel \overline{WZ}$	7. Def. of trap.
8. *XYVU* is a ▱.	8. Def. of ▱
9. $\overline{XU} \cong \overline{YV}$	9. ▱ → opp. sides ≅
10. △*XUZ* ≅ △*YVW*	10. HL
11. ∠*XZW* ≅ ∠*YWZ*	11. CPCTC
12. $\overline{WZ} \cong \overline{WZ}$	12. Reflex. Prop. of ≅
13. △*XZW* ≅ △*YWZ*	13. SAS
14. $\overline{XW} \cong \overline{YZ}$	14. CPCTC
15. *WXYZ* is an isosc. trap.	15. Def. of isosc. trap.

51. $BC + AD = 2(8.62) = 17.24$ and $CD = AB$

$P = AB + BC + CD + AD$

$27.4 = 2AB + 17.24$

$10.16 = 2AB$

$5.08 = AB$

$AB = CD = 5.08$ in.

$BC = 2AB$

$= 2(5.08) = 10.16$ in.

$AD = 17.24 - BC$

$= 17.24 - 10.16 = 7.08$ in.

SPIRAL REVIEW

52. $\dfrac{x}{20\%} = \dfrac{25}{10}$

$x = \dfrac{25}{10}(20\%)$

$= 50\% = \dfrac{1}{2}$

53. Think: Height of △ is min. dist. from apex to base.

$2x < x + 6$

$x < 6$

54. $3x - 10 < 30 + x$

$2x < 40$

$x < 20$

55. slope of $\overline{AB} = \frac{2}{2} = 1$; slope of $\overline{CD} = \frac{-2}{-2} = 1$

slope of $\overline{BC} = \frac{-2}{2} = -1$; slope of $\overline{AD} = \frac{-2}{2} = -1$

So ▱ is a rect.

$AB = BC = CD = AD = \sqrt{2^2 + 2^2} = 2\sqrt{2}$

So ▱ is a rhombus.

Rect., rhombus → square; so ▱ is a square.

56. slope of $\overline{AB} = \frac{4}{3}$; slope of $\overline{CD} = \frac{-4}{-3} = \frac{4}{3}$

$\overline{BC}$ and $\overline{AD}$ are vert.

Cons. edges are not ⊥, so ▱ is not a rect. and therefore not a square.

$\overline{AB} = \sqrt{3^2 + 4^2} = 5$; $\overline{CD} = \sqrt{3^2 + 4^2} = 5$

$\overline{BC} = |0 - 5| = 5$; $\overline{AD} = |-4 - 1| = 5$

Since all four sides are ≅., ▱ is a rhombus.

CONSTRUCTION

1. Choose *B* and *D* both above or both below $\overline{AC}$.

READY TO GO ON? PAGE 437

1. Think: $\overline{QS}$ and $\overline{RT}$ bisect each other at *T*.

$SP = \frac{1}{2}QS$

$= \frac{1}{2}(80.5) = 40.25$

2. $\overline{QT} \cong \overline{RS}$

$QT = RS = 36$

3. Think: Diags. are ≅.

$\overline{TR} \cong \overline{QS}$

$TR = QS = 80.5$

4. $TP = \frac{1}{2}TR$

$= \frac{1}{2}(80.5) = 40.25$

5. $\overline{GH} \cong \overline{GK} \cong \overline{HJ}$

$GH = GK$

$6a - 7 = 3a + 9$

$3a = 16$

$HJ = GK$

$= (16) + 9 = 25$

6. Think: Rhombus → diags. are ⊥

$m\angle JLH = 4b - 6$

$90 = 4b - 6$

$96 = 4b$

$24 = b$

Think: All 4 △s are ≅.

$m\angle HJG + m\angle JHK = 90$

$m\angle HJG + m\angle JKH = 90$

$m\angle HJG + 2(24) + 11 = 90$

$m\angle HJG = 31°$

$m\angle GHJ = m\angle GHK + m\angle JHK$

$= 2m\angle JHK$

$= 2(2(24) + 11) = 118°$

7.

