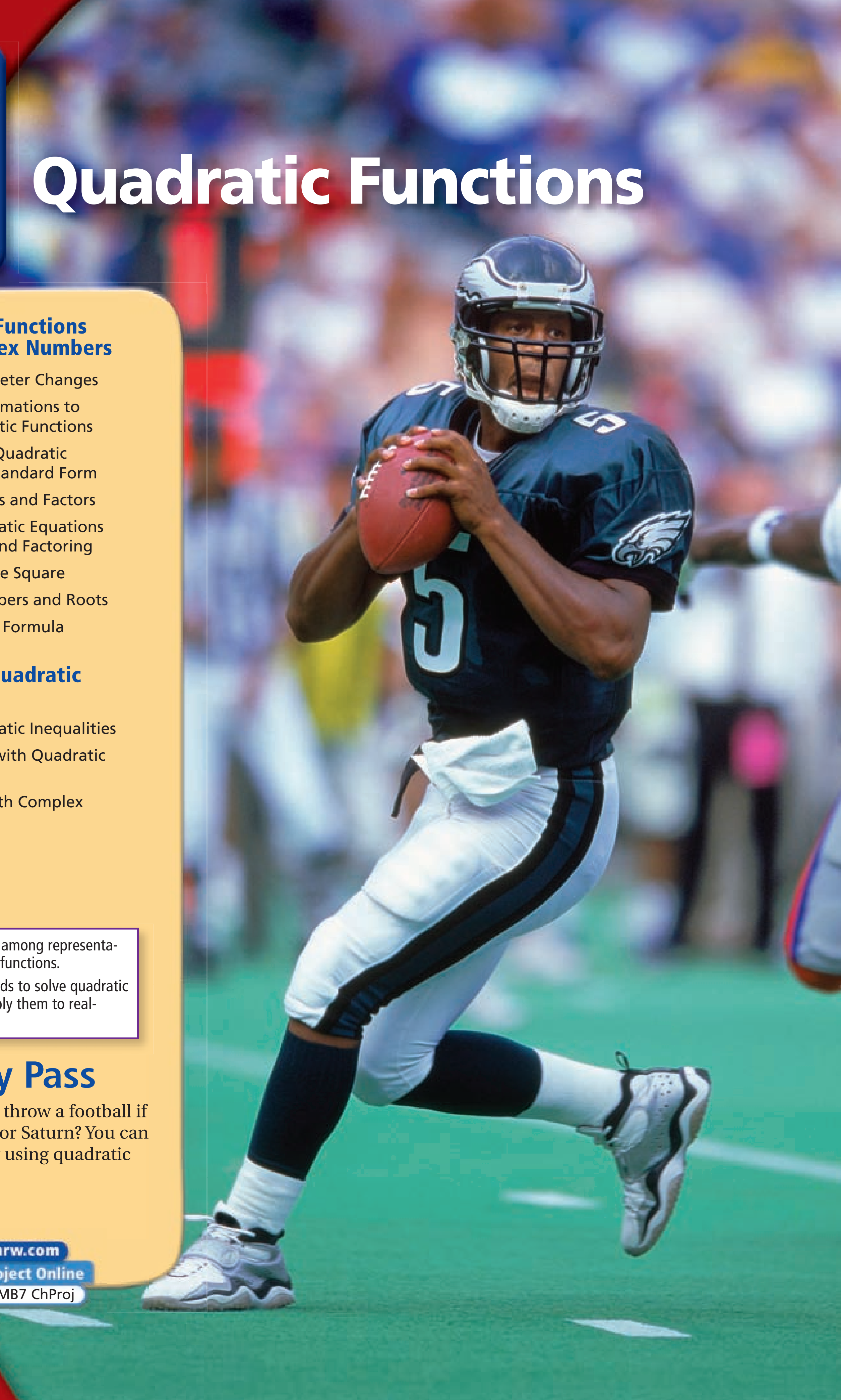
CHAPTER 5

Quadratic Functions



Chapter Focus

- Make connections among representations of quadratic functions.
- Use various methods to solve quadratic equations and apply them to real-world problems.

Planetary Pass

How far could you throw a football if you were on Mars or Saturn? You can find the answer by using quadratic functions.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. linear equation
2. solution set
3. transformation
4. x-intercept

A. a change in a function rule and its graph
B. the x-coordinate of the point where a graph crosses the x-axis
C. the group of values that make an equation or inequality true
D. a letter or symbol that represents a number
E. an equation whose graph is a line

Squares and Square Roots

Simplify each expression.

5. 3.2^2
6. $\left(\frac{2}{5}\right)^2$
7. $\sqrt{121}$
8. $\sqrt{\frac{1}{16}}$

Simplify Radical Expressions

Simplify each expression.

9. $\sqrt{72}$
10. $2\left(\sqrt{144} - 4\right)$
11. $\sqrt{33} \cdot \sqrt{75}$
12. $\frac{\sqrt{54}}{\sqrt{3}}$

Multiply Binomials

Multiply.

13. $(x - 2)(x - 6)$
14. $(x + 9)(x - 9)$
15. $(x + 2)(x + 7)$
16. $(2x - 3)(5x + 1)$

Solve Multi-Step Equations

Solve each equation.

17. $2x + 10 = -32$
18. $2x - (1 - x) = 2$
19. $\frac{2}{3}(x - 1) = 11$
20. $2(x + 5) - 5x = 1$

Graph Linear Functions

Graph each function.

21. $y = -x$
22. $y = 2x - 1$
23. $y = -3x + 6$
24. $y = \frac{1}{3}x + 2$

CHAPTER 5

Study Guide: Preview

Where You've Been

Previously, you

- graphed and transformed linear functions.
- solved linear equations and inequalities.
- fit data using linear models.
- used and performed operations with real numbers.

In This Chapter

You will study

- graphing and transforming quadratic functions.
- solving quadratic equations and inequalities.
- fitting data to quadratic models.
- using and performing operations with imaginary and other complex numbers.

Where You're Going

You can use the skills in this chapter

- in advanced math classes, including Precalculus.
- in other classes, such as Chemistry, Physics, and Economics.
- outside of school to identify patterns and model data.

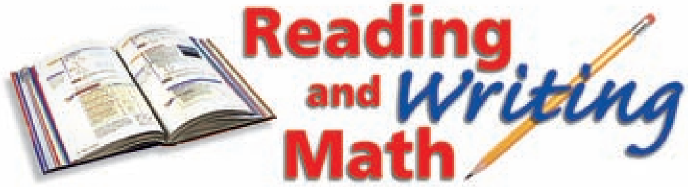
Key Vocabulary/Vocabulario

absolute value of a complex number	valor absoluto de un número complejo
complex conjugate	conjugado complejo
complex number	número complejo
imaginary number	número imaginario
maximum value	valor máximo
minimum value	valor mínimo
parabola	parábola
quadratic function	función cuadrática
vertex form	forma en vértice
zero of a function	cero de una función

Thinking About Vocabulary

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. **Quadratic** is from the Latin *quadrum,* which means "square." A quadratic function always contains a *square* of the variable, such as x^2. What is a quadrilateral, and how does it relate to a square? What are some other words that use the root *quad-*, and what do they mean?
2. The word **conjugate** can mean "joined together, especially in pairs." Name some mathematical relationships that involve pairs.
3. What might the terms **maximum value** or **minimum value** of a function refer to?
4. The word *vertex* can mean "highest point." What might the **vertex form** of a quadratic function indicate about the function's graph?



CHAPTER 5

Study Strategy: Use Multiple Representations

The explanation and example problems used to introduce new math concepts often include various representations of information. Different representations of the same idea help you fully understand the material. As you study, take note of the tables, lists, graphs, diagrams, symbols, and/or words used to clarify a concept.

From Lesson 3-2

EXAMPLE 1 **Solving Linear Systems by Substitution**

Use substitution to solve each system of equations.

A $\begin{cases} y = x + 2 \\ x + y = 8 \end{cases}$

Symbols

Step 1 Solve one equation for one variable.
The first equation is already solved for y: $y = x + 2$.

Step 2 Substitute the expression into the other equation.

$x + y = 8$
$x + (x + 2) = 8$ *Substitute $(x + 2)$ for y in the other equation.*
$2x + 2 = 8$ *Combine like terms.*
$2x = 6$
$x = 3$

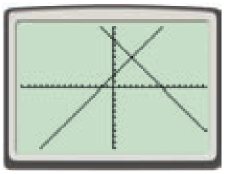
Step 3 Substitute the x-value into one of the original equations to solve for y.

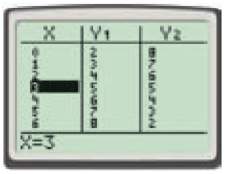
$y = x + 2$
$y = (3) + 2$ *Substitute $x = 3$.*
$y = 5$

The solution is the ordered pair $(3, 5)$.

Check A graph or table supports your answer.

Graph





Table

Caution!

The solution to an independent system of equations is an ordered pair. Do not stop working when you have found only one value.

Try This

Describe two representations you could use to solve each problem.

1. A triangle with coordinates $A(3, 5)$, $B(2, 2)$, and $C(3, -2)$ is translated 3 units left and 2 units up. Give the coordinates of the image.
2. A bottle of juice from a vending machine costs \$1.50. Hiroshi buys a bottle by inserting 8 coins in quarters and dimes. If Hiroshi receives 5 cents in change, how many quarters did he use? how many dimes?
3. What is the slope of the line that passes through the point (6, 9) and has a y-intercept of 3?

5-1 Technology LAB

Explore Parameter Changes

You can use a graphing calculator to explore how changes in the parameters of a quadratic function affect its graph. Recall from Lesson 1-9 that the quadratic parent function is $f(x) = x^2$ and that its graph is a parabola.

Use with Lesson 5-1

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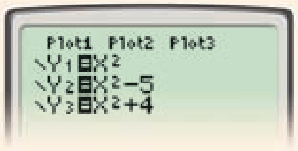
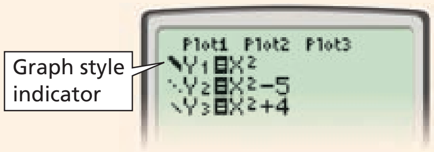
Activity

Describe what happens when you change the value of k in the quadratic function $g(x) = x^2 + k$.

1. Choose three values for k. Use 0, -5 (a negative value), and 4 (a positive value). Press Y= , and enter **X²** for **Y1**, **X² − 5** for **Y2**, and **X² + 4** for **Y3**.

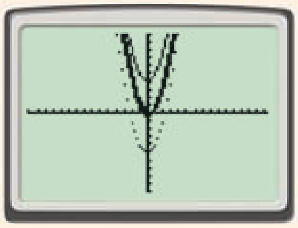


2. Change the style of the graphs of **Y1** and **Y2** so that you can tell which graph represents which function. To do this, move the cursor to the graph style indicator next to **Y1**. Press ENTER to cycle through the options. For **Y1**, which represents the parent function, choose the thick line.



3. Next, change the line style for **Y2** to the dotted line.

4. Graph the functions in the square window by pressing ZOOM and choosing **5 : ZSquare**.



Notice that the graphs are identical except that the graph of **Y2** is shifted 5 units down and the graph of **Y3** has been shifted 4 units up from the graph of **Y1**.

You can conclude that the parameter k in the function $g(x) = x^2 + k$ has the effect of translating the parent function $f(x) = x^2$ k units up if k is positive and $|k|$ units down if k is negative.

Try This

Use your graphing calculator to compare the graph of each function to the graph of $f(x) = x^2$. Describe how the graphs differ.

1. $g(x) = (x - 4)^2$ **2.** $g(x) = (x + 3)^2$ **3.** $g(x) = -x^2$

4. Make a Conjecture Use your graphing calculator to determine what happens when you change the value of h in the quadratic function $g(x) = (x - h)^2$. Check both positive and negative values of h.

5. Make a Conjecture Use your graphing calculator to determine what happens when you change the value of a in the quadratic function $g(x) = ax^2$. Check values of a that are greater than 1 and values of a that are between 0 and 1.

5-1 Using Transformations to Graph Quadratic Functions

Objectives

Transform quadratic functions.

Describe the effects of changes in the coefficients of $y = a(x - h)^2 + k$.

Vocabulary

quadratic function
parabola
vertex of a parabola
vertex form

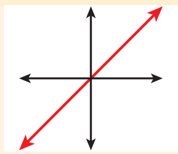
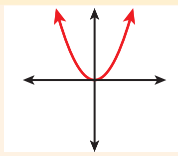
Why learn this?

You can use transformations of quadratic functions to analyze changes in braking distance. (See Example 5.)

In Chapters 2 and 3, you studied linear functions of the form $f(x) = mx + b$. A **quadratic function** is a function that can be written in the form $f(x) = a(x - h)^2 + k \ (a \neq 0)$. In a quadratic function, the variable is always squared. The table shows the linear and quadratic parent functions.

Know it! Note

Linear and Quadratic Parent Functions

ALGEBRA	NUMBERS	GRAPH
Linear Parent Function $f(x) = x$		
Quadratic Parent Function $f(x) = x^2$		

Linear parent function:

x	-2	-1	0	1	2
$f(x) = x$	-2	-1	0	1	2

Quadratic parent function:

x	-2	-1	0	1	2
$f(x) = x^2$	4	1	0	1	4

Notice that the graph of the parent function $f(x) = x^2$ is a U-shaped curve called a **parabola**. As with other functions, you can graph a quadratic function by plotting points with coordinates that make the equation true.

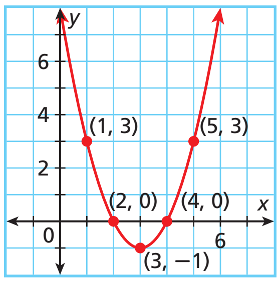
EXAMPLE 1

Graphing Quadratic Functions Using a Table

Graph $f(x) = x^2 - 6x + 8$ by using a table.

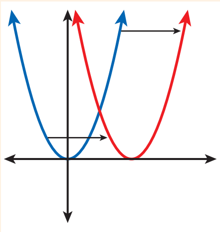
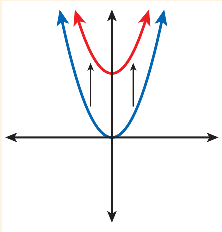
Make a table. Plot enough ordered pairs to see both sides of the curve.

x	$f(x) = x^2 - 6x + 8$	$(x, f(x))$
1	$f(1) = 1^2 - 6(1) + 8 = 3$	$(1, 3)$
2	$f(2) = 2^2 - 6(2) + 8 = 0$	$(2, 0)$
3	$f(3) = 3^2 - 6(3) + 8 = -1$	$(3, -1)$
4	$f(4) = 4^2 - 6(4) + 8 = 0$	$(4, 0)$
5	$f(5) = 5^2 - 6(5) + 8 = 3$	$(5, 3)$



1. Graph $g(x) = -x^2 + 6x - 8$ by using a table.

You can also graph quadratic functions by applying transformations to the parent function $f(x) = x^2$. Transforming quadratic functions is similar to transforming linear functions (Lesson 2-6).

Translations of Quadratic Functions	
Horizontal Translations	**Vertical Translations**
Horizontal Shift of $\lvert h \rvert$ Units	Vertical Shift of $\lvert k \rvert$ Units
 $f(x) = x^2$ $f(x - h) = (x - h)^2$ Moves left for $h < 0$ Moves right for $h > 0$	 $f(x) = x^2$ $f(x) + k = x^2 + k$ Moves down for $k < 0$ Moves up for $k > 0$

EXAMPLE 2 Translating Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

A $g(x) = (x + 3)^2 + 1$

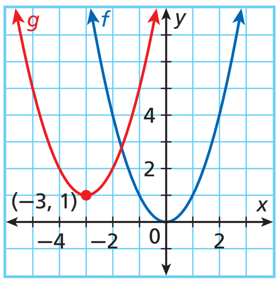
Identify h and k.

$$g(x) = (x - \underbrace{(-3)}_{h})^2 + \underbrace{1}_{k}$$

Because $h = -3$, the graph is translated **3 units left**.

Because $k = 1$, the graph is translated **1 unit up**.

Therefore, g is f translated 3 units left and 1 unit up.



B $g(x) = (x - 2)^2 - 1$

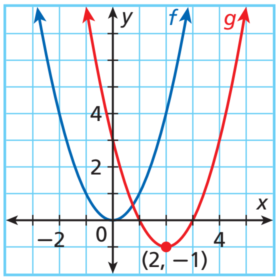
Identify h and k.

$$g(x) = (x - \underbrace{2}_{h})^2 + (\underbrace{-1}_{k})$$

Because $h = 2$, the graph is translated **2 units right**.

Because $k = -1$, the graph is translated **1 unit down**.

Therefore, g is f translated 2 units right and 1 unit down.



Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

2a. $g(x) = x^2 - 5$

2b. $g(x) = (x + 3)^2 - 2$

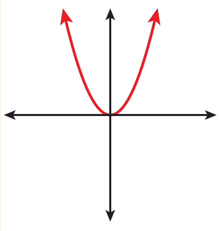
Recall that functions can also be reflected, stretched, or compressed.

Know it! Note

Reflections, Stretches, and Compressions of Quadratic Functions

Reflections

Reflection Across *y*-axis



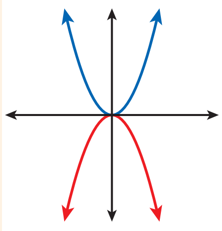
Input values change.

$f(x) = x^2$

$f(-x) = (-x)^2 = x^2$

The function $f(x) = x^2$ is its own reflection across the y-axis.

Reflection Across *x*-axis



Output values change.

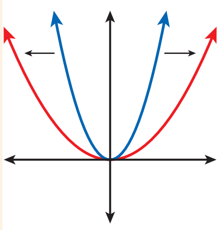
$f(x) = x^2$

$-f(x) = -(x^2)$
$= -x^2$

The function is flipped across the x-axis.

Stretches and Compressions

Horizontal Stretch/Compression by a Factor of $|b|$



Input values change.

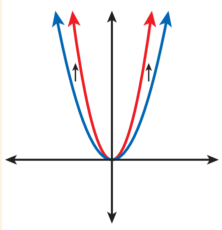
$f(x) = x^2$

$f\left(\frac{1}{b}x\right) = \left(\frac{1}{b}x\right)^2$

$|b| > 1$ stretches away from the y-axis.

$0 < |b| < 1$ compresses toward the y-axis.

Vertical Stretch/Compression by a Factor of $|a|$



Output values change.

$f(x) = x^2$

$a \cdot f(x) = ax^2$

$|a| > 1$ stretches away from the x-axis.

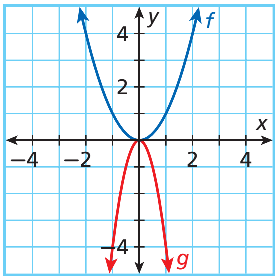
$0 < |a| < 1$ compresses toward the x-axis.

EXAMPLE 3 Reflecting, Stretching, and Compressing Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

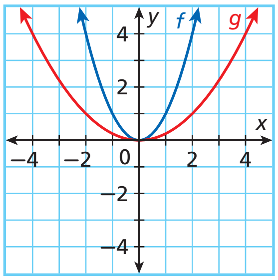
A $g(x) = -4x^2$

Because a is negative, g is a reflection of f across the x-axis. Because $|a| = 4$, g is a vertical stretch of f by a factor of 4.



B $g(x) = \left(\frac{1}{2}x\right)^2$

Because $b = 2$, g is a horizontal stretch of f by a factor of 2.



CHECK IT OUT!

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

3a. $g(x) = (2x)^2$

3b. $g(x) = -\frac{1}{2}x^2$

If a parabola opens upward, it has a lowest point. If a parabola opens downward, it has a highest point. This lowest or highest point is the **vertex of a parabola**.

The parent function $f(x) = x^2$ has its vertex at the origin. You can identify the vertex of other quadratic functions by analyzing the function in *vertex form*. The **vertex form** of a quadratic function is $f(x) = a(x - h)^2 + k$, where a, h, and k are constants.

Know it! Note

Vertex Form of a Quadratic Function

$$f(x) = a(x - h)^2 + k$$

a indicates a reflection across the x-axis and/or a vertical stretch or compression.

h indicates a horizontal translation.

k indicates a vertical translation.

Because the vertex is translated h horizontal units and k vertical units from the origin, the vertex of the parabola is at (h, k).

EXAMPLE 4 Writing Transformed Quadratic Functions

Use the description to write the quadratic function in vertex form.

The parent function $f(x) = x^2$ is reflected across the x-axis, vertically stretched by a factor of 6, and translated 3 units left to create g.

Step 1 Identify how each transformation affects the constants in vertex form.

reflection across x-axis:	a is negative	$a = -6$
vertical stretch by 6:	$\lvert a \rvert = 6$	
translation left 3 units:	$h = -3$	

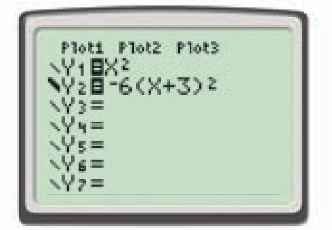
Step 2 Write the transformed function.

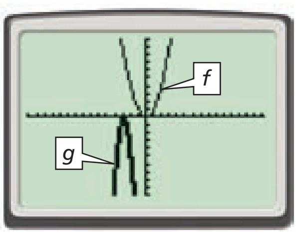
$g(x) = a(x - h)^2 + k$ — *Vertex form of a quadratic function*

$= -6(x - (-3))^2 + 0$ — *Substitute −6 for a, −3 for h, and 0 for k.*

$= -6(x + 3)^2$ — *Simplify.*

Check Graph both functions on a graphing calculator. Enter f as **Y1** and g as **Y2**. The graph indicates the identified transformations.





Helpful Hint

When the quadratic parent function $f(x) = x^2$ is written in vertex form, $y = a(x - h)^2 + k$, $a = 1$, $h = 0$, and $k = 0$.

CHECK IT OUT! **Use the description to write the quadratic function in vertex form.**

4a. The parent function $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{3}$ and translated 2 units right and 4 units down to create g.

4b. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 5 units left and 1 unit up to create g.

EXAMPLE 5 *Automotive Application*

The minimum braking distance d in feet for a vehicle on dry concrete is approximated by the function $d(v) = 0.045v^2$, where v is the vehicle's speed in miles per hour. If the vehicle's tires are in poor condition, the braking-distance function is $d_p(v) = 0.068v^2$. What kind of transformation describes this change, and what does the transformation mean?

Examine both functions in vertex form.

$$d(v) = 0.045(v - 0)^2 + 0 \qquad d_p(v) = 0.068(v - 0)^2 + 0$$

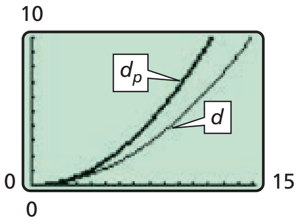
The value of a has increased from 0.045 to 0.068. The increase indicates a vertical stretch.

Find the stretch factor by comparing the new a-value to the old a-value:

$$\frac{a \text{ from } d_p(v)}{a \text{ from } d(v)} = \frac{0.068}{0.045} \approx 1.5$$

The function d_p represents a vertical stretch of d by a factor of approximately 1.5. Because the value of each function approximates braking distance, a vehicle with tires in poor condition takes about 1.5 times as many feet to stop as a vehicle with good tires does.

