Solutions Key
Extending Perimeter, Circumference, and Area

ARE YOU READY? PAGE 585

1. C **2.** D

3. E **4.** A

5. 12 mi = 12 • 1760 yd = 21,120 yd

6. 7.3 km = 7.3 • 1000 m = 7300 m

7. 6 in = (6 ÷ 12) ft = 0.5 ft

8. 15 m = 15 • 1000 mm = 15,000 mm

9. $x^2 = 3.1^2 + 5.8^2$
$x^2 = 43.25$
$x = \sqrt{43.25} \approx 6.6$ in.

10. $10^2 = x^2 + 8^2$
$x^2 = 100 - 64$
$x^2 = 36$
$x = 6$ cm

11. $9.9^2 = x^2 + 4.3^2$
$x^2 = 98.01 - 18.49$
$x^2 = 79.52$
$x = \sqrt{79.52} \approx 8.9$ m

12. $\frac{5}{8}$ in.; 1.5 cm **13.** $1\frac{1}{8}$ in.; 3 cm

14. $1\frac{3}{4}$ in.; 4.5 cm

15. $A = \frac{1}{2}bh$
$2A = bh$
$b = \frac{2A}{h}$

16. $P = 2b + 2h$
$P - 2b = 2h$
$h = \frac{P - 2b}{2}$

17. $A = \frac{1}{2}(b_1 + b_2)h$
$\frac{2A}{h} = b_1 + b_2$
$b_1 = \frac{2A}{h} - b_2$

18. $A = \frac{1}{2}d_1d_2$
$2A = d_1d_2$
$d_1 = \frac{2A}{d_2}$

CONNECTING GEOMETRY TO ALGEBRA: LITERAL EQUATIONS, PAGE 588

TRY THIS, PAGE 588

1. $P = 2\ell + 2w$
$P - 2\ell = 2w$
$\frac{P - 2\ell}{2} = w$
$w = \frac{P - 2\ell}{2} = \frac{24 - 2(2)}{2} = 10$ cm
$w = \frac{P - 2\ell}{2} = \frac{24 - 2(3)}{2} = 9$ cm
$w = \frac{P - 2\ell}{2} = \frac{24 - 2(4)}{2} = 8$ cm
$w = \frac{P - 2\ell}{2} = \frac{24 - 2(6)}{2} = 6$ cm
$w = \frac{P - 2\ell}{2} = \frac{24 - 2(8)}{2} = 4$ cm

2. $a^2 + b^2 = c^2$
$a^2 = c^2 - b^2$
$a = \sqrt{c^2 - b^2}$
$a = \sqrt{c^2 - b^2} = \sqrt{65^2 - 16^2} = \sqrt{3969} = 63$ ft
$a = \sqrt{c^2 - b^2} = \sqrt{65^2 - 25^2} = \sqrt{3600} = 60$ ft
$a = \sqrt{c^2 - b^2} = \sqrt{65^2 - 33^2} = \sqrt{3136} = 56$ ft
$a = \sqrt{c^2 - b^2} = \sqrt{65^2 - 39^2} = \sqrt{2704} = 52$ ft

3. $P = a + b + c$
$112 = a + b + c$
$a = 112 - b - c$

a	b	c
112 − 48 − 35 = 29	48	35
112 − 36 − 36 = 40	36	36
112 − 14 − 50 = 48	14	50

9-1 DEVELOPING FORMULAS FOR TRIANGLES AND QUADRILATERALS, PAGES 589–597

CHECK IT OUT! PAGES 590–592

1. $A = bh$
$28 = 56b$
$b = 0.5$ yd

2. $b^2 + h^2 = c^2$
$b^2 + 12^2 = 20^2$
$b^2 = 256 = 16^2$
$b = 16$ m
$A = \frac{1}{2}bh$
$= \frac{1}{2}(16)(12) = 96\text{ m}^2$

3. $A = \frac{1}{2}d_1d_2$
$12xy = \frac{1}{2}(3x)d_2$
$24xy = (3x)d_2$
$d_2 = 8y$ m

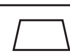
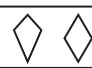
4. $P = 4 + 2\sqrt{2} + 2\sqrt{2}$
$= (4 + 4\sqrt{2})$ cm
$A = \frac{1}{2}bh$
$= \frac{1}{2}(4)(2) = 4\text{ cm}^2$

THINK AND DISCUSS, PAGE 593

1. Because 2 congruent copies of the triangle fit together to form a parallelogram with same base and height as the triangle

2. The area of a rectangle is the base times the height, and area of a trapezoid is the average of the bases times the height.

3.

Area Formula	Shape(s)	Example(s)
$A = bh$	rectangle, paralleogram	
$A = \frac{1}{2}bh$	triangle	
$A = \frac{1}{2}(b_1 + b_2)h$	trapezoid	
$A = \frac{1}{2}d_1d_2$	kite, rhombus	

EXERCISES, PAGES 593–597

GUIDED PRACTICE, PAGE 593

1. $A = bh$
$= (12)(10) = 120 \text{ cm}^2$

2. $A = bh$
$10x^2 = (2x)h$
$h = 5x$ ft

3. $A = s^2$
$169 = s^2$
$s = 13$
$P = 4s$
$= 4(13) = 52$ cm

4. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(9 + 15)(20)$
$= 240 \text{ m}^2$

5. $A = \frac{1}{2}bh$
$58.5 = \frac{1}{2}b(9) = 4.5b$
$b = 13$ in.

6. $A = \frac{1}{2}(b_1 + b_2)h$
$48x + 68 = \frac{1}{2}(b_1 + 9x + 12)(8)$
$48x + 68 = 4b_1 + 36x + 48$
$12x + 20 = 4b_1$
$b_1 = (3x + 5)$ in.

7. $\left(\frac{d_1}{2}\right)^2 + \left(\frac{d_2}{2}\right)^2 = c^2$
${d_1}^2 + {d_2}^2 = 4c^2$
${d_1}^2 + 14^2 = 4(25)^2$
${d_1}^2 = 2500 - 196 = 2304$
$d_1 = 48$ in.
$A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(48)(14) = 336 \text{ in.}^2$

8. $A = \frac{1}{2}d_1d_2$
$187.5 = \frac{1}{2}(15)d_2$
$= 7.5d_2$
$d_2 = 25$ m

9. $A = \frac{1}{2}d_1d_2$
$12x^2y^3 = \frac{1}{2}(3xy)d_2$
$24x^2y^3 = (3xy)d_2$
$d_2 = 8xy^2$ cm

10. Parallelogram: $A = bh = \sqrt{2}\left(1.5\sqrt{2}\right) \text{ ft}^2 = 3 \text{ ft}^2$
Rectangles: $A = bh = 1(2) = 2 \text{ ft}^2$
Triangles: $A = \frac{1}{2}bh = \frac{1}{2}(1)(2) = 1 \text{ ft}^2$
Trapezoids: $A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(1 + 2)(1) = 1.5 \text{ ft}^2$

PRACTICE AND PROBLEM SOLVING, PAGES 594–596

11. $A = bh$
$7.5 = 6h$
$h = 1.25$ m

12. $P = 2\ell + 2w$
$= 2(x + 2) + 2(x - 1)$
$= 2x + 4 + 2x - 2$
$= (4x + 2)$ in.

13. $A = bh$
$= (3x + 5)(7x - 1)$
$= (21x^2 + 32x - 5) \text{ ft}^2$

14. Let $b = x + y$
$x^2 + 15^2 = 17^2$
$x^2 = 64$
$x = 8$ in.
$y^2 + 15^2 = 25^2$
$y^2 = 400$
$y = 20$ in.
$A = \frac{1}{2}bh$
$= \frac{1}{2}(8 + 20)(15)$
$= 210 \text{ in.}^2$

15. $A = \frac{1}{2}(b_1 + b_2)h$
$280 = \frac{1}{2}(8 + 20)h = 14h$
$h = 20$ cm

16. $A = \frac{1}{2}bh$
$= \frac{1}{2}(x + 1)(8x)$
$= (4x^2 + 4x) \text{ ft}^2$

17. Right triangles are 30°-60°-90° triangles; so $d_1 = 7 + 21 = 28$ in.
$A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(28)\left(2\left(7\sqrt{3}\right)\right)$
$= 196\sqrt{3} \text{ in.}^2$

18. $A = \frac{1}{2}d_1d_2$
$3x^2 + 6x = \frac{1}{2}(x + 2)d_2$
$3x(x + 2) = \frac{1}{2}(x + 2)d_2$
$d_2 = 6x$ m

19. $A = \frac{1}{2}d_1d_2$
$= \frac{1}{2}(6x + 5)(4x + 8)$
$= (12x^2 + 34x + 20)$ ft

20. $A = bh = 6(3) = 18 \text{ in.}^2$

21. $A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5 \text{ in.}^2$

22. $A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(3 + 6)(3) = 13.5 \text{ in.}^2$

23. $h = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$
$A = bh$
$= (10)3\sqrt{3} = 30\sqrt{3} \text{ cm}^2$

24. $b^2 + 5^2 = 13^2$
$b = 12$ m
$A = \frac{1}{2}bh$
$= \frac{1}{2}(5)(12) = 30 \text{ m}^2$

25. $h^2 + 7^2 = 25^2$
$h = 24$
$(b - 7)^2 + 24^2 = 30^2$
$b - 7 = 18$
$b = 25$
$A = \frac{1}{2}bh$
$= \frac{1}{2}(25)(24) = 300 \text{ in.}^2$

26. $b = x,\ h = \frac{x\sqrt{3}}{2}$
$A = \frac{1}{2}(x)\frac{x\sqrt{3}}{2} = \frac{x^2\sqrt{3}}{4}$

27. $b = x\sqrt{3},\ h = x$
$A = \frac{1}{2}\left(x\sqrt{3}\right)x = \frac{x^2\sqrt{3}}{2}$

28. $b = h = x$
$A = \frac{1}{2}(x)(x) = \frac{x^2}{2}$

29a. $h = \frac{36\sqrt{3}}{2} \approx 31.2$ in.

b. $A \approx \frac{1}{2}(36)(31.18)$
$\approx 561.2 \text{ in.}^2$

c. Material left is
$\approx 36^2 - 561.2$
$\approx 734.8 \text{ in.}^2$

	Base b	Height h	Area A	Perimeter P
30.	12	16	12(16) = **192**	2(12) + 2(16) = **56**
31.	17	**8**	136 = 17(**8**)	2(17) + 2(8) = **50**
32.	**14**	11	**14**(11) = **154**	50 = 2(**14**) + 2(11)
33.	**9**	**24**	216 = **9**(**24**)	66 = 2(**9**) + 2(**24**)

34. $P = 2b + 2h$
$72 = 2(3h) + 2h = 8h$
$h = 9$ in.
$A = bh$
$= (3(9))9 = 243 \text{ in.}^2$

35. $A = \frac{1}{2}bh$
$50 = \frac{1}{2}(4h)h = 2h^2$
$25 = h^2$
$h = 5$ cm

36. $P = s_1 + b_1 + s_2 + b_2$
$40 = s + 11 + s + 19$
$10 = 2s$
$s = 5$ ft
Let $19 = 11 + 2x$, so $x = 4$ ft.
$x^2 + h^2 = s^2$
$16 + h^2 = 25$
$h = 3$ ft
$A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(11 + 19)(3) = 45 \text{ ft}^2$

37. $1 \text{ yd}^2 = (3 \text{ ft})^2 = 9 \text{ ft}^2$

38. $1 \text{ m}^2 = (100 \text{ cm})^2 = 10{,}000 \text{ cm}^2$

39. $1 \text{ cm}^2 = (10 \text{ mm})^2 = 100 \text{ mm}^2$

40. $1 \text{ mi}^2 = (5280(12) \text{ in.})^2 = 4{,}014{,}489{,}600 \text{ in.}^2$

41. $A = \frac{1}{2}(3)(8) = 12 \text{ yd}^2$
$= 12(9 \text{ ft}^2) = 108 \text{ ft}^2$

42. $A = \frac{1}{2}(500)(800) = 200{,}000 \text{ yd}^2$
$= \dfrac{200{,}000 \text{ yd}^2}{(1760 \text{ yd/mi})^2} \approx 0.065 \text{ mi}^2$

43a. $A = \dfrac{(a + b)}{2}(a + b)$
$= \frac{1}{2}(a + b)^2$

b. $\frac{1}{2}ab; \frac{1}{2}ab; \frac{1}{2}c^2$

c. $\frac{1}{2}(a + b)^2 = \frac{1}{2}ab + \frac{1}{2}ab + \frac{1}{2}c^2$
$(a + b)^2 = 2ab + c^2$
$a^2 + 2ab + b^2 = 2ab + c^2$
$a^2 + b^2 = c^2$

44. The area of the large square is $(b + h)^2$. The area of the medium square is b^2 and the area of the small square is h^2. The total area is the sum of the areas. Let A represent area of the rectangle.
$(b + h)^2 = b^2 + h^2 + 2A$
$b^2 + 2bh + h^2 = b^2 + h^2 + 2A$
$2bh = 2A$
$A = bh$

45. Opposite sides of a parallelogram are congruent, so the diagonal divides the parallelogram into 2 congruent triangles. Let A represent the area of each triangle. The sum of the triangles' areas is the area of the parallelogram.
$2A = bh$
$A = \frac{1}{2}bh$

46. Both triangles have height h. The area of the upper triangle is $\frac{1}{2}b_1h$ and the area of the lower triangle is $\frac{1}{2}b_2h$. The area of the trapezoid is the sum of the areas of the triangles.
$A = \frac{1}{2}b_1h + \frac{1}{2}b_2h$
$= \frac{1}{2}(b_1 + b_2)h$

47a. Possible answers:
A: $(2.1)(2.0) = 4.2 \text{ cm}^2$
B: $(1.2)(3.2) = 3.8 \text{ cm}^2$
C: $(2.7)(1.6) = 4.3 \text{ cm}^2$

b. C has greatest area.

48. $w = 40A + 20d_1 + 20d_1$
$= 40\frac{1}{2}d_1d_2 + 20(d_1 + d_2)$
$= 20(d_1d_2 + d_1 + d_2)$
$= 20(0.90(0.80) + 0.90 + 0.80) = 48.4$ g

49. There are 9 tiles per 1-ft square. So Tom needs $1.15[12(18)(9)] \approx 2236$ tiles, or 23 cases.

50. From the given measurements, the area is 12 cm^2. If the actual measurements were 5.9 cm and 1.9 cm, the area would be 11.21 cm^2. If the actual measurements were 6.1 cm and 2.1 cm, the area would be 12.81 cm^2. The maximum error is 0.81 cm^2.

51. A square is a parallelogram and a rectangle in which $b = h = s$, so $A = bh = (s)(s) = s^2$.
A square is a rhombus in which $d_1 = d_2 = s\sqrt{2}$, so $A = \frac{1}{2}\left(s\sqrt{2}\right)\left(s\sqrt{2}\right) = \frac{1}{2}s^2(2) = s^2$.

TEST PREP, PAGES 596–597

52. B

53. H

54. C
$\frac{1}{2}(16)\frac{1}{2}(18) = 8(9)$
$= 72$

55. H
$JK = \sqrt{6^2 + 10^2}$
$= \sqrt{136} \approx 11.7$ cm

56. \$1309
$C = 2.75A$
$= 2.75bh$
$= 2.75(28)(17) = 1309$

CHALLENGE AND EXTEND, PAGE 597

57. $A = 3h = 6(2)$
$3h = 12$
$h = 4$ in.

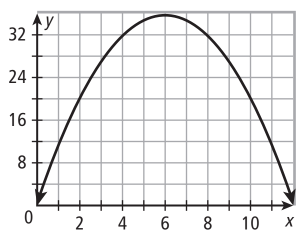
58. $A = 25h = 15(20)$
$25h = 300$
$h = 12$ m

59. $A = 42x^2 + 51x + 15$
$= (7x + 5)(6x + 3)$
$P = 26x + 16$
$= 2(13x + 8)$
$= 2(7x + 5) + 2(6x + 3)$
$b = (7x + 5)$ cm
$h = (6x + 3)$ cm

60. Let $ABCD$ be a quadrilateral with perpendicular diagonals. $\overline{AC}$ and $\overline{BD}$ that intersect at E. Let $d_1 = AC$, $d_2 = BD$, and $x = BE$. $\triangle ABC$ has $b = d_1$ and $h = x$, so $A = \frac{1}{2}d_1x$. $\triangle ADC$ has $b = d_1$ and $h = d_2 - x$, so $A = \frac{1}{2}d_1(d_2 - x)$. $ABCD$ has area $A = \frac{1}{2}d_1x + \frac{1}{2}d_1(d_2 - x)$
$= \frac{1}{2}d_1(x + d_2 - x) = \frac{1}{2}d_1d_2$.

61a. $2x + 2y = 24$
$2y = 24 - 2x$
$y = 12 - x$
$A = xy = x(12 - x)$

b.



D: $\{x \mid 0 < x < 12\}$
R: $\{y \mid 0 < y \leq 36\}$

c. Area is maximized when $x = 6$; therefore, the dimensions are 6 ft by 6 ft.

d. Solve the area formula for y and substitute the expression into the perimeter formula. Graph, and find the minimum value.