Statements	Reasons
1. $QSTV$ is a rhombus. $\overline{PT} \cong \overline{RT}$	1. Given
2. $\overrightarrow{TQ}$ bisects $\angle PTR$.	2. rhombus → each diag. bisects opp. ∠s
3. $\angle QTP \cong \angle QTR$	3. Def. of ∠ bisector
4. $\overline{QT} \cong \overline{QT}$	4. Reflex. Prop. of ≅
5. $\triangle PQT \cong \triangle RQT$	5. SAS
6. $\overline{PQ} \cong \overline{RQ}$	6. CPCTC

8. not valid; By Thm. 6-5-4, if the diags. of a ▱ are ⊥, then the ▱ is a rhombus. But you need to know that $ABCD$ is a ▱.

9. valid (By Thm. 6-3-1, $ABCD$ is a ▱. By Thm. 6-5-2, $ABCD$ is a rect.)

10. $WY = \sqrt{9^2 + 3^2} = 3\sqrt{10}$; $XZ = \sqrt{3^2 + 9^2} = 3\sqrt{10}$
The diags. are ≅, so $WXYZ$ is a rect.
slope of $\overline{WY} = \frac{-3}{9} = -\frac{1}{3}$; slope of $\overline{XZ} = \frac{-9}{3} = -3$
The diags. are not ⊥, so $WXYZ$ is not a rhombus. Therefore $WXYZ$ is not a square.

11. $MP = \sqrt{7^2 + 3^2} = \sqrt{58}$; $NQ = \sqrt{3^2 + 7^2} = \sqrt{58}$
The diags. are ≅, so $MNPQ$ is a rect.
slope of $\overline{MP} = -\frac{3}{7}$; slope of $\overline{NQ} = \frac{-7}{-3} = \frac{7}{3}$
The diags. are ⊥, so $MNPQ$ is a rhombus.
Rect., rhombus → $MNPQ$ is a square.

12. Possible answer: Since $\overline{VX}$ is a midseg. of $\triangle TWY$, $\overline{VX} \parallel \overline{TZ}$ by △ Midseg. Thm. Similarly, $\overline{XZ} \parallel \overline{TV}$. So $TVXZ$ is a ▱ by def. V is mdpt. of $\overline{TW}$; thus $TV = \frac{1}{2}TW$ by def. of mdpt. Similarly, $TZ = \frac{1}{2}TY$. It is given that $\overline{TW} \cong \overline{TY}$, so $TW = TY$ by def. of ≅ segs. By subst., $TZ = \frac{1}{2}TW$, and thus $TZ = TV$. By def. of ≅ segs., $\overline{TZ} \cong \overline{TV}$. Since $TVXZ$ is a ▱ with 1 pair of cons. sides ≅, $TVXZ$ is a rhombus.

13. Think: $\overrightarrow{EG}$ bisects $\angle FEH$.
$m\angle FEJ = \frac{1}{2}m\angle FEH$
$= \frac{1}{2}(62) = 31°$

14. Think: $\angle HEJ$ is ≅ to $\angle FEJ$ and comp. to $\angle EHJ$.
$m\angle HEJ + m\angle EHJ = 90$
$m\angle FEJ + m\angle EHJ = 90$
$31 + m\angle EHJ = 90$
$m\angle EHJ = 59°$

15. Think: $\angle HFG$ is comp. to $\angle FGJ$ and ≅ to $\angle FHG$.
$m\angle HFG + m\angle FGJ = 90$
$m\angle FHG + m\angle FGJ = 90$
$68 + m\angle FGJ = 90$
$m\angle FGJ = 22°$

16. $m\angle EHG = m\angle EHJ + m\angle FHG$
$= 59 + 68 = 127°$

17. Think: Use Same-Side Int. ∠s Thm., isosc. trap. → base ∠s ≅.
$m\angle U + m\angle T = 180$
$m\angle U + 77 = 180$
$m\angle U = 103°$
$\angle R \cong \angle U$
$m\angle R = m\angle U = 103°$

18. Think: Isosc. trap → diags. ≅
$WY \cong VX$
$WZ + YZ = VX$
$WZ + 34.2 = 53.4$
$WZ = 19.2$

19. length of midseg. $= \frac{1}{2}(43 + 23) = 33$ in.