Check Graph both functions on a graphing calculator. The graph of d_p appears to be vertically stretched compared with the graph of d.



CHECK IT OUT! **Use the information above to answer the following.**

5. The minimum braking distance d_n in feet for a vehicle with new tires at optimal inflation is $d_n(v) = 0.039v^2$, where v is the vehicle's speed in miles per hour. What kind of transformation describes this change from $d(v) = 0.045v^2$, and what does this transformation mean?

THINK AND DISCUSS

1. Explain how the values of a, h, and k in the vertex form of a quadratic function affect the function's graph.

2. Explain how to determine which of two quadratic functions expressed in vertex form has a narrower graph.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each row, write an equation that represents the indicated transformation of the quadratic parent function, and show its graph.

Transformation	Equation	Graph
Vertical translation		
Horizontal translation		
Reflection		
Vertical stretch		
Vertical compression		

5-1 Exercises

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Homework Help Online
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Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The highest or lowest point on the graph of a quadratic function is the __?__. (*vertex* or *parabola*)

SEE EXAMPLE 1 p. 315

Graph each function by using a table.

2. $f(x) = -2x^2 - 4$
3. $g(x) = -x^2 + 3x - 2$
4. $h(x) = x^2 + 2x$

SEE EXAMPLE 2 p. 316

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

5. $d(x) = (x - 4)^2$
6. $g(x) = (x - 3)^2 + 2$
7. $h(x) = (x + 1)^2 - 3$

SEE EXAMPLE 3 p. 317

8. $g(x) = 3x^2$
9. $h(x) = \left(\frac{1}{8}x\right)^2$
10. $p(x) = 0.25x^2$
11. $h(x) = -(5x)^2$
12. $g(x) = 4.2x^2$
13. $d(x) = -\frac{2}{3}x^2$

SEE EXAMPLE 4 p. 318

Use the description to write each quadratic function in vertex form.

14. The parent function $f(x) = x^2$ is vertically stretched by a factor of 2 and translated 3 units left to create g.
15. The parent function $f(x) = x^2$ is reflected across the x-axis and translated 6 units down to create h.

SEE EXAMPLE 5 p. 319

16. **Physics** The safe working load L in pounds for a natural rope can be estimated by $L(r) = 5920r^2$, where r is the radius of the rope in inches. For an old rope, the function $L_o(r) = 4150r^2$ is used to estimate its safe working load. What kind of transformation describes this change, and what does this transformation mean?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
17–19	1
20–25	2
26–28	3
29–30	4
31	5

Extra Practice

Skills Practice p. S12
Application Practice p. S36

Graph each function by using a table.

17. $f(x) = -x^2 + 4$
18. $g(x) = x^2 - 2x + 1$
19. $h(x) = 2x^2 + 4x - 1$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

20. $g(x) = x^2 - 2$
21. $h(x) = (x + 5)^2$
22. $j(x) = (x - 1)^2$
23. $g(x) = (x + 4)^2 - 3$
24. $h(x) = (x + 2)^2 + 2$
25. $j(x) = (x - 4)^2 - 9$
26. $g(x) = \frac{4}{7}x^2$
27. $h(x) = -20x^2$
28. $j(x) = \left(\frac{1}{3}x\right)^2$

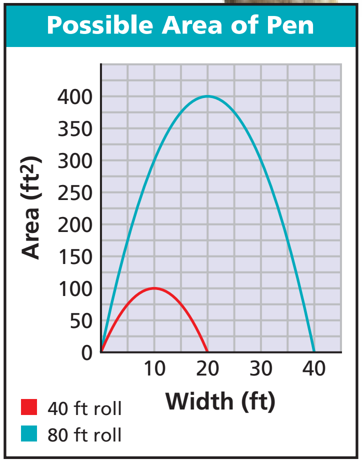
Use the description to write each quadratic function in vertex form.

29. The parent function $f(x) = x^2$ is reflected across the x-axis, vertically compressed by a factor of $\frac{1}{2}$, and translated 1 unit right to create g.
30. The parent function $f(x) = x^2$ is vertically stretched by a factor of 2.5 and translated 2 units left and 1 unit up to create h.
31. **Consumer Economics** The average gas mileage m in miles per gallon for a compact car is modeled by $m(s) = -0.015(s - 47)^2 + 33$, where s is the car's speed in miles per hour. The average gas mileage for an SUV is modeled by $m_u(s) = -0.015(s - 47)^2 + 15$. What kind of transformation describes this change, and what does this transformation mean?

32. **Pets** Keille is building a rectangular pen for a pet rabbit. She can buy wire fencing in a roll of 40 ft or a roll of 80 ft. The graph shows the area of pens she can build with each type of roll.

a. Describe the function for an 80 ft roll of fencing as a transformation of the function for a 40 ft roll of fencing.

b. Is the largest pen Keille can build with an 80 ft roll of fencing twice as large as the largest pen she can build with a 40 ft roll of fencing? Explain.



Using $f(x) = x^2$ as a guide, describe the transformations for each function.

33. $p(x) = -(x-4)^2$

34. $g(x) = 8(x+2)^2$

35. $h(x) = 4x^2 - 2$

36. $p(x) = \frac{1}{4}x^2 + 2$

37. $g(x) = (3x)^2 + 1$

38. $h(x) = -\left(\frac{1}{3}x\right)^2$

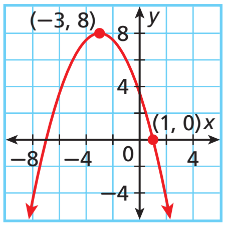
Match each graph with one of the following functions.

A. $a(x) = 4(x+8)^2 - 3$

B. $b(x) = -2(x-8)^2 + 3$

C. $c(x) = -\frac{1}{2}(x+3)^2 + 8$

39.



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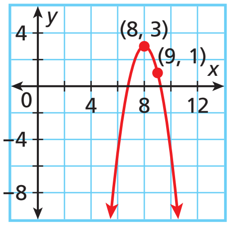

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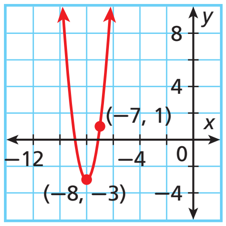
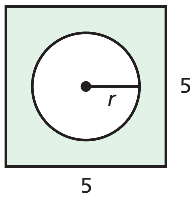


42. **Geometry** The area A of the circle in the figure can be represented by $A(r) = \pi r^2$, where r is the radius.

a. Write a function B in terms of r that represents the area of the shaded portion of the figure.

b. Describe B as a transformation of A.

c. What are the reasonable domain and range for each function? Explain.



43. **Critical Thinking** What type of graph would a function of the form $f(x) = a(x-h)^2 + k$ have if $a = 0$? What type of function would it be?

44. **Write About It** Describe the graph of $f(x) = 999{,}999(x+5)^2 + 5$ without graphing it.

Multi-Step Test Prep

45. This problem will prepare you for the Multi-Step Test Prep on page 364.

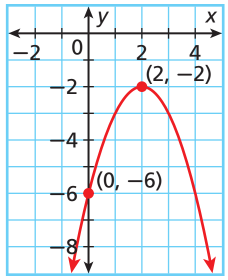
The height h in feet of a baseball on Earth after t seconds can be modeled by the function $h(t) = -16(t-1.5)^2 + 36$, where -16 is a constant in ft/s^2 due to Earth's gravity.

a. **What if…?** The gravity on Mars is only 0.38 times that on Earth. If the same baseball were thrown on Mars, it would reach a maximum height 59 feet higher and 2.5 seconds later than on Earth. Describe the transformations that must be applied to make the function model the height of the baseball on Mars.

b. Write a height function for the baseball thrown on Mars.

TEST PREP

Use the graph for Exercises 46 and 47.



46. Which best describes how the graph of the function $y = -x^2$ was transformed to produce the graph shown?

Ⓐ Translation 2 units right and 2 units up

Ⓑ Translation 2 units right and 2 units down

Ⓒ Translation 2 units left and 2 units up

Ⓓ Translation 2 units left and 2 units down

47. Which gives the function rule for the parabola shown?

Ⓕ $f(x) = (x + 2)^2 - 2$

Ⓗ $f(x) = (x - 2)^2 - 2$

Ⓖ $f(x) = -(x + 2)^2 - 2$

Ⓙ $f(x) = -(x - 2)^2 - 2$

48. Which shows the functions below in order from widest to narrowest of their corresponding graphs?

$m(x) = \frac{1}{6}x^2$ $\quad n(x) = 4x^2$ $\quad p(x) = 6x^2$ $\quad q(x) = -\frac{1}{2}x^2$

Ⓐ m, n, p, q

Ⓒ m, q, n, p

Ⓑ q, m, n, p

Ⓓ q, p, n, m

49. Which of the following functions has its vertex below the x-axis?

Ⓕ $f(x) = (x - 7)^2$

Ⓗ $f(x) = -2x^2$

Ⓖ $f(x) = x^2 - 8$

Ⓙ $f(x) = -(x + 3)^2$

50. **Gridded Response** What is the y-coordinate of the vertex of the graph of $f(x) = -3(x - 1)^2 + 5$?

CHALLENGE AND EXTEND

51. Identify the transformations of the graph of $f(x) = -3(x + 3)^2 - 3$ that would cause the graph's image to have a vertex at $(3, 3)$. Then write the transformed function.

52. Consider the functions $f(x) = (2x)^2 - 2$ and $g(x) = 4x^2 - 2$.

a. Describe each function as a transformation of the quadratic parent function.

b. Graph both functions on the coordinate plane.

c. Make a conjecture about the relationship between the two functions.

d. Write the rule for a horizontal compression of the parent function that would give the same graph as $f(x) = 9x^2$.

SPIRAL REVIEW

53. **Packaging** Peanuts are packaged in cylindrical containers. A small container is 7 in. tall and has a radius of 2 in. A large container is 5.5 in. tall and has a radius twice that of the small container. The price of the large container is three times the price of the small container. Is this price justified? Explain. *(Previous course)*

Identify the parent function for g from its function rule. *(Lesson 1-9)*

54. $g(x) = 4x + \sqrt{3}$

55. $g(x) = 3\sqrt{x + 4}$

Write each function in slope-intercept form. Then graph the function. *(Lesson 2-3)*

56. $2y + 5x = 14$

57. $x - \frac{1}{2}y + 4 = -1$

5-2 Properties of Quadratic Functions in Standard Form

Objectives

Define, identify, and graph quadratic functions.

Identify and use maximums and minimums of quadratic functions to solve problems.

Vocabulary

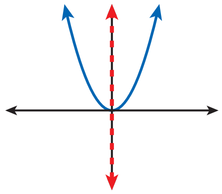
axis of symmetry
standard form
minimum value
maximum value

Why learn this?

Quadratic functions can be used to find the maximum power generated by the engine of a speedboat. (See Example 4.)

When you transformed quadratic functions in the previous lesson, you saw that reflecting the parent function across the y-axis results in the same function.

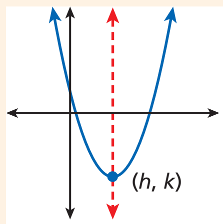
$f(x) = x^2$
$g(x) = (-x)^2 = x^2$



This shows that parabolas are symmetric curves. The **axis of symmetry** is the line through the vertex of a parabola that divides the parabola into two congruent halves.

Know it! Note

Axis of Symmetry — Quadratic Functions

WORDS	ALGEBRA	GRAPH
The axis of symmetry is a vertical line through the vertex of the function's graph.	The quadratic function $f(x) = a(x - h)^2 + k$ has the axis of symmetry $x = h$.	

EXAMPLE 1 **Identifying the Axis of Symmetry**

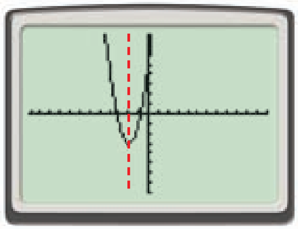
Identify the axis of symmetry for the graph of $f(x) = 2(x + 2)^2 - 3$.

Rewrite the function to find the value of h.

$f(x) = 2[x - (-2)]^2 - 3$

Because $h = -2$, the axis of symmetry is the vertical line $x = -2$.

Check Analyze the graph on a graphing calculator. The parabola is symmetric about the vertical line $x = -2$.



CHECK IT OUT!

1. Identify the axis of symmetry for the graph of $f(x) = (x - 3)^2 + 1$.

Another useful form of writing quadratic functions is the *standard form*. The **standard form** of a quadratic function is $f(x) = ax^2 + bx + c$, where $a \neq 0$.

The coefficients a, b, and c can show properties of the graph of the function. You can determine these properties by expanding the vertex form.

$$f(x) = a(x - h)^2 + k$$

$$f(x) = a(x^2 - 2xh + h^2) + k \qquad \textit{Multiply to expand } (x - h)^2.$$

$$f(x) = a(x^2) - a(2hx) + a(h^2) + k \qquad \textit{Distribute } a.$$

$$f(x) = ax^2 + (-2ah)x + (ah^2 + k) \qquad \textit{Simplify and group like terms.}$$

$a = a \qquad -2ah = b \qquad ah^2 + k = c$

$$f(x) = ax^2 + bx + c$$

$a = a$ — a in standard form is the same as in vertex form. It indicates whether a reflection and/or vertical stretch or compression has been applied.

$b = -2ah$ — Solving for h gives $h = \frac{b}{-2a} = -\frac{b}{2a}$. Therefore, the axis of symmetry, $x = h$, for a quadratic function in standard form is $x = -\frac{b}{2a}$.

$c = ah^2 + k$ — Notice that the value of c is the same value given by the vertex form of f when $x = 0$: $f(0) = a(0 - h)^2 + k = ah^2 + k$. So c is the y-intercept.

These properties can be generalized to help you graph quadratic functions.

Know it! Note

Properties of a Parabola

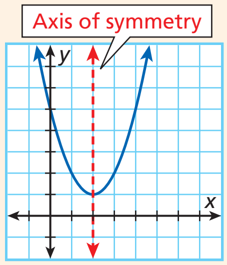
For $f(x) = ax^2 + bx + c$, where a, b, and c are real numbers and $a \neq 0$, the parabola has these properties:

The parabola **opens** upward if $a > 0$ and downward if $a < 0$.

The **axis of symmetry** is the vertical line $x = -\frac{b}{2a}$.

The **vertex** is the point $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.

The ***y*-intercept** is c.



EXAMPLE 2 Graphing Quadratic Functions in Standard Form

A **Consider the function $f(x) = x^2 - 4x + 6$.**

a. Determine whether the graph opens upward or downward.

Because a is positive, the parabola opens upward.

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$.

$$x = -\frac{(-4)}{2(1)} = 2 \qquad \textit{Substitute } -4 \textit{ for } b \textit{ and } 1 \textit{ for } a.$$

The axis of symmetry is the line $x = 2$.

c. Find the vertex.

The vertex lies on the axis of symmetry, so the x-coordinate is 2. The y-coordinate is the value of the function at this x-value, or $f(2)$.

$$f(2) = (2)^2 - 4(2) + 6 = 2$$

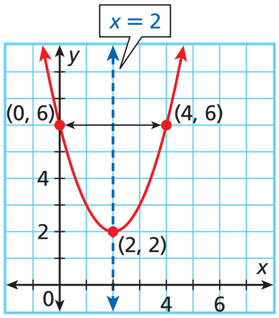
The vertex is $(2, 2)$.

d. Find the y-intercept.

Because $c = 6$, the y-intercept is 6.

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0, 6)$. Use the axis of symmetry to find another point on the parabola. Notice that $(0, 6)$ is 2 units left of the axis of symmetry. The point on the parabola symmetrical to $(0, 6)$ is 2 units right of the axis at $(4, 6)$.



B **Consider the function $f(x) = -4x^2 - 12x - 3$.**

a. Determine whether the graph opens upward or downward.

Because a is negative, the parabola opens downward.

Helpful Hint

When a is positive, the parabola is happy (∪). When a is negative, the parabola is sad (∩).

b. Find the axis of symmetry.

The axis of symmetry is given by $x = -\frac{b}{2a}$.

$$x = -\frac{(-12)}{2(-4)} = -\frac{3}{2}$$ *Substitute −12 for b and −4 for a.*

The axis of symmetry is the line $x = -\frac{3}{2}$, or $x = -1.5$.

c. Find the vertex.

The vertex lies on the axis of symmetry, so the x-coordinate is -1.5.

The y-coordinate is the value of the function at this x-value, or $f(-1.5)$.

$$f(-1.5) = -4(-1.5)^2 - 12(-1.5) - 3 = 6$$

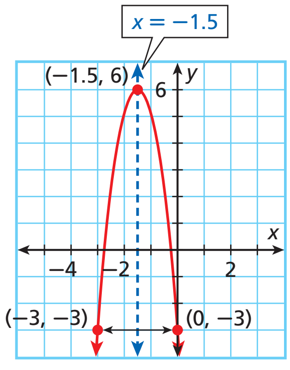
The vertex is $(-1.5, 6)$.

d. Find the y-intercept.

Because $c = -3$, the y-intercept is -3.

e. Graph the function.

Graph by sketching the axis of symmetry and then plotting the vertex and the intercept point, $(0, -3)$. Use the axis of symmetry to find another point on the parabola. Notice that $(0, -3)$ is 1.5 units right of the axis of symmetry. The point on the parabola symmetrical to $(0, -3)$ is 1.5 units left of the axis at $(-3, -3)$.



CHECK IT OUT! **For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.**

2a. $f(x) = -2x^2 - 4x$ **2b.** $g(x) = x^2 + 3x - 1$

Substituting any real value of x into a quadratic equation results in a real number. Therefore, the domain of any quadratic function is all real numbers, $\mathbb{R}$. The range of a quadratic function depends on its vertex and the direction that the parabola opens.

Know it! Note

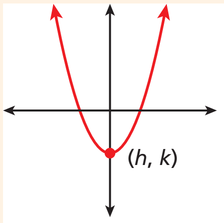
Minimum and Maximum Values

OPENS UPWARD

When a parabola opens upward, the y-value of the vertex is the **minimum value**.

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \geq k\}$



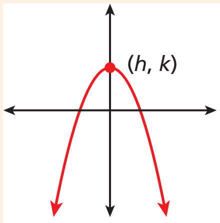
The domain is all real numbers, $\mathbb{R}$. The range is all values greater than or equal to the minimum.

OPENS DOWNWARD

When a parabola opens downward, the y-value of the vertex is the **maximum value**.

D: $\{x \mid x \in \mathbb{R}\}$

R: $\{y \mid y \leq k\}$



The domain is all real numbers, $\mathbb{R}$. The range is all values less than or equal to the maximum.

EXAMPLE 3 Finding Minimum or Maximum Values

Find the minimum or maximum value of $f(x) = 2x^2 - 2x + 5$. Then state the domain and range of the function.

Step 1 Determine whether the function has a minimum or maximum value. Because a is positive, the graph opens upward and has a minimum value.

Step 2 Find the x-value of the vertex.

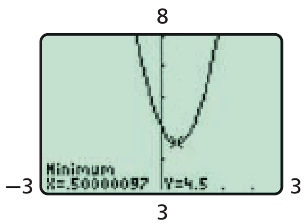
$$x = -\frac{b}{2a} = -\frac{(-2)}{2(2)} = \frac{2}{4} = \frac{1}{2}$$ *Substitute −2 for b and 2 for a.*

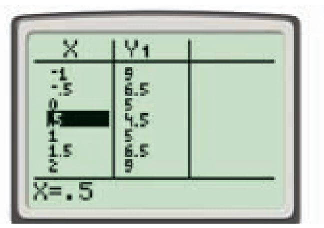
Step 3 Then find the y-value of the vertex, $f\left(-\frac{b}{2a}\right)$.

$$f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 2\left(\frac{1}{2}\right) + 5 = 4\frac{1}{2}$$

The minimum value is $4\frac{1}{2}$, or 4.5. The domain is all real numbers, $\mathbb{R}$. The range is all real numbers greater than or equal to 4.5, or $\{y \mid y \geq 4.5\}$.

Check Graph $f(x) = 2x^2 - 2x + 5$ on a graphing calculator. The graph and table support the answer.





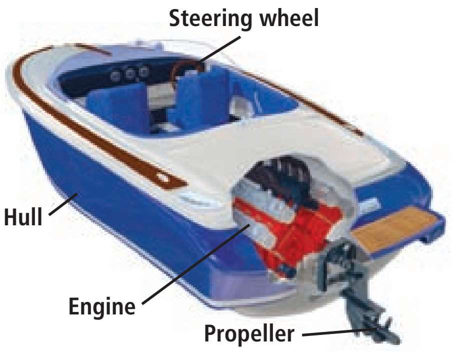
Caution!

The minimum (or maximum) value is the *y-value* of the vertex. It is *not* the ordered pair representing the vertex.

CHECK IT OUT! **Find the minimum or maximum value of each function. Then state the domain and range of the function.**

3a. $f(x) = x^2 - 6x + 3$ **3b.** $g(x) = -2x^2 - 4$

EXAMPLE 4 Transportation Application



The power p in horsepower (hp) generated by a high-performance speedboat engine operating at r revolutions per minute (rpm) can be modeled by the function $p(r) = -0.0000147r^2 + 0.18r - 251$. What is the maximum power of this engine to the nearest horsepower? At how many revolutions per minute must the engine be operating to achieve this power?

The maximum value will be at the vertex $\left(r, p(r)\right)$.

Step 1 Find the r-value of the vertex using $a = -0.0000147$ and $b = 0.18$.

$$r = -\frac{b}{2a} = -\frac{0.18}{2(-0.0000147)} \approx 6122$$

Step 2 Substitute this r-value into p to find the corresponding maximum, $p(r)$.

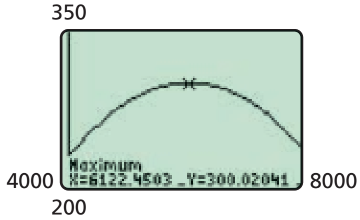
$$p(r) = -0.0000147r^2 + 0.18r - 251$$

$$p(6122) = -0.0000147(6122)^2 + 0.18(6122) - 251 \quad \textit{Substitute 6122 for r.}$$

$$p(6122) \approx 300 \quad \textit{Use a calculator.}$$

The maximum power is about 300 hp at 6122 rpm.

Check Graph the function on a graphing calculator. Use the **maximum** feature under the **CALCULATE** menu to approximate the maximum. The graph supports your answer.



CHECK IT OUT!

4. The highway mileage m in miles per gallon for a compact car is approximated by $m(s) = -0.025s^2 + 2.45s - 30$, where s is the speed in miles per hour. What is the maximum mileage for this compact car to the nearest tenth of a mile per gallon? What speed results in this mileage?

THINK AND DISCUSS

1. Explain whether a quadratic function can have both a maximum value and a minimum value.

2. Explain why the value of $f(x) = x^2 + 2x - 1$ increases as the value of x decreases from -1 to -10.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, write the criteria or equation to find each property of the parabola for $f(x) = ax^2 + bx + c$.

Opens upward or downward	Axis of symmetry
Properties of Parabolas	
y-intercept	Vertex

5-2 Exercises

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Homework Help Online
KEYWORD: MB7 5-2
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** If the graph of a quadratic function opens upward, the y-value of the vertex is a __?__ value. (*maximum* or *minimum*)

SEE EXAMPLE 1 p. 323

Identify the axis of symmetry for the graph of each function.