SPIRAL REVIEW, PAGE 597

62. $-4 \leq x \leq 6$
$-4 - 3 \leq x - 3 \leq 6 - 3$
$-7 \leq y \leq 3$

63. $-2 \leq x \leq 2$
$0 \leq x^2 \leq 4$
$-4 \leq -x^2 \leq 0$
$-4 + 2 \leq -x^2 + 2 \leq 0 + 2$
$-2 \leq y \leq 2$

64. $P = 2(x + 2) + 2(2x)$
$= 2x + 4 + 4x$
$= 6x + 4$

$A = (x + 2)(2x)$
$= 2x^2 + 4x$

65. $P = x + 7 + (x + 1)$
$= 2x + 8$

$A = \frac{1}{2}(x)(7)$
$= \frac{7x}{2}$

66. $\overrightarrow{LM} = (5 - 4, 10 - 3) = \langle 1, 7 \rangle$

67. $\overrightarrow{ST} = (4 - (-2), 6 - (-2)) = \langle 6, 8 \rangle$

9-2 GEOMETRY LAB: DEVELOP π, PAGES 598–599

ACTIVITY 1, TRY THIS, PAGE 598

1. No; possible answer: all circles are similar, so the ratio of circumference to diameter is always the same.

2. Solving the relationship for C gives a formula in terms of d and π.

3. If the circumference is $n\pi$, then the diameter is n. Check students' measurements.

ACTIVITY 2, TRY THIS, PAGE 599

4. Outer hexagon: let r and s represent the radius and hexagon side length respectively. Then $r = \frac{s\sqrt{3}}{2} = 1$, so $s = \frac{2\sqrt{3}}{3}$ and $P = 6s = 4\sqrt{3}$.
Inner hexagon: $s = r = 1$, so $P = 6$
$6 < C < 4\sqrt{3}$
$6 < \pi(2) < 4\sqrt{3}$
$3 < \pi < 3.46$

5. Possible answer: The second inequality values are closer together. With more sides, the values would be even closer together. You can estimate π by averaging the upper and lower values.

6. Possible answer: Average the areas of the inscribed and circumscribed polygons.

9-2 DEVELOPING FORMULAS FOR CIRCLES AND REGULAR POLYGONS, PAGES 600–605

CHECK IT OUT! PAGES 601–602

1. Step 1 Use given circumference to solve for r.
$C = 2\pi r$
$(4x - 6)\pi = 2\pi r$
$2x - 3 = r$
Step 2 Use expression for r to find area.
$A = \pi r^2$
$A = \pi(2x - 3)^2$
$A = (4x^2 - 12x + 9)\pi$ m^2

2. $C = \pi(10) \approx 31.4$ in.; $C = \pi(12) \approx 37.7$ in.; $C = \pi(14) \approx 44.0$ in.

3. Step 1 Find perimeter.
$P = 8(4) = 32$ cm
Step 2 Use tangent ratio to find apothem.
$\tan 22.5° = \frac{2}{a}$
$a = \frac{2}{\tan 22.5°}$
Step 3 Use apothem and perimeter to find area.
$A = \frac{1}{2}aP$
$A = \frac{1}{2}\left(\frac{2}{\tan 22.5°}\right)(32)$
$A \approx 77.3$ cm^2

THINK AND DISCUSS, PAGE 602

1. Circumference of a circle is π times diameter.
2. Divide 360° by n.
3.

Regular Polygons (side length = 1)					
Polygon	Number of Sides	Perimeter	Central Angle	Apothem	Area
Triangle	3	3	120°	$\frac{\sqrt{3}}{6}$	$\frac{\sqrt{3}}{4}$
Square	4	4	90°	$\frac{1}{2}$	1
Hexagon	6	6	60°	$\frac{\sqrt{3}}{2}$	$\frac{3\sqrt{3}}{2}$

EXERCISES, PAGES 603–605

GUIDED PRACTICE, PAGE 603

1. Draw a segment perpendicular to a side with one endpoint at the center. The apothem is $\frac{1}{2}s$.
2. $C = \pi d = \pi\left(\frac{10}{\pi}\right) = 10$ cm
3. $A = \pi r^2 = \pi(3x)^2 = 9x^2\pi$ in.^{2}
4. **Step 1** Use given area to solve for r.
$$A = \pi r^2$$
$$36\pi = \pi r^2$$
$$36 = r^2$$
$$r = 6 \text{ in.}$$
Step 2 Use value of r to find circumference.
$$C = 2\pi r$$
$$C = 2\pi(6) = 12\pi \text{ in.}$$
5. $A = \pi\left(\frac{8}{2}\right)^2 \approx 50.3$ in.^{2}; $A = \pi\left(\frac{10}{2}\right)^2 \approx 78.5$ in.^{2}; $A = \pi\left(\frac{12}{2}\right)^2 \approx 113.1$ in.^{2}
6. **Step 1** Find perimeter.
$P = 6(10) = 60$ in.
Step 2 Use properties of 30°-60°-90° $\triangle$ to find apothem.
$a = 5\sqrt{3}$ in.
Step 3 Use apothem and perimeter to find area.
$$A = \frac{1}{2}aP$$
$$A = \frac{1}{2}(5\sqrt{3})(60) \approx 259.8 \text{ in.}^2$$
7. **Step 1** Find perimeter.
$P = 7(3) = 21$ cm
Step 2 Use tangent ratio to find apothem.
$$\tan\left(\frac{360}{14}\right)^\circ = \frac{1.5}{a}$$
$$a = \frac{1.5}{\tan\left(\frac{360}{14}\right)^\circ}$$
Step 3 Use apothem and perimeter to find area.
$$A = \frac{1}{2}aP$$
$$A = \frac{1}{2}\left(\frac{1.5}{\tan\left(\frac{360}{14}\right)^\circ}\right)(21) \approx 32.7 \text{ cm}^2$$
8. **Step 1** Find side length.
$s = 2a\sqrt{3} = 4\sqrt{3}$ ft
Step 2 Find perimeter.
$P = 3s = 34\sqrt{3} = 12\sqrt{3}$ ft
Step 3 Find area.
$$A = \frac{1}{2}aP$$
$$A = \frac{1}{2}(2)(12\sqrt{3}) \approx 20.8 \text{ ft}^2$$
9. **Step 1** Find perimeter.
$P = 12(5) = 60$ m
Step 2 Use tangent ratio to find apothem.
$$\tan 15° = \frac{2.5}{a}$$
$$a = \frac{2.5}{\tan 15°}$$
Step 3 Use apothem and perimeter to find area.
$$A = \frac{1}{2}aP$$
$$A = \frac{1}{2}\left(\frac{2.5}{\tan 15°}\right)(60) \approx 279.9 \text{ m}^2$$

PRACTICE AND PROBLEM SOLVING, PAGES 603–605

10. $A = \pi(7)^2 = 49\pi$ yd^2
11. $C = \pi(5) = 5\pi$ m
12. $C = \pi d$
$10 = \pi d$
$d = \frac{10}{\pi}$ ft
13. $A = \pi\left(\frac{35}{2}\right)^2 \approx 962.1$ ft^2;
$A = \pi\left(\frac{50}{2}\right)^2 \approx 1963.5$ ft^2;
$A = \pi\left(\frac{66}{2}\right)^2 \approx 3421.2$ ft^2
14. $A = (2(12))^2 = 576$ cm^2
15. **Step 1** Find side length and perimeter.
$$\tan 22.5° = \frac{\frac{s}{2}}{a} = \frac{s}{4}$$
$$s = 4\tan 22.5°$$
$$P = 8s = 32\tan 22.5°$$
Step 2 Find area.
$$A = \frac{1}{2}aP$$
$$= \frac{1}{2}(2)(32\tan 22.5°) \approx 13.3 \text{ ft}^2$$
16. **Step 1** Find side length and apothem.
$$P = 9s$$
$$144 = 9s$$
$$s = 16$$
$$\tan 20° = \frac{8}{a}$$
$$a = \frac{8}{\tan 20°}$$
Step 2 Find area.
$$A = \frac{1}{2}aP$$
$$= \frac{1}{2}\left(\frac{8}{\tan 20°}\right)(144) \approx 1582.5 \text{ in.}^2$$

17. **Step 1** Find side length and perimeter.

$$\tan 36° = \frac{\frac{s}{2}}{a} = \frac{s}{4}$$
$$s = 4\tan 36°$$
$$P = 5s = 20\tan 36°$$

Step 2 Find area.

$$A = \frac{1}{2}aP$$
$$= \frac{1}{2}(2)(20\tan 36°) \approx 14.5 \text{ ft}^2$$

18. $\frac{360°}{3} = 120°$ **19.** $\frac{360°}{4} = 90°$

20. $\frac{360°}{5} = 72°$ **21.** $\frac{360°}{6} = 60°$

22. $\frac{360°}{7} \approx 51.4°$ **23.** $\frac{360°}{8} = 45°$

24. $\frac{360°}{9} = 40°$ **25.** $\frac{360°}{10} = 36°$

26. $s = 2a\tan 30° = 28\tan 30°$
$P = 6s = 168\tan 30°$
$A = \frac{1}{2}(14)168\tan 30° \approx 679.0 \text{ in.}^2$

27. $s = 2a\tan 25.71° = 10\tan 25.71°$
$P = 7s = 70\tan 25.71°$
$A = \frac{1}{2}(5)70\tan 25.71° \approx 84.3 \text{ cm}^2$

28. $a = \frac{s}{2\tan 20°} = \frac{3}{\tan 20°}$
$P = 9s = 54$
$A = \frac{1}{2}\left(\frac{3}{\tan 20°}\right)(54) \approx 222.5 \text{ in.}^2$

29. $s = 2a\tan 60° = 6\tan 60°$
$P = 3s = 18\tan 60°$
$A = \frac{1}{2}(3)18\tan 60° \approx 46.8 \text{ m}^2$

30. $a = r\cos 22.5° = 2\cos 22.5°$
$s = 2r\sin 22.5° = 4\sin 22.5°$
$P = 8s = 32\sin 22.5°$
$A = \frac{1}{2}(2\cos 22.5°)(32\sin 22.5°) \approx 11.3 \text{ cm}^2$

31. $a = \frac{s}{2\tan 25.71°} = \frac{2.5}{\tan 25.71°}$
$P = 7s = 35$
$A = \frac{1}{2}\left(\frac{2.5}{\tan 25.71°}\right)(35) \approx 90.8 \text{ ft}^2$

32. $C = 100 = 2\pi(r + 0.5)$
$r = \frac{50}{\pi} - 0.5$
$a = \frac{r}{w} = \frac{\frac{50}{\pi} - 0.5}{0.2} \approx 77 \text{ yr}$

33. A is incorrect because the diameter, instead of the radius, is used to find the area.

	Diam. d	Radius r	Area A	Circ. C
34.	6	**3**	$\pi(3)^2 = 9\pi$	6π
35.	$\frac{20\sqrt{\pi}}{\pi}$	$\sqrt{\frac{100}{\pi}} = \frac{10\sqrt{\pi}}{\pi}$	100	$20\sqrt{\pi}$
36.	**34**	17	289π	34π
37.	**36**	**18**	324π	36π

38. $A_{garden} = A_{circle} - A_{gazebo}$
$= \pi r^2 - \frac{1}{2}aP$
$= \pi(10 + 6)^2 - \frac{1}{2}\left(\frac{6\sqrt{3}}{2}\right)(36) \approx 711 \text{ ft}^2$

39a. **Step 1** Find side length and perimeter.

$30 = \frac{s\sqrt{2}}{2} + s + \frac{s\sqrt{2}}{2}$
$s = \frac{30}{1 + \sqrt{2}} = 30\left(\sqrt{2} - 1\right)$
$P = 8s = 240\left(\sqrt{2} - 1\right)$

Step 2 Find apothem.

$2a = 30$
$a = 15$

Step 3 Find area.

$A = \frac{1}{2}aP$
$= \frac{1}{2}(15)240\left(\sqrt{2} - 1\right) \approx 745.6 \text{ in.}^2$

b. **Step 1** Find side length and perimeter.

$s = \frac{36}{1 + \sqrt{2}} = 36\left(\sqrt{2} - 1\right)$
$P = 8s = 288\left(\sqrt{2} - 1\right)$

Step 2 Find apothem.

$2a = 36$
$a = 18$

Step 3 Find area.

$A = \frac{1}{2}aP$
$= \frac{1}{2}(18)288\left(\sqrt{2} - 1\right) \approx 1073.6 \text{ in.}^2$

c. s, P, and a are all proportional to the given sign height. Therefore the area is proportional to the square of the height.

Percent increase $= \frac{A(36)}{A(30)} - 100\%$
$= \left(\frac{36}{30}\right)^2 - 100\% = 44\%$

40. $C = 1 = \pi d$
$d = \frac{1}{\pi} \approx 0.318 \text{ m}$

41. Possible answer: The circular table would fit at least as many people as the rectangle table. At the rectangle table, 2 people would fit at each of the 4-ft sides and 3 people would fit at each of the 6-ft sides, for a total of 10 people. Each person would have 2 ft of space. Between 10 and 12 people would fit around the circular table, with about 1 ft 9 in of space per person.

42. The circumference is proportional to the diameter. So, the circumference of the largest circle is the sum of the circumferences of the 4 smaller circles.

TEST PREP, PAGE 605

43. B
$s\left(1 + \sqrt{2}\right) = 2(6)$
$s = 12\left(\sqrt{2} - 1\right)$
$P = 8s$
$= 96\left(\sqrt{2} - 1\right) \approx 40 \text{ cm}$

44. F **45.** B

CHALLENGE AND EXTEND, PAGE 605

46. $C_{large} - C_{small} = 2\pi r_{large} - 2\pi r_{small}$
$= 2\pi(r_{large} - r_{small})$
$= 2\pi(5) = 10\pi$ units

47. $C = 2\pi r$
$r = \dfrac{C}{2\pi}$
$A = \pi r^2$
$= \pi\left(\dfrac{C}{2\pi}\right)^2 = \dfrac{C^2}{4\pi}$

48. As n gets very large, the regular n-gon begins to look like a circle with $C \approx P$ and $r \approx a$. The area of the polygon is $A = \frac{1}{2}aP$, which is close to $\frac{1}{2}rC$ or $\frac{1}{2}r(2\pi r) = \pi r^2$.

SPIRAL REVIEW, PAGE 605

49. $m = \dfrac{17 - 2}{10 - 5} = 3$
$y = 3x - 13$

50. $m = \dfrac{-1 - 2}{0 - (-3)} = -1$
$y = -x - 1$

51. By Isosceles Triangle Theorem, $\angle A \cong \angle C$
$m\angle B = 180 - (m\angle A + m\angle C)$
$= 180 - 2(28) = 124°$

52. $AB = BC$
$6x = 3x + 15$
$3x = 15$
$x = 5$
$AB = 6(5) = 30$

53. $A = \frac{1}{2}d_1d_2$
$14 = \frac{1}{2}(20)d_2$
$d_2 = 1.4$ cm

54. $A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(3 + 6)(4) = 18 \text{ yd}^2$

9-3 COMPOSITE FIGURES, PAGES 606–612

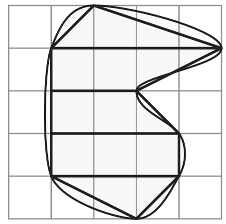
CHECK IT OUT! PAGES 606–608

1. Divide the figure into a rectangle and a triangle.
Area of rectangle (on left):
$A = bh = (37.5)(22.5) = 843.75 \text{ m}^2$
Area of triangle (on right):
$b = 75 - 37.5 = 37.5$ m
$h = \sqrt{62.5^2 - 37.5^2} = 50$ m
$A = \frac{1}{2}bh = \frac{1}{2}(37.5)(50) = 937.5 \text{ m}^2$
Shaded area:
$A = 843.75 + 937.5 = 1781.25 \text{ m}^2$

2. Area of circle:
$A = \pi r^2 = 9\pi \text{ in.}^2$
Area of square:
$A = s^2 = \left(3\sqrt{2}\right)^2 = 18 \text{ in.}^2$
Shaded area:
$A = 9\pi - 18 \approx 10.3 \text{ in.}^2$

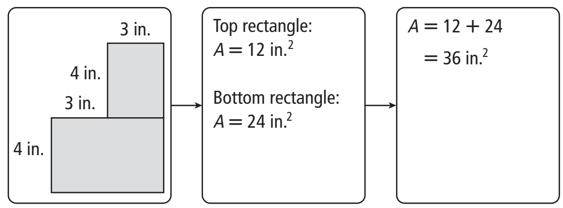
3. Xeriscape garden will save
$375.75(79 - 17) = 23{,}296.5$ gal per year.

4. Area of triangle a:
$A = \frac{1}{2}bh = \frac{1}{2}(4)(1) = 2$
Area of trapezoid b:
$A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(2 + 4)(1) = 3$
Area of trapezoid c:
$A = \frac{1}{2}(b_1 + b_2)h$
$= \frac{1}{2}(3 + 2)(1) = 2.5$
Area of rectangle d:
$A = bh = (3)(1) = 3$
Area of triangle e:
$A = \frac{1}{2}bh = \frac{1}{2}(3)(1) = 1.5$
Shaded area of composite figure is
$= 2 + 3 + 2.5 + 3 + 1.5 = 12 \text{ ft}^2$
So the shaded area is : $A \approx 12 \text{ ft}^2$.



THINK AND DISCUSS, PAGE 608

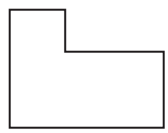
1. Possible answer: figure with a hole in the middle
2. Draw a composite figure with an area close to the area of the irregular shape. Divide the composite figure into simpler shapes, such as triangles, rectangles, and trapezoids. Find the sum of the areas of the simpler figures.
3.