STUDY GUIDE: REVIEW, PAGES 438–441

1. vertex of a polygon
2. convex
3. rhombus
4. base of a trapezoid

LESSON 6-1

5. not a polygon
6. polygon; triangle
7. polygon; dodecagon
8. irregular; concave
9. irregular; convex
10. regular; convex

11. $(n - 2)180$
$(12 - 2)180$
$1800°$

12. $(n)m\angle = (n - 2)180$
$20m\angle = (18)180$
$20m\angle = 3240$
$m\angle = 162°$

13. $(n)m(\text{ext. } \angle) = 360$
$4m(\text{ext. } \angle) = 360$
$m(\text{ext. } \angle) = 90°$

14. $m\angle A + \ldots + m\angle F = (6 - 2)180$
$8s + 7s + 5s + 8s + 7s + 5s = 720$
$40s = 720$
$s = 18$
$m\angle A = m\angle D = 8(18) = 144°$;
$m\angle B = m\angle E = 7(18) = 126°$;
$m\angle C = m\angle F = 5(18) = 90°$

LESSON 6-2

15. Think: Diags. bisect each other.
$BE = \frac{1}{2}BD$
$= \frac{1}{2}(75) = 37.5$

16. $\overline{AD} \cong \overline{BC}$
$AD = BC = 62.4$

17. $ED = BE = 37.5$

18. $\angle CDA \cong \angle ABC$
$m\angle CDA = m\angle ABC$
$= 79°$

19. Think: Cons. ∠s supp.
$m\angle ABC + m\angle BCD = 180$
$79 + m\angle BCD = 180$
$m\angle BCD = 101°$

20. $\angle BCD \cong \angle DAB$
$m\angle BCD = m\angle DAB = 101°$

21. $WX = YZ$
$b + 6 = 5b - 8$
$14 = 4b$
$3.5 = b$
$WX = 3.5 + 6 = 9.5$

22. $YZ = 5(3.5) - 8 = 9.5$

23. $m\angle W + m\angle X = 180$
$6a + 14a = 180$
$20a = 180$
$a = 9$
$m\angle W = 6(9) = 54°$

24. $m\angle X = 14(9) = 126°$

25. $\angle Y \cong \angle W$
$m\angle Y = m\angle W = 54°$

26. $\angle Z \cong \angle X$
$m\angle Z = m\angle X = 126°$

27. Slope from R to S is rise of 2 and run of 10; rise of 2 from V to T is $-7 + 2 = -5$; run of 10 from V to T is $-4 + 10 = 6$; $T = (6, -5)$

28.

Statements	Reasons
1. $GHLM$ is a ▱. $\angle L \cong \angle JMG$	1. Given
2. $\angle G \cong \angle L$	2. ▱ → opp. ∡ ≅
3. $\angle G \cong \angle JMG$	3. Trans. Prop. of ≅
4. $\overline{GJ} \cong \overline{MJ}$	4. Conv. Isosc. △ Thm.
5. $\triangle GJM$ is isosc.	5. Def. of isosc. △

LESSON 6-3

29. $m = 13 \rightarrow m\angle G = 9(13) = 117°$; $n = 27 \rightarrow m\angle A = 2(27) + 9 = 63°$, $m\angle E = 3(27) - 18 = 63°$ Since $117° + 63° = 180°$ $\angle G$ is supp. to $\angle A$ and $\angle E$, so one ∠ of $ACEG$ is supp. to both of its cons. ∡. $ACEG$ is a ▱ by Thm. 6-3-4.

30. $x = 25 \rightarrow m\angle Q = 4(25) + 4 = 104°$, $m\angle R = 3(25) + 1 = 76°$; so $\angle Q$ and $\angle R$ are supp. $y = 7 \rightarrow QT = 2(7) + 11 = 25$, $RS = 5(7) - 10 = 25$ By Conv. of Same-Side Int. ∡ Thm., $\overline{QT} \parallel \overline{RS}$; since $\overline{QT} \cong \overline{RS}$, $QRST$ is a ▱ by Thm. 6-3-1.

31. Yes; The diags. bisect each other. By Thm. 6-3-5 the quad. is a ▱.

32. No; By Conv. of Alt. Int. ∡ Thm., one pair of opp. sides is ∥, but other pair is ≅. None of conditions for a ▱ are met.

33. slope of $\overline{BD} = \frac{2}{10} = \frac{1}{5}$; slope of $\overline{FH} = \frac{-2}{-10} = \frac{1}{5}$
slope of $\overline{BH} = \frac{-6}{1} = -6$; slope of $\overline{DF} = \frac{-6}{1} = -6$
Both pairs of opp. sides have the same slope, so $\overline{BD} \parallel \overline{FH}$ and $\overline{BH} \parallel \overline{DF}$; by def., $BDFH$ is a ▱.