2. $f(x) = -2(x-2)^2 - 4$
3. $g(x) = 3x^2 + 4$
4. $h(x) = (x+5)^2$

SEE EXAMPLE 2 p. 324

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

5. $f(x) = -x^2 - 2x - 8$
6. $g(x) = x^2 - 3x + 2$
7. $h(x) = 4x - x^2 - 1$

SEE EXAMPLE 3 p. 326

Find the minimum or maximum value of each function. Then state the domain and range of the function.

8. $f(x) = x^2 - 1$
9. $g(x) = -x^2 + 3x - 2$
10. $h(x) = -16x^2 + 32x + 4$

SEE EXAMPLE 4 p. 327

11. **Sports** The path of a soccer ball is modeled by the function $h(x) = -0.005x^2 + 0.25x$, where h is the height in meters and x is the horizontal distance that the ball travels in meters. What is the maximum height that the ball reaches?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–14	1
15–23	2
24–29	3
30	4

Extra Practice

Skills Practice p. S12
Application Practice p. S36

Identify the axis of symmetry for the graph of each function.

12. $f(x) = -x^2 + 4$
13. $g(x) = (x-1)^2$
14. $h(x) = 2(x+1)^2 - 3$

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

15. $f(x) = x^2 + x - 2$
16. $g(x) = -3x^2 + 6x$
17. $h(x) = 0.5x^2 - 2x - 4$
18. $f(x) = -2x^2 + 8x + 5$
19. $g(x) = 3x^2 + 2x - 8$
20. $h(x) = 2x - 1 + x^2$
21. $f(x) = -(2 + x^2)$
22. $g(x) = 0.5x^2 + 3x - 5$
23. $h(x) = \frac{1}{4}x^2 + x + 2$

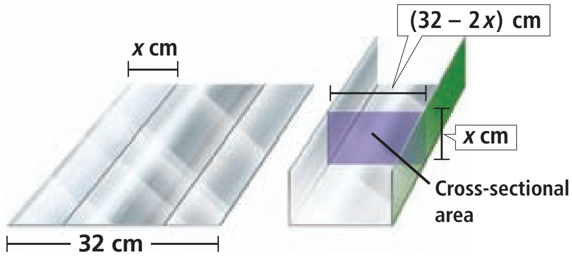
Find the minimum or maximum value of each function. Then state the domain and range of the function.

24. $f(x) = -2x^2 + 7x - 3$
25. $g(x) = 6x - x^2$
26. $h(x) = x^2 - 4x + 3$
27. $f(x) = -\frac{1}{2}x^2 - 4$
28. $g(x) = -x^2 - 6x + 1$
29. $h(x) = x^2 + 8x + 16$
30. **Weather** The daily high temperature in Death Valley, California, in 2003 can be modeled by $T(d) = -0.0018d^2 + 0.657d + 50.95$, where T is temperature in degrees Fahrenheit and d is the day of the year. What was the maximum temperature in 2003 to the nearest degree?
31. **Sports** The height of a golf ball over time can be represented by a quadratic function. Graph the data in the table. What is the maximum height that the ball will reach? Explain your answer in terms of the axis of symmetry and vertex of the graph.

Golf Ball Height

Time (s)	0	0.5	1	2	3
Height (ft)	0	28	48	64	48

32. **Manufacturing** A roll of aluminum with a width of 32 cm is to be bent into rain gutters by folding up two sides at 90° angles. A rain gutter's greatest capacity, or volume, is determined by the gutter's greatest cross-sectional area, as shown.



a. Write a function C to describe the cross-sectional area in terms of the width of the bend x.

b. Make a table, and graph the function.

c. Identify the meaningful domain and range of the function.

d. Find the value of x that maximizes the cross-sectional area.

LINK Biology

Spittlebugs are insects that feed on the sap of plants. Young spittlebug larvae use sap to produce a layer of bubbles around themselves. The bubbles help to keep the larvae from drying out and may protect them from predators.

33. **Biology** The spittlebug is the world's highest jumping animal relative to its body length of about 6 mm. The height h of a spittlebug's jump in millimeters can be modeled by the function $h(t) = -4000t^2 + 3000t$, where t is the time in seconds.

a. What is the maximum height that the spittlebug will reach?

b. What is the ratio of a spittlebug's maximum jumping height to its body length? In the best human jumpers, this ratio is about 1.38. Compare the ratio for spittlebugs with the ratio for the best human jumpers.

c. **What if...?** Suppose humans had the same ratio of maximum jumping height to body length as spittlebugs. How high would a person with a height of 1.8 m be able to jump?

34. **Gardening** The function $A(x) = x(10 - x)$ describes the area A of a rectangular flower garden, where x is its width in yards. What is the maximum area of the garden?

Graphing Calculator Once you have graphed a function, the graphing calculator can automatically find the minimum or maximum value. From the **CALC** menu, choose the **minimum** or **maximum** feature.

Use a graphing calculator to find the approximate minimum or maximum value of each function.

35. $f(x) = 5.23x^2 - 4.84x - 1.91$

36. $g(x) = -12.8x^2 + 8.73x + 11.69$

37. $h(x) = \frac{1}{12}x^2 - \frac{4}{5}x + \frac{2}{3}$

38. $j(x) = -\frac{5}{3}x^2 + \frac{9}{10}x + \frac{21}{4}$

39. **Critical Thinking** Suppose you are given a parabola with two points that have the same y-value, such as $(-7, 11)$ and $(3, 11)$. Explain how to find the equation for the axis of symmetry of this parabola, and then determine this equation.

40. **Write About It** Can a maximum value for a quadratic function be negative? Can a minimum value for a quadratic function be positive? Explain by using examples.

MULTI-STEP TEST PREP

41. This problem will prepare you for the Multi-Step Test Prep on page 364.

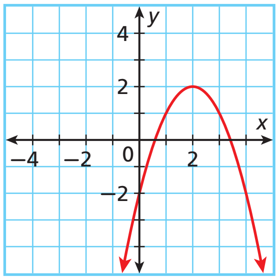
A baseball is thrown with a vertical velocity of 50 ft/s from an initial height of 6 ft. The height h in feet of the baseball can be modeled by $h(t) = -16t^2 + 50t + 6$, where t is the time in seconds since the ball was thrown.

a. Approximately how many seconds does it take the ball to reach its maximum height?

b. What is the maximum height that the ball reaches?

TEST PREP

Use the graph for exercises 42 and 43.



42. What is the range of the function graphed?

Ⓐ All real numbers
Ⓑ $y \geq -2$
Ⓒ $y \leq 2$
Ⓓ $-2 \leq y \leq 2$

43. The graph shown represents which quadratic function?

Ⓕ $f(x) = x^2 + 2x - 2$
Ⓖ $f(x) = -x^2 + 4x - 2$
Ⓗ $f(x) = x^2 - 4x - 2$
Ⓙ $f(x) = -x^2 - 2x + 2$

44. Which of the following is NOT true of the graph of the function $f(x) = -x^2 - 6x + 5$?

Ⓐ Its vertex is at $(-3, 14)$.
Ⓑ Its axis of symmetry is $x = 14$.
Ⓒ Its maximum value is 14.
Ⓓ Its y-intercept is 5.

45. Which equation represents the axis of symmetry for $f(x) = 2x^2 - 4x + 5$?

Ⓕ $x = -4$ Ⓖ $x = 1$ Ⓗ $x = 2$ Ⓙ $x = 5$

46. **Short Response** Explain how to find the maximum value or minimum value of a quadratic function such as $f(x) = -x^2 - 8x + 4$.

CHALLENGE AND EXTEND

47. Write the equations in standard form for two quadratic functions that have the same vertex but open in different directions.

48. The graph of a quadratic function passes through the point $(-5, 8)$, and its axis of symmetry is $x = 3$.

a. What are the coordinates of another point on the graph of the function? Explain how you determined your answer.

b. Can you determine whether the graph of the function opens upward or downward? Explain.

49. **Critical Thinking** What conclusions can you make about the axis of symmetry and the vertex of a quadratic function of the form $f(x) = ax^2 + c$?

50. **Critical Thinking** Given the quadratic function f and the fact that $f(-1) = f(2)$, how can you find the axis of symmetry of this function?

SPIRAL REVIEW

Simplify each expression. *(Lesson 1-3)*

51. $\sqrt{40} \cdot \sqrt{180}$ **52.** $2\sqrt{8} \cdot 4\sqrt{3}$ **53.** $\sqrt{54} \div \sqrt{30}$ **54.** $\sqrt{304}$

For each function, evaluate $f(0)$, $f\left(\frac{1}{2}\right)$, and $f(-2)$. *(Lesson 1-7)*

55. $f(x) = (x - 3)^2 + 1$ **56.** $g(x) = 2\left(x - \frac{1}{2}\right)^2$

57. $f(x) = -4(x + 5)$ **58.** $g(x) = x^3 - 4x + 8$

Write the equation of each line with the given properties. *(Lesson 2-4)*

59. a slope of 3 passing through $(1, -4)$ **60.** passing through $(-3, 5)$ and $(-1, -7)$

61. a slope of -2 passing through $(3, 5)$ **62.** passing through $(4, 6)$ and $(-2, 1)$

Connecting Algebra to Previous Courses

Factoring Quadratic Expressions

Review the methods of factoring quadratic expressions in the examples below. Recall that the standard form of a quadratic expression is $ax^2 + bx + c$.

Examples

Factor each expression.

1 $x^2 - 3x - 10$

Because $a = 1$, use a table to find the factors of -10 that have a sum of -3. These factors are 2 and -5.

Factors of -10	Sum
-2 and 5	3 ✘
-1 and 10	9 ✘
1 and -10	-9 ✘
2 and -5	-3 ✔

Rewrite the expression as a product of binomial factors with 2 and -5 as constants.

$$x^2 - 3x - 10 = (x + 2)(x - 5)$$

Check your answer by multiplying.

$$(x + 2)(x - 5) = x^2 - 5x + 2x - 10$$
$$= x^2 - 3x - 10 \checkmark$$

2 $6x^2 - 15x$

Find the greatest common factor (GCF) of the terms.

$$6x^2 = 2 \cdot 3 \cdot x \cdot x$$
$$15x = 3 \cdot 5 \cdot x$$ *The GCF is 3x.*

Factor $3x$ from both terms.

$$6x^2 - 15x = 3x(2x - 5)$$

Check your answer by multiplying.

$$3x(2x - 5) = 3x(2x) - 3x(5)$$
$$= 6x^2 - 15x \checkmark$$

3 $-x^2 + 3x + 4$

Because a is negative, factor out -1.

$$-x^2 + 3x + 4 = -1(x^2 - 3x - 4)$$

Use the method from Example 1 to factor the expression in parentheses.

$$-(x^2 - 3x - 4) = -(x + 1)(x - 4)$$

Check your answer by multiplying.

$$-(x + 1)(x - 4) = -(x^2 - 3x - 4)$$
$$= -x^2 + 3x + 4 \checkmark$$

Try This

Factor each expression.

1. $4x^2 + 10x$
2. $16x - 2x^2$
3. $x^2 - 6x + 8$
4. $x^2 + 4x + 3$
5. $x^2 - 8x + 15$
6. $x^2 + 10x - 24$
7. $x^2 - x - 56$
8. $x^2 - 6x + 9$
9. $x^2 + 48x - 100$
10. $-x^2 + 12x - 32$
11. $-x^2 + x + 20$
12. $-x^2 - 14x - 13$
13. $4x^2 + 6x$
14. $x^2 + 14x + 24$
15. $x^2 - 16$
16. $2x^2 - x - 3$
17. $3x^2 + 16x + 5$
18. $2x^2 - 9x + 7$

5-3 Technology LAB

Explore Graphs and Factors

You can use graphs and linear factors to find the x-intercepts of a parabola.

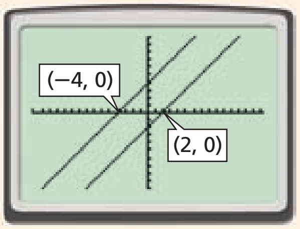
Use with Lesson 5-3

go.hrw.com
Lab Resources Online
KEYWORD: MB7 Lab5

Activity

Graph the lines $y = x + 4$ and $y = x - 2$.

1. Press **Y=**, and enter **X + 4** for **Y1** and **X − 2** for **Y2**. Graph the functions in the square window by pressing **ZOOM** and choosing **5 : ZSquare**.
2. Identify the x-intercept of each line. The x-intercepts are -4 and 2.
3. Find the x-value halfway between the two x-intercepts. This x-value is the average of the x-intercepts: $\frac{-4+2}{2} = -1$.

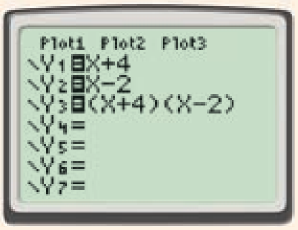


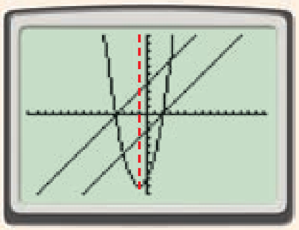
Graph the quadratic function $y = (x + 4)(x - 2)$, which is the product of the two linear factors graphed above.

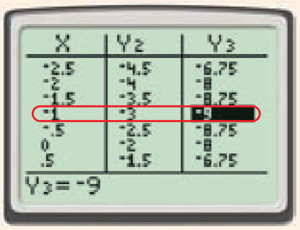
4. Press **Y=** and enter **(X + 4)(X − 2)** for **Y3**. Press **GRAPH**.

5. Identify the x-intercepts of the parabola. The x-intercepts are -4 and 2. Notice that they are the same as those of the two linear factors.

6. Examine the parabola at $x = -1$ (the x-value that is halfway between the x-intercepts). The axis of symmetry and the vertex of the parabola occur at this x-value.







Try This

Graph each quadratic function and each of its linear factors. Then identify the x-intercepts and the axis of symmetry of each parabola.

1. $y = (x - 2)(x - 6)$
2. $y = (x + 3)(x - 1)$
3. $y = (x - 5)(x + 2)$
4. $y = (x + 4)(x - 4)$
5. $y = (x - 5)(x - 5)$
6. $y = (2x - 1)(2x + 3)$
7. **Critical Thinking** Use a graph to determine whether the quadratic function $y = 2x^2 + 5x - 12$ is the product of the linear factors $2x - 3$ and $x + 4$.
8. **Make a Conjecture** Make a conjecture about the linear factors, x-intercepts, and axis of symmetry of a quadratic function.

5-3 Solving Quadratic Equations by Graphing and Factoring

Objectives

Solve quadratic equations by graphing or factoring.

Determine a quadratic function from its roots.

Vocabulary

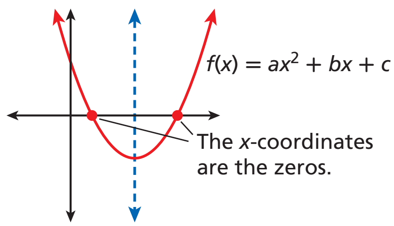
zero of a function
root of an equation
binomial
trinomial

Why learn this?

You can use quadratic functions to model the height of a football, baseball, or soccer ball. (See Example 3.)

When a soccer ball is kicked into the air, how long will the ball take to hit the ground? The height h in feet of the ball after t seconds can be modeled by the quadratic function $h(t) = -16t^2 + 32t$. In this situation, the value of the function represents the height of the soccer ball. When the ball hits the ground, the value of the function is zero.

A **zero of a function** is a value of the input x that makes the output $f(x)$ equal zero. The zeros of a function are the x-intercepts.



Unlike linear functions, which have no more than one zero, quadratic functions can have two zeros, as shown at right. These zeros are always symmetric about the axis of symmetry.

EXAMPLE 1 Finding Zeros by Using a Graph or Table

Find the zeros of $f(x) = x^2 + 2x - 3$ by using a graph and table.

Method 1 Graph the function $f(x) = x^2 + 2x - 3$.

The graph opens upward because $a > 0$. The y-intercept is -3 because $c = -3$.

Find the vertex: $x = -\frac{b}{2a} = -\frac{2}{2(1)} = -1$ *The x-coordinate of the vertex is $-\frac{b}{2a}$.*

Find $f(-1)$: $f(x) = x^2 + 2x - 3$

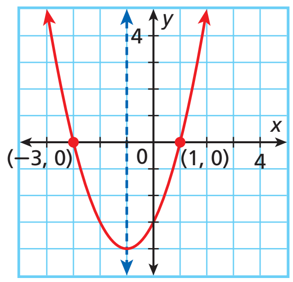
$f(-1) = (-1)^2 + 2(-1) - 3$ *Substitute -1 for x.*

$f(-1) = -4$

The vertex is $(-1, -4)$.

Plot the vertex and the y-intercept. Use symmetry and a table of values to find additional points.

x	-3	-2	-1	0	1
$f(x)$	0	-3	-4	-3	0



The table and the graph indicate that the zeros are -3 and 1.

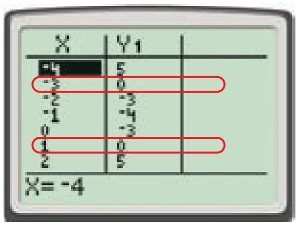
Helpful Hint

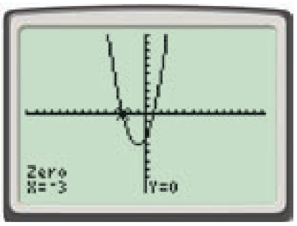
Recall that for the graph of a quadratic function, *any* pair of points with the same y-value are symmetric about the axis of symmetry.

Find the zeros of $f(x) = x^2 + 2x - 3$ by using a graph and table.

Method 2 Use a calculator.

Enter $y = x^2 + 2x - 3$ into a graphing calculator.





Both the table and the graph show that $y = 0$ at $x = -3$ and $x = 1$. These are the zeros of the function.

CHECK IT OUT!

1. Find the zeros of $g(x) = -x^2 - 2x + 3$ by using a graph and a table.

Reading Math

- Functions have *zeros* or *x-intercepts*.
- Equations have *solutions* or *roots*.

You can also find zeros by using algebra. For example, to find the zeros of $f(x) = x^2 + 2x - 3$, you can set the function equal to zero. The solutions to the related equation $x^2 + 2x - 3 = 0$ represent the zeros of the function.

The solutions to a quadratic equation of the form $ax^2 + bx + c = 0$ are *roots*. The **roots of an equation** are the values of the variable that make the equation true.

You can find the roots of some quadratic equations by factoring and applying the Zero Product Property.

Know it! Note

Zero Product Property

For all real numbers a and b,

WORDS	NUMBERS	ALGEBRA
If the product of two quantities equals zero, at least one of the quantities equals zero.	$3(0) = 0$ $0(4) = 0$	If $ab = 0$, then $a = 0$ or $b = 0$.

EXAMPLE 2 Finding Zeros by Factoring

Find the zeros of each function by factoring.

A $f(x) = x^2 - 8x + 12$

$$x^2 - 8x + 12 = 0 \quad \textit{Set the function equal to 0.}$$

$$(x - 2)(x - 6) = 0 \quad \textit{Factor: Find factors of 12 that add to } -8.$$

$$x - 2 = 0 \text{ or } x - 6 = 0 \quad \textit{Apply the Zero Product Property.}$$

$$x = 2 \text{ or } x = 6 \quad \textit{Solve each equation.}$$

Check

$$\begin{array}{r|l} \multicolumn{2}{c}{x^2 - 8x + 12 = 0} \\ \hline (2)^2 - 8(2) + 12 & 0 \\ 4 - 16 + 12 & 0 \\ 0 & 0 \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{x^2 - 8x + 12 = 0} \\ \hline (6)^2 - 8(6) + 12 & 0 \\ 36 - 48 + 12 & 0 \\ 0 & 0 \checkmark \end{array}$$

Substitute each value into the original equation.

Find the zeros of each function by factoring.

B $g(x) = 3x^2 + 12x$

$3x^2 + 12x = 0$ — *Set the function equal to 0.*

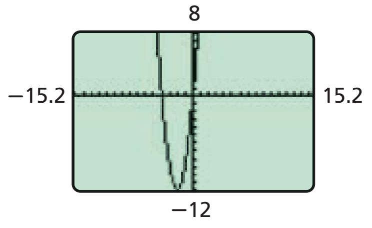
$3x(x + 4) = 0$ — *Factor: The GCF is 3x.*

$3x = 0$ or $x + 4 = 0$ — *Apply the Zero Product Property.*

$x = 0$ or $x = -4$ — *Solve each equation.*

Check Check algebraically and by graphing.

$$\begin{array}{r|l} \multicolumn{2}{c}{3x^2 + 12x = 0} \\ \hline 3(0)^2 + 12(0) & 0 \\ 0 + 0 & 0 \checkmark \end{array} \qquad \begin{array}{r|l} \multicolumn{2}{c}{3x^2 + 12x = 0} \\ \hline 3(-4)^2 + 12(-4) & 0 \\ 48 - 48 & 0 \checkmark \end{array}$$



CHECK IT OUT! **Find the zeros of each function by factoring.**

2a. $f(x) = x^2 - 5x - 6$ **2b.** $g(x) = x^2 - 8x$

Any object that is thrown or launched into the air, such as a baseball, basketball, or soccer ball, is a *projectile*. The general function that approximates the height h in feet of a projectile on Earth after t seconds is given below.

$$h(t) = -16t^2 + v_0 t + h_0$$

Constant due to Earth's gravity in ft/s^2 — Initial vertical velocity in ft/s (at $t = 0$) — Initial height in ft (at $t = 0$)

Note that this model has limitations because it does not account for air resistance, wind, and other real-world factors.

EXAMPLE 3 *Sports Application*

A soccer ball is kicked from ground level with an initial vertical velocity of 32 ft/s. After how many seconds will the ball hit the ground?

$h(t) = -16t^2 + v_0 t + h_0$ — *Write the general projectile function.*

$h(t) = -16t^2 + 32t + 0$ — *Substitute 32 for v_0 and 0 for h_0.*

The ball will hit the ground when its height is zero.

$-16t^2 + 32t = 0$ — *Set $h(t)$ equal to 0.*

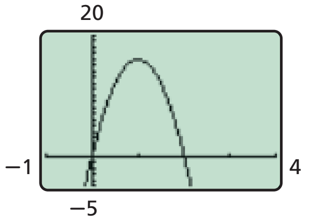
$-16t(t - 2) = 0$ — *Factor: The GCF is $-16t$.*

$-16t = 0$ or $(t - 2) = 0$ — *Apply the Zero Product Property.*

$t = 0$ or $t = 2$ — *Solve each equation.*

The ball will hit the ground in 2 seconds. Notice that the height is also zero when $t = 0$, the instant that the ball is kicked.

Check The graph of the function $h(t) = -16t^2 + 32t$ shows its zeros at 0 and 2.



3. A football is kicked from ground level with an initial vertical velocity of 48 ft/s. How long is the ball in the air?

Quadratic expressions can have one, two, or three terms, such as $-16t^2$, $-16t^2 + 25t$, or $-16t^2 + 25t + 6$. Quadratic expressions with two terms are **binomials**. Quadratic expressions with three terms are **trinomials**. Some quadratic expressions with perfect squares have special factoring rules.