EXERCISES, PAGES 609–612

GUIDED PRACTICE, PAGE 609

1. Possible answer:



2. Area of top rectangle:
$A = bh = (12 - 5 - 3)(4) = 16 \text{ cm}^2$
Area of bottom rectangle:
$A = bh = (12)(2) = 24 \text{ cm}^2$
Shaded area:
$A = 16 + 24 = 40 \text{ cm}^2$

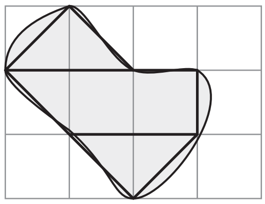
3. Area of semicircle:
$A = \frac{1}{2}\pi r^2 = \frac{1}{2}\pi(2)^2 = 2\pi \text{ ft}^2$
Area of triangle:
$A = \frac{1}{2}bh = \frac{1}{2}(4)(5) = 10 \text{ ft}^2$
Shaded area:
$A = 2\pi + 10 \approx 16.3 \text{ ft}^2$

4. Area of rectangle:
$A = bh = (18)(8) = 144 \text{ in.}^2$
Area of circle:
$A = \pi r^2 = \pi(3)^2 = 9\pi \text{ in.}^2$
Shaded area:
$A = 144 - 9\pi \approx 115.7 \text{ in.}^2$

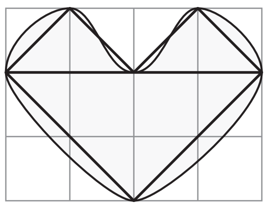
5. Area of rectangle:
$A = bh = (6)(3) = 18 \text{ m}^2$
Area of triangle at corner:
$A = \frac{1}{2}bh = \frac{1}{2}(6 - 5)(3 - 2) = 0.5 \text{ m}^2$
Shaded area:
$A = 18 - 0.5 = 17.5 \text{ m}^2$

6. Area of rectangle:
$A = bh = (4.5)(7) = 31.5 \text{ yd}^2$
Area of middle rectangle:
$A = bh = (2)(7 - 5.5) = 3 \text{ yd}^2$
Area of right rectangle:
$A = bh = (1.5)(7) = 10.5 \text{ yd}^2$
Area of carpet:
$A = 31.5 + 3 + 10.5 = 45 \text{ yd}^2$
Cost to install: 45(6) = \$270

7. Area of triangle a:
$A = \frac{1}{2}(2)(1) = 1$
Area of trapezoid b:
$A = \frac{1}{2}(2 + 3)(1) = 2.5$
Area of triangle c:
$A = \frac{1}{2}(2)(1) = 1$
Shaded area is:
$A = 1 + 2.5 + 1 = 4.5 \text{ in.}^2$



8. Area of triangle a:
$A = \frac{1}{2}(2)(1) = 1$
Area of triangle b:
$A = \frac{1}{2}(2)(1) = 1$
Area of triangle c:
$A = \frac{1}{2}(4)(2) = 4$
Shaded area is:
$A = 1 + 1 + 4 = 6 \text{ in.}^2$



PRACTICE AND PROBLEM SOLVING, PAGES 609–611

9. Area of rectangle (on left):
$A = (7)(6) = 42 \text{ mm}^2$
Area of triangle (on right):
$A = \frac{1}{2}(12 - 7)(6 - 3) = 7.5 \text{ mm}^2$
Shaded area:
$A = 42 + 7.5 = 49.5 \text{ mm}^2$

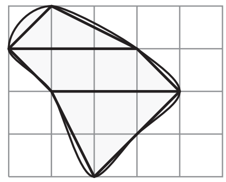
10. Area of each semicircle:
$A = \frac{1}{2}\pi(10)^2 = 50\pi \text{ yd}^2$
Area of rectangle:
$A = (40)(20) = 800 \text{ yd}^2$
Shaded area:
$A = 800 + 2(50\pi) \approx 1114.2 \text{ yd}^2$

11. Area of square:
$A = 2^2 = 4 \text{ m}^2$
Area of missing triangle:
$A = \frac{1}{2}(2)\sqrt{3} = \sqrt{3} \text{ m}^2$
Shaded area:
$A = 4 - \sqrt{3} \approx 2.3 \text{ m}^2$

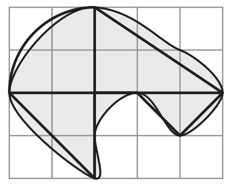
12. Area of trapzoid:
$A = \frac{1}{2}(51 + 24)(18) = 675 \text{ in.}^2$
Area of missing triangle:
$A = \frac{1}{2}(9)(12) = 54 \text{ in.}^2$
Shaded area:
$A = 675 - 54 = 621 \text{ in.}^2$

13. Area of each triangle:
$A = \frac{1}{2}(10)(22 - 15) = 35 \text{ ft}^2$
Area of rectangle:
$A = (30)(15) = 450 \text{ ft}^2$
Area of backdrop:
$A = 450 + 3(35) = 555 \text{ ft}^2$
Paint required $= 555 \div 90 \approx 6.2$ qt
Pat must buy 7 qt of paint.

14. $A \approx \frac{1}{2}(3)(1) + (3)(1) + \frac{1}{2}(3)(2)$
$\approx 1.5 + 3 + 3$
$\approx 7.5 \text{ m}^2$



15. $A \approx \frac{1}{4}\pi(2)^2 + \frac{1}{2}(3)(2) + \frac{1}{2}(2)(2) + \frac{1}{2}(2)(1)$
$\approx 9 \text{ m}^2$



16. Adding:
Upper rectangle: $A = (16)(3) = 48 \text{ cm}^2$
Middle rectangle: $A = (16 - 8)(3) = 24 \text{ cm}^2$
Lower rectangle: $A = (16)(4) = 64 \text{ cm}^2$
Composite:
$A = 48 + 24 + 64 = 136 \text{ cm}^2$
Subtracting:
Outer rectangle: $A = (16)(10) = 160 \text{ cm}^2$
Missing rectangle: $A = (8)(3) = 24 \text{ cm}^2$
Composite:
$A = (16)(10) - (8)(3) = 136 \text{ cm}^2$;
the answers are the same.

17. Adding:
Left trapezoid: $A = \frac{1}{2}(21 + 9)(18) = 270 \text{ in.}^2$
Right trapezoid: $A = \frac{1}{2}(21 + 9)(18) = 270 \text{ in.}^2$
Composite: $A = 270 + 270 = 540 \text{ in.}^2$
Subtracting:
Outer rectangle: $A = (18 + 18)(21) = 756 \text{ in.}^2$
Missing triangle: $A = \frac{1}{2}(18 + 18)(21 - 9) = 216 \text{ in.}^2$
Composite:
$A = 756 - 216 = 540 \text{ in.}^2$;
the answers are the same.

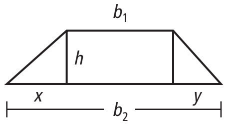
18. Divide composite into kite with diagonals $16 + 32 = 48$ m and $2(30) = 60$ m, and semicircles with radius $\frac{1}{2}\sqrt{16^2 + 30^2} = 17$ m. Area of composite is $A = \frac{1}{2}(48)(60) + 2\left(\frac{1}{2}\pi(17)^2\right) = (1440 + 289\pi) \text{ m}^2$.

19. Area of triangle:
$A = \frac{1}{2}(10)\left(5\sqrt{3}\right) = 25\sqrt{3} \text{ in.}^2$
Area of each semicircle:
$A = 3\left(\frac{1}{2}\pi(5)^2\right) = \frac{75\pi}{2} \text{ in.}^2$
Area of composite:
$A = \left(25\sqrt{3} + \frac{75\pi}{2}\right) \text{ in.}^2$

20. Outer circle: $A = \pi(8)^2 = 64\pi$ cm^2
Inner circle: $A = \pi(5)^2 = 25\pi$ cm^2
Composite:
$A = 64\pi - 25\pi = 39\pi$ cm^2

21. Possible answer: 35,000 mi^2

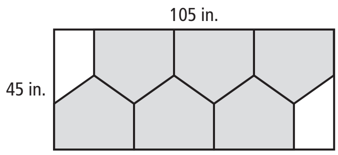
22. Let b_1 and b_2 be the bases of the trapezoid; let h be the height of the trapezoid, triangles, and rectangle; let x and y be the bases of the triangles. Then $x + b_1 + y = b_2$. The area of the trapezoid is:



$$A = \tfrac{1}{2}xh + b_1h + \tfrac{1}{2}yh$$
$$= \tfrac{1}{2}h(x + 2b_1 + y)$$
$$= \tfrac{1}{2}h(b_1 + x + b_1 + y)$$
$$= \tfrac{1}{2}h(b_1 + b_2).$$

23a. Lower rectangle:
$A = (30)(15) = 450$ in.^{2}
Upper triangle:
$A = \frac{1}{2}(30)(30 - 15) = 225$ in.^{2}
Composite:
$A = 450 + 225 = 675$ in.^{2}

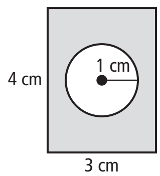
b.



c. Area of metal left:
$A = (105)(45) - 6(675) = 675$ in.^{2}

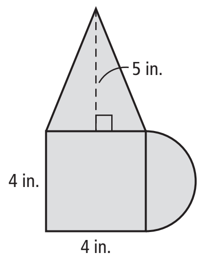
24. $A = (3)(4) - \pi(1)^2$
$= (12 - \pi)$ cm^2

Possible drawing:



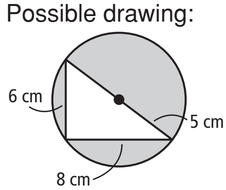
25. $A = (4)^2 + \frac{1}{2}(4)(5)$
$+ \frac{1}{2}\pi(2)^2$
$= (26 + 2\pi)$ in.^{2}

Possible drawing:



26. $A = \pi(5)^2 - \frac{1}{2}(8)(6)$
$= (25\pi - 24)$ cm^2

Possible drawing:



27. $A_1 = \frac{1}{4}\pi(2)^2 = \pi$
$A_2 = \frac{1}{2}(2)(2) = 2$
$A_3 = \frac{1}{2}\pi\left(\sqrt{2}\right)^2 = \pi$
$A_4 = A_3 - (A_2 - A_1)$
$= \pi - (\pi - 2) = 2$

28. Possible answer:
$A \approx 13.4$ cm^2

29. Possible answer:
$A \approx 10$ cm^2

30. Possible answer: Use addition to find the area of a figure that can be divided into triangles, rectangles, trapezoids, and semicircles. Use subtraction to find the area of a figure that has a shape removed from its interior.

TEST PREP, PAGES 611–612

31. A

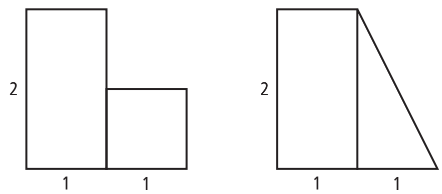
32. G
Upper trapezoid:
$A = \frac{1}{2}(7.8 + 5.4)(2.5) = 16.5$ cm^2
Lower triangle:
$A = \frac{1}{2}(2.2)(2) = 2.2$ cm^2
Composite:
$A = 16.5 + 2.2 \approx 19$ cm^2

33. C
$A = 105(45) - 45(30) - \frac{1}{2}(20)(45) = 2925$ m^2

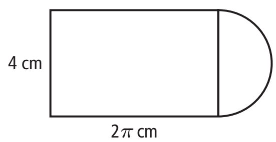
CHALLENGE AND EXTEND, PAGE 612

34. $A = \pi R^2 - \pi r^2 = \pi\left(R^2 - r^2\right)$

35. Possible answer:



36. Possible answer:



SPIRAL REVIEW, PAGE 612

37. Sale price = (100% − 20%)(19.95)
= 0.8(19.95) = $15.96

38. Sale price = (100% − 15%)34.60
= 0.85(34.60) = $29.41

39. $\frac{BC}{AB} = \frac{GF}{AG}$
$\frac{BC}{2.8} = \frac{1}{2}$
$2BC = 2.8$
$BC = 1.4$

40. $\frac{CD}{AB} = \frac{FE}{AG}$
$\frac{CD}{2.8} = \frac{0.5}{2}$
$2CD = 0.5(2.8) = 1.4$
$CD = 0.7$

41. $b = 3$ cm, $h = \frac{3\sqrt{3}}{2}$ cm
$A = \frac{1}{2}(3)\left(\frac{3\sqrt{3}}{2}\right) \approx 3.9$ cm^2

42. $\dfrac{s\sqrt{3}}{2} = a = 4\sqrt{3}$

$s\sqrt{3} = 8\sqrt{3}$

$s = 8$ m

$P = 6(8) = 48$ m

$A = \frac{1}{2}aP = \frac{1}{2}\left(4\sqrt{3}\right)(48) \approx 166.3$ m^2

9-3 GEOMETRY LAB: DEVELOP PICK'S THEOREM FOR AREA OF LATTICE POLYGONS, PAGE 613

TRY THIS, PAGE 613

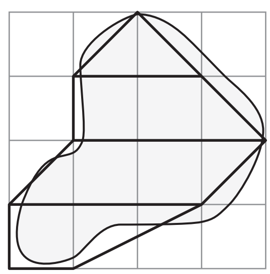
1. Possible answer: $A = \frac{1}{2}B + I - 1$
2. Check students' work.
3. Possible answer: $A \approx \frac{1}{2}(6) + 3 - 1 = 5$ units^2
4. Split outer figure into 2 trapezoids and 2 triangles
   Upper left trapezoid: $A = \frac{1}{2}(2 + 1)(2) = 3$
   Upper right trapezoid: $A = \frac{1}{2}(2 + 1)(1) = 1.5$
   Lower left triangle: $A = \frac{1}{2}(2)(1) = 1$
   Lower right triangle: $A = \frac{1}{2}(2)(2) = 2$
   Missing square: $A = 1^2 = 1$
   Shaded area: $A = 3 + 1.5 + 1 + 2 - 1 = 6.5$ units^2
   $A_{formula} = \frac{1}{2}B + I - 1 = \frac{1}{2}(11) + (1) - 1 = 5.5$; no

9A MULTI-STEP TEST PREP, PAGE 614

1. Area of each circle: $A = \pi(15)^2 = 225\pi$ in.^{2}
   Area of sheet: $A = (90)(60) = 5400$ in.^{2}
   Area left over: $A = 5400 - 6(225\pi) \approx 1159$ in.^{2}
2. For stop sign, $30 = 2a = s(\sqrt{2} + 1)$
   $a = 15$; $s = 30(\sqrt{2} - 1)$; $P = 8s = 240(\sqrt{2} - 1)$
   Area of each sign:
   $A = \frac{1}{2}aP = \frac{1}{2}(15)(240(\sqrt{2} - 1)) = 1800(\sqrt{2} - 1)$ in.^{2}
   Area left over: $A = 5400 - 1800(\sqrt{2} - 1) \approx 926$ in.^{2}
3. For yield sign, $b = 30$, $h = 15\sqrt{3}$
   Area of each sign: $A = \frac{1}{2}(30)15\sqrt{3} = 225\sqrt{3}$ in.^{2}
   Area left over: $A = 5400 - 10(225R3) \approx 1503$ in.^{2}
4. Stop sign results in least amount of waste.

9A READY TO GO ON? PAGE 615

1. $A = bh = (10)(5) = 50$ ft^2
2. $A = bh$
   $(24x^2 + 8x) = b(4x)$
   $4x(6x + 2) = b(4x)$
   $b = (6x + 2)$ m
3. $A = \frac{1}{2}d_1d_2$
   $126 = \frac{1}{2}d_1(12) = 6d_1$
   $d_1 = 21$ ft
4. $d_1 = 18$ cm, $d_2 = 2\sqrt{15^2 - 9^2} = 24$ cm
   $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(18)(24) = 216$ cm^2
5. Green triangle:
   $P = 2 + 1 + \sqrt{2^2 + 1^2} = 3 + \sqrt{5} \approx 5.2$ cm
   $A = \frac{1}{2}(2)(1) = 1$ cm^2
   Blue trapezoid:
   $P = 2 + 1 + 1 + \sqrt{2} = 4 + \sqrt{2} \approx 5.4$ cm
   $A = \frac{1}{2}(2 + 1)(1) = 1.5$ cm^2
   Yellow parallelogram:
   $P = \sqrt{2} + \sqrt{5} + \sqrt{2} + \sqrt{5} = 2\left(\sqrt{2} + \sqrt{5}\right) \approx 7.3$ cm
   $A = \left(\sqrt{2}\right)\left(1.5\sqrt{2}\right) = 3$ cm^2
6. $C = \pi d = 18\pi$ in.
7. $A = \pi r^2 = \pi(6x)^2 = 36x^2\pi$ ft^2
8. $\dfrac{s\sqrt{3}}{2} = a = 6$
   $3s = 12\sqrt{3}$
   $s = 4\sqrt{3}$
   $P = 6s = 24\sqrt{3}$
   $A = \frac{1}{2}aP = \frac{1}{2}(6)24\sqrt{3} \approx 124.7$ ft^2
9. $\tan 36° = \dfrac{\frac{s}{2}}{a} = \dfrac{6}{a}$
   $a = \dfrac{6}{\tan 36°}$
   $P = 5(12) = 60$
   $A = \frac{1}{2}aP = \dfrac{1}{2}\left(\dfrac{6}{\tan 36°}\right)(60) \approx 247.7$ m^2
10. Triangle: $A = \frac{1}{2}(12)(12) = 72$ cm^2
    Square: $A = 12^2 = 144$ cm^2
    Missing semicircle: $A = \frac{1}{2}\pi(6)^2 = 18\pi$ cm^2
    Shaded area: $A = 72 + 144 - 18\pi \approx 159.5$ cm^2
11. Outer rectangle: $A = (16)(12) = 192$ ft^2
    Missing rectangle: $A = (12)(4) = 48$ ft^2
    Shaded area: $A = 192 - 48 = 144$ ft^2
12. Triangle a:
    $\frac{1}{2}(2)(1) = 1$
    Trapezoid b:
    $\frac{1}{2}(3 + 2)(1) = 2.5$
    Parallelogram c:
    $(3)(1) = 3$
    Trapezoid d:
    $\frac{1}{2}(1 + 3)(1) = 2$

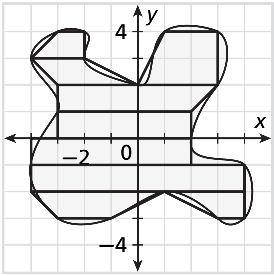
    

    Area of garden:
    $A \approx 1 + 2.5 + 3 + 2 = 8.5$ yd^2
    Cost of grass:
    $8.5(\$6.50) \approx \$55$

9-4 PERIMETER AND AREA IN THE COORDINATE PLANE, PAGES 616–621

CHECK IT OUT! PAGES 616–618

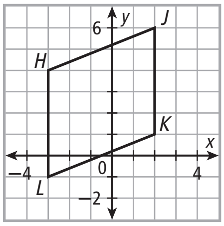
1. Method 1



The area is about $1.5 + 2.5 + 4.5 + 5.5 + 5 + 6 + 8 + 3.5 + 1.5 \approx 38$ units^2.