LESSON 6-4

34. $\overline{AB} \cong \overline{CD}$
$AB = CD = 18$

35. $AC = 2CE$
$= 2(19.8) = 39.6$

36. $\overline{BD} \cong \overline{AC}$
$BD = AC = 39.6$

37. $\overline{BE} \cong \overline{CE}$
$BE = CE = 19.8$

38. $WX = WZ$
$7a + 1 = 9a - 6$
$7 = 2a$
$3.5 = a$
$WX = 7(3.5) + 1 = 25.5$

39. $\overline{XV} \cong \overline{VZ}$
$XV = VZ$
$= 3(3.5) = 10.5$

40. $\overline{XY} \cong \overline{WX}$
$XY = WX = 25.5$

41. $XZ = 2XV$
$= 2(10.5) = 21$

42. $m\angle TZV = 90$
$8n + 18 = 90$
$8n = 72$
$n = 9$
Think: $\overrightarrow{RT}$ bisects $\angle SRV$.
$m\angle TRS = \frac{1}{2}m\angle SRV$
$= \frac{1}{2}(9(9) + 1) = 41°$

43. $m\angle RSV + m\angle TRS = 90$
$m\angle RSV + 41 = 90$
$m\angle RSV = 49°$

44. $\angle STV \cong \angle SRV$
$m\angle STV = m\angle SRV$
$= 9(9) + 1 = 82°$

45. $m\angle TVR + m\angle STV = 180$
$m\angle TVR + 82 = 180$
$m\angle TVR = 98°$

46. Think: All 4 ⟁ are isosc. ⟁.
$2m\angle 1 + m\angle 2 = 180$
$2m\angle 3 + m\angle 4 = 180$
By Lin. Pair Thm.,
$m\angle 2 + m\angle 4 = 180$
By Alt. Int ∡ Thm.,
$m\angle 3 = 33°$
$2(33) + m\angle 4 = 180$
$m\angle 4 = 114°$
$m\angle 2 + 114 = 180$
$m\angle 2 = 66°$
$2m\angle 1 + 66 = 180$
$2m\angle 1 = 114$
$m\angle 1 = 57°$
By Alt. Int ∡ Thm.,
$\angle 1 \cong \angle 5$
$m\angle 5 = m\angle 1 = 57°$

47. Think: All 4 ⟁ are ≅ rt. ⟁.
$m\angle 2 = m\angle 5 = 53°$
$m\angle 3 = 90°$
$m\angle 4 + m\angle 5 = 90$
$m\angle 4 + 53 = 90$
$m\angle 4 = 37°$
$\angle 1 \cong \angle 4$
$m\angle 1 = m\angle 4 = 37°$

48. **Step 1** Show that $\overline{RT}$ and $\overline{SU}$ are congruent.
$RT = \sqrt{((-3) - (-5))^2 + (-6 - 0)^2} = 2\sqrt{10}$
$SU = \sqrt{((-7) - (-1))^2 + ((-4) - (-2))^2} = 2\sqrt{10}$
Since $RT = SU$, $\overline{RT} \cong \overline{SU}$

Step 2 Show that $\overline{RT}$ and $\overline{SU}$ are perpendicular.
slope of RT: $\frac{-6 - 0}{-3 - (-5)} = -3$
slope of SU: $\frac{-4 - (-2)}{-7 - (-1)} = \frac{1}{3}$
since $-3\left(\frac{1}{3}\right) = -1$, $\overline{RT} \perp \overline{SU}$

Step 3 Show that $\overline{RT}$ and $\overline{SU}$ bisect each other.
mdpt. of RT: $\left(\frac{-5 + (-3)}{2}, \frac{0 + (-6)}{2}\right) = (-4, -3)$
mdpt. of SU: $\left(\frac{-1 + (-7)}{2}, \frac{-2 + (-4)}{2}\right) = (-4, -3)$
Since $\overline{RT}$ and $\overline{SU}$ have the same midpoint, they bisect each other. The diagonals are congruent perpendicular biectors of each other.