Special Products and Factors

Difference of Two Squares	Perfect-Square Trinomial
$a^2 - b^2 = (a+b)(a-b)$	$a^2 - 2ab + b^2 = (a-b)^2$ $a^2 + 2ab + b^2 = (a+b)^2$

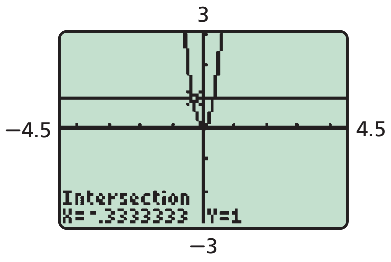
EXAMPLE 4 Finding Roots by Using Special Factors

Find the roots of each equation by factoring.

A $9x^2 = 1$

$9x^2 - 1 = 0$	*Rewrite in standard form.*
$(3x)^2 - (1)^2 = 0$	*Write the left side as $a^2 - b^2$.*
$(3x+1)(3x-1) = 0$	*Factor the difference of squares.*
$3x + 1 = 0$ or $3x - 1 = 0$	*Apply the Zero Product Property.*
$x = -\frac{1}{3}$ or $x = \frac{1}{3}$	*Solve each equation.*

Check Graph each side of the equation on a graphing calculator. Let **Y1** equal $9x^2$, and let **Y2** equal 1. The graphs appear to intersect at $x = -\frac{1}{3}$ and at $x = \frac{1}{3}$.



B $40x = 8x^2 + 50$

$8x^2 - 40x + 50 = 0$	*Rewrite in standard form.*
$2(4x^2 - 20x + 25) = 0$	*Factor. The GCF is 2.*
$4x^2 - 20x + 25 = 0$	*Divide both sides by 2.*
$(2x)^2 - 2(2x)(5) + (5)^2 = 0$	*Write the left side as $a^2 - 2ab + b^2$.*
$(2x - 5)^2 = 0$	*Factor the perfect-square trinomial: $(a-b)^2$.*
$2x - 5 = 0$ or $2x - 5 = 0$	*Apply the Zero Product Property.*
$x = \frac{5}{2}$ or $x = \frac{5}{2}$	*Solve each equation.*

Check Substitute the root $\frac{5}{2}$ into the original equation.

$40x$	$= 8x^2 + 50$
$40\left(\frac{5}{2}\right)$	$8\left(\frac{5}{2}\right)^2 + 50$
100	100 ✔

Helpful Hint

A quadratic equation can have two roots that are equal, such as $x = \frac{5}{2}$ and $x = \frac{5}{2}$. Two equal roots are sometimes called a double root.

Find the roots of each equation by factoring.

4a. $x^2 - 4x = -4$ **4b.** $25x^2 = 9$

If you know the zeros of a function, you can work backward to write a rule for the function.

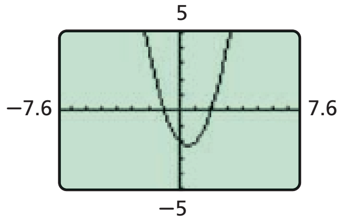
EXAMPLE 5

Using Zeros to Write Function Rules

Write a quadratic function in standard form with zeros 2 and −1.

$x = 2$ or $x = -1$	*Write the zeros as solutions for two equations.*
$x - 2 = 0$ or $x + 1 = 0$	*Rewrite each equation so that it equals 0.*
$(x - 2)(x + 1) = 0$	*Apply the converse of the Zero Product Property to write a product that equals 0.*
$x^2 - x - 2 = 0$	*Multiply the binomials.*
$f(x) = x^2 - x - 2$	*Replace 0 with $f(x)$.*

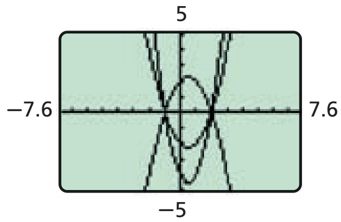
Check Graph the function $f(x) = x^2 - x - 2$ on a calculator. The graph shows the original zeros of 2 and −1.



CHECK IT OUT!

5. Write a quadratic function in standard form with zeros 5 and −5.

Note that there are many quadratic functions with the same zeros. For example, the functions $f(x) = x^2 - x - 2$, $g(x) = -x^2 + x + 2$, and $h(x) = 2x^2 - 2x - 4$ all have zeros at 2 and −1.



THINK AND DISCUSS

1. Describe the zeros of a function whose terms form a perfect square trinomial.

2. Compare the x- and y-intercepts of a quadratic function with those of a linear function.

3. A quadratic equation has no real solutions. Describe the graph of the related quadratic function.

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. In each box, give information about special products and factors.

Name	Rule	Example	Graph
Difference of Two Squares			
Perfect-Square Trinomial			

5-3 Exercises

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Homework Help Online
KEYWORD: MB7 5-3
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The solutions of the equation $3x^2 + 2x + 5 = 0$ are its __?__. (*roots* or *zeros*)

SEE EXAMPLE 1 p. 333

Find the zeros of each function by using a graph and table.

2. $f(x) = x^2 + 4x - 5$
3. $g(x) = -x^2 + 6x - 8$
4. $f(x) = x^2 - 1$

SEE EXAMPLE 2 p. 334

Find the zeros of each function by factoring.

5. $f(x) = x^2 - 7x + 6$
6. $g(x) = 2x^2 - 5x + 2$
7. $h(x) = x^2 + 4x$
8. $f(x) = x^2 + 9x + 20$
9. $g(x) = x^2 - 6x - 16$
10. $h(x) = 3x^2 + 13x + 4$

SEE EXAMPLE 3 p. 335

11. **Archery** The height h of an arrow in feet is modeled by $h(t) = -16t^2 + 63t + 4$, where t is the time in seconds since the arrow was shot. How long is the arrow in the air?

SEE EXAMPLE 4 p. 336

Find the roots of each equation by factoring.

12. $x^2 - 6x = -9$
13. $5x^2 + 20 = 20x$
14. $x^2 = 49$

SEE EXAMPLE 5 p. 337

Write a quadratic function in standard form for each given set of zeros.

15. 3 and 4
16. −4 and −4
17. 3 and 0

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–20	1
21–26	2
27	3
28–33	4
34–36	5

Extra Practice

Skills Practice p. S12
Application Practice p. S36

Find the zeros of each function by using a graph and table.

18. $f(x) = -x^2 + 4x - 3$
19. $g(x) = x^2 + x - 6$
20. $f(x) = x^2 - 9$

Find the zeros of each function by factoring.

21. $f(x) = x^2 + 11x + 24$
22. $g(x) = 2x^2 + x - 10$
23. $h(x) = -x^2 + 9x$
24. $f(x) = x^2 - 15x + 54$
25. $g(x) = x^2 + 7x - 8$
26. $h(x) = 2x^2 - 12x + 18$

27. **Biology** A bald eagle snatches a fish from a lake and flies to an altitude of 256 ft. The fish manages to squirm free and falls back down into the lake. Its height h in feet can be modeled by $h(t) = 256 - 16t^2$, where t is the time in seconds. How many seconds will the fish fall before hitting the water?

Find the roots of each equation by factoring.

28. $x^2 + 8x = -16$
29. $4x^2 = 81$
30. $9x^2 + 12x + 4 = 0$
31. $36x^2 - 9 = 0$
32. $x^2 - 10x + 25 = 0$
33. $49x^2 = 28x - 4$

Write a quadratic function in standard form for each given set of zeros.

34. 5 and −1
35. 6 and 2
36. 3 and 3

Find the zeros of each function.

37. $f(x) = 6x - x^2$
38. $g(x) = x^2 - 25$
39. $h(x) = x^2 - 12x + 36$
40. $f(x) = 3x^2 - 12$
41. $g(x) = x^2 - 22x + 121$
42. $h(x) = 30 + x - x^2$
43. $f(x) = x^2 - 11x + 30$
44. $g(x) = x^2 - 8x - 20$
45. $h(x) = 2x^2 + 18x + 28$

46. **Movies** A stuntwoman jumps from a building 73 ft high and lands on an air bag that is 9 ft tall. Her height above ground h in feet can be modeled by $h(t) = 73 - 16t^2$, where t is the time in seconds.

a. **Multi-Step** How many seconds will the stuntwoman fall before touching the air bag? (*Hint:* Find the time t when the stuntwoman's height above ground is 9 ft.)

b. **What if...?** Suppose the stuntwoman jumps from a building that is half as tall. Will she be in the air for half as long? Explain.

LINK

Entertainment

The Guinness world record for the greatest number of people juggling at one time was set in 1998 by 1508 people, each of whom juggled at least 3 objects for 10 seconds.

47. **Entertainment** A juggler throws a ball into the air from a height of 5 ft with an initial vertical velocity of 16 ft/s.

a. Write a function that can be used to model the height h of the ball in feet t seconds after the ball is thrown.

b. How long does the juggler have to catch the ball before it hits the ground?

Find the roots of each equation.

48. $x^2 - 2x + 1 = 0$ 49. $x^2 + 6x = -5$ 50. $25x^2 + 40x = -16$

51. $9x^2 + 6x = -1$ 52. $5x^2 = 45$ 53. $x^2 - 6 = x$

For each function, (a) find its vertex, (b) find its y-intercept, (c) find its zeros, and (d) graph it.

54. $f(x) = x^2 + 2x - 8$ 55. $g(x) = x^2 - 16$ 56. $h(x) = x^2 - x - 12$

57. $f(x) = -2x^2 + 4x$ 58. $g(x) = x^2 - 5x - 6$ 59. $h(x) = 3x^2 + x - 4$

60. **Geometry** The hypotenuse of a right triangle is 2 cm longer than one leg and 4 cm longer than the other leg.

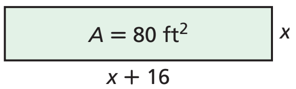
a. Let x represent the length of the hypotenuse. Use the Pythagorean Theorem to write an equation that can be solved for x.

b. Find the solutions of the equation from part **a.**

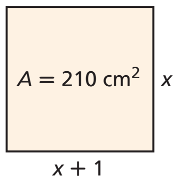
c. Are both solutions reasonable in the context of the problem situation? Explain.

Geometry **Find the dimensions of each rectangle.**

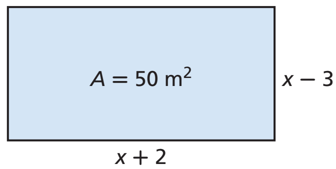
61.



62.



63.



64. **Critical Thinking** Will a function whose rule can be factored as a binomial squared ever have two different zeros? Explain.

65. **Write About It** Explain how the Zero Product Property can be used to help determine the zeros of quadratic functions.

MULTI-STEP TEST PREP

66. This problem will prepare you for the Multi-Step Test Prep on page 364.

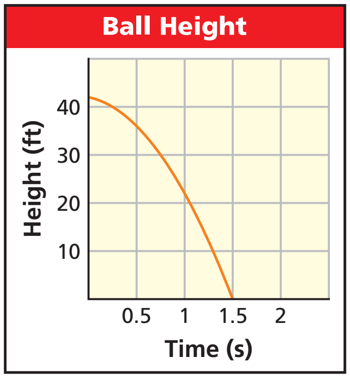
A baseball player hits a ball toward the outfield. The height h of the ball in feet is modeled by $h(t) = -16t^2 + 22t + 3$, where t is the time in seconds. In addition, the function $d(t) = 85t$ models the horizontal distance d traveled by the ball.

a. If no one catches the ball, how long will it stay in the air?

b. What is the horizontal distance that the ball travels before it hits the ground?

TEST PREP

67. Use the graph provided to choose the best description of what the graph represents.



Ⓐ A ball is dropped from a height of 42 feet and lands on the ground after 3 seconds.

Ⓑ A ball is dropped from a height of 42 feet and lands on the ground after 1.5 seconds.

Ⓒ A ball is shot up in the air and reaches a height of 42 feet after 1 second.

Ⓓ A ball is shot up in the air, reaches a height of 42 feet, and lands on the ground after 1.5 seconds.

68. Which function has -7 as its only zero?

Ⓕ $f(x) = x(x - 7)$ Ⓗ $g(x) = (x + 1)(x + 7)$

Ⓖ $h(x) = (x - 7)^2$ Ⓙ $j(x) = (x + 7)^2$

69. Which expression is a perfect square trinomial?

Ⓐ $25y^2 - 16$ Ⓒ $25y^2 - 40y + 16$

Ⓑ $25y^2 - 20y + 16$ Ⓓ $25y^2 - 10y + 16$

70. **Gridded Response** Find the positive root of $x^2 + 4x - 21 = 0$.

CHALLENGE AND EXTEND

Find the roots of each equation by factoring.

71. $3(x^2 - x) = x^2$ **72.** $x^2 = \frac{1}{3}x$

73. $x^2 - \frac{3}{4}x + \frac{1}{8} = 0$ **74.** $x^2 + x + 0.21 = 0$

75. Another special factoring case involves perfect cubes. The sum of two cubes can be factored by using the formula $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$.

a. Verify the formula by multiplying the right side of the equation.

b. Factor the expression $8x^3 + 27$.

c. Use multiplication and guess and check to find the factors of $a^3 - b^3$.

d. Factor the expression $x^3 - 1$.

SPIRAL REVIEW

Evaluate each expression. Write the answer in scientific notation. *(Lesson 1-5)*

76. $(1.4 \times 10^8)(6.1 \times 10^{-3})$ **77.** $(2.7 \times 10^{10})(3.2 \times 10^2)$

78. $\dfrac{(3.5 \times 10^6)}{(1.4 \times 10^{-4})}$ **79.** $\dfrac{(3.12 \times 10^{-6})}{(4.8 \times 10^3)}$

Solve each proportion. *(Lesson 2-2)*

80. $\frac{12}{7.5} = \frac{n}{5}$ **81.** $\frac{1.2}{4.8} = \frac{w}{8.8}$ **82.** $\frac{6.8}{4.5} = \frac{r}{90}$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function. *(Lesson 5-1)*

83. $h(x) = 0.5x^2$ **84.** $d(x) = x^2 + 2$ **85.** $g(x) = (x + 1)^2$

5-4 Completing the Square

Objectives

Solve quadratic equations by completing the square.

Write quadratic equations in vertex form.

Vocabulary

completing the square

Why learn this?

You can solve quadratic equations to find how long water takes to fall from the top to the bottom of a waterfall. (See Exercise 39.)

Many quadratic equations contain expressions that cannot be easily factored. For equations containing these types of expressions, you can use square roots to find roots.

Know it! Note

Square-Root Property

WORDS	NUMBERS	ALGEBRA
To solve a quadratic equation, you can take the square root of both sides. Be sure to consider the positive and negative square roots.	$x^2 = 15$ $\lvert x \rvert = \sqrt{15}$ $x = \pm\sqrt{15}$	If $x^2 = a$ and a is a nonnegative real number, then $x = \pm\sqrt{a}$.

EXAMPLE 1 **Solving Equations by Using the Square Root Property**

Solve each equation.

A $3x^2 - 4 = 68$

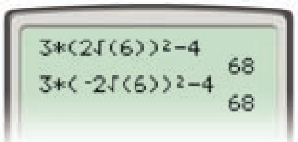
$3x^2 = 72$ *Add 4 to both sides.*

$x^2 = 24$ *Divide both sides by 3 to isolate the squared term.*

$x = \pm\sqrt{24}$ *Take the square root of both sides.*

$x = \pm 2\sqrt{6}$ *Simplify.*

Check Use a graphing calculator.



Reading Math

Read $\pm\sqrt{a}$ as "plus or minus square root of a."

B $x^2 - 10x + 25 = 27$

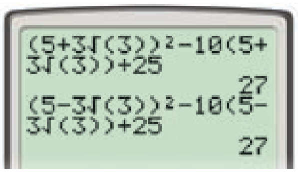
$(x - 5)^2 = 27$ *Factor the perfect square trinomial.*

$x - 5 = \pm\sqrt{27}$ *Take the square root of both sides.*

$x = 5 \pm \sqrt{27}$ *Add 5 to both sides.*

$x = 5 \pm 3\sqrt{3}$ *Simplify.*

Check Use a graphing calculator.



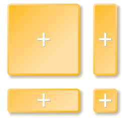
CHECK IT OUT! **Solve each equation.**

1a. $4x^2 - 20 = 5$

1b. $x^2 + 8x + 16 = 49$

The methods in the previous examples can be used only for expressions that are perfect squares. However, you can use algebra to rewrite any quadratic expression as a perfect square.

You can use algebra tiles to model a perfect square trinomial as a perfect square. The area of the square at right is $x^2 + 2x + 1$. Because each side of the square measures $x + 1$ units, the area is also $(x + 1)(x + 1)$, or $(x + 1)^2$. This shows that $(x + 1)^2 = x^2 + 2x + 1$.



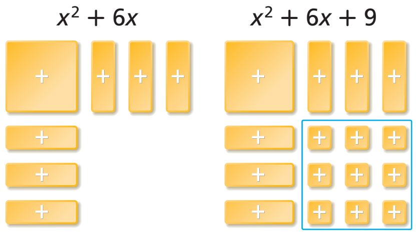
If a quadratic expression of the form $x^2 + bx$ *cannot* model a square, you can add a term to form a perfect square trinomial. This is called **completing the square**.

Know it! Note

Completing the Square

WORDS	NUMBERS	ALGEBRA
To complete the square of $x^2 + bx$, add $\left(\frac{b}{2}\right)^2$.	$x^2 + 6x + \blacksquare$ $x^2 + 6x + \left(\frac{6}{2}\right)^2$ $x^2 + 6x + 9$ $(x + 3)^2$	$x^2 + bx + \blacksquare$ $x^2 + bx + \left(\frac{b}{2}\right)^2$ $\left(x + \frac{b}{2}\right)^2$

The model shows completing the square for $x^2 + 6x$ by adding 9 unit tiles. The resulting perfect square trinomial is $x^2 + 6x + 9$. Note that completing the square does not produce an equivalent expression.



$b = 6$

$\left(\frac{b}{2}\right)^2 = \left(\frac{6}{2}\right)^2 = 9$

EXAMPLE 2

Completing the Square

Complete the square for each expression. Write the resulting expression as a binomial squared.

A $x^2 - 2x + \square$

$\left(\frac{-2}{2}\right)^2 = (-1)^2 = 1$ *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 - 2x + 1$ *Add.*

$(x - 1)^2$ *Factor.*

Check Find the square of the binomial.

$$(x - 1)^2 = (x - 1)(x - 1)$$
$$= x^2 - 2x + 1$$

B $x^2 + 5x + \square$

$\left(\frac{5}{2}\right)^2 = \frac{25}{4}$ *Find $\left(\frac{b}{2}\right)^2$.*

$x^2 + 5x + \frac{25}{4}$ *Add.*

$\left(x + \frac{5}{2}\right)^2$ *Factor.*

Check Find the square of the binomial.

$$\left(x + \frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)\left(x + \frac{5}{2}\right)$$
$$= x^2 + 5x + \frac{25}{4}$$

CHECK IT OUT!

Complete the square for each expression. Write the resulting expression as a binomial squared.

2a. $x^2 + 4x + \square$ **2b.** $x^2 - 4x + \square$ **2c.** $x^2 + 3x + \square$

You can complete the square to solve quadratic equations.

Know it! Note

Solving Quadratic Equations $ax^2 + bx + c = 0$ by Completing the Square
1. Collect variable terms on one side of the equation and constants on the other.
2. As needed, divide both sides by a to make the coefficient of the x^2-term 1.
3. Complete the square by adding $\left(\frac{b}{2}\right)^2$ to both sides of the equation.
4. Factor the variable expression as a perfect square.
5. Take the square root of both sides of the equation.
6. Solve for the values of the variable.

EXAMPLE 3 Solving a Quadratic Equation by Completing the Square

Caution!

To keep the equation balanced, you must add $\left(\frac{b}{2}\right)^2$ to both sides of the equation.

Solve each equation by completing the square.

A $x^2 = 27 - 6x$

$x^2 + 6x = 27$	*Collect variable terms on one side.*
$x^2 + 6x + \square = 27 + \square$	*Set up to complete the square.*
$x^2 + 6x + \left(\frac{6}{2}\right)^2 = 27 + \left(\frac{6}{2}\right)^2$	*Add $\left(\frac{b}{2}\right)^2$ to both sides.*
$x^2 + 6x + 9 = 27 + 9$	*Simplify.*
$(x+3)^2 = 36$	*Factor.*
$x + 3 = \pm\sqrt{36}$	*Take the square root of both sides.*
$x + 3 = \pm 6$	*Simplify.*
$x + 3 = 6$ or $x + 3 = -6$	*Solve for x.*
$x = 3$ or $x = -9$	

B $2x^2 + 8x = 12$

$x^2 + 4x = 6$	*Divide both sides by 2.*
$x^2 + 4x + \square = 6 + \square$	*Set up to complete the square.*
$x^2 + 4x + \left(\frac{4}{2}\right)^2 = 6 + \left(\frac{4}{2}\right)^2$	*Add $\left(\frac{b}{2}\right)^2$ to both sides.*
$x^2 + 4x + 4 = 6 + 4$	*Simplify.*
$(x+2)^2 = 10$	*Factor.*
$x + 2 = \pm\sqrt{10}$	*Take the square root of both sides.*
$x = -2 \pm \sqrt{10}$	*Solve for x.*

CHECK IT OUT! **Solve each equation by completing the square.**

3a. $x^2 - 2 = 9x$ **3b.** $3x^2 - 24x = 27$

Recall the vertex form of a quadratic function from Lesson 5-1: $f(x) = a(x-h)^2 + k$, where the vertex is (h, k).

You can complete the square to rewrite any quadratic function in vertex form.

EXAMPLE 4 Writing a Quadratic Function in Vertex Form

Helpful Hint

In Example 3, the equation was balanced by adding $\left(\frac{b}{2}\right)^2$ to *both* sides. Here, the equation is balanced by adding and subtracting $\left(\frac{b}{2}\right)^2$ on *one* side.

Write each function in vertex form, and identify its vertex.

A $f(x) = x^2 + 10x - 13$

$f(x) = \left(x^2 + 10x + \blacksquare\right) - 13 - \blacksquare$ *Set up to complete the square.*

$f(x) = \left[x^2 + 10x + \left(\frac{10}{2}\right)^2\right] - 13 - \left(\frac{10}{2}\right)^2$ *Add and subtract $\left(\frac{b}{2}\right)^2$.*

$f(x) = (x + 5)^2 - 38$ *Simplify and factor.*

Because $h = -5$ and $k = -38$, the vertex is $(-5, -38)$.