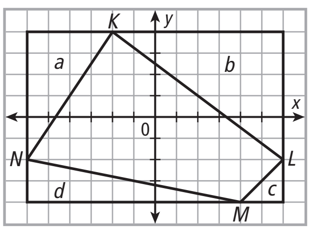
Method 2
There are about 32 whole squares and 8 half-squares, so the area is about $32 + \frac{1}{2}(11) \approx 38$ units^2.

2. Step 1 Draw the polygon.



Step 2 *HJKL* appears to be a parallelogram.
$\overline{HL}$ and $\overline{JK}$ are vertical, therefore parallel.
slope of $\overline{HJ} = \dfrac{6-4}{2-(-3)} = \dfrac{2}{5}$
slope of $\overline{LK} = \dfrac{1-(-1)}{2-(-3)} = \dfrac{2}{5}$
$\overline{HJ}$ and $\overline{LK}$ are also $\parallel$, so *HJKL* is a parallelogram.
Step 3 Let $\overline{HL}$ be the base; let $c = HJ$
$b = 4 - (-1) = 5$; $h = 2 - (-3) = 5$;
$c = \sqrt{(2-(-3))^2 + (6-4)^2} = \sqrt{29}$
$P = 2b + 2c = \left(20 + 2\sqrt{29}\right) \approx 20.8$ units
$A = bh = (5)(5) = 25$ units^2.

3.



Area of rectangle: $A = bh = (12)(8) = 96$ units^2
Area of triangles:
a: $A = \frac{1}{2}bh = \frac{1}{2}(4)(6) = 12$ units^2
b: $A = \frac{1}{2}bh = \frac{1}{2}(8)(6) = 24$ units^2
c: $A = \frac{1}{2}bh = \frac{1}{2}(2)(2) = 2$ units^2
d: $A = \frac{1}{2}bh = \frac{1}{2}(10)(2) = 10$ units^2
Area of polygon: $96 - 12 - 24 - 2 - 10 = 48$ units^2

4. Check students' work.

THINK AND DISCUSS, PAGE 619

1. One way: draw a composite figure that approximates the irregular shape and then find its area. Another way: count grid squares, estimating half squares.

2. If the quadrilateral is a parallelogram, rectangle, or trapezoid, use the Distance Formula to find the height and base or bases. If it is a rhombus or kite, use the Distance Formula to find the lengths of the diagonals.

3.

Finding the Area

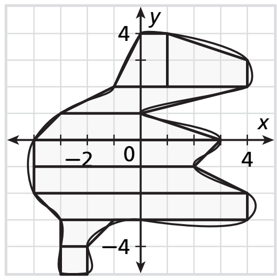
Using the formula:
The base of the parallelogram is $3\sqrt{5}$ units, and the height is $2\sqrt{5}$ units, so the area is $A = (3\sqrt{5})(2\sqrt{5}) = 30$ units^2.

By subtracting:
Draw a rectangle with an area of 72 units^2 around the figure. Subtract the areas of the triangles in the corners:
$A = 72 - \frac{1}{2}(9)(2) - \frac{1}{2}(8)(3) - \frac{1}{2}(9)(2) - \frac{1}{2}(8)(3) = 30$ units^2.

EXERCISES, PAGES 619–621

GUIDED PRACTICE, PAGE 619

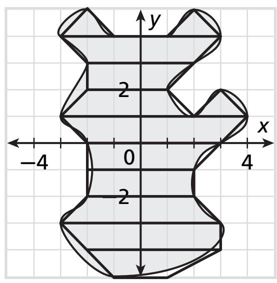
1. Method 1



The area is about $3.25 + 4.75 + 4 + 5 + 6.5 + 7 + 7.5 + 1.5 + 1 \approx 40.5$ units^2.

Method 2
There are about 33 whole squares and 15 half-squares, so the area is about $33 + \frac{1}{2}(15) \approx 40.5$ units^2.

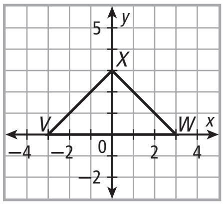
2. Method 1



The area is about $1 + 1 + 5 + 3.5 + 4 + 1 + 5.5 + 4.5 + 4 + 4.5 + 5.5 + 3.5 \approx 43$ units^2.

Method 2
There are about 32 whole squares and 22 half-squares, so the area is about $32 + \frac{1}{2}(22) \approx 43$ units^2.

3. Step 1 Draw the polygon.



Step 2 *VWX* appears to be an isosceles triangle.

$VX = \sqrt{(0-(-3))^2 + (3-0)^2} = \sqrt{18} = 3\sqrt{2}$

$WX = \sqrt{(3-0)^2 + (0-3)^2} = \sqrt{18} = 3\sqrt{2}$

$VX = WX$, so *VWX* is an isosceles triangle.

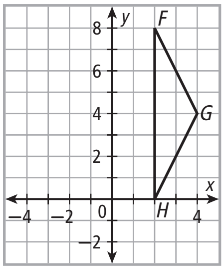
Step 3 Let $\overline{VW}$ be base; let $a = VX$, $c = WX$.

$b = 3 - (-3) = 6$; $h = 3 - 0 = 3$; $a = c = 3\sqrt{2}$

$P = a + b + c = 3\sqrt{2} + 6 + 3\sqrt{2} = (6 + 6\sqrt{2})$ units

$A = \frac{1}{2}bh = \frac{1}{2}(6)(3) = 9 \text{ units}^2$

4. Step 1 Draw the polygon.



Step 2 *FGH* appears to be an isosceles triangle.

$FG = \sqrt{(4-2)^2 + (4-8)^2} = \sqrt{20} = 2\sqrt{5}$

$GH = \sqrt{(4-2)^2 + (4-0)^2} = \sqrt{20} = 2\sqrt{5}$

$FG = GH$, so *FGH* is an isosceles triangle.

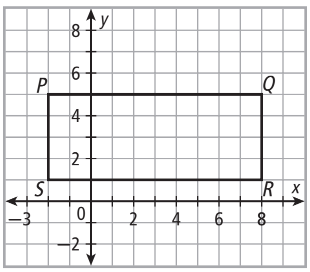
Step 3 Let $\overline{FH}$ be the base; let $a = FG$, $c = GH$.

$b = 8 - 0 = 8$; $h = 4 - 2 = 2$; $a = c = 2\sqrt{5}$

$P = a + b + c = 2\sqrt{5} + 8 + 2\sqrt{5} = (8 + 4\sqrt{5})$ units

$A = \frac{1}{2}bh = \frac{1}{2}(8)(2) = 8 \text{ units}^2$

5. Step 1 Draw the polygon.



Step 2 *PQRS* appears to be a rectangle.

$\overline{PQ}$ and $\overline{RS}$ are horizontal; $\overline{QR}$ and $\overline{SP}$ are vertical.

Consecutive sides are perpendicular, so *PQRS* is a rectangle.

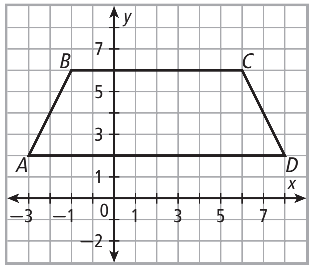
Step 3 Let $\overline{PQ}$ be the base; let $\overline{QR}$ be height.

$b = 8 - (-2) = 10$; $h = 5 - 1 = 4$

$P = 2b + 2h = 2(10) + 2(4) = 28$ units

$A = bh = (10)(4) = 40 \text{ units}^2$

6. Step 1 Draw the polygon.



Step 2 *ABCD* appears to be an isosceles trapezoid.

$\overline{AD}$ and $\overline{BC}$ are horizontal, so *ABCD* is a trapezoid.

$AB = \sqrt{(-2-(-4))^2 + (6-2)^2} = \sqrt{20} = 2\sqrt{5}$

$CD = \sqrt{(8-6)^2 + (2-6)^2} = \sqrt{20} = 2\sqrt{5}$

$AB = CD$, so trapezoid *ABCD* is isosceles.

Step 3 Let $\overline{AD}$ and $\overline{BC}$ be the bases;

let $a = AB$, $c = CD$

$b_1 = 8 - (-4) = 12$; $b_2 = 6 - (-2) = 8$;

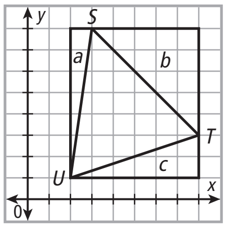
$h = 6 - 2 = 4$; $a = c = 2\sqrt{5}$

$P = a + b_1 + b_2 + c$

$= 2\sqrt{5} + 8 + 12 + 2\sqrt{5} = (20 + 4\sqrt{5})$ units

$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(12 + 8)(4) = 40 \text{ units}^2$

7.



Area of rectangle: $A = bh = (6)(7) = 42 \text{ units}^2$

Area of triangles:

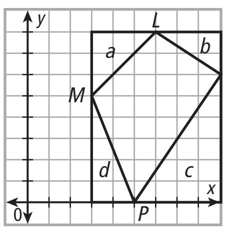
a: $A = \frac{1}{2}bh = \frac{1}{2}(1)(7) = 3.5 \text{ units}^2$

b: $A = \frac{1}{2}bh = \frac{1}{2}(5)(5) = 12.5 \text{ units}^2$

c: $A = \frac{1}{2}bh = \frac{1}{2}(6)(2) = 6 \text{ units}^2$

Area of $\triangle STU$: $42 - 3.5 - 12.5 - 6 = 20 \text{ units}^2$

8.



Area of rectangle: $A = bh = (6)(8) = 48 \text{ units}^2$

Area of triangles:

a: $A = \frac{1}{2}bh = \frac{1}{2}(3)(3) = 4.5 \text{ units}^2$

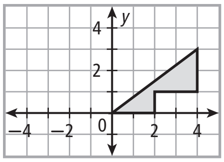
b: $A = \frac{1}{2}bh = \frac{1}{2}(3)(2) = 3 \text{ units}^2$

c: $A = \frac{1}{2}bh = \frac{1}{2}(4)(6) = 12 \text{ units}^2$

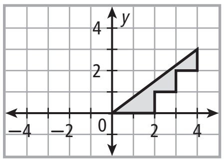
d: $A = \frac{1}{2}bh = \frac{1}{2}(2)(5) = 5 \text{ units}^2$

Area of polygon: $48 - 4.5 - 3 - 12 - 5 = 23.5 \text{ units}^2$

9. First polygon:
$A = \frac{1}{2}bh = \frac{1}{2}(4)(3) = 6$ units^2
$P = 4 + 3 + \sqrt{4^2 + 3^2} = 12$ units
Second polygon:
The area is reduced by 1 unit^2:
$A = 6 - 1 = 5$ units^2
The perimeter is unchanged: $P = 12$ units
Third Polygon: remove one square at edge:

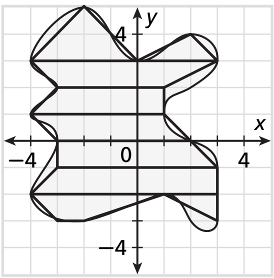


Fourth Polygon: remove one more square at edge:



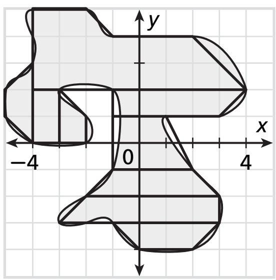
PRACTICE AND PROBLEM SOLVING, PAGE 620

10. Method 1



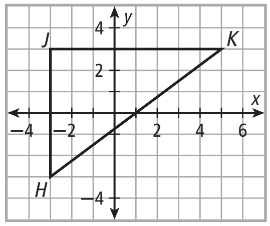
The area is about $4 + 1.5 + 5.5 + 4.5 + 5.5 + 5.5 + 6.5 + 3.5 + 1.5 + 0.5 \approx 38.5$ units^2.
Method 2
There are about 28 whole squares and 21 half-squares, so the area is about $28 + \frac{1}{2}(21) \approx 38.5$ units^2.

11. Method 1



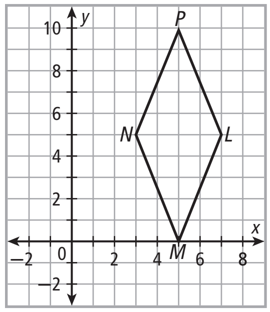
The area is about $2.5 + 15.5 + 2 + 2 + 1.5 + 4.5 + 4 + 4 + 4.5 + 3 \approx 43.5$ units^2.
Method 2
There are about 35 whole squares and 17 half-squares, so the area is about $35 + \frac{1}{2}(17) \approx 43.5$ units^2.

12. Step 1 Draw the polygon.



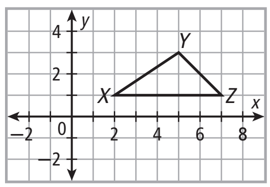
Step 2 *HJK* is a right triangle.
Step 3 Let $\overline{HJ}$ be the height and $\overline{JK}$ be the base; let $c = HK$.
$h = 3 - (-3) = 6$; $b = 5 - (-3) = 8$;
$c = \sqrt{6^2 + 8^2} = 10$
$P = b + h + c = 6 + 8 + 10 = 24$ units
$A = \frac{1}{2}bh = \frac{1}{2}(6)(8) = 24$ units^2

13. Step 1 Draw the polygon.



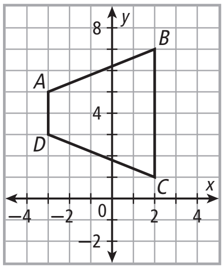
Step 2 *LMNP* appears to be a rhombus.
$LM = \sqrt{(7-5)^2 + (5-0)^2} = \sqrt{29}$
$MN = \sqrt{(3-5)^2 + (5-0)^2} = \sqrt{29}$
$NP = \sqrt{(5-3)^2 + (10-5)^2} = \sqrt{29}$
$LP = \sqrt{(5-7)^2 + (10-5)^2} = \sqrt{29}$
All 4 sides are congruent, so *LMNP* is a rhombus.
Step 3 Let $d_1 = LN$ and $d_2 = MP$.
$d_1 = 7 - 3 = 4$; $d_2 = 10 - 0 = 10$
$P = 4\sqrt{29}$ units
$A = \frac{1}{2}d_1d_2 = \frac{1}{2}(4)(10) = 20$ units^2

14. Step 1 Draw the polygon.



Step 2 *XYZ* appears to be a scalene triangle.
$XY = \sqrt{(5-2)^2 + (3-1)^2} = \sqrt{13}$
$YZ = \sqrt{(5-7)^2 + (3-1)^2} = \sqrt{8} = 2\sqrt{2}$
$XZ = 7 - 2 = 5$
The 3 sides are different in length, so *XYZ* is a scalene triangle.
Step 3 $b = XZ = 5$; $h = 3 - 1 = 2$
$P = \left(5 + 2\sqrt{2} + \sqrt{13}\right)$ units
$A = \frac{1}{2}bh = \frac{1}{2}(5)(2) = 5$ units^2

15. Step 1 Draw the polygon.



Step 2 *TUVW* appears to be an isosceles trapezoid.

$AB = \sqrt{(-3-2)^2 + (5-7)^2} = \sqrt{25+4} = \sqrt{29}$

$CD = \sqrt{(2-(-3))^2 + (1-3)^2} = \sqrt{25+4} = \sqrt{29}$

$AB = CD$, so trapezoid $ABCD$ is isosceles.

Step 3 Let $\overline{AD}$ and $\overline{BC}$ be the bases;
let $a = AB$, $c = CD$.

$b_1 = 5 - 3 = 2$; $b_2 = 7 - 1 = 6$;

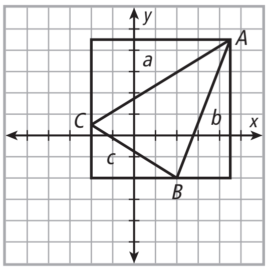
$h = 2 - (-3) = 5$; $a = c = \sqrt{29}$

$P = a + b_1 + b_2 + c$

$= \sqrt{29} + 2 + 6 + \sqrt{29} = \left(8 + 2\sqrt{29}\right)$units

$A = \frac{1}{2}(b_1 + b_2)h = \frac{1}{2}(2 + 6)(5) = 20$ units^2

16.



Area of rectangle: $A = bh = (13)(13) = 169$ units^2

Area of triangles:

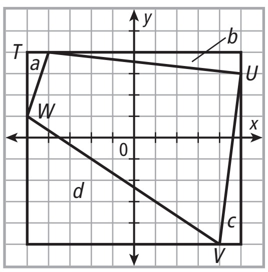
a: $A = \frac{1}{2}bh = \frac{1}{2}(13)(8) = 52$ units^2

b: $A = \frac{1}{2}bh = \frac{1}{2}(5)(13) = 32.5$ units^2

c: $A = \frac{1}{2}bh = \frac{1}{2}(8)(5) = 20$ units^2

Area of $\triangle ABC$: $169 - 52 - 32.5 - 20 = 64.5$ units^2

17.



Area of rectangle: $A = bh = (10)(9) = 90$ units^2

Area of triangles:

a: $A = \frac{1}{2}bh = \frac{1}{2}(1)(3) = 1.5$ units^2

b: $A = \frac{1}{2}bh = \frac{1}{2}(9)(1) = 4.5$ units^2

c: $A = \frac{1}{2}bh = \frac{1}{2}(1)(8) = 4$ units^2

d: $A = \frac{1}{2}bh = \frac{1}{2}(9)(6) = 27$ units^2

Area of polygon: $90 - 1.5 - 4.5 - 4 - 27 = 53$ units^2

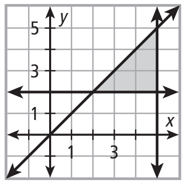
18. Figure A: $A = 1(2) + 3(2) = 8$ units^2

Figure B: $A = \left(2\sqrt{2}\right)\left(\sqrt{2}\right) + 1(1) + 4(1) = 9$ units^2

Figure C: $A = 1(3) + 4(1) + \left(\sqrt{2}\right)\left(\sqrt{2}\right) = 9$ units^2

Figures B and C have same area.