49. Step 1 Show that $\overline{EG}$ and $\overline{FH}$ are congruent.

$EG = \sqrt{(5-2)^2 + (-2-1)^2} = 3\sqrt{2}$

$FH = \sqrt{(2-5)^2 + (-2-1)^2} = 3\sqrt{2}$

Since $EG = FH$, $\overline{EG} \cong \overline{FH}$

Step 2 Show that $\overline{EG}$ and $\overline{FH}$ are perpendicular.

slope of EG: $\dfrac{-2-1}{5-2} = -1$

slope of FH: $\dfrac{-2-1}{2-5} = 1$

since $-1(1) = -1$, $\overline{EG} \perp \overline{FH}$

Step 3 Show that $\overline{EG}$ and $\overline{FH}$ bisect each other.

mdpt. of RT: $\left(\dfrac{2+5}{2}, \dfrac{1+(-2)}{2}\right) = \left(\dfrac{7}{2}, -\dfrac{1}{2}\right)$

mdpt. of SU: $\left(\dfrac{5+2}{2}, \dfrac{1+(-2)}{2}\right) = \left(\dfrac{7}{2}, -\dfrac{1}{2}\right)$

Since $\overline{EG}$ and $\overline{FH}$ have the same midpoint, they bisect each other. The diagonals are congruent perpendicular biectors of each other.

LESSON 6-5

50. Not valid; by Thm. 6-5-2, if the diags. of a ▱ are ≅, then the ▱ is a rect. By Thm. 6-5-4, if the diags. of a ▱ are ⊥, then the ▱ is a rhombus. If a quad. is a rect. and a rhombus, then it is a square. But to apply this line of reasoning, you must first know that *EFRS* is a ▱.

51. valid (diags. bisect each other → ▱: ▱ with diags. ≅ → rect.)

52. valid (*EFRS* is a ▱ by def.; ▱ with 1 pair cons. sides ≅ → rhombus)

53. $BJ = \sqrt{8^2 + 8^2} = 8\sqrt{2}$; $FN = \sqrt{6^2 + 6^2} = 6\sqrt{2}$
Diags. are ≇, so ▱ is not a rect. Therefore ▱ is not a square.
slope of $\overline{BJ} = \dfrac{8}{8} = 1$; slope of $\overline{FN} = \dfrac{-6}{6} = -1$
Diags. are ⊥, so ▱ is a rhombus.

54. $DL = \sqrt{12^2 + 6^2} = 6\sqrt{5}$; $HP = \sqrt{6^2 + 12^2} = 6\sqrt{5}$
Diags. are ≅, so ▱ is a rect.
slope of $\overline{DL} = \dfrac{6}{12} = \dfrac{1}{2}$; slope of $\overline{HP} = \dfrac{-12}{-6} = 2$
Diags. are not ⊥, so ▱ is not a rhombus. Therefore ▱ is not a square.

55. $QW = \sqrt{12^2 + 8^2} = 4\sqrt{13}$; $TZ = \sqrt{8^2 + 12^2} = 4\sqrt{13}$
Diags. are ≅, so ▱ is a rect.
slope of $\overline{QW} = \dfrac{8}{12} = \dfrac{2}{3}$; slope of $\overline{TZ} = \dfrac{-12}{8} = -\dfrac{3}{2}$
Diags. are ⊥, so ▱ is a rhombus.
Rect., rhombus → ▱ is a square.

LESSON 6-6

56. Think: All 4 $\triangle$s are rt. $\triangle$s; left pair of $\triangle$s is ≅, as is right pair.

$$\begin{aligned} m\angle XYZ &= 2m\angle XYV \\ &= 2(90 - m\angle VXY) \\ &= 2(90 - 58) \\ &= 64° \end{aligned}$$

57. $$\begin{aligned} m\angle ZWV &= \tfrac{1}{2}m\angle ZWX \\ &= \tfrac{1}{2}(50) = 25° \end{aligned}$$

58. $$\begin{aligned} m\angle VZW &= 90 - m\angle ZWV \\ &= 90 - 25 = 65° \end{aligned}$$

59. $$\begin{aligned} m\angle WZY &= m\angle VZW + m\angle VZY \\ &= m\angle VZW + m\angle VXY \\ &= 65 + 58 = 123° \end{aligned}$$

60. Think: Use Same-Side Int. $\angle$s Thm., isosc. trap. → base $\angle$s ≅.

$$\begin{aligned} m\angle V + m\angle T &= 180 \\ m\angle V + 54 &= 180 \\ m\angle V &= 126° \\ \angle R &\cong \angle V \\ m\angle R = m\angle V &= 126° \\ \angle S &\cong \angle T \\ m\angle S = m\angle T &= 54° \end{aligned}$$