Check Use the axis of symmetry formula to confirm the vertex.

$$x = -\frac{b}{2a} = -\frac{10}{2(1)} = -5 \qquad y = f(-5) = (-5)^2 + 10(-5) - 13 = -38 \checkmark$$

B $g(x) = 2x^2 - 8x + 3$

$g(x) = 2\left(x^2 - 4x\right) + 3$ *Factor so the coefficient of x^2 is 1.*

$g(x) = 2\left(x^2 - 4x + \blacksquare\right) + 3 - \blacksquare$ *Set up to complete the square.*

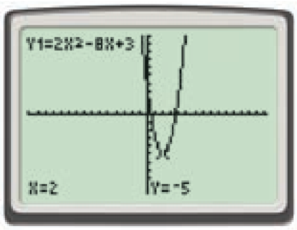
$g(x) = 2\left(x^2 - 4x + \left(\frac{-4}{2}\right)^2\right) + 3 - 2\left(\frac{-4}{2}\right)^2$ *Add $\left(\frac{b}{2}\right)^2$. Because $\left(\frac{b}{2}\right)^2$ is multiplied by 2, you must subtract $2\left(\frac{b}{2}\right)^2$.*

$g(x) = 2\left(x^2 - 4x + 4\right) - 5$ *Simplify.*

$g(x) = 2(x - 2)^2 - 5$ *Factor.*

Because $h = 2$ and $k = -5$, the vertex is $(2, -5)$.

Check A graph of the function on a graphing calculator supports your answer.



CHECK IT OUT! **Write each function in vertex form, and identify its vertex.**

4a. $f(x) = x^2 + 24x + 145$ **4b.** $g(x) = 5x^2 - 50x + 128$

THINK AND DISCUSS

1. Explain two ways to solve $x^2 = 25$.

2. Describe how to change a quadratic function from standard form to vertex form by completing the square.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Compare and contrast two methods of solving quadratic equations.

Using Square-Root Property vs. Completing the Square

Similarities | Differences

5-4 Exercises

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Homework Help Online
KEYWORD: MB7 5-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** What must you add to the expression $x^2 + bx$ to *complete the square*?

SEE EXAMPLE 1
p. 341

Solve each equation.

2. $(x - 2)^2 = 16$
3. $x^2 - 10x + 25 = 16$
4. $x^2 - 2x + 1 = 3$

SEE EXAMPLE 2
p. 342

Complete the square for each expression. Write the resulting expression as a binomial squared.

5. $x^2 + 14x + \blacksquare$
6. $x^2 - 12x + \blacksquare$
7. $x^2 - 9x + \blacksquare$

SEE EXAMPLE 3
p. 343

Solve each equation by completing the square.

8. $x^2 - 6x = -4$
9. $x^2 + 8 = 6x$
10. $2x^2 - 20x = 8$
11. $x^2 = 24 - 4x$
12. $10x + x^2 = 42$
13. $2x^2 + 8x - 15 = 0$

SEE EXAMPLE 4
p. 344

Write each function in vertex form, and identify its vertex.

14. $f(x) = x^2 + 6x - 3$
15. $g(x) = x^2 - 10x + 11$
16. $h(x) = 3x^2 - 24x + 53$
17. $f(x) = x^2 + 8x - 10$
18. $g(x) = x^2 - 3x + 16$
19. $h(x) = 3x^2 - 12x - 4$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
20–22	1
23–25	2
26–31	3
32–37	4

Extra Practice

Skills Practice p. S12
Application Practice p. S36

Solve each equation.

20. $(x + 2)^2 = 36$
21. $x^2 - 6x + 9 = 100$
22. $(x - 3)^2 = 5$

Complete the square for each expression. Write the resulting expression as a binomial squared.

23. $x^2 - 18x + \blacksquare$
24. $x^2 + 10x + \blacksquare$
25. $x^2 - \frac{1}{2}x + \blacksquare$

Solve each equation by completing the square.

26. $x^2 + 2x = 7$
27. $x^2 - 4x = -1$
28. $2x^2 - 8x = 22$
29. $8x = x^2 + 12$
30. $x^2 + 3x - 5 = 0$
31. $3x^2 + 6x = 1$

Write each function in vertex form, and identify its vertex.

32. $f(x) = x^2 - 4x + 13$
33. $g(x) = x^2 + 14x + 71$
34. $h(x) = 9x^2 + 18x - 3$
35. $f(x) = x^2 + 4x - 7$
36. $g(x) = x^2 - 16x + 2$
37. $h(x) = 2x^2 + 6x + 25$

38. **Engineering** The height h above the roadway of the main cable of the Golden Gate Bridge can be modeled by the function $h(x) = \frac{1}{9000}x^2 - \frac{7}{15}x + 500$, where x is the distance in feet from the left tower.

a. Complete the square, and write the function in vertex form.

b. What is the vertex, and what does it represent?

c. **Multi-Step** The left and right towers have the same height. What is the distance in feet between them?

39. **Waterfalls** Angel Falls in Venezuela is the tallest waterfall in the world. Water falls uninterrupted for 2421 feet before entering the river below. The height h above the river in feet of water going over the edge of the waterfall is modeled by $h(t) = -16t^2 + 2421$, where t is the time in seconds after the initial fall.

a. Estimate the time it takes for the water to reach the river.

b. **Multi-Step** Ribbon Falls in California has a height of 1612 ft. Approximately how much longer does it take water to reach the bottom when going over Angel Falls than when going over Ribbon Falls?

40. **Sports** A basketball is shot with an initial vertical velocity of 24 ft/s from 6 ft above the ground. The ball's height h in feet is modeled by $h(t) = -16t^2 + 24t + 6$, where t is the time in seconds after the ball is shot. What is the maximum height of the ball, and when does the ball reach this height?

Solve each equation using square roots.

41. $x^2 - 1 = 2$

42. $25x^2 = 0$

43. $8x^2 - 200 = 0$

44. $-3x^2 + 6 = -1$

45. $(x + 13)^2 = 7$

46. $\left(x + \frac{1}{4}\right)^2 - \frac{9}{16} = 0$

47. $\left(x + \frac{3}{2}\right)^2 = \frac{25}{2}$

48. $x^2 + 14x + 49 = 64$

49. $9x^2 + 18x + 9 = 5$

50. **ERROR ANALYSIS** Two attempts to write $f(x) = 2x^2 - 8x$ in vertex form are shown. Which is incorrect? Explain the error.

A

$f(x) = 2x^2 - 8x$
$f(x) = 2(x^2 - 4x)$
$f(x) = 2(x^2 - 4x + 4) - 4$
$f(x) = 2(x - 2)^2 - 4$

B

$f(x) = 2x^2 - 8x$
$f(x) = 2(x^2 - 4x)$
$f(x) = 2(x^2 - 4x + 4) - 8$
$f(x) = 2(x - 2)^2 - 8$

Solve each equation by completing the square.

51. $x^2 + 8x = -15$

52. $x^2 + 22x = -21$

53. $3x^2 + 4x = 1$

54. $2x^2 = 5x + 12$

55. $x^2 - 7x - 2 = 0$

56. $x^2 = 4x + 11$

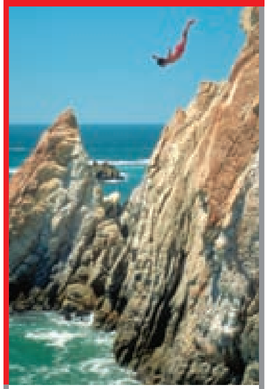
57. $x^2 + 6x + 4 = 0$

58. $5x^2 + 10x - 7 = 0$

59. $x^2 - 8x = 24$

LINK

Sports



Acapulco, Mexico, is famous for its cliff-diving shows. Divers perform complicated acrobatic dives from heights of up to 80 feet.

60. **Sports** A diver's height h in meters above the water is approximated by $h(t) = h_0 - 5t^2$, where h_0 is the initial height in meters, -5 is a constant based on the acceleration due to gravity in m/s^2, and t is the time in seconds that the diver falls through the air.

a. Find the total time that the diver falls through the air for each type of dive in the table.

b. How high is a dive that keeps the diver in the air twice as long as a 5-meter dive?

c. The speed of a diver entering the water can be approximated by $s = 18t$, where s is the speed in kilometers per hour and t is the time in seconds. Using your results from part **a**, find the speed of the diver entering the water for each dive height.

d. How many times as high is a dive that results in a speed that is twice as fast?

Dive Heights

Type	Height (m)
Platform	5
Platform	10
Cliff	20
Cliff	30

MULTI-STEP TEST PREP

61. This problem will prepare you for the Multi-Step Test Prep on page 364.

The height h in feet of a baseball hit from home plate can be modeled by the function $h(t) = -16t^2 + 32t + 5.5$, where t is the time in seconds since the ball was hit. The ball is descending when it passes 7.5 ft over the head of a 6 ft player standing on the ground.

a. To the nearest tenth of a second, how long after the ball is hit does it pass over the player's head?

b. The horizontal distance between the player and home plate is 120 ft. Use your answer from part **a** to determine the horizontal speed of the ball to the nearest foot per second.

62. Estimation A bag of grass seed will cover 525 square feet. Twenty bags of seed are used to cover an area shaped like a square. Estimate the side length of the square. Check your answer with a calculator.

63. Critical Thinking The functions f and g are defined by $f(x) = x^2 + 2x - 2$ and $g(x) = (x + 1)^2 - 3$. Use algebra to prove that f and g represent the same function.

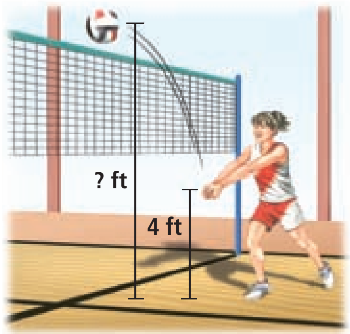
64. Sports A player bumps a volleyball with an initial vertical velocity of 20 ft/s.

a. Write a function h in standard form for the ball's height in feet in terms of the time t in seconds after the ball is hit.

b. Complete the square to rewrite h in vertex form.

c. What is the maximum height of the ball?

d. What if...? Suppose the volleyball were hit under the same conditions, but with an initial velocity of 32 ft/s. How much higher would the ball go?



Graphing Calculator **Use a graphing calculator to approximate the roots of each equation to the nearest thousandth.**

65. $x^2 - 15 = 40$

66. $x^2 = 2.85$

67. $1.4x^2 = 24.6$

68. $(x + 0.6)^2 = 7.4$

69. $\frac{x^2}{7} = \frac{1}{3}$

70. $\left(x + \frac{1}{4}\right)^2 = \frac{5}{6}$

71. Critical Thinking Why do equations of the form $x^2 = k$ have no real solution when $k < 0$?

72. Write About It Compare the methods of factoring and completing the square for solving quadratic equations.

TEST PREP

73. Which gives the solution to $3x^2 = 33$?

Ⓐ $\pm\sqrt{3}$ Ⓑ $\pm\sqrt{11}$ Ⓒ 11 Ⓓ 121

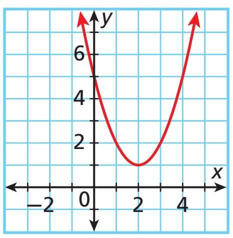
74. Which equation represents the graph at right?

Ⓕ $y = (x - 2)^2 + 1$

Ⓖ $y = (x - 2)^2 - 1$

Ⓗ $y = (x + 2)^2 + 1$

Ⓙ $y = (x + 2)^2 - 1$



75. Which gives the vertex of the graph of $y = 3(x - 1)^2 - 22$?

Ⓐ $(1, -22)$　Ⓑ $(-1, -22)$　Ⓒ $(3, -22)$　Ⓓ $(-3, -22)$

76. Which number should be added to $x^2 + 14x$ to make a perfect square trinomial?

Ⓕ 7　Ⓖ 14　Ⓗ 49　Ⓙ 196

77. Gridded Response What is the positive root of the equation $2x^2 - x = 10$?

78. Extended Response Solve the quadratic equation $x^2 - 6x = 16$ by completing the square. Explain each step of the solution process, and check your answer.

CHALLENGE AND EXTEND

Find the value of b in each perfect square trinomial.

79. $x^2 - bx + 144$

80. $4x^2 - bx + 16$

81. $3x^2 + bx + 27$

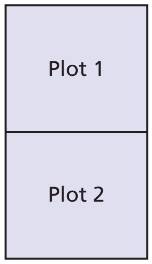
82. $ax^2 + bx + c$

Find the zeros of each function.

83. $f(x) = x^2 - 4x\sqrt{5} + 19$

84. $f(x) = x^2 + 6x\sqrt{3} + 23$

85. Farming To create a temporary grazing area, a farmer is using 1800 feet of electric fencing to enclose a rectangular field and then to subdivide the field into two plots. The fence that divides the field into two plots is parallel to the field's shorter sides.



a. What is the largest area of the field that the farmer can enclose?

b. What are the dimensions of the field with the largest area?

c. What if...? What would be the largest area of a square field that the farmer could enclose and divide into two plots?

SPIRAL REVIEW

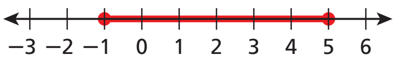
Express each set of numbers using set-builder notation. *(Lesson 1-1)*

86. $(72, \infty)$

87. numbers within 10 units of 4

88. positive multiples of 4

89.



Use the table for Exercises 90–93. *(Lesson 4-1)*

Monthly Budget	Food	Housing	Auto
Aboline family	\$352	\$895	\$426
Hernandez family	\$675	\$1368	\$642
Walker family	\$185	\$615	\$295

90. Display the data in the form of a matrix B.

91. What are the dimensions of the matrix?

92. What is the address of the entry that has the value 185?

93. What is the value of the matrix entry with the address b_{22}? What does it represent?

Identify the axis of symmetry and the vertex of the graph of each function. *(Lesson 5-2)*

94. $f(x) = 3(x - 2)^2$

95. $g(x) = \frac{2}{5}x^2 - 1$

96. $h(x) = 6x^2 + 2.5$

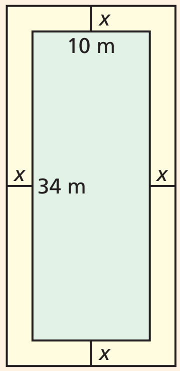
Connecting Algebra to **Geometry**

Areas of Composite Figures

Quadratic equations can be used to solve problems involving the areas of composite figures. Write an equation that represents the information given in the problem. Then solve the equation.

Example

The diagram shows a rectangular garden surrounded by a walkway. The garden measures 10 m by 34 m. The total area of the garden and walkway is 640 m^2. What is the width x of the walkway?



Total area = 640 m^2

The total area is equal to the total length multiplied by the total width. The total length is $2x + 34$ m, and the total width is $2x + 10$ m.

$A = \ell \times w$	*Write the formula for total area.*
$640 = (2x + 34)(2x + 10)$	*Substitute.*
$640 = 4x^2 + 88x + 340$	*Multiply the binomials.*
$0 = 4x^2 + 88x - 300$	*Subtract 640 from both sides.*
$0 = x^2 + 22x - 75$	*Divide both sides by 4.*
$0 = (x - 3)(x + 25)$	*Factor.*
$x - 3 = 0$ or $x + 25 = 0$	*Use the Zero Product Property.*
$x = 3$ or $x = -25$	*Solve for x.*

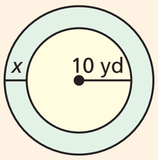
The width cannot be negative. Therefore, the width of the walkway is 3 m.

Try This

Write an equation that represents each problem. Then solve.

1. Use figure 1 below. A ring of grass with an area of 314 yd^2 surrounds a circular flower bed. Find the width x of the ring of grass.
2. Use figure 2 below. Sid cuts four congruent squares from the corners of a 30-in.-by-50-in. rectangular piece of cardboard so that it can be folded to make a box. Find the side length s of the squares, given that the area of the bottom of the box is 200 in^2.
3. Use figure 3 below. Harriet has 80 m of fencing materials to enclose three sides of a rectangular garden. She will use the side of her garage as a border for the fourth side. Find the width x of the garden if its area is to be 700 m^2.

Figure 1



Grass area = 314 yd^2

Figure 2

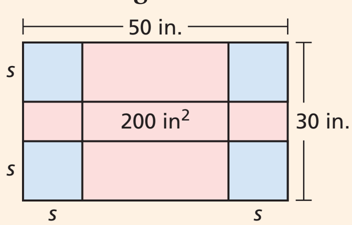
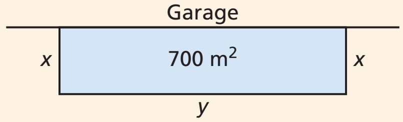


Figure 3



Fencing material = 80 m

5-5 Complex Numbers and Roots

Objectives
Define and use imaginary and complex numbers.

Solve quadratic equations with complex roots.

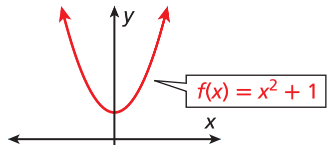
Vocabulary
imaginary unit
imaginary number
complex number
real part
imaginary part
complex conjugate

Why learn this?
Complex numbers can be used to describe the zeros of quadratic functions that have no real zeros. (See Example 4.)

Andrew Toos/CartoonResource.com

You can see in the graph of $f(x) = x^2 + 1$ below that f has no real zeros. If you solve the corresponding equation $0 = x^2 + 1$, you find that $x = \pm\sqrt{-1}$, which has no *real* solutions.

However, you can find solutions if you define the square root of negative numbers, which is why *imaginary numbers* were invented. The **imaginary unit** i is defined as $\sqrt{-1}$. You can use the imaginary unit to write the square root of any negative number.



No x-intercepts

Know it! Note

Imaginary Numbers

WORDS	NUMBERS	ALGEBRA
An **imaginary number** is the square root of a negative number.	$\sqrt{-1} = i$	If b is a positive real number,
Imaginary numbers can be written in the form bi, where b is a real number and i is the imaginary unit.	$\sqrt{-2} = \sqrt{-1}\sqrt{2} = i\sqrt{2}$ $\sqrt{-4} = \sqrt{-1}\sqrt{4} = 2i$	then $\sqrt{-b} = i\sqrt{b}$ and $\sqrt{-b^2} = bi$.
The square of an imaginary number is the original negative number.	$\left(\sqrt{-1}\right)^2 = i^2 = -1$	$\left(\sqrt{-b}\right)^2 = -b$

EXAMPLE 1 Simplifying Square Roots of Negative Numbers

Express each number in terms of i.

A $3\sqrt{-16}$

$3\sqrt{(16)(-1)}$	*Factor out –1.*
$3\sqrt{16}\,\sqrt{-1}$	*Product Property*
$3 \cdot 4\sqrt{-1}$	*Simplify.*
$12\sqrt{-1}$	*Multiply.*
$12i$	*Express in terms of i.*

B $-\sqrt{-75}$

$-\sqrt{(75)(-1)}$	*Factor out –1.*
$-\sqrt{75}\,\sqrt{-1}$	*Product Property*
$-\sqrt{25}\,\sqrt{3}\,\sqrt{-1}$	*Product Property*
$-5\sqrt{3}\,\sqrt{-1}$	*Simplify.*
$-5\sqrt{3}i = -5i\sqrt{3}$	*Express in terms of i.*

CHECK IT OUT! **Express each number in terms of i.**

1a. $\sqrt{-12}$ **1b.** $2\sqrt{-36}$ **1c.** $-\frac{1}{3}\sqrt{-63}$

EXAMPLE 2 Solving a Quadratic Equation with Imaginary Solutions

Solve each equation.

A $x^2 = -81$

$x = \pm\sqrt{-81}$ *Take square roots.*

$x = \pm 9i$ *Express in terms of i.*

Check

x^2	$=-81$
$(9i)^2$	-81
$81i^2$	-81
$81(-1)$	-81 ✔

x^2	$=-81$
$(-9i)^2$	-81
$81i^2$	-81
$81(-1)$	-81 ✔

B $3x^2 + 75 = 0$

$3x^2 = -75$ *Add −75 to both sides.*

$x^2 = -25$ *Divide both sides by 3.*

$x = \pm\sqrt{-25}$ *Take square roots.*

$x = \pm 5i$ *Express in terms of i.*

Check

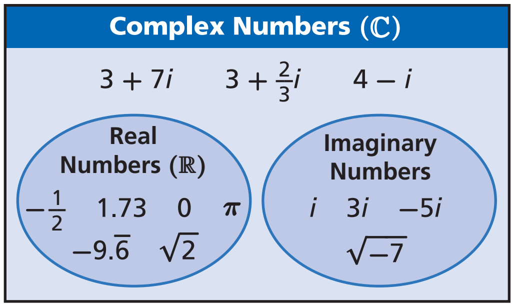
$3x^2 + 75$	$= 0$
$3(\pm 5i)^2 + 75$	0
$3(25)i^2 + 75$	0
$75(-1) + 75$	0 ✔

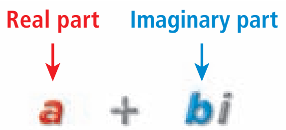
CHECK IT OUT! **Solve each equation.**

2a. $x^2 = -36$ **2b.** $x^2 + 48 = 0$ **2c.** $9x^2 + 25 = 0$

A **complex number** is a number that can be written in the form $a + bi$, where a and b are real numbers and $i = \sqrt{-1}$. The set of real numbers is a subset of the set of complex numbers $\mathbb{C}$.

Every complex number has a **real part** a and an **imaginary part** b.





Real numbers are complex numbers where $b = 0$. Imaginary numbers are complex numbers where $a = 0$ and $b \neq 0$. These are sometimes called *pure imaginary numbers.*

Two complex numbers are equal if and only if their real parts are equal and their imaginary parts are equal.

EXAMPLE 3 Equating Two Complex Numbers

Find the values of x and y that make the equation $3x - 5i = 6 - (10y)i$ true.

Real parts

$$3x - 5i = 6 - (10y)i$$

Imaginary parts

$3x = 6$ *Equate the real parts.*

$x = 2$ *Solve for x.*

$-5 = -(10y)$ *Equate the imaginary parts.*

$\frac{1}{2} = y$ *Solve for y.*

CHECK IT OUT! **Find the values of x and y that make each equation true.**

3a. $2x - 6i = -8 + (20y)i$ **3b.** $-8 + (6y)i = 5x - i\sqrt{6}$

EXAMPLE 4 Finding Complex Zeros of Quadratic Functions

Find the zeros of each function.

A $f(x) = x^2 - 2x + 5$

$x^2 - 2x + 5 = 0$	*Set equal to 0.*
$x^2 - 2x + \square = -5 + \square$	*Rewrite.*
$x^2 - 2x + 1 = -5 + 1$	*Add $\left(\frac{b}{2}\right)^2$.*
$(x-1)^2 = -4$	*Factor.*
$x - 1 = \pm\sqrt{-4}$	*Take square roots.*
$x = 1 \pm 2i$	*Simplify.*

B $g(x) = x^2 + 10x + 35$

$$x^2 + 10x + 35 = 0$$
$$x^2 + 10x + \square = -35 + \square$$
$$x^2 + 10x + 25 = -35 + 25$$
$$(x+5)^2 = -10$$
$$x + 5 = \pm\sqrt{-10}$$
$$x = -5 \pm i\sqrt{10}$$

CHECK IT OUT! **Find the zeros of each function.**

4a. $f(x) = x^2 + 4x + 13$ **4b.** $g(x) = x^2 - 8x + 18$

Helpful Hint

When given one complex root, you can always find the other by finding its conjugate.

The solutions $-5 + i\sqrt{10}$ and $-5 - i\sqrt{10}$ in Example 4B are related. These solutions are a *complex conjugate* pair. Their real parts are equal and their imaginary parts are opposites. The **complex conjugate** of any complex number $a + bi$ is the complex number $a - bi$.

If a quadratic equation with real coefficients has nonreal roots, those roots are complex conjugates.