19.

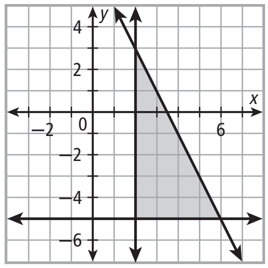


$b = 3$, $h = 3$, $c = \sqrt{18} = 3\sqrt{2}$

$P = 3 + 3 + 3\sqrt{2} = \left(6 + 3\sqrt{2}\right)$ units

$A = \frac{1}{2}(3)(3) = 4.5$ units^2

20.



$b = 4$, $h = 8$, $c = \sqrt{4^2 + 8^2} = \sqrt{80} = 4\sqrt{5}$

$P = 4 + 8 + 4\sqrt{5} = \left(12 + 4\sqrt{5}\right)$ units

$A = \frac{1}{2}(4)(8) = 16$ units^2

21a. $A = bh = (1 \text{ h})(20 \text{ mi/h}) = 20$ mi^2

b. Upper trapezoid: $A = \frac{1}{2}(4 + 2)(1)(20 \text{ mi}) = 60$ mi

Lower trapezoid: $A = \frac{1}{2}(5 + 4)(1)(20 \text{ mi}) = 90$ mi

Shaded area: $A \approx 60 + 90 \approx 150$ mi^2

c. The area represents the distance the boat traveled in 5 h.

22. Possible answer: Draw polygon *ABCDE*. Draw a rectangle with base 6 and height 5 around polygon. The rectangle has area 30 units^2, and regions not included in *ABCDE* have areas 6, 3, 1, and 3.5 units^2; so the area of *ABCDE* is $30 - 6 - 3 - 1 - 3.5 = 16.5$ units^2.

23a. $A = bh = (3)(2) = 6$ units^2

b. Possible answer:

ABC: $A = \frac{1}{2}bh$

$6 = \frac{1}{2}(3)h = 1.5h$

$h = 4$

y-coordinate of A, B is 5, so y-coordinate of C is $5 - 4 = 1$ or $5 + 4 = 9$. Therefore, let the *y*-coordinate of *C* be 1 or 9. A possible coordinate for *C* is $C = (2, 1)$.

DEFG: the *x*-coordinate of *D* must be 8.

$A = \frac{1}{2}d_1d_2$

$6 = \frac{1}{2}(2)d_2$

$d_2 = 6$

The *y*-coordinate of *F* is 8, so the *y*-coordinate of *D* is $8 - 6 = 2$; $D = (8, 2)$.

TEST PREP, PAGE 621

24. D

$r = \sqrt{(3-0)^2 = (4-0)^2} = 5$

$A = \pi r^2 = \pi(5)^2 \approx 78.5 \text{ units}^2$

25. J

The area of ABC would be:

$\frac{1}{2}bh = \frac{1}{2}(3-1)(5-(-3)) = \frac{1}{2}(2)(8) = 8 \text{ units}^2$.

26a. Mike estimated the area by using a square with vertices at (−4, 4), (4, 4), (4, −4), and (−4, −4). This does not include the area at corners of the graph.

b. The composite figure is made of a square with area 64 units^2, 4 triangles each with area 2.5 units^2, and 4 other triangles each with area 2 units^2. The area is $64 + 4(2.5) + 4(2) = 82$ units^2.

c. The irregular shape encloses a square with area 64 units^2 and is enclosed in a square with area 100 units^2. The average of the areas is $\frac{64+100}{2} = 82$ units^2.

CHALLENGE AND EXTEND, PAGE 621

27. Split the area into a triangle to the left of the *y*-axis and 3 trapezoids to the right of the *y*-axis.

$A \approx \frac{1}{2}(2)(1) + \frac{1}{2}(1+2)(1) + \frac{1}{2}(2+4)(1) + \frac{1}{2}(1+2)(1)$

$= 1 + 1.5 + 3 + 6 \approx 10.5 \text{ units}^2$

28. Split the area into a triangle and 2 trapezoids.

$A \approx \frac{1}{2}(1)(1) + \frac{1}{2}(1+4)(1) + \frac{1}{2}(4+9)(1)$

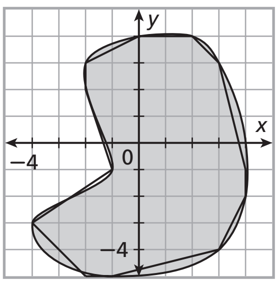
$= 0.5 + 2.5 + 6.5 \approx 9.5 \text{ units}^2$

29. Split the area into a triangle and 2 trapezoids.

$A \approx \frac{1}{2}(1)(1) + \frac{1}{2}(1+2)(3) + \frac{1}{2}(2+3)(5)$

$= 0.5 + 4.5 + 12.5 \approx 17.5 \text{ units}^2$

30.



Starting at (−2, 3),

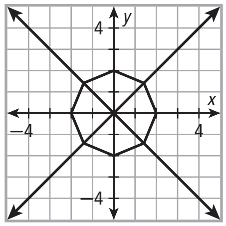
$P \approx \sqrt{5} + 2 + \sqrt{2} + \sqrt{17} + 1 + \sqrt{5} + \sqrt{17} + 1$

$+ \sqrt{2} + \sqrt{2} + \sqrt{13} + \sqrt{10} + 1$

$\approx 28.7 \text{ units}^2$

SPIRAL REVIEW, PAGE 621

31.



The vertices of the octagon are (0, 1), $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, (1, 0),

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, (0, −1), $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, (−1, 0), $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$s = \sqrt{\left(0 - \frac{\sqrt{2}}{2}\right)^2 + \left(1 - \frac{\sqrt{2}}{2}\right)^2}$

$= \sqrt{\frac{1}{2} + 1 - \sqrt{2} + \frac{1}{2}} = \sqrt{2 - \sqrt{2}}$

$P = 8s = 8\sqrt{2 - \sqrt{2}}$ units

The coordinates of the midpoint of the upper-right side are $\left(\frac{\sqrt{2}}{4}, \frac{1}{2} + \frac{\sqrt{2}}{4}\right)$. Therefore,

$a = \sqrt{\left(\frac{\sqrt{2}}{4}\right)^2 + \left(\frac{1}{2} + \frac{\sqrt{2}}{4}\right)^2}$

$= \sqrt{\frac{1}{8} + \frac{1}{4} + \frac{\sqrt{2}}{4} + \frac{1}{8}}$

$= \sqrt{\frac{1}{2} + \frac{\sqrt{2}}{4}} = \frac{1}{2}\sqrt{2 + \sqrt{2}}$

$A = \frac{1}{2}aP$

$= \frac{1}{2}\left(\frac{1}{2}\sqrt{2 + \sqrt{2}}\right)\left(8\sqrt{2 - \sqrt{2}}\right)$

$= 2\sqrt{\left(2 - \sqrt{2}\right)\left(2 + \sqrt{2}\right)}$

$= 2\sqrt{2^2 - \left(\sqrt{2}\right)^2} = 2\sqrt{2} \text{ units}^2$

32. $-4 < x + 4 < 7$

$-4 - 3 < x < 7 - 3$

$-7 < x < 4$

33. $0 < 2a + 4 < 10$

$0 - 4 < 2a < 10 - 4$

$-4 < 2a < 6$

$-2 < a < 3$

34. $12 \le -2m + 10 \le 20$

$2 \le -2m \le 10$

$-10 \le 2m \le -2$

$-5 \le m \le -1$

35.

Statements	Reasons
1. $\overline{DC} \cong \overline{BC}$, $\angle DCA \cong \angle ACB$	1. Given
2. $\overline{AC} \cong \overline{AC}$	2. Reflex Prop. of $\cong$
3. $\triangle DCA \cong \triangle BCA$	3. SAS
4. $\angle DAC \cong \angle BAC$	4. CPCTC

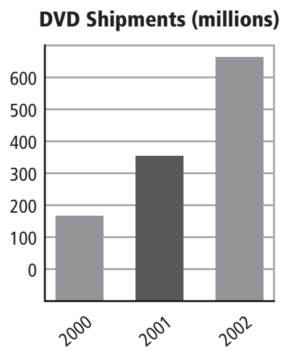
36. $C = 2\pi r$
$16\pi = 2\pi r$
$r = 8$ cm
$A = \pi r^2$
$= \pi(8)^2 = 64\pi$ cm^2

37. $A = \pi r^2$
$121\pi = \pi r^2$
$121 = r^2$
$r = 11$ ft
$d = 2r$
$= 2(11) = 22$ ft

9-5 EFFECTS OF CHANGING DIMENSIONS PROPORTIONALLY, PAGES 622–627

CHECK IT OUT! PAGES 622–624

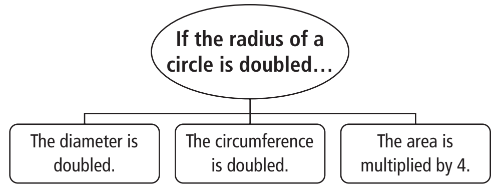
1. Original dimensions: $A = bh$ $= 7(4) = 28$ ft^2
   Triple the height: $A = bh$ $= 7(12) = 84$ ft^2
   $84 = 3(28)$; if height is tripled, area is also tripled.
2. Original dimensions:
   $b = 7 - 2 = 5$, $h = 5 - 1 = 4$, $c = \sqrt{5^2 + 4^2} = \sqrt{41}$
   $P = 5 + 4 + \sqrt{41} = \left(9 + \sqrt{41}\right)$ units
   $A = \frac{1}{2}(5)(4) = 10$ units^2
   Dimensions multiplied by 3:
   $b = 3(5) = 15$, $h = 3(4) = 12$, $c = 3\sqrt{41}$
   $P = 15 + 12 + 3\sqrt{41} = \left(27 + 3\sqrt{41}\right)$ units
   $A = \frac{1}{2}(15)(12) = 90$ units^2
   Perimeter is multiplied by 3. Area is multiplied by 3^2, or 9.
3. Original perimeter is $P = 4s = 36$ mm; side length is 9 mm, and area is $A = 9^2 = 81$ mm^2. If the area is multiplied by $\frac{1}{2}$, the new area is 40.5 mm.
   $s^2 = 40.5$
   $s = \sqrt{40.5} = \frac{9}{\sqrt{2}}$
   $4.5 = \frac{1}{\sqrt{2}}(9)$; side length is multiplied by $\frac{1}{\sqrt{2}}$.
4. Possible answer:



THINK AND DISCUSS, PAGE 624

1. If one dimension of a rectangle is multiplied by *a*, the area is also multiplied by *a*. If both dimensions of a rectangle are multiplied by *a* the perimeter is multiplied by *a*.

2.



EXERCISES, PAGES 625–627

GUIDED PRACTICE, PAGE 625

1. Original dimensions: $A = \frac{1}{2}bh$ $= \frac{1}{2}(21)(12) = 126$ m^2
   Double the height: $A = \frac{1}{2}bh$ $= \frac{1}{2}(21)(24) = 252$ m^2
   $252 = 2(126)$; if the height is doubled, the area is also doubled.
2. Original dimensions: $A = \frac{1}{2}(b_1 + b_2)h$ $= \frac{1}{2}(12 + 18)(5)$ $= 75$ cm^2
   Multiply height by $\frac{1}{3}$: $A = \frac{1}{2}(b_1 + b_2)h$ $= \frac{1}{2}(12 + 18)\left(\frac{5}{3}\right)$ $= 25$ cm^2
   $25 = \frac{1}{3}(75)$; if the height is multiplied by $\frac{1}{3}$, the area is also multiplied by $\frac{1}{3}$.
3. Original dimensions:
   $b = 12$, $h = 6$, $c = \sqrt{12^2 + 6^2} = \sqrt{180} = 6\sqrt{5}$
   $P = 12 + 6 + 6\sqrt{5} = \left(18 + 6\sqrt{5}\right)$ in.
   $A = \frac{1}{2}(12)(6) = 36$ in.^{2}
   Dimensions multiplied by 3:
   $b = 3(12) = 36$, $h = 3(6) = 18$, $c = 18\sqrt{5}$
   $P = 36 + 18 + 18\sqrt{5} = \left(54 + 18\sqrt{5}\right)$ in.
   $A = \frac{1}{2}(36)(18) = 324$ in.^{2}
   Perimeter is multiplied by 3. Area is multiplied by 3^2, or 9.
4. Original dimensions:
   $P = 2(18) + 2(6) = 48$ ft
   $A = (18)(6) = 108$ ft^2
   Dimensions multiplied by $\frac{1}{2}$:
   $P = 2(9) + 2(3) = 24$ ft
   $A = (9)(3) = 27$ ft^2
   Perimeter is multiplied by $\frac{1}{2}$. Area is multiplied by $\left(\frac{1}{2}\right)^2$, or $\frac{1}{4}$.
5. The original area is $A = s^2 = 36$ m^2; the side length is 6 m. If the area is doubled, the new area is 72 m.
   $s^2 = 72$
   $s = \sqrt{72} = 6\sqrt{2}$ m
   The side length is multiplied by $\sqrt{2}$.
6. The original diameter is $d = 2r = 14$ ft; the radius is 7 ft, area is $A = \pi(7)^2 = 49\pi$ ft^2, and circumference is 14π ft. If the area is tripled, the new area is 147π ft^2.
   $\pi r^2 = 147\pi$
   $r^2 = 147$
   $r = \sqrt{147} = 7\sqrt{3}$
   $C = 2\pi 7\sqrt{3} = 14\pi\sqrt{3}$ ft
   The circumference is multiplied by $\sqrt{3}$.

7. Old area $= (2)(4) = 8$ in.^{2}
New area $= (4)(8) = 32$ in.^{2}
$32 = 4(8)$, so the area is multiplied by 4. Therefore the cost is multiplied by 4:
Cost of new ad $= 4(\$36.75) = \$147$

PRACTICE AND PROBLEM SOLVING, PAGES 625–626

8. Original dimensions:
$A_{original} = \frac{1}{2}bh$
Multiply height by 4:
$A_{new} = \frac{1}{2}b(4h) = 2bh$
$2bh = 4\left(\frac{1}{2}bh\right)$; if the height is multiplied by 4, the area is also multiplied by 4.

9. Original dimensions:
$A = bh$
$= (24)(9) = 216$ in.^{2}
Double the height:
$A = bh$
$= (16)(9) = 144$ in.^{2}
$144 = \frac{2}{3}(216)$; if the base is multiplied by $\frac{2}{3}$, the area is also multiplied by $\frac{2}{3}$.

10. Original dimensions:
$a = b = c = 10$, $h = 5\sqrt{3}$
$P = 10 + 10 + 10 = 30$ cm
$A = \frac{1}{2}(10)5\sqrt{3} = 25\sqrt{3}$ cm^2
Dimensions doubled:
$a = b = c = 20$, $h = 10\sqrt{3}$
$P = 20 + 20 + 20 = 60$ cm
$A = \frac{1}{2}(20)10\sqrt{3} = 1000\sqrt{3}$ cm^2
Perimeter is doubled. Area is multiplied by 2^2, or 4.

11. Original dimensions:
$r = 5 - 0 = 5$
$C = 2\pi(5) = 10\pi$ units
$A = \pi(5)^2 = 25\pi$ units^2
Dimensions multiplied by $\frac{3}{5}$:
$r = \frac{3}{5}(5) = 3$
$C = 2\pi(3) = 6\pi$ units
$A = \pi(3)^2 = 9\pi$ units^2
Circumference is multiplied by $\frac{3}{5}$. Area is multiplied by
$\left(\frac{3}{5}\right)^2$, or $\frac{9}{25}$.

12. Original circumference is $C = 2\pi r = 16\pi$ mm; radius is 8 mm, and area is $A = \pi(8)^2 = 64\pi$ ft^2.
If the area is multiplied by $\frac{1}{3}$, the new area is $\frac{64\pi}{3}$ ft^2.
$\pi r^2 = \frac{64\pi}{3}$
$r^2 = \frac{64}{3}$
$r = \sqrt{\frac{64}{3}} = \frac{8\sqrt{3}}{3}$
$\frac{8\sqrt{3}}{3} = \frac{81}{\sqrt{3}}$; radius is multiplied by $\frac{1}{\sqrt{3}}$.

13. The original side length is $8 - 3 = 5$ units, and the original area is $5^2 = 25$ units^2. The new area is $3(25) = 75$ units^2.
$s^2 = 75$
$s = \sqrt{75} = 5\sqrt{3}$ units
The side length is multiplied by $\sqrt{3}$.

14a. Smaller screen: $32^2 = b^2 + h^2$
Larger screen:
$36^2 = (kb)^2 + (kh)^2$
$= k^2(b^2 + h^2)$
$= k^2(32)^2$
$36 = k(32)$
$kh : h = 36 : 32 = 9 : 8$

b. Ratio of areas:
$(kb)(kh) : bh$
$= k^2(bh) : bh$
$= k^2 : 1$
$= \left(\frac{9}{8}\right)^2 : 1$
$= 9^2 : 8^2 = 81 : 64$

15. Original dimensions: $A = \frac{1}{2}d_1d_2$
New dimensions: $A = \frac{1}{2}(8d_1)(8d_2) = 32d_1d_2$
$32d_1d_2 = 64\left(\frac{1}{2}d_1d_2\right)$, so area is multiplied by 64.

16. Original dimensions: $C = 2\pi r$, $A = \pi r$^2
New dimensions: $C = 2.4(2\pi r) = 2\pi(2.4r)$;
Therefore the new radius is $2.4r$, and
$A = \pi(2.4r)^2 = 5.76(\pi r^2)$. So area is multiplied by 5.76.