61. Think: Isosc. trap. → diags. ≅

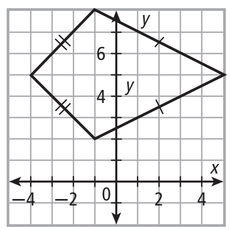
$$\begin{aligned} \overline{BH} &\cong \overline{EK} \\ BZ + ZH &= EK \\ BZ + 70 &= 121.6 \\ BZ &= 51.6 \end{aligned}$$

62. $$\begin{aligned} MN &= \tfrac{1}{2}(AD + JG) \\ &= \tfrac{1}{2}(67 + 30) \\ &= 48.5 \end{aligned}$$

63. $$\begin{aligned} ST &= \tfrac{1}{2}(FP + EQ) \\ 2ST &= FP + EQ \\ 2(3.1) &= 2.7 + EQ \\ 3.5 &= EQ \end{aligned}$$

64. $$\begin{aligned} \angle P &\cong \angle Y \\ m\angle P &= m\angle Y \\ 8n^2 - 11 &= 6n^2 + 7 \\ 2n^2 &= 18 \\ n^2 &= 9 \\ n &= \pm 3 \end{aligned}$$

65.

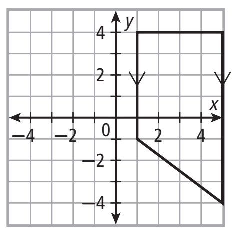


$AB = DA = \sqrt{3^2 + 3^2} = 3\sqrt{2}$

$BC = CD = \sqrt{6^2 + 3^2} = 3\sqrt{5}$

Exactly 2 pairs of cons. sides ≅ → kite.

66.

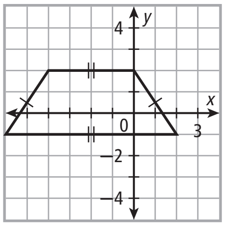


$\overline{AD}$ and $\overline{BC}$ are vert., so $\overline{AD} \parallel \overline{BC}$

$AB = |5 - 1| = 4$, $CD = \sqrt{4^2 + 3^2} = 5$

$\overline{AB} \ncong \overline{CD}$ and hence $\overline{AD} \nparallel \overline{BC}$

Exactly 1 pair opp. sides ∥, other pair ≇ → trap. (not isosc.)

67. 

$\overline{AD}$ and $\overline{BC}$ are horiz., but $AD = 2 + 6 = 8 \neq 4 = BC$
$\overline{AD} \parallel \overline{BC}$, $\overline{AD} \not\cong \overline{BC} \rightarrow$ trap. (not ▱)
$AB = \sqrt{2^2 + 3^2} = \sqrt{13}$; $CD = \sqrt{2^2 + 3^2} = \sqrt{13}$
$\overline{AB} \cong \overline{CD}$, not ▱ $\rightarrow$ isosc. trap

CHAPTER TEST, PAGE 442

1. not a polygon

2. polygon; decagon

3. Think: Use Quad. $\angle$ Sum Thm.

$$\begin{aligned} m\angle A + m\angle B + m\angle C + m\angle D &= 360 \\ 12n + 14n + 8n + 11n &= 360 \\ 45n &= 360 \\ n &= 8 \end{aligned}$$

$m\angle A = 12(8) = 96°$, $m\angle B = 14(8) = 112°$, $m\angle C = 8(8) = 64°$, $m\angle D = 11(8) = 88°$

4. $(9 - 2)180$
$(7)180$
$1260°$

5. $15m(\text{ext. } \angle) = 360$
$m(\text{ext. } \angle) = 24°$

6. Think: Z is mdpt. of $\overline{FH}$.

$$\begin{aligned} FH &= 2HZ \\ &= 2(9) = 18 \end{aligned}$$

Think: Cons. $\angle$s are supp.

$$\begin{aligned} m\angle FEH + m\angle EHG &= 180 \\ m\angle FEH + 145 &= 180 \\ m\angle FEH &= 35° \end{aligned}$$

7. Think: Opp. sides are $\cong$.

$$\begin{aligned} JM &= KL \\ 4y - 7 &= y + 11 \\ 3y &= 18 \\ y &= 6 \end{aligned}$$

$KL = 6 + 11 = 17$

Think: Cons. $\angle$s are supp.

$$\begin{aligned} m\angle M + m\angle L &= 180 \\ 6x - 1 + 2x + 9 &= 180 \\ 8x &= 172 \\ x &= 21.5 \end{aligned}$$

$m\angle L = 2(21.5) + 9 = 52°$

8. Slope from S to R is rise of 4 and run of 1; from P to Q: rise of 4 is -3 to $-3 + 4 = 1$, run of 1 is -2 to $-2 + 1 = -1$; so coords. of $Q = (-1, 1)$.