EXAMPLE 5 Finding Complex Conjugates

Find each complex conjugate.

A $2i - 15$

$-15 + 2i$	*Write as $a + bi$.*
$-15 - 2i$	*Find $a - bi$.*

B $-4i$

$0 + (-4)i$	*Write as $a + bi$.*
$0 - (-4)i$	*Find $a - bi$.*
$4i$	*Simplify.*

CHECK IT OUT! **Find each complex conjugate.**

5a. $9 - i$ **5b.** $i + \sqrt{3}$ **5c.** $-8i$

THINK AND DISCUSS

1. Given that one solution of a quadratic equation is $3 + i$, explain how to determine the other solution.

2. Describe a number of the form $a + bi$ in which $a \neq 0$ and $b = 0$. Then describe a number in which $a = 0$ and $b \neq 0$. Are both numbers complex? Explain.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box or oval, give a definition and examples of each type of number.

Complex Numbers

Real Numbers | Imaginary Numbers

5-5 Exercises

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Homework Help Online
KEYWORD: MB7 5-5
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** The number 7 is the __?__ part of the complex number $\sqrt{5} + 7i$. (*real* or *imaginary*)

SEE EXAMPLE 1 p. 350

Express each number in terms of *i*.

2. $5\sqrt{-100}$
3. $\frac{1}{2}\sqrt{-16}$
4. $-\sqrt{-32}$
5. $\sqrt{-144}$

SEE EXAMPLE 2 p. 351

Solve each equation.

6. $x^2 = -9$
7. $2x^2 + 72 = 0$
8. $4x^2 = -16$
9. $x^2 + 121 = 0$

SEE EXAMPLE 3 p. 351

Find the values of *x* and *y* that make each equation true.

10. $-2x + 6i = (-24y)i - 14$
11. $-4 + (y)i = -12x - i + 8$

SEE EXAMPLE 4 p. 352

Find the zeros of each function.

12. $f(x) = x^2 - 12x + 45$
13. $g(x) = x^2 + 6x + 34$

SEE EXAMPLE 5 p. 352

Find each complex conjugate.

14. $-9i$
15. $\sqrt{5} + 5i$
16. $8i - 3$
17. $6 + i\sqrt{2}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–21	1
22–25	2
26–27	3
28–31	4
32–35	5

Extra Practice

Skills Practice p. S13
Application Practice p. S36

Express each number in terms of *i*.

18. $8\sqrt{-4}$
19. $-\frac{1}{3}\sqrt{-90}$
20. $6\sqrt{-12}$
21. $\sqrt{-50}$

Solve each equation.

22. $x^2 + 49 = 0$
23. $5x^2 = -80$
24. $3x^2 + 27 = 0$
25. $\frac{1}{2}x^2 = -32$

Find the values of *x* and *y* that make each equation true.

26. $9x + (y)i - 5 = -12i + 4$
27. $5(x - 1) + (3y)i = -15i - 20$

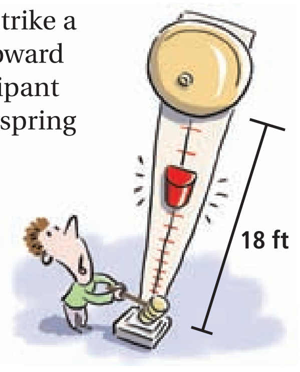
Find the zeros of each function.

28. $f(x) = x^2 + 2x + 3$
29. $g(x) = 4x^2 - 3x + 1$
30. $f(x) = x^2 + 4x + 8$
31. $g(x) = 3x^2 - 6x + 10$

Find each complex conjugate.

32. i
33. $-\frac{\sqrt{3}}{2} - 2i$
34. $-2.5i + 1$
35. $\frac{i}{10} - 1$

36. **What if...?** A carnival game asks participants to strike a spring with a hammer. The spring shoots a puck upward toward a bell. If the puck strikes the bell, the participant wins a prize. Suppose that a participant strikes the spring and shoots the puck according to the model $d(t) = 16t^2 - 32t + 18$, where d is the distance in feet between the puck and the bell and t is the time in seconds since the puck was struck. Is it possible for the participant to win a prize? Explain your answer.



Given each solution to a quadratic equation, find the other solution.

37. $1 + 14i$ **38.** $\frac{5}{7}i$ **39.** $4i - 2\sqrt{5}$

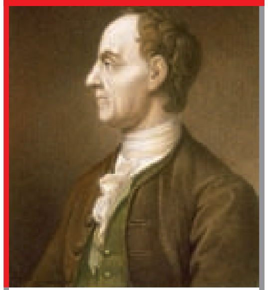
40. $-12 - i$ **41.** $9 - i\sqrt{2}$ **42.** $-\frac{17i}{3}$

Find the values of *c* and *d* that make each equation true.

43. $2ci + 1 = -d + 6 - ci$ **44.** $c + 3ci = 4 + di$ **45.** $c^2 + 4i = d + di$

LINK

Math History



The Granger Collection, New York

The Swiss mathematician Leonhard Euler (1707–1783) was the first to use the notation i to represent $\sqrt{-1}$. He also introduced the notation $f(x)$ to represent the value of a function f at x.

Solve each equation.

46. $8x^2 = -8$ **47.** $\frac{1}{3}x^2 = -27$ **48.** $2x^2 + 12.5 = 0$

49. $\frac{1}{2}x^2 + 72 = 0$ **50.** $x^2 = -30$ **51.** $2x^2 + 16 = 0$

52. $x^2 - 4x + 8 = 0$ **53.** $x^2 + 10x + 29 = 0$ **54.** $x^2 - 12x + 44 = 0$

55. $x^2 + 2x = -5$ **56.** $x^2 + 18 = -6x$ **57.** $-149 = x^2 - 24x$

Tell whether each statement is always, sometimes, or never true. If sometimes true, give examples to support your answer.

58. A real number is an imaginary number.

59. An imaginary number is a complex number.

60. A rational number is a complex number.

61. A complex number is an imaginary number.

62. An integer is a complex number.

63. Quadratic equations have no real solutions.

64. Quadratic equations have roots that are real and complex.

65. Roots of quadratic equations are conjugate pairs.

Find the zeros of each function.

66. $f(x) = x^2 - 10x + 26$ **67.** $g(x) = x^2 + 2x + 17$ **68.** $h(x) = x^2 - 10x + 50$

69. $f(x) = x^2 + 16x + 73$ **70.** $g(x) = x^2 - 10x + 37$ **71.** $h(x) = x^2 - 16x + 68$

72. **Critical Thinking** Can you determine the zeros of $f(x) = x^2 + 64$ by using a graph? Explain why or why not.

73. **Critical Thinking** What is the complex conjugate of a real number?

74. **Write About It** Explain the procedures you can use to solve for nonreal complex roots.

MULTI-STEP TEST PREP

75. This problem will prepare you for the Multi-Step Test Prep on page 364.

A player throws a ball straight up toward the roof of an indoor baseball stadium. The height h in feet of the ball after t seconds can be modeled by the function $h(t) = -16t^2 + 112t$.

a. The height of the roof is 208 ft. Solve the equation $208 = -16t^2 + 112t$.

b. Based on your answer to part **a**, does the ball hit the roof? Explain your answer.

c. Based on the function model, what is the maximum height that the ball will reach?

TEST PREP

76. What is the complex conjugate of $-2 + i$?

Ⓐ $2 + i$ Ⓑ $2 - i$ Ⓒ $i - 2$ Ⓓ $-2 - i$

77. Express $\sqrt{-225}$ in terms of i.

Ⓕ $15i$ Ⓖ $-15i$ Ⓗ $i\sqrt{15}$ Ⓙ $-i\sqrt{15}$

78. Find the zeros of $f(x) = x^2 - 2x + 17$.

Ⓐ $1 \pm 4i$ Ⓑ $4 \pm i$ Ⓒ $-1 \pm 4i$ Ⓓ $-4 \pm i$

79. What value of c makes the equation $3 - 4i - 5 = (9 + ci) - 11$ true?

Ⓕ -2 Ⓖ -4 Ⓗ 2 Ⓙ 4

80. Which of the following equations has roots of $-6i$ and $6i$?

Ⓐ $-\frac{1}{6}x^2 = 6$ Ⓒ $\frac{1}{4}x^2 = 9$

Ⓑ $x^2 - 30 = 6$ Ⓓ $20 - x^2 = -16$

81. **Short Response** Explain the types of solutions that equations of the form $x^2 = a$ have when $a < 0$ and when $a > 0$.

CHALLENGE AND EXTEND

82. Find the complex number $a + bi$ such that $5a + 3b = 1$ and $-5b = 7 + 4a$.

83. Can a quadratic equation have only one real number root? only one imaginary root? only one complex root? Explain.

84. Given the general form of a quadratic equation $x^2 + bx + c = 0$, determine the effect of each condition on the solutions.

a. $b = 0$ **b.** $c \leq 0$ **c.** $c > 0$

d. What is needed for the solutions to have imaginary parts?

SPIRAL REVIEW

Use the following matrices for Exercises 85–88. Evaluate, if possible. *(Lesson 4-2)*

$$S = \begin{bmatrix} 1 & -5 \\ -2 & 0 \end{bmatrix} \quad T = \begin{bmatrix} -4 & 1 & -2 \\ 0 & -3 & 1 \\ 2 & -2 & 2 \end{bmatrix} \quad V = \begin{bmatrix} 10 & 1 \\ 0 & -1 \\ -5 & 5 \end{bmatrix}$$

85. T^2 **86.** TV **87.** ST **88.** S^2

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function. *(Lesson 5-2)*

89. $f(x) = \frac{1}{5}x^2 + x - 10$ **90.** $f(x) = -x^2 + 3$

91. $f(x) = 2x^2 + 4x - 3$ **92.** $f(x) = -\frac{1}{2}x^2 + 3x + 1$

Find the roots of each equation by factoring. *(Lesson 5-3)*

93. $x^2 + 5x = 14$ **94.** $6x^2 = -x + 2$

95. $4x^2 + 9 = 15x$ **96.** $4x^2 = 1$

97. $x^2 + 11x = -24$ **98.** $x^2 = -7x$

5-6 The Quadratic Formula

Objectives

Solve quadratic equations using the Quadratic Formula.

Classify roots using the discriminant.

Vocabulary

discriminant

Who uses this?

Firefighting pilots can use the Quadratic Formula to estimate when to release water on a fire. (See Example 4.)

You have learned several methods for solving quadratic equations: graphing, making tables, factoring, using square roots, and completing the square. Another method is to use the *Quadratic Formula*, which allows you to solve a quadratic equation in standard form.

By completing the square on the standard form of a quadratic equation, you can determine the Quadratic Formula.

Numbers		Algebra
$3x^2 + 5x + 1 = 0$		$ax^2 + bx + c = 0\ (a \neq 0)$
$x^2 + \frac{5}{3}x + \frac{1}{3} = 0$	*Divide by a.*	$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$
$x^2 + \frac{5}{3}x = -\frac{1}{3}$	*Subtract $\frac{c}{a}$.*	$x^2 + \frac{b}{a}x = -\frac{c}{a}$
$x^2 + \frac{5}{3}x + \left(\frac{5}{2(3)}\right)^2 = -\frac{1}{3} + \left(\frac{5}{2(3)}\right)^2$	*Complete the square.*	$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$
$\left(x + \frac{5}{6}\right)^2 = \frac{25}{36} - \frac{1}{3}$	*Factor.*	$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$
$x + \frac{5}{6} = \pm\sqrt{\frac{13}{36}}$	*Take square roots.*	$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$
$x = -\frac{5}{6} \pm \frac{\sqrt{13}}{6}$	*Subtract $\frac{b}{2a}$.*	$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$
$x = \frac{-5 \pm \sqrt{13}}{6}$	*Simplify.*	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Remember!

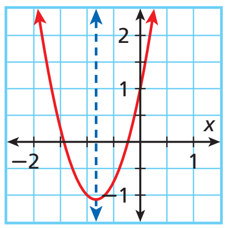
To subtract fractions, you need a common denominator.

$$\frac{b^2}{4a^2} - \frac{c}{a}$$

$$\frac{b^2}{4a^2} - \frac{c}{a}\left(\frac{4a}{4a}\right)$$

$$\frac{b^2 - 4ac}{4a^2}$$

The symmetry of a quadratic function is evident in the next to last step, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$. These two zeros are the same distance, $\frac{\sqrt{b^2 - 4ac}}{2a}$, away from the axis of symmetry, $x = -\frac{b}{2a}$, with one zero on either side of the vertex.



Know it! Note

The Quadratic Formula

If $ax^2 + bx + c = 0\ (a \neq 0)$, then the solutions, or roots, are

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

You can use the Quadratic Formula to solve any quadratic equation that is written in standard form, including equations with real solutions or complex solutions.

EXAMPLE 1 Quadratic Functions with Real Zeros

Find the zeros of $f(x) = x^2 + 10x + 2$ by using the Quadratic Formula.

$$x^2 + 10x + 2 = 0$$ *Set $f(x) = 0$.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *Write the Quadratic Formula.*

$$x = \frac{-10 \pm \sqrt{(10)^2 - 4(1)(2)}}{2(1)}$$ *Substitute 1 for a, 10 for b, and 2 for c.*

$$x = \frac{-10 \pm \sqrt{100 - 8}}{2} = \frac{-10 \pm \sqrt{92}}{2}$$ *Simplify.*

$$x = \frac{-10 \pm 2\sqrt{23}}{2} = -5 \pm \sqrt{23}$$ *Write in simplest form.*

Check Solve by completing the square.

$$x^2 + 10x + 2 = 0$$
$$x^2 + 10x = -2$$
$$x^2 + 10x + 25 = -2 + 25$$
$$(x + 5)^2 = 23$$
$$x = -5 \pm \sqrt{23} \checkmark$$

CHECK IT OUT!

Find the zeros of each function by using the Quadratic Formula.

1a. $f(x) = x^2 + 3x - 7$ **1b.** $g(x) = x^2 - 8x + 10$

EXAMPLE 2 Quadratic Functions with Complex Zeros

Find the zeros of $f(x) = 2x^2 - x + 2$ by using the Quadratic Formula.

$$2x^2 - x + 2 = 0$$ *Set $f(x) = 0$.*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ *Write the Quadratic Formula.*

$$x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(2)(2)}}{2(2)}$$ *Substitute 2 for a, −1 for b, and 2 for c.*

$$x = \frac{1 \pm \sqrt{1 - 16}}{4} = \frac{1 \pm \sqrt{-15}}{4}$$ *Simplify.*

$$x = \frac{1 \pm i\sqrt{15}}{4} = \frac{1}{4} \pm \frac{\sqrt{15}}{4}i$$ *Write in terms of i.*

CHECK IT OUT!

2. Find the zeros of $g(x) = 3x^2 - x + 8$ by using the Quadratic Formula.

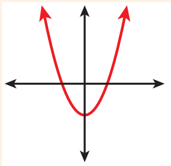
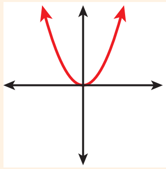
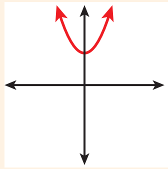
The **discriminant** is part of the Quadratic Formula that you can use to determine the number of real roots of a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \leftarrow \text{Discriminant}$$

Know it! Note

Discriminant

The discriminant of the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ is $b^2 - 4ac$.

$b^2 - 4ac > 0$	$b^2 - 4ac = 0$	$b^2 - 4ac < 0$
two distinct real solutions	one distinct real solution	two distinct nonreal complex solutions
		

EXAMPLE 3 Analyzing Quadratic Equations by Using the Discriminant

Caution!

Make sure the equation is in standard form before you evaluate the discriminant, $b^2 - 4ac$.

Find the type and number of solutions for each equation.

A $x^2 - 6x = -7$

$x^2 - 6x + 7 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(1)(7)$

$36 - 28 = 8$

$b^2 - 4ac > 0$; the equation has two distinct real solutions.

B $x^2 - 6x = -9$

$x^2 - 6x + 9 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(1)(9)$

$36 - 36 = 0$

$b^2 - 4ac = 0$; the equation has one distinct real solution.

C $x^2 - 6x = -11$

$x^2 - 6x + 11 = 0$

$b^2 - 4ac$

$(-6)^2 - 4(1)(11)$

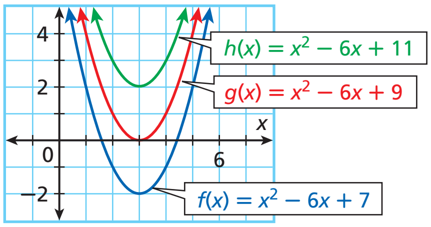
$36 - 44 = -8$

$b^2 - 4ac < 0$; the equation has two distinct nonreal complex solutions.

CHECK IT OUT! **Find the type and number of solutions for each equation.**

3a. $x^2 - 4x = -4$ **3b.** $x^2 - 4x = -8$ **3c.** $x^2 - 4x = 2$

The graph shows the related functions for Example 3. Notice that the number of real solutions for the equation can be changed by changing the value of the constant c.



Student to Student

Double-Checking Roots

Terry Cannon,
Carver High School

If I get integer roots when I use the Quadratic Formula, I know that I can quickly factor to check the roots. Look at my work for the equation $x^2 - 7x + 10 = 0$.

Quadratic Formula:

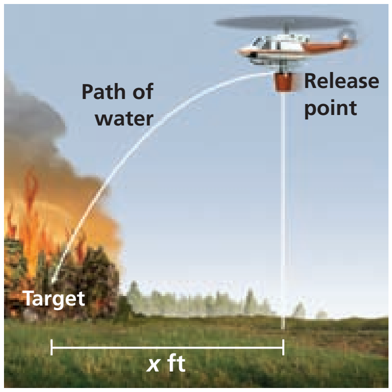
$$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(1)(10)}}{2(1)}$$

$$= \frac{7 \pm \sqrt{9}}{2} = \frac{10}{2} \text{ or } \frac{4}{2} = 5 \text{ or } 2$$

Factoring:

$$x^2 - 7x + 10 = 0$$
$$(x - 5)(x - 2) = 0$$
$$x = 5 \text{ or } x = 2$$

EXAMPLE 4 Aviation Application



The pilot of a helicopter plans to release a bucket of water on a forest fire. The height y in feet of the water t seconds after its release is modeled by $y = -16t^2 - 2t + 500$. The horizontal distance x in feet between the water and its point of release is modeled by $x = 91t$. At what horizontal distance from the fire should the pilot start releasing the water in order to hit the target?

Step 1 Use the first equation to determine how long it will take the water to hit the ground. Set the height of the water equal to 0 feet, and use the quadratic formula to solve for t.

$$y = -16t^2 - 2t + 500$$

$$0 = -16t^2 - 2t + 500 \qquad \textit{Set y equal to 0.}$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \qquad \textit{Use the Quadratic Formula.}$$

$$t = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-16)(500)}}{2(-16)} \qquad \textit{Substitute for a, b, and c.}$$

$$t = \frac{2 \pm \sqrt{32{,}004}}{-32} \qquad \textit{Simplify.}$$

$$t \approx -5.65 \text{ or } t \approx 5.53$$

The time cannot be negative, so the water lands on the target about 5.5 seconds after it is released.

Step 2 Find the horizontal distance that the water will have traveled in this time.

$$x = 91t$$

$$x = 91(5.5) \qquad \textit{Substitute 5.5 for t.}$$

$$x = 500.5 \qquad \textit{Simplify.}$$

The water will have traveled a horizontal distance of about 500 feet. Therefore, the pilot should start releasing the water when the horizontal distance between the helicopter and the fire is 500 feet.

Check Use substitution to check that the water hits the ground after about 5.53 seconds.

$$y = -16t^2 - 2t + 500$$

$$y = -16(5.53)^2 - 2(5.53) + 500$$

$$y \approx -0.3544 \checkmark \qquad \textit{The height is approximately equal to 0 when } t = 5.53.$$

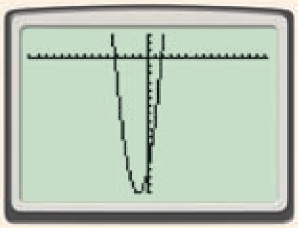
Caution!

Once you have found the value of t, you have solved only part of the problem. You will use this value to find the answer you are looking for.

CHECK IT OUT! **Use the information given above to answer the following.**

4. The pilot's altitude decreases, which changes the function describing the water's height to $y = -16t^2 - 2t + 400$. To the nearest foot, at what horizontal distance from the target should the pilot begin releasing the water?

Summary of Solving Quadratic Equations

Method	When to Use	Examples
Graphing	Only approximate solutions or the number of real solutions is needed.	$2x^2 + 5x - 14 = 0$  $x \approx -4.2$ or $x \approx 1.7$
Factoring	$c = 0$ or the expression is easily factorable.	$x^2 + 4x + 3 = 0$ $(x + 3)(x + 1) = 0$ $x = -3$ or $x = -1$
Square roots	The variable side of the equation is a perfect square.	$(x - 5)^2 = 24$ $\sqrt{(x - 5)^2} = \pm\sqrt{24}$ $x - 5 = \pm 2\sqrt{6}$ $x = 5 \pm 2\sqrt{6}$
Completing the square	$a = 1$ and b is an even number.	$x^2 + 6x = 10$ $x^2 + 6x + \blacksquare = 10 + \blacksquare$ $x^2 + 6x + \left(\frac{6}{2}\right)^2 = 10 + \left(\frac{6}{2}\right)^2$ $(x + 3)^2 = 19$ $x = -3 \pm \sqrt{19}$
Quadratic Formula	Numbers are large or complicated, and the expression does not factor easily.	$5x^2 - 7x - 8 = 0$ $x = \dfrac{-(-7) \pm \sqrt{(-7)^2 - 4(5)(-8)}}{2(5)}$ $x = \dfrac{7 \pm \sqrt{209}}{10}$

Helpful Hint

No matter which method you use to solve a quadratic equation, you should get the same answer.

THINK AND DISCUSS

1. Describe how the graphs of quadratic functions illustrate the type and number of zeros.

2. Describe the values of c for which the equation $x^2 + 8x + c = 0$ will have zero, one, or two distinct solutions.

3. GET ORGANIZED Copy and complete the graphic organizer. Describe the possible solution methods for each value of the discriminant.

Value of Discriminant	Type of Solutions	Possible Solution Methods
Negative		
Zero		
Positive		

5-6 Exercises

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Homework Help Online
KEYWORD: MB7 5-6
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** What information does the value of the *discriminant* give about a quadratic equation?

SEE EXAMPLE 1 p. 357

Find the zeros of each function by using the Quadratic Formula.

2. $f(x) = x^2 + 7x + 10$
3. $g(x) = 3x^2 - 4x - 1$
4. $h(x) = 3x^2 - 5x$
5. $g(x) = -x^2 - 5x + 6$
6. $h(x) = 4x^2 - 5x - 6$
7. $f(x) = 2x^2 - 19$

SEE EXAMPLE 2 p. 357

8. $f(x) = 2x^2 - 2x + 3$
9. $r(x) = x^2 + 6x + 12$
10. $h(x) = 3x^2 + 4x + 3$
11. $p(x) = x^2 + 4x + 10$
12. $g(x) = -5x^2 + 7x - 3$
13. $f(x) = 10x^2 + 7x + 4$

SEE EXAMPLE 3 p. 358

Find the type and number of solutions for each equation.