17. Original dimensions: $A = bh$
New dimensions: $A = (4b)(7h) = 28(bh)$
The area is multiplied by 28.

18. Original dimensions:
$s = 2a\tan 22.5°$, $P = 8s = 16a\tan 22.5°$
$A = \frac{1}{2}aP$
$= \frac{1}{2}a(16a\tan 22.5°)$
$= 8a^2\tan 22.5°$
New dimensions:
$P = 16(3a)\tan 22.5°$
$= 48a\tan 22.5°$
$A = \frac{1}{2}aP$
$= \frac{1}{2}(3a)(48a\tan 22.5°)$
$= 72a^2\tan 22.5°$
$72a^2\tan 22.5° = 9(8a^2\tan 22.5°)$. So the area is multiplied by 9.

19. Original dimensions: $d = s\sqrt{2}$, $A = s^2$
New dimensions: $d = s\sqrt{2} \div 4 = \left(\frac{s}{4}\right)\sqrt{2}$;
Therefore the new side length is $\frac{s}{4}$, and
$A = \left(\frac{s}{4}\right)^2 = s^2 \div 16$. So the area is divided by 16.

20. Original dimensions: $A = \frac{1}{2}d_1d_2$
New dimensions: $A = \frac{1}{2}\left(\frac{1}{7}d_1\right)(8d_2) = \frac{1}{14}d_1d_2$
$\frac{1}{14}d_1d_2 = \frac{1}{7}\left(\frac{1}{2}d_1d_2\right)$, so the area is multiplied by $\frac{1}{7}$.

21. Original dimensions: $P = 3s$, $h = s\frac{\sqrt{3}}{2}$,
$A = \frac{1}{2}s\left(s\frac{\sqrt{3}}{2}\right) = s^2\frac{\sqrt{3}}{4}$
New dimensions: $P = 2(3s) = 3(2s)$;
Therefore the new side length is $2s$, and $A = \left(\frac{s}{2}\right)^2\frac{\sqrt{3}}{4}$
$= s^2\frac{\sqrt{3}}{16} = \frac{1}{4}\left(s^2\frac{\sqrt{3}}{4}\right)$. So the area is multiplied by 4.

22a. $A = \frac{1}{2}(42 + 24)(15) = 495 \text{ cm}^2$
top base doubled:
$A = \frac{1}{2}(42 + 2(24))(15) = 675 \text{ cm}^2$
$675 \approx 1.4(495)$, so the area is multiplied by about 1.4.

b. $A = \frac{1}{2}(2(42) + 2(24))(15) = 990 \text{ cm}^2$
$990 = 2(495)$, so the area is doubled.

c. $A = \frac{1}{2}(42 + 24)(2(15)) = 990 \text{ cm}^2$
$990 = 2(495)$, so the area is doubled.

d. $A = \frac{1}{2}(2(42) + 2(24))(2(15)) = 1980 \text{ cm}^2$
$1980 = 4(495)$, so the area is multiplied by 4.

23. 1 square inch $= 10^2 \text{ mi}^2 = 100 \text{ mi}^2$
$= 100(640 \text{ acres}) = 64{,}000$ acres
12.5 sq. in. $= 12.5(64{,}000 \text{ acres}) = 800{,}000$ acres

24. If the dimensions are multiplied by x, the area is multiplied by x^2.
$x^2 = 50\% = \frac{1}{2}$
$x = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$

25a. Original dimensions:
$b = 2 - (-2) = 4$, $h = 3 - (-2) = 5$
$A = \frac{1}{2}(4)(5) = 10 \text{ units}^2$
New dimensions:
$b = 6 - (-6) = 12$, $h = 3 - (-2) = 5$
$A = \frac{1}{2}(12)(5) = 30 \text{ units}^2$
$30 = 3(10)$, so the area is multiplied by 3.

b. New dimensions:
$b = 2 - (-2) = 4$, $h = 9 - (-6) = 15$
$A = \frac{1}{2}(4)(15) = 30 \text{ units}^2$
$30 = 3(10)$, so the area is multiplied by 3.

c. New dimensions:
$b = 6 - (-6) = 12$, $h = 9 - (-6) = 15$
$A = \frac{1}{2}(12)(15) = 90 \text{ units}^2$
$90 = 9(10)$, so the area is multiplied by 9.

26a. Original dimensions:
Outer rectangle: $A = (4 - (-4))(4 - (-3)) = 48$
Missing upper left, lower right triangles:
$A = \frac{1}{2}(-1 - (-4))(4 - 0) = 6$
Missing lower left, upper right triangles:
$A = \frac{1}{2}(4 - (-1))(4 - 1) = 7.5$
Area of figure:
$A = 48 - 2(6) - 2(7.5) = 21 \text{ units}^2$
New dimensions:
Outer rectangle:
$A = (12 - (-12))(4 - (-3)) = 144$
Missing upper left, lower right triangles:
$A = \frac{1}{2}(-3 - (-12))(4 - 0) = 18$
Missing lower left, upper right triangles:
$A = \frac{1}{2}(12 - (-3))(4 - 1) = 22.5$
Area of figure:
$A = 144 - 2(18) - 2(22.5) = 63 \text{ units}^2$
$63 = 3(21)$, so the area is multiplied by 3.

b. New dimensions:
Outer rectangle: $A = (4 - (-4))(12 - (-9)) = 144$
Missing upper left, lower right triangles:
$A = \frac{1}{2}(-1 - (-4))(12 - 0) = 18$
Missing lower left, upper right triangles:
$A = \frac{1}{2}(4 - (-1))(12 - 3) = 22.5$
Area of figure:
$A = 144 - 2(18) - 2(22.5) = 63 \text{ units}^2$
$63 = 3(21)$, so the area is multiplied by 3.

c. New dimensions:
Outer rectangle: $A = (12 - (-12))(12 - (-9)) = 432$
Missing upper left, lower right triangles:
$A = \frac{1}{2}(-3 - (-12))(12 - 0) = 54$
Missing lower left, upper right triangles:
$A = \frac{1}{2}(12 - (-3))(12 - 3) = 67.5$
Area of figure:
$A = 432 - 2(54) - 2(67.5) = 189 \text{ units}^2$
$189 = 9(21)$, so the area is multiplied by 9.

27a. Original dimensions:
Left rectangle: $A = (-1 - (-3))(3 - (-2)) = 10$
Middle square: $A = (0 - (-1))(-1 - (-2)) = 1$
Right rectangle: $A = (2 - (0))(1 - (-2)) = 6$
Area of figure: $A = 10 + 1 + 6 = 17 \text{ units}^2$
New dimensions:
Left rectangle: $A = (-3 - (-9))(3 - (-2)) = 30$
Middle square: $A = (0 - (-3))(-1 - (-2)) = 3$
Right rectangle: $A = (6 - (0))(1 - (-2)) = 18$
Area of figure: $A = 30 + 3 + 18 = 51 \text{ units}^2$
$51 = 3(17)$, so the area is multiplied by 3.

b. New dimensions:
Left rectangle: $A = (-1 - (-3))(9 - (-6)) = 30$
Middle square: $A = (0 - (-1))(-3 - (-6)) = 3$
Right rectangle: $A = (2 - (0))(3 - (-6)) = 18$
Area of figure: $A = 30 + 3 + 18 = 51 \text{ units}^2$
$51 = 3(17)$, so the area is multiplied by 3.

c. New dimensions:
Left rectangle: $A = (-3 - (-9))(9 - (-6)) = 90$
Middle square: $A = (0 - (-3))(-3 - (-6)) = 9$
Right rectangle: $A = (6 - (0))(3 - (-6)) = 54$
Area of figure: $A = 90 + 9 + 54 = 153 \text{ units}^2$
$153 = 9(17)$, so the area is multiplied by 9.

28. Possible answers:
Multiply the base or height by 5.
Multiply the base and height by $\sqrt{5}$.

29a. Original area is $\pi\left(\frac{8}{2}\right)^2 = 16\pi$

Now we want $2\ (16)\pi = 32\ \pi$, which means the new diameter $= (2)\sqrt{32} = 8\sqrt{2}$ in.

b. Now we want the area to be $0.5\ (16\ \pi) = 8\ \pi$.

So now the new diameter $= 2\ (\sqrt{8}) = 4\sqrt{2}$ in.

TEST PREP, PAGE 627

30. D
$A = (2s)^2 = 4(s^2)$

31. G
$A = 4(\pi r^2) = \pi(2r)^2$;
$d = 2(2r)$

32. C
$bh = 60 \text{ ft}^2$
$A = (1.5b)(1.5h) = 2.25bh = 2.25(60) = 135 \text{ ft}^2$

33. 36
Old dimensions: $P = a + b + c = 18$ in.
New dimensions:
$P = 2a + 2b + 2c = 2(a + b + c) = 2(18) = 36$ in.

CHALLENGE AND EXTEND, PAGE 627

34. $A = (5(2x + 5))^2$
$= 25(4x^2 + 20x + 25)$
$= (100x^2 + 500x + 625) \text{ cm}^2$

35. Old dimensions: $C = 6\pi$ in.
New dimensions:
$C = 6\pi(x + 3) = 2\pi(3(x + 3))$ in.
$r = 3(x + 3) = 3x + 9$ in.
$A = \pi(3x + 9)^2 = (9\pi x^2 + 54\pi x + 81\pi)$ in.^{2}

36. Possible answers:
Multiply all lengths of the horizontal segments by 2.
Multiply all side lengths by $\sqrt{2}$.

SPIRAL REVIEW, PAGE 627

37. $\dfrac{t \text{ tortillas}}{2 \text{ tortillas/min}} = 36 \text{ min}$
$\dfrac{t}{2} = 36$

38. $\dfrac{m \text{ mi}}{25 \text{ mi/gal}} = (13 - 8) \text{ gal}$
$\dfrac{m}{25} = 13 - 8$

39. $7\text{m}(\text{int. } \angle) = (7 - 2)180° = 900°$
$\text{m}(\text{int. } \angle) \approx 128.6°$
$7\text{m}(\text{ext. } \angle) = 360°$
$\text{m}(\text{ext. } \angle) \approx 51.4°$

40. $10\text{m}(\text{int. } \angle) = (10 - 2)180° = 1440°$
$\text{m}(\text{int. } \angle) = 144°$
$10\text{m}(\text{ext. } \angle) = 360°$
$\text{m}(\text{ext. } \angle) = 36°$

41. $14\text{m}(\text{int. } \angle) = (14 - 2)180° = 2160°$
$\text{m}(\text{int. } \angle) \approx 154.3°$
$14\text{m}(\text{ext. } \angle) = 360°$
$\text{m}(\text{ext. } \angle) \approx 25.7°$

42. Outer rectangle: $A = (6)(7) = 42$
Missing triangles:
$A = \frac{1}{2}(6)(1) = 3$
$A = \frac{1}{2}(4)(7) = 14$
$A = \frac{1}{2}(2)(6) = 6$
Area of figure: $A = 42 - 3 - 14 - 6 = 19 \text{ units}^2$

43. Outer rectangle: $A = (8)(8) = 64$
Missing triangles:
$A = \frac{1}{2}(2)(2) = 2$
$A = \frac{1}{2}(6)(2) = 6$
$A = \frac{1}{2}(2)(6) = 6$
$A = \frac{1}{2}(6)(6) = 18$
Area of figure: $A = 64 - 2 - 6 - 6 - 18 = 32 \text{ units}^2$

CONNECTING GEOMETRY TO PROBABILITY, PAGES 628–629

TRY THIS, PAGE 629

1. The event “choosing a circle” contains only 1 outcome. The probability is:
$P(\text{circle}) = \dfrac{\text{\# outcomes in event}}{\text{\# possible outcomes}} = \dfrac{1}{6}$

2. The event “choosing a shape with area 36 cm^2” contains 2 outcomes: the square has an area $6^2 = 36 \text{ cm}^2$, and the rectangle has an area $(9)(4) = 36 \text{ cm}^2$. The probability is:
$P(\text{circle}) = \dfrac{\text{\# outcomes in event}}{\text{\# possible outcomes}} = \dfrac{2}{6} = \dfrac{1}{3}$

3. The event “choosing a triangle or quadrilateral” contains 5 outcomes. The probability is:
$P(\text{circle}) = \dfrac{\text{\# outcomes in event}}{\text{\# possible outcomes}} = \dfrac{5}{6}$

4. The event “not choosing a triangle” contains 4 outcomes. The probability is:
$P(\text{circle}) = \dfrac{\text{\# outcomes in event}}{\text{\# possible outcomes}} = \dfrac{4}{6} = \dfrac{2}{3}$

9-6 GEOMETRIC PROBABILITY, PAGES 630–636

CHECK IT OUT! PAGES 631–632

1. $P(\overline{BD}) = P(\overline{BC}) + P(\overline{CD}) = \dfrac{3}{12} + \dfrac{5}{12} = \dfrac{8}{12} = \dfrac{2}{3}$

2. $P(\text{not red}) = P(\text{green or yellow})$
$= P(\text{green}) + \text{P}(\text{yellow})$
$= \dfrac{25}{60} + \dfrac{5}{60} = \dfrac{30}{60} = \dfrac{1}{2}$

3. $P = \dfrac{80 + 100}{360} = \dfrac{180}{360} = \dfrac{1}{2}$

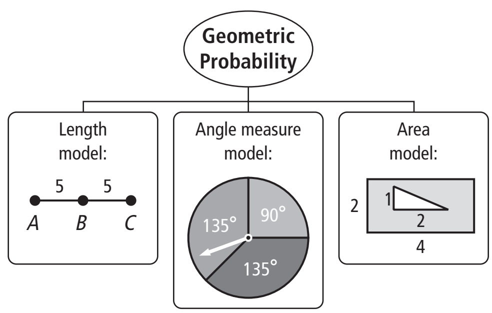
4. Area of the triangle (which contains the circle) is $\approx 187 \text{ m}^2$.
Area of the trapezoid is 75 m^2.
Area of the rectangle is 900 m^2
$P \approx \dfrac{900 - (187 + 75)}{900} = \dfrac{638}{900} \approx 0.71$

THINK AND DISCUSS, PAGE 633

1. In a geometric model, there are an infinite number of outcomes in each event.

2. Subtract $\frac{1}{2}$ and $\frac{1}{3}$ from 1 to find what part of the spinner is yellow.

3.



EXERCISES, PAGES 633–636

GUIDED PRACTICE, PAGE 633

1. Possible answer: a spinner
2. $P(\overline{XZ}) = P(\overline{XY}) + P(\overline{YZ}) = \frac{5}{10} + \frac{3}{10} = \frac{8}{10} = \frac{4}{5}$
3. $P(\text{not } \overline{XY}) = P(\overline{WX}) + P(\overline{YZ}) = \frac{2}{10} + \frac{3}{10} = \frac{5}{10} = \frac{1}{2}$
4. $P(\overline{WX} \text{ or } \overline{YZ}) = P(\overline{WX}) + P(\overline{YZ}) = \frac{1}{2}$
5. $P(\overline{WY}) = P(\overline{WX}) + P(\overline{XY}) = \frac{2}{10} + \frac{5}{10} = \frac{7}{10}$
6. $P = \frac{1.5 \text{ min}}{10 \text{ min}} = 0.15$
7. $P(\text{wait} < 3 \text{ min, once}) = \frac{3 + 1.5}{10} = 0.45$
   In 20 times, expect to wait < 3 min, $0.45(20)$ $= 9$ times.
8. $P = \frac{45}{360} = \frac{1}{8}$
9. $P = \frac{45 + 90}{360} = \frac{135}{360} = \frac{3}{8}$
10. $P = \frac{360 - 120}{360} = \frac{240}{360} = \frac{2}{3}$
11. $P = \frac{60 + 90}{360} = \frac{150}{360} = \frac{5}{12}$
12. Area of the triangle is $A = \frac{1}{2}(10)(10) = 50 \text{ ft}^2$
    Area of the rectangle is $A = (48)(24) = 1152 \text{ ft}^2$
    $P = \frac{50}{1152} \approx 0.04$
13. Area of the trapezoid is $A = \frac{1}{2}(18 + 12)(6) = 90 \text{ ft}^2$
    $P = \frac{90}{1152} \approx 0.08$
14. Area of the square is $A = (10)^2 = 100 \text{ ft}^2$
    $P = \frac{100}{1152} \approx 0.09$
15. The combined area of the smaller shapes is $A = 50 + 90 + 100 = 240 \text{ ft}^2$.
    $P = \frac{1152 - 240}{1152} \approx 0.79$