9. $a = 4 \rightarrow XN = 3(4) = 12$, $NZ = 4 + 8 = 12$
$b = 3 \rightarrow WN = 4(3) + 3 = 15$, $NY = 5(3) = 15$
So N is mdpt. of $\overline{XZ}$ and $\overline{WY}$, and therefore diags. bisect each other. By Thm. 6-3-5, $WXYZ$ is a ▱.

10. No; one pair of opp. sides of the quad. are $\parallel$. A pair of vert. $\angle$s formed by the diags. are $\cong$. None of conditions for a ▱ are met.

11. Possible answer: slope of $\overline{KL} = \frac{3}{9} = \frac{1}{3}$; slope of $\overline{ST} = \frac{-3}{-9} = \frac{1}{3}$
slope of $\overline{KT} = -\frac{4}{3}$; slope of $\overline{LS} = -\frac{4}{3}$
$\overline{KL} \parallel \overline{ST}$ and $\overline{KT} \parallel \overline{LS} \rightarrow KLST$ is a ▱.

12.

$$\begin{aligned} PT &= \tfrac{1}{2}PC \\ &= \tfrac{1}{2}LM \\ &= \tfrac{1}{2}(23) = 11.5 \end{aligned}$$

$\overline{PM} \cong \overline{LC}$
$PM = LC = 19$

13. Think: All 4 $\triangle$s are $\cong$ rt. $\triangle$s.

$$\begin{aligned} m\angle NQK &= 90 \\ 7z + 6 &= 90 \\ 7z &= 84 \\ z &= 12 \end{aligned}$$

$$\begin{aligned} m\angle HEQ &= 90 - m\angle EHQ \\ &= 90 - m\angle ENQ \\ &= 90 - (5(12) + 1) = 29° \end{aligned}$$

$$\begin{aligned} m\angle EHK &= 2m\angle EHQ \\ &= 2m\angle ENQ \\ &= 2(5(12) + 1) = 122° \end{aligned}$$

14. Not valid; possible answer: $MNPQ$ is a rhombus by def. However, to show that $MNPQ$ is a square, you need to know that $MNPQ$ is also a rect.

15. valid ($MNPQ$ is a ▱ by def.; diags $\cong \rightarrow MNPQ$ is a rect.)

16. $AE = \sqrt{12^2 + 8^2} = 4\sqrt{13}$; $CG = \sqrt{4^2 + 6^2} = 2\sqrt{13}$
Diags. are $\not\cong$, so $ACEG$ is not a rect. Therefore $ACEG$ is not a square.
slope of $\overline{AE} = \frac{-8}{12} = -\frac{2}{3}$; slope of $\overline{CG} = \frac{-6}{-4} = \frac{3}{2}$
Diags. are $\perp$, so $ACEG$ is a rhombus.

17. $PR = \sqrt{7^2 + 1^2} = \sqrt{50}$; $QS = \sqrt{5^2 + 5^2} = \sqrt{50}$
Diags. are $\cong$, so $PQRS$ is a rect.
slope of $\overline{PR} = \frac{1}{-7} = -\frac{1}{7}$; slope of $\overline{QS} = \frac{-5}{-5} = 1$
Diags. are not $\perp$, so $PQRS$ is not a rhombus. Therefore $PQRS$ is not a square.

18.

$$\begin{aligned} m\angle FBN &= m\angle FBR + m\angle RBN \\ &= 90 - m\angle BFR + 90 - m\angle RNB \\ &= 90 - m\angle JFR + 90 - \tfrac{1}{2}m\angle JNB \\ &= 180 - 43 - \tfrac{1}{2}(68) = 103° \end{aligned}$$

19.

$$\begin{aligned} \overline{MS} &\cong \overline{PV} \\ MY + YS &= PV \\ MY + 24.7 &= 61.1 \\ MY &= 36.4 \end{aligned}$$

20.

$$\begin{aligned} XY &= \tfrac{1}{2}(HR + GS) \\ 2XY &= HR + GS \\ 2(25.5) &= HR + 24 \\ HR &= 27 \text{ in.} \end{aligned}$$