14. $4x^2 + 1 = 4x$
15. $x^2 + 2x = 10$
16. $2x - x^2 = 4$

SEE EXAMPLE 4 p. 359

17. **Geometry** One leg of a right triangle is 6 in. longer than the other leg. The hypotenuse of the triangle is 25 in. What is the length of each leg to the nearest inch?

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
18–23	1
24–29	2
30–35	3
36	4

Extra Practice

Skills Practice p. S13
Application Practice p. S36

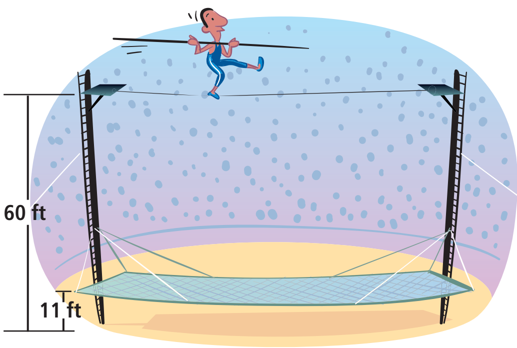
Find the zeros of each function by using the Quadratic Formula.

18. $f(x) = 3x^2 - 10x + 3$
19. $g(x) = x^2 + 6x$
20. $h(x) = x(x - 3) - 4$
21. $g(x) = -x^2 - 2x + 9$
22. $p(x) = 2x^2 - 7x - 8$
23. $f(x) = 7x^2 - 3$
24. $r(x) = x^2 + x + 1$
25. $h(x) = -x^2 - x - 1$
26. $f(x) = 2x^2 + 8$
27. $f(x) = 2x^2 + 7x - 13$
28. $g(x) = x^2 - x - 5$
29. $h(x) = -3x^2 + 4x - 4$

Find the type and number of solutions for each equation.

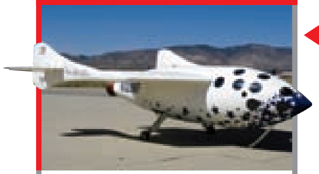
30. $2x^2 + 5 = 2x$
31. $2x^2 - 3x = 8$
32. $2x^2 - 16x = -32$
33. $4x^2 - 28x = -49$
34. $3x^2 - 8x + 8 = 0$
35. $3.2x^2 - 8.5x + 1.3 = 0$

36. **Safety** If a tightrope walker falls, he will land on a safety net. His height h in feet after a fall can be modeled by $h(t) = 60 - 16t^2$, where t is the time in seconds. How many seconds will the tightrope walker fall before landing on the safety net?



37. **Physics** A bicyclist is riding at a speed of 20 mi/h when she starts down a long hill. The distance d she travels in feet can be modeled by the function $d(t) = 5t^2 + 20t$, where t is the time in seconds.
    a. The hill is 585 ft long. To the nearest second, how long will it take her to reach the bottom?
    b. **What if...?** Suppose the hill were only half as long. To the nearest second, how long would it take the bicyclist to reach the bottom?

Aerospace



SpaceShipOne was the winner of the Ansari X Prize competition. The X Prize was awarded to the first nongovernmental spacecraft to reach an altitude of at least 100 km twice within a 2 week period.

Find the zeros of each function. Then graph the function.

38. $f(x) = 3x^2 - 4x - 2$ **39.** $g(x) = 2x^2 - 2x - 1$ **40.** $h(x) = 2x^2 + 6x + 5$

41. $p(x) = 2x^2 + 3x - 1$ **42.** $h(x) = 3x^2 - 5x - 4$ **43.** $r(x) = x^2 - x + 22$

44. **Aerospace** In 2004, the highest spaceplane flight was made by Brian Binnie in *SpaceShipOne*. A flight with this altitude can be modeled by the function $h(t) = -0.17t^2 + 187t + 61{,}000$, where h is the altitude in meters and t is flight time in seconds.

a. Approximately how long did the flight last?

b. What was the highest altitude to the nearest thousand meters?

c. The table shows the altitudes of layers of Earth's atmosphere. According to the model, which of these layers did *SpaceShipOne* enter, and at what time(s) did the spaceplane enter them?

Earth's Atmosphere

Layer	Altitude (in km)
Troposphere	0 to 10
Stratosphere	10 to 50
Mesosphere	50 to 85
Thermosphere	85 to 600

Solve each equation by any method.

45. $x^2 - 3x = 10$ **46.** $x^2 - 16 = 0$ **47.** $4x^2 + 4x = 15$

48. $x^2 + 2x - 2 = 0$ **49.** $x^2 - 4x - 21 = 0$ **50.** $4x^2 - 4x - 1 = 0$

51. $6x^2 = 150$ **52.** $x^2 = 7$ **53.** $x^2 - 16x + 64 = 0$

54. **Critical Thinking** If you are solving a real-world problem involving a quadratic equation, and the discriminant is negative, what can you conclude?

55. **Multi-Step** The outer dimensions of a picture frame are 25 inches by 20 inches. If the area inside the picture frame is 266 square inches, what is the width w of the frame?

Critical Thinking **Find the values of c that make each equation have one real solution.**

56. $x^2 + 8x + c = 0$ **57.** $x^2 + 12x = c$ **58.** $x^2 + 2cx + 49 = 0$

59. **Write About It** What method would you use to solve the equation $-14x^2 + 6x = 2.7$? Why would this method be easier to use than the other methods?

MULTI-STEP TEST PREP

60. This problem will prepare you for the Multi-Step Test Prep on page 364.

An outfielder throws a baseball to the player on third base. The height h of the ball in feet is modeled by the function $h(t) = -16t^2 + 19t + 5$, where t is time in seconds. The third baseman catches the ball when it is 4 ft above the ground.

a. To the nearest tenth of a second, how long was the ball in the air before it was caught?

b. A player on the opposing team starts running from second base to third base 1.2 s before the outfielder throws the ball. The distance between the bases is 90 ft, and the runner's average speed is 27 ft/s. Will the runner reach third base before the ball does? Explain.

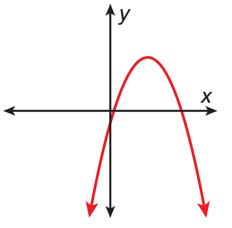
TEST PREP

61. Which best describes the graph of a quadratic function with a discriminant of −3?
   (A) Parabola with two x-intercepts
   (B) Parabola with no x-intercepts
   (C) Parabola that opens upward
   (D) Parabola that opens downward

62. What is the discriminant of the equation $2x^2 - 8x = 14$?
   (F) 48
   (H) 176
   (G) −48
   (J) −176

63. Which function has zeros of $3 \pm i$?
   (A) $f(x) = x^2 + 6x + 10$
   (C) $g(x) = x^2 - 6x + 10$
   (B) $f(x) = x^2 + 6x - 10$
   (D) $h(x) = x^2 - 6x - 10$

64. Which best describes the discriminant of the function whose graph is shown?
   (F) Positive
   (H) Negative
   (G) Zero
   (J) Undefined



CHALLENGE AND EXTEND

65. **Geometry** The perimeter of a right triangle is 40 cm, and its hypotenuse measures 17 cm. Find the length of each leg.

66. **Geometry** The perimeter of a rectangle is 88 cm.
   a. Find the least possible value of the length of the diagonal. Round to the nearest tenth of a centimeter.
   b. What are the dimensions of the rectangle with this diagonal?

Write a quadratic equation whose solutions belong to the indicated sets.

67. integers
68. irrational real numbers
69. complex numbers

70. A quadratic equation has the form $ax^2 + bx + c = 0 \ (a \neq 0)$.
   a. What is the sum of the roots of the equation? the product of the roots?
   b. Determine the standard form of a quadratic equation whose roots have a sum of 2 and a product of −15.

71. Describe the solutions to a quadratic equation for which $a = b = c$.

SPIRAL REVIEW

72. **Biology** The length of a human hair is a linear function of time. Juan's hair grows 2.1 cm in 60 days. Express the growth in centimeters of Juan's hair as a function of the number of days since his last haircut. *(Lesson 2-4)*

Write the augmented matrix, and use row reduction to solve. *(Lesson 4-6)*

73. $\begin{cases} 3y = 2x + 7 \\ x - 6y = 1 \end{cases}$
74. $\begin{cases} 2x = -3y + 12 \\ x + y = 14 \end{cases}$
75. $\begin{cases} 4x + 5y = -1 \\ 9 + 7y = 2x \end{cases}$

Solve each equation by completing the square. *(Lesson 5-4)*

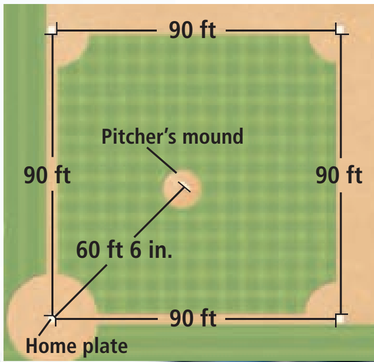
76. $x^2 - 5x = 1$
77. $2x^2 = 16x - 4$
78. $3x = 5x^2 - 12$

MULTI-STEP TEST PREP

Quadratic Functions and Complex Numbers

Ballpark Figures When a baseball is thrown or hit into the air, its height h in feet after t seconds can be modeled by $h(t) = -16t^2 + v_y t + h_0$, where v_y is the initial vertical velocity of the ball in feet per second and h_0 is the ball's initial height. The horizontal distance d in feet that the ball travels in t seconds can be modeled by $d(t) = v_x t$, where v_x is the ball's initial horizontal velocity in feet per second.

1. A short stop makes an error by dropping the ball. As the ball drops, its height h in feet is modeled by $h(t) = -16t^2 + 3$. A slow-motion replay of the error shows the play at half speed. What function describes the height of the ball in the replay?

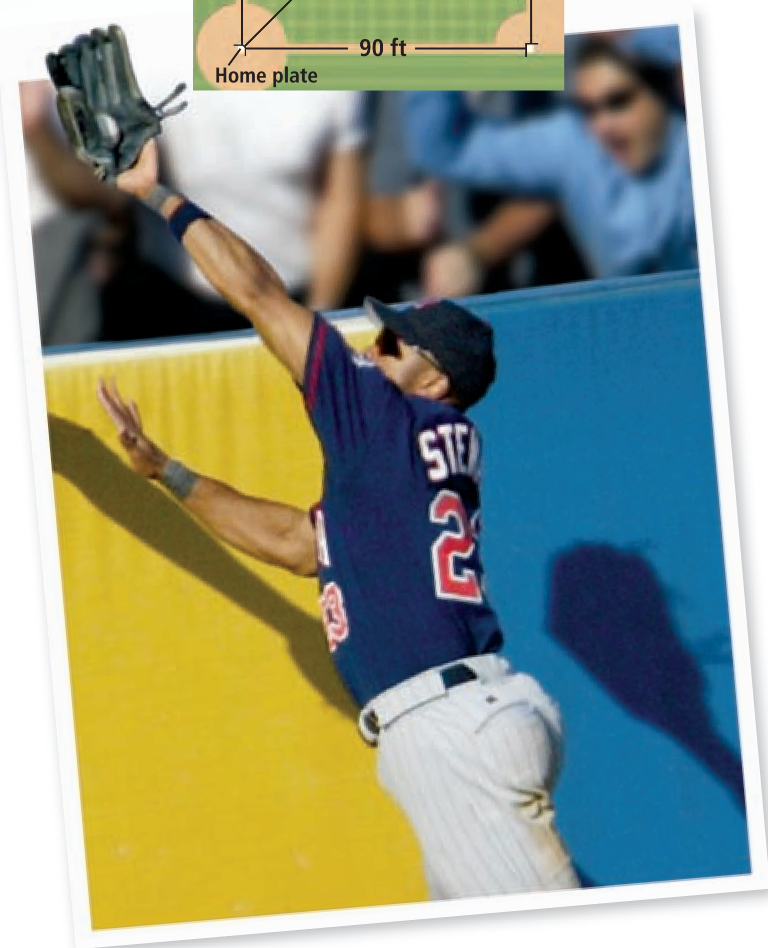


2. A player hits a foul ball with an initial vertical velocity of 70 ft/s and an initial height of 5 ft. To the nearest foot, what is the maximum height reached by the ball?

3. A pitch will be a strike if its height is between 2.5 ft and 5 ft when it crosses home plate. The pitcher throws the ball from a height of 6 ft with an initial vertical velocity of 5 ft/s and a horizontal velocity of 116 ft/s. Could this pitch be a strike? Explain.

4. The next pitch crosses home plate 1 ft too high to be a strike. The pitch is thrown from a height of 6 ft with an initial vertical velocity of 8 ft/s. What is the initial horizontal velocity of this pitch?

5. A player throws the ball home from a height of 5.5 ft with an initial vertical velocity of 28 ft/s. The ball is caught at home plate at a height of 5 ft. Three seconds before the ball is thrown, a runner on third base starts toward home plate at an average speed of 25 ft/s. Does the runner reach home plate before the ball does? Explain.



READY TO GO ON?

CHAPTER 5

SECTION 5A

Quiz for Lessons 5-1 Through 5-6

5-1 Using Transformations to Graph Quadratic Functions

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

1. $g(x) = (x + 2)^2 - 4$
2. $g(x) = -4(x - 1)^2$
3. $g(x) = \frac{1}{2}x^2 + 1$

Use the description to write each quadratic function in vertex form.

4. $f(x) = x^2$ is vertically stretched by a factor of 9 and translated 2 units left to create g.
5. $f(x) = x^2$ is reflected across the x-axis and translated 4 units up to create g.

5-2 Properties of Quadratic Functions in Standard Form

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

6. $f(x)= x^2 - 4x + 3$
7. $g(x) = -x^2 + 2x - 1$
8. $h(x) = x^2 - 6x$
9. A football kick is modeled by the function $h(x) = -0.0075x^2 + 0.5x + 5$, where h is the height of the ball in feet and x is the horizontal distance in feet that the ball travels. Find the maximum height of the ball to the nearest foot.

5-3 Solving Quadratic Equations by Graphing and Factoring

Find the roots of each equation by factoring.

10. $x^2 - 100 = 0$
11. $x^2 + 5x = 24$
12. $4x^2 + 8x = 0$

5-4 Completing the Square

Solve each equation by completing the square.

13. $x^2 - 6x = 40$
14. $x^2 + 18x = 15$
15. $x^2 + 14x = 8$

Write each function in vertex form, and identify its vertex.

16. $f(x) = x^2 + 24x + 138$
17. $g(x) = x^2 - 12x + 39$
18. $h(x) = 5x^2 - 20x + 9$

5-5 Complex Numbers and Roots

Solve each equation.

19. $3x^2 = -48$
20. $x^2 - 20x = -125$
21. $x^2 - 8x + 30 = 0$

5-6 The Quadratic Formula

Find the zeros of each function by using the Quadratic Formula.

22. $f(x) = (x + 6)^2 + 2$
23. $g(x) = x^2 + 7x + 15$
24. $h(x) = 2x^2 - 5x + 3$
25. A bicyclist is riding at a speed of 18 mi/h when she starts down a long hill. The distance d she travels in feet can be modeled by $d(t) = 4t^2 + 18t$, where t is the time in seconds. How long will it take her to reach the bottom of a 400-foot-long hill?

5-7 Solving Quadratic Inequalities

Objectives
Solve quadratic inequalities by using tables and graphs.

Solve quadratic inequalities by using algebra.

Vocabulary
quadratic inequality in two variables

Who uses this?
Tour companies and other businesses use quadratic inequalities to make predictions of profits. (See Example 4.)

Many business profits can be modeled by quadratic functions. To ensure that the profit is above a certain level, financial planners may need to graph and solve *quadratic inequalities*.

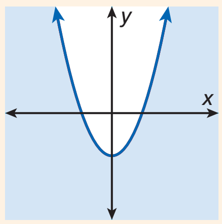
A **quadratic inequality in two variables** can be written in one of the following forms, where a, b, and c are real numbers and $a \neq 0$. Its solution set is a set of ordered pairs (x, y).

$$y < ax^2 + bx + c \qquad y > ax^2 + bx + c$$
$$y \leq ax^2 + bx + c \qquad y \geq ax^2 + bx + c$$

In Lesson 2-5, you solved linear inequalities in two variables by graphing. You can use a similar procedure to graph quadratic inequalities.

Know it! Note

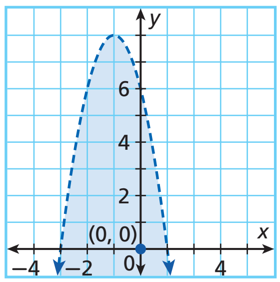
Graphing Quadratic Inequalities

To graph a quadratic inequality	
1. Graph the parabola that defines the boundary.	
2. Use a solid parabola for $y \leq$ and $y \geq$ and a dashed parabola for $y <$ and $y >$.	
3. Shade above the parabola for $y >$ or $\geq$ and below the parabola for $y \leq$ or $<$.	

EXAMPLE 1

Graphing Quadratic Inequalities in Two Variables

Graph $y < -2x^2 - 4x + 6$.



Step 1 Graph the boundary of the related parabola $y = -2x^2 - 4x + 6$ with a dashed curve.

Its y-intercept is 6, its vertex is $(-1, 8)$, and its x-intercepts are -3 and 1.

Step 2 Shade below the parabola because the solution consists of y-values less than those on the parabola for corresponding x-values.

Check Use a test point to verify the solution region.

$$y < -2x^2 - 4x + 6$$
$$0 < -2(0)^2 - 4(0) + 6 \qquad \textit{Try } (0, 0).$$
$$0 < 6 \checkmark$$

CHECK IT OUT! **Graph each inequality.**

1a. $y \geq 2x^2 - 5x - 2$ **1b.** $y < -3x^2 - 6x - 7$

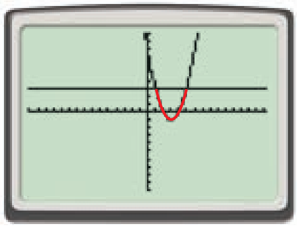
Quadratic inequalities in one variable, such as $ax^2 + bx + c > 0 \ (a \neq 0)$, have solutions in one variable that are graphed on a number line.

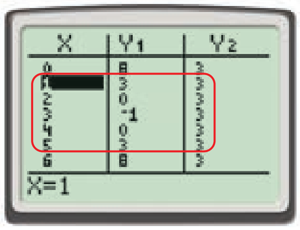
EXAMPLE 2 Solving Quadratic Inequalities by Using Tables and Graphs

Solve each inequality by using tables or graphs.

A $x^2 - 6x + 8 \leq 3$

Use a graphing calculator to graph each side of the inequality. Set **Y1** equal to $x^2 - 6x + 8$ and **Y2** equal to 3. Identify the values of x for which **Y1** $\leq$ **Y2**.



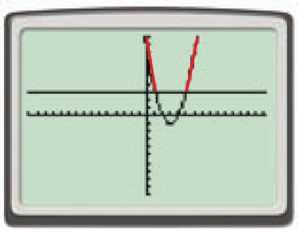


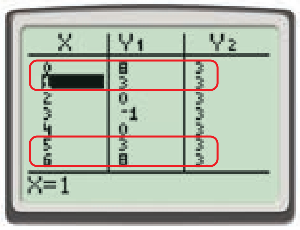
The parabola is at or below the line when x is between 1 and 5 inclusive. So, the solution set is $1 \leq x \leq 5$, or $[1, 5]$. The table supports your answer.

The number line shows the solution set.

B $x^2 - 6x + 8 > 3$

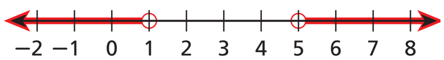
Use a graphing calculator to graph each side of the inequality. Set **Y1** equal to $x^2 - 6x + 8$ and **Y2** equal to 3. Identify the values of x for which **Y1** $>$ **Y2**.





The parabola is above the line $y = 3$ when x is less than 1 or greater than 5. So the solution set is $x < 1$ or $x > 5$, or $(-\infty, 1) \cup (5, \infty)$.

The number line shows the solution set.



Reading Math

For **and** statements, *both* of the conditions must be true. For **or** statements, *at least one* of the conditions must be true.

CHECK IT OUT! **Solve each inequality by using tables or graphs.**

2a. $x^2 - x + 5 < 7$ **2b.** $2x^2 - 5x + 1 \geq 1$

The number lines showing the solution sets in Example 2 are divided into three distinct regions by the points 1 and 5. These points are called *critical values*. By finding the critical values, you can solve quadratic inequalities algebraically.

EXAMPLE 3 Solving Quadratic Inequalities by Using Algebra

Solve the inequality $x^2 - 4x + 1 > 6$ by using algebra.

Step 1 Write the related equation.

$$x^2 - 4x + 1 = 6$$

Step 2 Solve the equation for x to find the critical values.

$x^2 - 4x - 5 = 0$ *Write in standard form.*

$(x - 5)(x + 1) = 0$ *Factor.*

$x - 5 = 0$ or $x + 1 = 0$ *Zero Product Property*

$x = 5$ or $x = -1$ *Solve for x.*

The critical values are 5 and -1. The critical values divide the number line into three intervals: $x < -1$, $-1 < x < 5$, and $x > 5$.

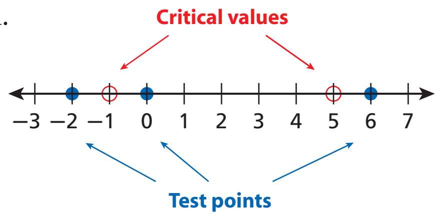
Step 3 Test an x-value in each interval.

$x^2 - 4x + 1 > 6$

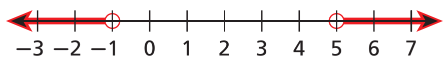
$(-2)^2 - 4(-2) + 1 > 6$ ✔ *Try x = −2.*

$(0)^2 - 4(0) + 1 > 6$ ✘ *Try x = 0.*

$(6)^2 - 4(6) + 1 > 6$ ✔ *Try x = 6.*



Shade the solution regions on the number line. Use open circles for the critical values because the inequality does not contain *or equal to*.



The solution is $x < -1$ or $x > 5$, or $(-\infty, -1) \cup (5, \infty)$.

CHECK IT OUT! **Solve each inequality by using algebra.**

3a. $x^2 - 6x + 10 \geq 2$ **3b.** $-2x^2 + 3x + 7 < 2$

EXAMPLE 4 *Problem-Solving Application*

PROBLEM SOLVING

A business offers tours to the Amazon. The profit P that the company earns for x number of tourists can be modeled by $P(x) = -25x^2 + 1000x - 3000$. How many people are needed for a profit of at least \$5000?

1 Understand the Problem

The **answer** will be the number of people required for a profit that is greater than or equal to \$5000.

List the important information:

- The profit must be at least \$5000.
- The function for the profit is $P(x) = -25x^2 + 1000x - 3000$.

2 Make a Plan

Write an inequality showing profit greater than or equal to \$5000. Then solve the inequality by using algebra.