PRACTICE AND PROBLEM SOLVING, PAGES 634–635

16. $HM = 16.4 + 21.9 + 15.3 + 14.8 = 68.4$
    $P(\overline{JK}) = \frac{21.9}{68.4} \approx 0.32$
17. $P(\text{not } \overline{LM}) = \frac{68.4 - 14.8}{68.4} \approx 0.78$
18. $P(\overline{HJ} \text{ or } \overline{KL}) = P(\overline{HJ}) + P(\overline{KL})$
    $= \frac{16.4}{68.4} + \frac{15.3}{68.4} \approx 0.46$
19. $P(\text{not } \overline{JK} \text{ or } \overline{LM}) = P(\overline{HJ} \text{ or } \overline{KL}) \approx 0.46$
20. $P = \frac{0.75 \text{ min}}{15 \text{ min}} = 0.05$
21. $P = \frac{15 - (5 + 0.75)}{15} \approx 0.62$
22. Assume that if the report has started, you must wait to hear the whole next one.
    $P = \frac{1}{15}$
    In 50 times, expect to wait < 1 min, $\frac{1}{15}(50)$ ≈ 3 times.
23. $P(\text{red}) = \frac{180}{360} = \frac{1}{2}$
24. $P(\text{yellow or blue}) = \frac{45 + 45}{360} = \frac{90}{360} = \frac{1}{4}$
25. $P(\text{not green}) = \frac{360 - 90}{360} = \frac{270}{360} = \frac{3}{4}$
26. $P(\text{red or green}) = \frac{180 + 90}{360} = \frac{270}{360} = \frac{3}{4}$
27. Area of the triangle is
    $A = \frac{1}{2}\left(\frac{30}{\sqrt{3}/2}\right)(30) = \frac{1}{2}(20\sqrt{3})(30) = 300\sqrt{3} \text{ m}^2$
    Area of the rectangle is
    $A = (20\sqrt{3})(30) = 600\sqrt{3} \text{ m}^2$
    $P = \frac{300\sqrt{3}}{600\sqrt{3}} = \frac{1}{2}$ or 0.5
28. Area of the square is $A = (10\sqrt{2})^2 = 200 \text{ m}^2$
    $P = \frac{200}{600\sqrt{3}} \approx 0.19$
29. Area of the circle is $A = \pi(10)^2 = 100\pi \text{ m}^2$
    $P = \frac{100\pi - 200}{600\sqrt{3}} \approx 0.11$
30. The circle and square are inside the triangle, so the remaining area is $A = 600\sqrt{3} - 300\sqrt{3} = 300\sqrt{3} \text{ m}^2$
    $P = \frac{300\sqrt{3}}{600\sqrt{3}} = \frac{1}{2}$ or 0.5
31. The value in A is incorrect because the sectors have different angle measures, so they are not equally likely outcomes.
32. Area of rectangle: $A = (15 - 2)(8 - 1) = 91 \text{ units}^2$
    Area of triangle:
    $A = (5)(4) - \frac{1}{2}(1)(4) - \frac{1}{2}(4)(2) - \frac{1}{2}(5)(2)$
    $= 20 - 2 - 4 - 5 = 9 \text{ units}^2$
    $P = \frac{9}{91} \approx 0.10$
33. Only half of $\odot P$ lies inside $ABCD$. The area of the semicircle is $A = \frac{1}{2}\pi(3)^2 = 4.5\pi \text{ units}^2$.
    $P = \frac{91 - 4.5\pi}{91} \approx 0.84$
34. Area of outer square: $A = (10)^2 = 100 \text{ units}^2$
    Area of parallelogram: $A = (2)(3) = 6 \text{ units}^2$
    $P = \frac{6}{100} = 0.06$
35. Area of circle: $A = \pi(2)^2 = 4\pi \text{ units}^2$
    $P = \frac{4\pi}{100} \approx 0.13$
36. Area of triangle is $A = \frac{1}{2}(3)(3) = 4.5 \text{ units}^2$
    $P = \frac{4.5 + 4\pi}{100} \approx 0.17$
37. $P = \frac{100 - (4.5 + 6 + 4\pi)}{100} \approx 0.77$

38a. Area of central region is $A = \pi(6.1)^2$ cm^2
Area of target is $A = \pi(61)^2$ cm^2
$$P = \frac{\pi(6.1)^2}{\pi(61)^2} = \left(\frac{6.1}{61}\right)^2 = (0.1)^2 = 0.01$$

b. Inner radius of blue rings: $r = 4(6.1)$ cm
Outer radius of black rings: $r = 8(6.1)$ cm
Area of blue and black rings:
$A = \pi(8(6.1))^2 - \pi(4(6.1))^2$
$= \pi(6.1)^2(64 - 16) = (48)\pi(6.1)^2$ cm^2
$$P = \frac{48\pi(6.1)^2}{\pi(61)^2} = \frac{48}{100} = 0.48$$

c. Area of 5 inner rings:
$A = \pi(5(6.1))^2 = 25\pi(6.1)^2$ cm^2
$$P = \frac{25\pi(6.1)^2}{\pi(61)^2} = \frac{25}{100} = 0.25$$

d. The probabilities might be different because an archer would be aiming for the center, not a random point.

39. Possible answer: The point lies on $\overline{AC}$.

40. Possible answer: The point lies in the red or yellow region.

41. Possible answer: The point lies in the blue or green triangle.

42a. Area of blue parallelogram: $A = (2)(1) = 2$ units^2
Area of tangram: $A = (4)^2 = 16$ units^2
$P = \frac{2}{16} = \frac{1}{8}$

b. Area of purple triangle: $A = \frac{1}{2}(2)(2) = 2$ units^2
$P = \frac{2}{16} = \frac{1}{8}$

c. Area of large yellow triangle:
$A = \frac{1}{2}(4)(2) = 4$ units^2
$P = \frac{4}{16} = \frac{1}{4}$

d. No, because areas are the same.

43. $P = \frac{4}{8} = \frac{1}{2}$; it does not matter which regions are shaded because they all have the same area.

44a. Area of each balloon: $A = \pi(1.5)^2 = 2.25\pi$ in.^{2}
Area of board: $A = (50)(30) = 1500$ in.^{2}
For 40 balloons,
$$P = \frac{40(2.25\pi)}{1500} \approx 0.19$$

b. For n balloons, if probability is ≥ 0.25,
$$P = \frac{n(2.25\pi)}{1500} \geq 0.25$$
$$n \geq \frac{1500}{2.25\pi}(0.25) \approx 53.1$$
$n \geq 54$ balloons

TEST PREP, PAGE 636

45. A
$$P = \frac{2(1.5)}{6(3.5)} = \frac{3}{21} \approx 0.14$$

46. G
$$P(\overline{AB}) = \frac{18}{18 + 24} = \frac{18}{42} \approx 0.43$$

47. D
Area of triangle: $A = \frac{1}{2}(10)(20) = 100$ m^2
Area of circle: $A = \pi(20)^2 = 400\pi$ m^2
Area of square: $A = (25)^2 = 625$ m^2
Area of field: $A = 100(70) = 7000$ m^2
$$P = \frac{7000 - (100 + 400\pi + 625)}{7000} \approx 0.717$$

48a. Let $P(r)$, $P(b)$, $P(g)$ be the probabilities of each color. From the given info:
$P(r) = 2P(b)$, $P(g) = P(b)$
Substitute into this eqation:
$P(r) + P(b) + P(g) = 1$
$2P(b) + P(b) + P(b) = 1$
$4P(b) = 1$
$P(b) = \frac{1}{4}$
$P(g) = P(b) = \frac{1}{4}$ or 0.25

b. 3; the probability of landing on green is 0.25, so the number of green regions is $0.25(12) = 3$.

CHALLENGE AND EXTEND, PAGE 636

49. Area of each red region:
$A = 1^2 - 4\left(\frac{1}{4}\pi(0.5)^2\right) = 1 - 0.25\pi$
$$P = \frac{1 - 0.25\pi}{1} = \frac{4 - \pi}{4} \approx 0.21$$

50. $P = \frac{s^2}{(18)(24)} = \frac{s^2}{432} = \frac{1}{3}$
$s^2 = 144$
$s = 12$ ft
The square will be 12 ft by 12 ft.
$P = \frac{s^2}{432} = \frac{3}{4}$
$s^2 = 324$
$s = 18$ ft
The square will be 18 ft by 18 ft.

51. Possible answer: The probabilities must add to 1, so P(yellow) + P(blue) + P(red) = 1. I would make the regions different sizes, and I would want each region to be worth more points the smaller it is. The point value for red is 6 times the point value for yellow, so I would make $6 \cdot P(\text{red}) = P(\text{yellow})$.

The point value for blue is 3 times the point value for yellow, so I would make $3 \cdot P(\text{blue}) = P(\text{yellow})$.

Then $P(\text{yellow}) + \frac{1}{3}P(\text{yellow}) + \frac{1}{6}P(\text{yellow}) = 1$.

This means $P(\text{yellow}) = \frac{2}{3}$, $P(\text{blue}) = \frac{2}{9}$,

and $P(\text{red}) = \frac{1}{9}$.

The angle measure for the yellow region would be 240°, for the blue region would be 80°, and for the red region would be 40°.

SPIRAL REVIEW, PAGE 636

52. $\left(3x^2y\right)\left(4x^3y^2\right)$
$= 3(4)x^{(2+3)}y^{(1+2)}$
$= 12x^5y^3$

53. $\left(2m^5\right)^2$
$= 2^2m^{5(2)}$
$= 4m^{10}$

54. $\dfrac{-8a^4b^6}{2ab^3} = -\dfrac{8}{2}a^{(4-1)}b^{(6-3)}$
$= -4a^3b^3$

55. By Distribution Formula, $AB = AC = 2\sqrt{5}$, $BC = 4$, $AD = AE = 4\sqrt{5}$, $DE = 8$.
$\dfrac{AB}{AD} = \dfrac{AC}{AE} = \dfrac{BC}{DE} = \dfrac{1}{2}$, so $\triangle ABC \sim \triangle ADE$ by SSS .

56. Each circle: $A = \pi(2)^2 = 4\pi$ cm^2
Square: $A = (8)^2 = 64$ cm^2
Shaded area: $A = 64 - 2(4\pi) \approx 38.9$ cm^2

57. Triangle: $A = \frac{1}{2}(2)(2) = 2$ in.^{2}
Circle: $A = \pi(2)^2 = 4\pi$ in.^{2}
Shaded area: $A = 4\pi - 2 \approx 10.6$ in.^{2}

9-6 GEOMETRY LAB: USE GEOMETRIC PROBABILITY TO ESTIMATE π, PAGE 637

TRY THIS, PAGE 637

1. Check students' work.

2a. $A = 4\left(\frac{1}{4}\pi r^2\right) = \pi r^2$ **2b.** $A = (2r)^2 = 4r^2$

2c. $P = \dfrac{\pi r^2}{4r^2} = \dfrac{\pi}{4}$

3. The probability is $\frac{\pi}{4}$, so 4 times the probability is π.

9B MULTI-STEP TEST PREP, PAGE 638

1. Area of each balloon: $A = \pi(2)^2 = 4\pi$ in.^{2}
Area of board: $A = (48)(24) = 1152$ in.^{2}
For 15 balloons,
$P = \dfrac{15(4\pi)}{1152} \approx 0.16$

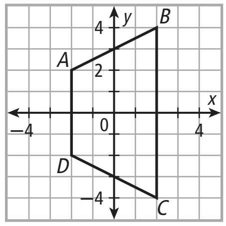
2. New area of each balloon: $A = \pi(4)^2 = 16\pi$ in.^{2}
$P = \dfrac{15(16\pi)}{1152} = 4\left(\dfrac{15(4\pi)}{1152}\right)$
The probability is 4 times as great.

3. Area of board: $A = (100)(60) = 6000$ units^2
Area of missing triangles: $A = \frac{1}{2}(40)(30) = 600$ units^2
$A = \frac{1}{2}(60)(30) = 900$ units^2
$A = \frac{1}{2}(40)(20) = 400$ units^2
$A = \frac{1}{2}(60)(40) = 1200$ units^2
Area of *ABCD*:
$A = 6000 - (200 + 900 + 400 + 1200)$
$= 2900$ units^2
$P = \dfrac{2900}{1600} \approx 0.48$

4. $0.16 < 0.48 < 4(0.16) = 0.64$, so balloon game in Problem 2 gives the best chance.

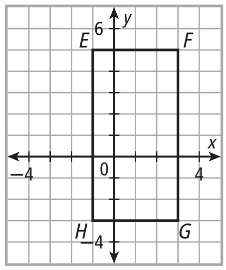
9B READY TO GO ON? PAGE 639

1.



The figure is a trapezoid.
$AB = \sqrt{(2+2)^2 + (4-2)^2} = \sqrt{20} = 2\sqrt{5}$
$BC = 4 + 4 = 8$
$CD = \sqrt{(-2-2)^2 + (-2+4)^2} = \sqrt{20} = 2\sqrt{5}$
$AD = 2 + 2 = 4$
$P = 2\sqrt{5} + 8 + 2\sqrt{5} + 4 = \left(12 + 4\sqrt{5}\right)$ units
$A = \frac{1}{2}(8 + 4)(2 + 2) = 24$ units^2

2.



The figure is a rectangle.
$EF = GH = 3 + 1 = 4$, $FG = EH = 3 + 5 = 8$
$P = 4 + 8 + 4 + 8 = 24$ units
$A = (8)(4) = 32$ units^2

3. Outer rectangle: $A = (6)(6) = 36$ units^2
Missing rectangles: $A = \frac{1}{2}(5)(1) = 2.5$ units^2,
$A = \frac{1}{2}(3)(5) = 7.5$ units^2, $A = \frac{1}{2}(3)(2) = 3$ units^2,
$A = \frac{1}{2}(1)(4) = 2$ units^2
Area of *JKLM*:
$A = 36 - (2.5 + 7.5 + 3 + 2) = 21$ units^2

4. Outer rectangle: $A = (8)(7) = 56$ units^2
Missing rectangles: $A = \frac{1}{2}(6)(2) = 6$ units^2,
$A = \frac{1}{2}(2)(2) = 2$ units^2, $A = \frac{1}{2}(3)(5) = 7.5$ units^2,
$A = \frac{1}{2}(5)(5) = 12.5$ units^2
Area of *NPQR*:
$A = 56 - (6 + 2 + 7.5 + 12.5) = 28$ units^2

5. Old dimensions: $P = 4(7) = 28$ m
$A = 7^2 = 49$ m^2
New dimensions: $P = 4(21) = 84$ m
$A = 21^2 = 441$ m^2
$84 = 3(21)$, so the perimeter is tripled.
$441 = 9(49)$, so the area is multiplied by 9.

6. Old dimensions:
Side length of rhombus:
$s^2 = 1.5^2 + 4.5^2 = 22.5$
$s = \sqrt{22.5} = 1.5\sqrt{10}$ ft
$P = 4s = 6\sqrt{10}$ ft
$A = \frac{1}{2}d_1d_2 = \frac{1}{2}(3)(9) = 13.5$ ft^2
New dimensions:
$s^2 = 0.5^2 + 1.5^2 = 2.5$
$s = \sqrt{2.5} = 0.5\sqrt{10}$ ft
$P = 4s = 2\sqrt{10}$ ft
$A = \frac{1}{2}d_1d_2 = \frac{1}{2}(1)(3) = 1.5$ ft^2
$2\sqrt{10} = \frac{1}{3}\left(6\sqrt{10}\right)$,
so the perimeter is multiplied by $\frac{1}{3}$.
$1.5 = \frac{1}{9}(13.5)$, so the area is multiplied by $\frac{1}{9}$.

7. Old dimensions: $P = 2(15) + 2(9) = 48$ cm
$A = (15)(9) = 135$ cm^2
New dimensions: $P = 2(30) + 2(18) = 96$ cm
$A = (30)(18) = 540$ cm^2
$96 = 2(48)$, so the perimeter is doubled.
$540 = 4(135)$, so the area is multiplied by 4.

8. Old dimensions: $c = \sqrt{15^2 + 8^2} = 17$ in.
$P = 15 + 8 + 17 = 40$ in.
$A = \frac{1}{2}(15)(8) = 60$ in.^{2}
New dimensions: $c = \sqrt{3^2 + \left(\frac{8}{3}\right)^2} = \frac{17}{3}$ in.
$P = 5 + \frac{8}{3} + \frac{17}{3} = \frac{40}{3}$ in.
$A = \frac{1}{2}(5)\left(\frac{8}{3}\right) = \frac{20}{3}$ in.^{2}
$\frac{40}{3} = \frac{1}{3}(40)$, so the perimeter is multiplied by $\frac{1}{3}$.
$\frac{20}{3} = \frac{1}{9}(60)$, so the area is multiplied by $\frac{1}{9}$.

9. Old dimensions: $s = 4$ units, $A = 4$^2 $= 16$ units^2
new dimensions:
$A = s^2$
$4(16) = s^2$
$64 = s^2$
$s = 8$ units
$8 = 2(4)$, so the side length is doubled.

10. Assume the batter required, B, is proportional to area.
$$\frac{B_{reg}}{B_{silver}} = \frac{A_{reg}}{A_{silver}}$$
$$\frac{B_{reg}}{1/8} = \frac{\pi(2.5(4))^2}{\pi(4)^2} = \frac{100}{16} = \frac{25}{4}$$
$$B_{reg} = \frac{25}{4}\left(\frac{1}{8}\right) \approx 0.78 \text{ cup}$$

11. $P = \frac{120}{360} = \frac{1}{3}$

12. $P = \frac{120 + 100}{360} = \frac{220}{360} = \frac{11}{18}$

13. $P = \frac{360 - 95}{360} = \frac{265}{360} = \frac{53}{72}$

14. $P = \frac{100 + 45}{360} = \frac{145}{360} = \frac{29}{72}$

15. Commercials play for 12 min out of every 60 min.
$P = \frac{12}{60} = \frac{1}{5}$ or 0.2

STUDY GUIDE: REVIEW, PAGES 640–643

VOCABULARY, PAGE 640

1. apothem

2. center of the circle

3. geometric probability

LESSON 9-1, PAGE 640

4. $P = 36 = 4s$
$s = 9$
$A = 9^2 = 81$ in.^{2}

5. $A = bh$
$28 = (4)h$
$h = 7$
$P = 2(4) + 2(7) = 22$ cm

6. $A = \frac{1}{2}bh$
$6x^3y = \frac{1}{2}(4xy)h$
$6x^2 = 2h$
$h = 3x^2$ in.

7. $A = \frac{1}{2}(b_1 + b_2)h$
$48xy = \frac{1}{2}(9xy + 3xy)h$
$48 = 6h$
$h = 8$ ft

8. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(21)(24) = 252$ yd^2

9. $A = \frac{1}{2}d_1d_2$
$630x^3y^7 = \frac{1}{2}(30x^2y^3)d_2$
$630xy^4 = 15d_2$
$d_2 = 42xy^4$ in.

10. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(32)(18) = 288$ m^2

LESSON 9-2, PAGE 641

11. $C = \pi d = \pi\left(\frac{2}{\pi}\right) = 2$ ft

12. $C = 2\pi r$
$14\pi = 2\pi r$
$r = 7$
$A = \pi r^2$
$= \pi(7)^2$
$= 49\pi$ yd^2
≈ 153.9 yd^2

13. $A = \pi r^2$
$64x^2\pi = \pi r^2$
$64x^2 = r^2$
$r = 8x$ m
$d = 2r = 16x$ m

14. $\tan 36° = \frac{\frac{s}{2}}{a} = \frac{5}{a}$
$a = \frac{5}{\tan 36°}$ ft
$P = 5s = 5(10) = 50$ ft
$A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{5}{\tan 36°}\right)(50) \approx 172.0$ ft^2

15. $b = 4 = 2(2)$, so $h = 2\sqrt{3}$
$A = \frac{1}{2}bh = \frac{1}{2}(4)\left(2\sqrt{3}\right) = 4\sqrt{3}$ in.$^2 \approx 6.9$ in.^{2}

16. $\tan 22.5° = \frac{s/2}{a} = \frac{4}{a}$
$a = \frac{4}{\tan 22.5°}$ cm
$P = 8s = 8(8) = 64$ cm
$A = \frac{1}{2}aP = \frac{1}{2}\left(\frac{4}{\tan 22.5°}\right)(64) \approx 309.0$ cm^2

17. $d^2 = s^2 + s^2 = 2s^2$
$12^2 = 2A$
$144 = 2A$
$A = 72$ m^2

LESSON 9-3, PAGE 641

18. Area of triangle: $A = \frac{1}{2}(15)(15) = 112.5 \text{ ft}^2$
Area of semicircle: $A = \frac{1}{2}\pi(7.5)^2 = 28.125\pi \text{ ft}^2$
Shaded area: $A = 112.5 + 28.125\pi \approx 200.9 \text{ ft}^2$

19. Left rectangle: $A = (8)(6) = 48 \text{ cm}^2$
Middle rectangle: $A = (6)(6 + 6) = 72 \text{ cm}^2$
Right rectangle: $A = (4)(18) = 72 \text{ cm}^2$
Shaded area: $A = 48 + 72 + 72 = 192 \text{ cm}^2$

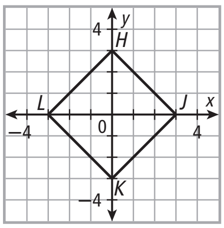
20. Triangle: $b = 8 = 2(4)$, *so* $h = 4\sqrt{3}$
$A = \frac{1}{2}(8)4\sqrt{3} = 16\sqrt{3} \text{ mm}^2$
Missing semicircle: $A = \frac{1}{2}\pi(2)^2 = 2\pi \text{ mm}^2$
Shaded area: $A = 16\sqrt{3} - 2\pi \approx 21.4 \text{ mm}^2$

LESSON 9-4, PAGE 642

21. The shape has approximately 41 whole squares and 17 half squares.
Total area is $\approx 41 + \frac{1}{2}(17) = 49.5 \text{ units}^2$.

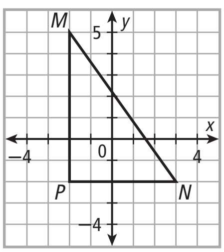
22. The shape has approximately 35 whole squares and 18 half squares.
Total area is $\approx 35 + \frac{1}{2}(18) = 44 \text{ units}^2$.

23.



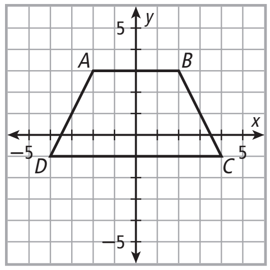
The figure is a square.
$s = \sqrt{3^2 + 3^2} = \sqrt{18} = 3\sqrt{2}$ units
$P = 4\left(3\sqrt{2}\right) = 12\sqrt{2}$ units
$A = \left(3\sqrt{2}\right)^2 = 9(2) = 18 \text{ units}^2$

24.



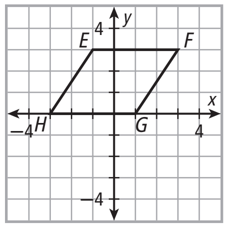
The figure is a right triangle.
$b = 5$, $h = 7$, so $c = \sqrt{5^2 + 7^2} = \sqrt{74}$ units
$P = 5 + 7 + \sqrt{74} = \left(12 + \sqrt{74}\right)$ units
$A = \frac{1}{2}(5)(7) = 17.5 \text{ units}^2$

25.



The figure is an isosceles trapezoid.
$b_1 = CD = 4 + 4 = 8$, $b_2 = AB = 2 + 2 = 4$,
$h = 3 + 1 = 4$, $AD = BC = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}$
$P = 8 + 2\sqrt{5} + 4 + 2\sqrt{5} = \left(12 + 4\sqrt{5}\right)$ units
$A = \frac{1}{2}(8 + 4)(4) = 24 \text{ units}^2$

26.



The figure is a parallelogram.
$b = EF = GH = 3 + 1 = 4$, $h = 3 - 0 = 3$,
$EH = FG = \sqrt{2^2 + 3^2} = \sqrt{13}$
$P = 4 + \sqrt{13} + 4 + \sqrt{13} = \left(8 + 2\sqrt{13}\right)$ units
$A = (4)(3) = 12 \text{ units}^2$

27. Outer rectangle: $A = (7)(8) = 56 \text{ units}^2$
Missing triangles: $A = \frac{1}{2}(3)(1) = 1.5 \text{ units}^2$,
$A = \frac{1}{2}(2)(7) = 7 \text{ units}^2$, $A = \frac{1}{2}(5)(2) = 5 \text{ units}^2$,
$A = \frac{1}{2}(4)(6) = 12 \text{ units}^2$
Area of *QRST*:
$A = 56 - (1.5 + 7 + 5 + 12) = 30.5 \text{ units}^2$

28. Outer rectangle: $A = (7)(5) = 35 \text{ units}^2$
Missing triangles: $A = \frac{1}{2}(6)(2) = 6 \text{ units}^2$,
$A = \frac{1}{2}(2)(3) = 3 \text{ units}^2$, $A = \frac{1}{2}(5)(3) = 7.5 \text{ units}^2$,
$A = \frac{1}{2}(1)(2) = 1 \text{ unit}^2$
Area of *VWXY*:
$A = 35 - (6 + 3 + 7.5 + 1) = 17.5 \text{ units}^2$

29. Outer rectangle: $A = (4)(7) = 28 \text{ units}^2$
Missing triangles: $A = \frac{1}{2}(1)(1) = 0.5 \text{ units}^2$,
$A = \frac{1}{2}(2)(6) = 6 \text{ units}^2$, $A = \frac{1}{2}(2)(2) = 2 \text{ units}^2$,
$A = \frac{1}{2}(3)(5) = 7.5 \text{ units}^2$
Area of *ABCD*:
$A = 28 - (0.5 + 6 + 2 + 7.5) = 12 \text{ units}^2$

30. Outer rectangle: $A = (6)(5) = 30 \text{ units}^2$
Missing triangles: $A = \frac{1}{2}(3)(2) = 3 \text{ units}^2$,
$A = \frac{1}{2}(1)(3) = 1.5 \text{ units}^2$, $A = \frac{1}{2}(5)(2) = 5 \text{ units}^2$,
$A = \frac{1}{2}(3)(3) = 4.5 \text{ units}^2$
Area of *EFGH*:
$A = 30 - (3 + 1.5 + 5 + 4.5) = 16 \text{ units}^2$

LESSON 9-5, PAGE 643

31. Original: $P = 5 + \sqrt{2^2 + 5^2} + \sqrt{3^2 + 5^2}$
$= \left(5 + \sqrt{29} + \sqrt{34}\right)$ units
$A = \frac{1}{2}bh = \frac{1}{2}(5)(5) = 12.5$ units^2
Tripled: $P = 15 + \sqrt{6^2 + 15^2} + \sqrt{9^2 + 15^2}$
$= \left(15 + 3\sqrt{29} + 3\sqrt{34}\right)$ units
$A = \frac{1}{2}bh = \frac{1}{2}(15)(15) = 112.5$ units^2
$15 + 3\sqrt{29} + 3\sqrt{34} = 3\left(15 + 3\sqrt{29} + 3\sqrt{34}\right)$, so the perimeter is tripled. $112.5 = 9(12.5)$, so the area is multiplied by 9.

32. Original: $P = 4s = 4(4) = 16$ units
$A = s^2 = (4)^2 = 16$ units^2
Doubled: $P = 4(8) = 32$ units
$A = (8)^2 = 64$ units^2
$32 = 2(16)$, so the perimeter is doubled.
$64 = 4(16)$, so the area is multiplied by 4.

33. Original: $C = 2\pi r = 2\pi(11) = 22\pi$ m
$A = \pi r^2 = \pi(11)^2 = 121\pi$ m^2
Halved: $C = 2\pi(5.5) = 11\pi$ m
$A = \pi(5.5)^2 = 30.25\pi$ m^2
$11\pi = \frac{1}{2}(22\pi)$, so the circumference is multiplied by $\frac{1}{2}$.
$30.25\pi = \frac{1}{4}(121\pi)$, so the area is multiplied by $\frac{1}{4}$.

34. Let the other 2 sides of the triangle (besides its base) have lengths x and y. Assume these side lengths are also multiplied by 4.
Original: $P = b + x + y = (8 + x + y)$ ft
$A = \frac{1}{2}bh = \frac{1}{2}(8)(20) = 80$ ft^2
New: $P = (32 + 4x + 4y)$ ft
$A = \frac{1}{2}(32)(80) = 1280$ ft^2
$32 + 4x + 4y = 4(8 + x + y)$, so the perimeter is multiplied by 4. $1280 = 16(80)$, so the area is multiplied by 16.

LESSON 9-6, PAGE 643

35. $AD = 7 + 1 + 5 = 13$
$P\left(\overline{AB}\right) = \frac{7}{13}$

36. $P\left(\text{not } \overline{CD}\right) = P\left(\overline{AB} \text{ or } \overline{BC}\right)$
$= P\left(\overline{AB}\right) + P\left(\overline{BC}\right)$
$= \frac{7}{13} + \frac{1}{13} = \frac{8}{13}$

37. $P\left(\overline{AB} \text{ or } \overline{CD}\right) = P\left(\overline{AB}\right) + P\left(\overline{CD}\right) = \frac{7}{13} + \frac{5}{13} = \frac{12}{13}$

38. $P\left(\overline{BC} \text{ or } \overline{CD}\right) = P\left(\overline{BC}\right) + P\left(\overline{CD}\right) = \frac{1}{13} + \frac{5}{13} = \frac{6}{13}$

39. Outer rectangle: $A = (40)(24) = 960$ m^2
Hexagon: $s = 8 = 2(4)$, so $a = 4\sqrt{3}$; $P = 6(8) = 48$ m
$A = \frac{1}{2}aP = \frac{1}{2}\left(4\sqrt{3}\right)(48) = 96\sqrt{3}$ m^2
$P = \frac{96\sqrt{3}}{960} \approx 0.17$

40. Triangle: $A = \frac{1}{2}(10)(10) = 50$ m^2
$P = \frac{50}{960} \approx 0.05$

41. Circle: $A = \pi(6)^2 = 36\pi$ m^2
$P = \frac{36\pi + 50}{960} \approx 0.17$

42. $P = \frac{960 - (96\sqrt{3} + 50 + 36\pi)}{960} \approx 0.66$

CHAPTER TEST, PAGE 644

1. $A = \frac{1}{2}bh$
$12x^2y = \frac{1}{2}(3x)h$
$24xy = 3h$
$h = 8xy$ ft

2. $A = \frac{1}{2}(b_1 + b_2)h$
$161.5 = \frac{1}{2}(b_1 + 13)(17)$
$323 = 17(b_1 + 13)$
$19 = b_1 + 13$
$b_1 = 6$ cm

3. $A = \frac{1}{2}d_1d_2 = \frac{1}{2}(25)(12) = 150$ in.^{2}

4. $C = \pi d = 12\pi$ in.
$r = \frac{1}{2}d = \frac{1}{2}(12) = 6$ in.
$A = \pi r^2 = \pi(6)^2 = 36\pi$ in.^{2}

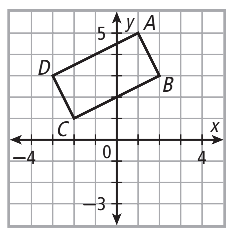
5. $s = 14 = 2(7)$, so $a = 7\sqrt{3}$ m
$P = 6(14) = 84$ m
$A = \frac{1}{2}aP$
$= \frac{1}{2}\left(7\sqrt{3}\right)(84)$
$= 294\sqrt{3}$ m$^2 \approx 509.2$ m^2

6. Rectangle: $A = (15)(8) = 120$ cm^2
Missing triangle: $A = \frac{1}{2}(6)(8) = 24$ cm^2
Missing semicircle: $A = \frac{1}{2}\pi(4)^2 = 8\pi$ cm^2
Shaded area: $A = 120 - (24 + 8\pi)$
$= (96 - 8\pi)$ cm$^2 \approx 70.9$ cm^2

7. Lower rectangle: $A = (26)(10) = 260$ in.^{2}
Upper triangle:
$A = \frac{1}{2}(26 - 16)(16 - 10) = \frac{1}{2}10)(6) = 30$ in.^{2}
Shaded area: $A = 260 + 30 = 290$ in.^{2}

8. Triangle (row 1): $A = \frac{1}{2}(2)(1) = 1$ yd^2
Parallelogram (row 2): $A = (2)(1) = 2$ yd^2
Trapezoid (row 3): $A = \frac{1}{2}(2 + 4)(1) = 3$ yd^2
Triangle (row 4): $A = \frac{1}{2}(4)(1) = 2$ yd^2
Pond: $A \approx 1 + 2 + 3 + 2 = 8$ yd^2

9.



The figure is a rectangle.
$AB = CD = \sqrt{1^2 + 2^2} = \sqrt{5}$
$AD = BC = \sqrt{4^2 + 2^2} = \sqrt{20} = 2\sqrt{5}$
$P = 2\left(\sqrt{5}\right) + 2\left(2\sqrt{5}\right) = 6\sqrt{5}$ units
$A = \left(2\sqrt{5}\right)\left(\sqrt{5}\right) = 2(5) = 10$ units^2

10. Outer rectangle: $A = (5)(8) = 40$ units^2
Missing triangles: $A = \frac{1}{2}(4)(3) = 6$ units^2,
$A = \frac{1}{2}(1)(5) = 2.5$ units^2, $A = \frac{1}{2}(4)(5) = 10$ units^2,
$A = \frac{1}{2}(1)(3) = 1.5$ units^2
Area of *EFGH*:
$A = 40 - (6 + 2.5 + 10 + 1.5) = 20$ units^2

11. Outer rectangle: $A = (7)(8) = 56$ units^2
Missing triangles: $A = \frac{1}{2}(1)(5) = 2.5$ units^2,
$A = \frac{1}{2}(6)(3) = 9$ units^2, $A = \frac{1}{2}(1)(7) = 3.5$ units^2,
$A = \frac{1}{2}(6)(1) = 3$ units^2
Area of *JKLM*:
$A = 56 - (2.5 + 9 + 3.5 + 3) = 38$ units^2

12. Let the other 2 sides of the triangle (besides its base) have lengths x and y. Assume these side lengths are also multiplied by 3.
Original: $P = b + x + y = (10 + x + y)$ cm
$A = \frac{1}{2}bh = \frac{1}{2}(10)(12) = 60$ cm^2
New: $P = (30 + 3x + 3y)$ cm
$A = \frac{1}{2}(30)(36) = 540$ cm^2
$30 + 3x + 3y = 3(10 + x + y)$, so the perimeter is multiplied by 3; $540 = 9(60)$, so the area is multiplied by 9.

13. Original: $C = 2\pi r = 2\pi(12) = 24\pi$ m
$A = \pi r^2 = \pi(12)^2 = 144\pi$ m^2
New: $C = 2\pi(6) = 12\pi$ m
$A = \pi(6)^2 = 36\pi$ m^2
$12\pi = \frac{1}{2}(24\pi)$ so the circumference is multiplied by $\frac{1}{2}$.
$36\pi = \frac{1}{4}(144\pi)$, so the area is multiplied by $\frac{1}{4}$.

14. Original: $C = 9\pi$ ft, $A = \pi(4.5)^2 = 20.25\pi$ ft^2
New: $A = \frac{1}{9}(20.25\pi)$
$\pi r^2 = 2.25\pi$
$r^2 = 2.25$
$r = 1.5$ ft
$C = 2\pi(1.5) = 3\pi$ ft
$3\pi = \frac{1}{3}(9\pi)$, so the circumference will be $\frac{1}{3}$ as long.

15. $NS = 12 + 6 + 8 = 26$
$P(\overline{NQ}) = \frac{12}{26} = \frac{6}{13}$

16. $P(\text{not } \overline{QR}) = \frac{26-6}{26} = \frac{10}{13}$

17. $P(\overline{NQ} \text{ or } \overline{RS}) = P(\overline{NQ}) + P(\overline{NS}) = \frac{12}{26} + \frac{8}{26} = \frac{10}{13}$

18. $P = \dfrac{2 \text{ min}}{18 \text{ min}} = \dfrac{1}{9}$

COLLEGE ENTRANCE EXAM PRACTICE, PAGE 645

1. 36
The 3rd angle measures 60°, so the triangle is equiangular and therefore equilateral.
The remaining side lengths are also 12, so $P = 12 + 12 + 12 = 36$.

2. 36
Let the shaded square have side length s.
$2 + s + 2 = 10$
$s = 6$
$A = 6^2 = 36$

3. 101
$m\angle U = m\angle R = 180 - (m\angle P + m\angle R)$
$x = 180 - (22 + 57) = 101$

4. 10
Points $(0 + 5, 5) = (5, 5)$ and $(2 + 4, 4) = (6, 4)$ lie on ℓ.
Slope of $\ell = \dfrac{4-5}{6-5} = -1$
Equation of ℓ: $y - 5 = -1(x - 5)$
$y - 5 = -x + 5$
$y = -x + 10$
y-intercept $= 10$

5. 135
By Linear Pair Post.,
$x + 3x = 180$
$4x = 180$
$x = 45$
By Vertical Angles Theorem, $y = 3x = 3(45) = 135$.

6. 5
$12x + 3x + 7y = 180$
$15x = 180 - 7y$
$7y > 60$, and y is an integer, so $y \geq 9$.
$180 - 7(9) = 117$; not divisible by 15
$180 - 7(10) = 110$; not divisible by 15
⋮
$180 - 7(15) = 75 = 5(15)$
⋮
$180 - 7(25) = 5$; not divisible by 15
Only solution is $x = 5$, $y = 15$.