3 Solve

Write the inequality.

$$-25x^2 + 1000x - 3000 \geq 5000$$

Find the critical values by solving the related equation.

$$-25x^2 + 1000x - 3000 = 5000 \qquad \textit{Write as an equation.}$$

$$-25x^2 + 1000x - 8000 = 0 \qquad \textit{Write in standard form.}$$

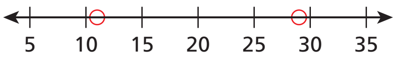
$$-25\left(x^2 - 40x + 320\right) = 0 \qquad \textit{Factor out } -25 \textit{ to simplify.}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(1)(320)}}{2(1)} \qquad \textit{Use the Quadratic Formula.}$$

$$= \frac{40 \pm \sqrt{320}}{2} \qquad \textit{Simplify.}$$

$$x \approx 28.94 \text{ or } x \approx 11.06$$

Test an x-value in each of the three regions formed by the critical x-values.



$$-25(10)^2 + 1000(10) - 3000 \overset{?}{\geq} 5000 \qquad \textit{Try } x = 10.$$

$$4500 \geq 5000 \text{ ✘}$$

$$-25(20)^2 + 1000(20) - 3000 \overset{?}{\geq} 5000 \qquad \textit{Try } x = 20.$$

$$7000 \geq 5000 \text{ ✓}$$

$$-25(30)^2 + 1000(30) - 3000 \overset{?}{\geq} 5000 \qquad \textit{Try } x = 30.$$

$$4500 \geq 5000 \text{ ✘}$$

Remember!

A compound inequality such as $12 \leq x \leq 28$ can be written as $\{x \mid x \geq 12 \cup x \leq 28\}$, or $x \geq 12$ **and** $x \leq 28$. (See Lesson 2-8.)

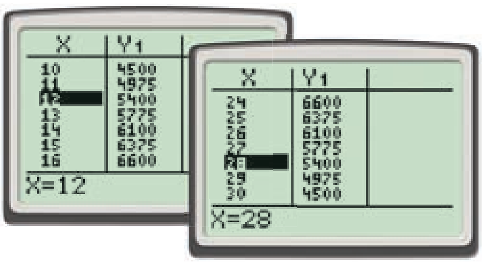
Write the solution as an inequality. The solution is approximately $11.06 \leq x \leq 28.94$. Because you cannot have a fraction of a person, round each critical value to the appropriate whole number.

$$12 \leq x \leq 28$$

For a profit of at least \$5000, from 12 to 28 people are needed.

4 Look Back

Enter $y = -25x^2 + 1000x - 3000$ into a graphing calculator, and create a table of values. The table shows that integer values of x between 12 and 28 inclusive result in y-values greater than or equal to 5000.



CHECK IT OUT!

4. The business also offers educational tours to Patagonia, a region of South America that includes parts of Chile and Argentina. The profit P for x number of persons is $P(x) = -25x^2 + 1250x - 5000$. The trip will be rescheduled if the profit is less than \$7500. How many people must have signed up if the trip is rescheduled?

THINK AND DISCUSS

1. Compare graphing a quadratic inequality with graphing a linear inequality.

2. Explain how to determine if the intersection point(s) is/are included in the solution set when you solve a quadratic inequality by graphing.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. Compare the solutions of quadratic equations and inequalities.

	Equation (=)	"Less Than" Inequality (< or ≤)	"Greater Than" Inequality (> or ≥)
Example			
Graph			
Solution Set			

5-7 Exercises

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GUIDED PRACTICE

1. Vocabulary Give an example of a *quadratic inequality in two variables*.

SEE EXAMPLE 1 p. 366

Graph each inequality.

2. $y > -(x+1)^2 + 5$ **3.** $y \leq 2x^2 - 4x - 1$ **4.** $y \leq -3x^2 + x + 3$

SEE EXAMPLE 2 p. 367

Solve each inequality by using tables or graphs.

5. $x^2 - 5x + 3 \leq 3$ **6.** $3x^2 - 3x - 1 > -1$ **7.** $2x^2 - 9x + 5 \leq -4$

SEE EXAMPLE 3 p. 368

Solve each inequality by using algebra.

8. $x^2 + 10x + 1 \geq 12$ **9.** $x^2 + 13x + 45 < 5$ **10.** $-2x^2 + 3x + 12 > 10$

SEE EXAMPLE 4 p. 368

11. Business A consultant advises the owners of a beauty salon that their profit p each month can be modeled by $p(x) = -50x^2 + 3500x - 2500$, where x is the average cost that a customer is charged. What range of costs will bring in a profit of at least $50,000?

PRACTICE AND PROBLEM SOLVING

Graph each inequality.

12. $y < x^2 + 2x - 5$ **13.** $y > -\frac{1}{2}x^2 + 3$ **14.** $y \leq 2(x-1)^2 - 3$

15. $y \geq x^2 + 6$ **16.** $y < (x+1)(x+4)$ **17.** $y \leq x^2 - 2x + 6$

Solve each inequality by using tables or graphs.

18. $x^2 - x + 5 < 11$ **19.** $2x^2 + 3x + 6 \geq 5$ **20.** $x^2 - 5x + 12 > 6$

21. $x^2 - 2x - 8 > 0$ **22.** $x^2 + 7x + 6 \leq 6$ **23.** $x^2 - 12x + 32 < 12$

Independent Practice	
For Exercises	See Example
12–17	1
18–23	2
24–26	3
27	4

Extra Practice

Skills Practice p. S13
Application Practice p. S36

Solve each inequality by using algebra.

24. $x^2 - 11x + 13 \leq 25$

25. $-2x^2 + 3x + 4 \geq -1$

26. $x^2 - 5x - 4 < -9$

27. Sports A football thrown by a quarterback follows a path given by $h(x) = -0.0095x^2 + x + 7$, where h is the height of the ball in feet and x is the horizontal distance the ball has traveled in feet. If any height less than 10 feet can be caught or knocked down, at what distances from the quarterback can the ball be knocked down?

Graph each quadratic inequality.

28. $y \leq 2x^2 + 4x - 3$

29. $y < 3x^2 - 12x - 4$

30. $y \geq -3x^2 + 4x$

31. $y > -2(x + 3)^2 + 1$

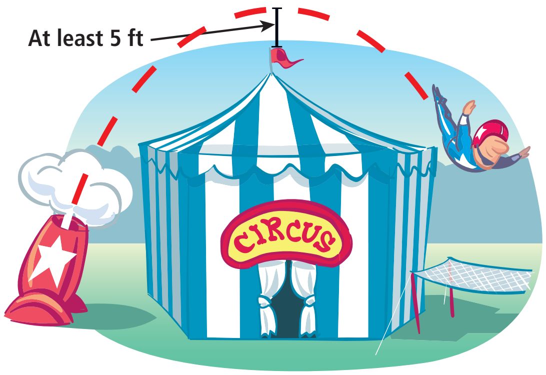
32. $y > -x^2 - 2x - 1$

33. $y \leq \frac{1}{3}x^2 + 2x - 1$

34. Circus The human cannonball is an act where a performer is launched through the air. The height of the performer can be modeled by $h(x) = -0.007x^2 + x + 20$, where h is the height in feet and x is the horizontal distance traveled in feet. The circus act is considering a flight path directly over the main tent.

a. If the performer wants at least 5 ft of vertical height clearance, how tall can the tent be?

b. How far from the central pole should the "cannon" be placed?



Solve each inequality by using any method.

35. $x^2 - 5x - 24 \leq 0$

36. $x^2 - 14 \geq 2$

37. $-2x^2 - x + 8 > 6$

38. $x^2 - 4x - 5 \leq -9$

39. $3x^2 + 6x + 11 < 10$

40. $4x^2 - 9 > 0$

41. $3x^2 + 5x + 13 \leq 16$

42. $-2x^2 + 3x + 17 \geq 11$

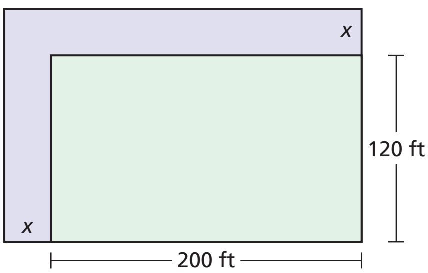
43. $5x^2 - 2x - 1 \geq 0$

44. $(x - 2)(x + 11) \geq 2$

45. $x^2 + 27 > 12x$

46. $-2x^2 + 3x + 6 > 0$

47. Multi-Step A medical office has a rectangular parking lot that measures 120 ft by 200 ft. The owner wants to expand the size of the parking lot by adding an equal distance to two sides as shown. If zoning restrictions limit the total size of the parking lot to 35,000 ft^2, what range of distances can be added?



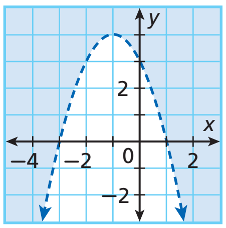
Match each graph with one of the following inequalities.

A. $y < x^2 + 2x - 3$

B. $y > -x^2 - 2x + 3$

C. $y < x^2 - 2x + 3$

48.



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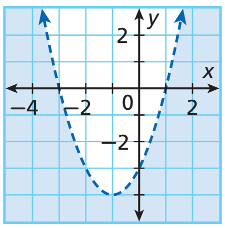

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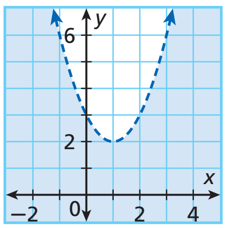
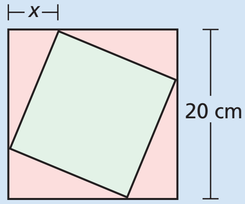


MULTI-STEP TEST PREP

51. This problem will prepare you for the Multi-Step Test Prep on page 390.
A small square tile is placed on top of a larger square tile as shown. This creates four congruent triangular regions.



a. Write a function for the area A of one of the triangular regions in terms of x.

b. For what values of x, to the nearest tenth, is the area of each triangular region at least 30 cm^2?

c. For what values of x, to the nearest tenth, is the area of each triangular region less than 40 cm^2?

52. Music A manager estimates a band's profit p for a concert by using the function $p(t) = -200t^2 + 2500t - c$, where t is the price per ticket and c is the band's operating cost. The table shows the band's operating cost at three different concert locations. What range of ticket prices should the band charge at each location in order to make a profit of at least \$1000 at each concert?

Band's Costs

Location	Operating Cost
Freemont Park	\$900
Saltillo Plaza	\$1500
Riverside Walk	\$2500

53. Gardening Lindsey has 40 feet of metal fencing material to fence three sides of a rectangular garden. A tall wooden fence serves as her fourth side.

a. Write a function for the area of the garden A in terms of x, the width in feet.

b. What measures for the width will give an area of at least 150 square feet?

c. What measures for the width will give an area of at least 200 square feet?

Graphing Calculator Use the intersect feature of a graphing calculator to solve each inequality to the nearest tenth.

54. $x^2 + 6x - 13 > 4$

55. $x^2 - 15x + 20 \leq 7$

56. $x^2 - 24 < 28$

57. $2x^2 + 3x + 5 \geq 8$

58. Business A wholesaler sells snowboards to sporting-good stores. The price per snowboard varies based on the number purchased in each order. The function $r(x) = -x^2 + 125x$ models the wholesaler's revenue r in dollars for an order of x snowboards.

a. To the nearest dollar, what is the maximum revenue per order?

b. How many snowboards must the wholesaler sell to make at least \$1500 in revenue in one order?

59. Critical Thinking Explain whether the solution to a quadratic inequality in one variable is always a compound inequality.

60. Critical Thinking Can a quadratic inequality have a solution set that is all real numbers? Give an example to support your answer.

61. Write About It Explain how the solutions of $x^2 - 3x - 4 \leq 6$ differ from the solutions of $x^2 - 3x - 4 = 6$.

TEST PREP

62. Which is the solution set of $x^2 - 9 < 0$?

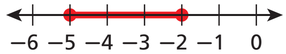
Ⓐ $-3 < x < 3$

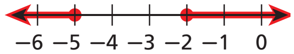
Ⓑ $-9 < x < 9$

Ⓒ $x < -3$ or $x > 3$

Ⓓ $x < -9$ or $x > 9$

63. Which is the graph of the solution to $x^2 - 7x + 10 \geq 0$?

Ⓕ 

Ⓗ 

Ⓖ 

Ⓙ 

64. Which is the solution set of $x^2 - 7x \leq 0$?

Ⓐ $0 < x < 7$

Ⓑ $0 \leq x \leq 7$

Ⓒ $x < 0$ or $x > 7$

Ⓓ $x \leq 0$ or $x \geq 7$

65. **Short Response** Demonstrate the process for solving $x^2 + 4x + 4 > 1$ algebraically. Justify each step in the solution process.

CHALLENGE AND EXTEND

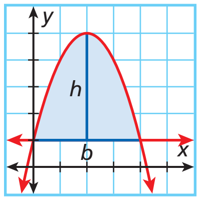
Graph each system of inequalities.

66. $\begin{cases} y \leq x^2 \\ y \geq -x^2 + 5 \end{cases}$

67. $\begin{cases} y \geq x^2 - 3 \\ y \leq -x^2 - 2x + 9 \end{cases}$

68. $\begin{cases} y \geq 2x^2 - 12x + 20 \\ y \geq \frac{1}{3}x^2 - 2x + 8 \end{cases}$

Geometry The area inside a parabola bounded from above or below by a horizontal line segment is $\frac{2}{3}bh$, where b is the length of the line segment and h is the vertical distance from the vertex of the parabola to the line segment. Find the area bounded by the graphs of each pair of inequalities.



69. $y > x^2 + 5x - 6;\ y < 8$

70. $y < -2x^2 + 3x + 9;\ y > -5$

SPIRAL REVIEW

71. **Community** Once a month, four teams of teens (lawn team, shopping team, cleaning team, and laundry team) spend a day assisting elderly residents of their neighborhood. Lynnette started the assignment chart for June but was interrupted. Complete the chart. Each home has only one team helping during each shift. *(Previous course)*

Shifts	Reed Home	Brown Home	Sondi Home	Clem Home
7:00 A.M.–9:30 A.M.	Lawn	Cleaning	?	?
10:00 A.M.–12:30 P.M.	?	Shopping	?	Lawn
1:00 P.M.–3:30 P.M.	?	?	Laundry	?
4:00 P.M.–6:30 P.M.	Cleaning	?	?	?

Graph each inequality by using intercepts. *(Lesson 2-5)*

72. $4x - 3y > 15$

73. $6x - y \leq 8$

74. $8x + 5y < 40$

Find the values of c that make each equation true. *(Lesson 5-5)*

75. $4 - 2c + 7i = 7i - 14$

76. $4c + 2 - 3i + 2(i - 5) = 4(2i - 6) - 9i$

5-8 Curve Fitting with Quadratic Models

The Granger Collection, New York

Objectives
Use quadratic functions to model data.

Use quadratic models to analyze and predict.

Vocabulary
quadratic model
quadratic regression

Who uses this?
Film preservationists use quadratic relationships to estimate film run times. (See Example 3.)

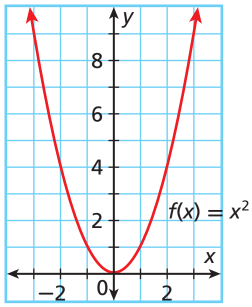
Recall that you can use differences to analyze patterns in data. For a set of ordered pairs with equally spaced x-values, a quadratic function has constant nonzero **second** differences, as shown below.

Equally spaced x-values

x	−3	−2	−1	0	1	2	3
$f(x) = x^2$	9	4	1	0	1	4	9

1st differences −5 −3 −1 1 3 5
2nd differences 2 2 2 2 2

Constant 2nd differences



EXAMPLE 1 Identifying Quadratic Data

Determine whether each data set could represent a quadratic function. Explain.

A

x	0	2	4	6	8
y	12	10	9	9	10

Find the first and second differences.

Equally spaced x-values

x	0	2	4	6	8
y	12	10	9	9	10

1st −2 −1 0 1
2nd 1 1 1

Quadratic function; **second** differences are constant for equally spaced x-values.

B

x	−2	−1	0	1	2
y	1	2	4	8	16

Find the first and second differences.

Equally spaced x-values

x	−2	−1	0	1	2
y	1	2	4	8	16

1st 1 2 4 8
2nd 1 2 4

Not a quadratic function; **second** differences are not constant for equally spaced x-values.

CHECK IT OUT! **Determine whether each data set could represent a quadratic function. Explain.**

1a.

x	3	4	5	6	7
y	11	21	35	53	75

1b.

x	10	9	8	7	6
y	6	8	10	12	14

Just as two points define a linear function, three noncollinear points define a quadratic function. You can find the three coefficients, a, b, and c, of $f(x) = ax^2 + bx + c$ by using a system of three equations, one for each point. The points do not need to have equally spaced x-values.

Reading Math

Collinear points lie on the same line. *Noncollinear* points *do not* all lie on the same line.

EXAMPLE 2 Writing a Quadratic Function from Data

Write a quadratic function that fits the points (0, 5), (2, 1), and (3, 2).

Use each point to write a system of equations to find a, b, and c in $f(x) = ax^2 + bx + c$.

(x, y)	$f(x) = ax^2 + bx + c$	System in a, b, c
$(0, 5)$	$5 = a(0)^2 + b(0) + c$	$c = 5$ ❶
$(2, 1)$	$1 = a(2)^2 + b(2) + c$	$4a + 2b + c = 1$ ❷
$(3, 2)$	$2 = a(3)^2 + b(3) + c$	$9a + 3b + c = 2$ ❸

Substitute $c = 5$ from equation ❶ into both equation ❷ and equation ❸.

❷ $4a + 2b + c = 1$ ❸ $9a + 3b + c = 2$

$4a + 2b + 5 = 1$ $9a + 3b + 5 = 2$

$4a + 2b = -4$ ❹ $9a + 3b = -3$ ❺

Solve equation ❹ and equation ❺ for a and b using elimination.

❹ $3(4a + 2b) = 3(-4) \rightarrow 12a + 6b = -12$ *Multiply by 3.*

❺ $-2(9a + 3b) = -2(-3) \rightarrow -18a - 6b = 6$ *Multiply by −2.*

$-6a = -6$ *Add the equations.*

$a = 1$

Substitute 1 for a into equation ❹ or equation ❺ to find b.

❹ $4a + 2b = -4 \rightarrow 4(1) + 2b = -4$

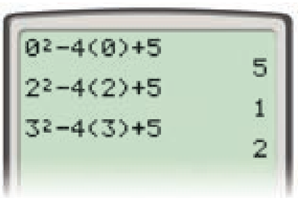
$2b = -8$

$b = -4$

Write the function using $a = 1$, $b = -4$, and $c = 5$.

$$f(x) = ax^2 + bx + c \rightarrow f(x) = 1x^2 - 4x + 5, \text{ or } f(x) = x^2 - 4x + 5$$

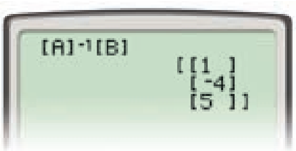
Check Substitute or create a table to verify that (0, 5), (2, 1), and (3, 2) satisfy the function rule.



CHECK IT OUT! **2.** Write a quadratic function that fits the points (0, −3), (1, 0), and (2, 1).

You may use any method that you studied in Chapters 3 or 4 to solve the system of three equations in three variables. For example, you can use a matrix equation as shown.

$$\begin{cases} c = 5 \\ 4a + 2b + c = 1 \\ 9a + 3b + c = 2 \end{cases} \rightarrow \begin{bmatrix} 0 & 0 & 1 \\ 4 & 2 & 1 \\ 9 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix} \rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -4 \\ 5 \end{bmatrix}$$



A **quadratic model** is a quadratic function that represents a real data set. Models are useful for making estimates.

In Chapter 2, you used a graphing calculator to perform a *linear regression* and make predictions. You can apply a similar statistical method to make a quadratic model for a given data set using **quadratic regression.**

EXAMPLE 3 *Film Application*

The table shows approximate run times for 16 mm films, given the diameter of the film on the reel. Find a quadratic model for the run time given the diameter. Use the model to estimate the run time for a reel of film with a diameter of 15 in.

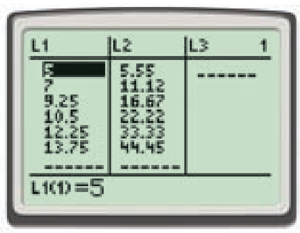
Film Run Times (16 mm)

Diameter (in.)	Reel Length (ft)	Run Time (min)
5	200	5.55
7	400	11.12
9.25	600	16.67
10.5	800	22.22
12.25	1200	33.33
13.75	1600	44.45

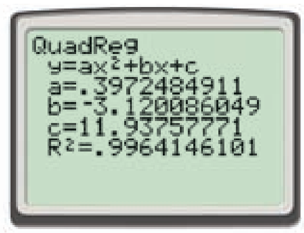
Helpful Hint

The coefficient of determination R^2 shows how well a quadratic model fits the data. The closer R^2 is to 1, the better the fit. In this model, $R^2 \approx 0.996$, which is very close to 1, so the quadratic model is a good fit.

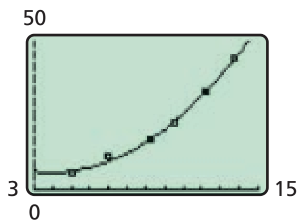
Step 1 Enter the data into two lists in a graphing calculator.



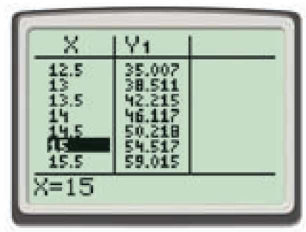
Step 2 Use the quadratic regression feature.



Step 3 Graph the data and function model to verify that the model fits the data.



Step 4 Use the table feature to find the function value at $x = 15$.



A quadratic model is $T(d) \approx 0.397d^2 - 3.12d + 11.94$, where T is the run time in minutes and d is the film diameter in inches.

For a 15 in. diameter, the model predicts a run time of about 54.5 min, or 54 min 30 s.

CHECK IT OUT! **Use the information given above to answer the following.**

3. Find a quadratic model for the reel length given the diameter of the film. Use the model to estimate the reel length for an 8-inch-diameter film.

THINK AND DISCUSS

1. Describe how to determine if a data set is quadratic.
2. Explain whether a quadratic function is a good model for the path of an airplane that ascends, descends, and rises again out of view.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Compare the different quadratic models presented in the lesson.

Know it! Note

Quadratic Model	When Appropriate	Procedure
Exact model		
Approximate model		

5-8 Exercises

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Homework Help Online
KEYWORD: MB7 5-8
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** How does a *quadratic model* differ from a linear model?

SEE EXAMPLE 1 p. 374

Determine whether each data set could represent a quadratic function. Explain.

2.

x	−2	−1	0	1	2
y	16	8	0	−8	−16

3.

x	1	2	3	4	5
y	1	3	9	27	81

4.

x	2	4	6	8	10
y	4	−5	−8	−5	4

SEE EXAMPLE 2 p. 375

Write a quadratic function that fits each set of points.

5. $(-2, 5)$, $(0, -3)$, and $(3, 0)$
6. $(0, 1)$, $(2, -1)$, and $(3, -8)$
7. $(-1, 8)$, $(0, 4)$, and $(2, 2)$
8. $(-4, 9)$, $(0, -7)$, and $(1, -1)$
9. $(2, 3)$, $(6, 3)$, and $(8, -3)$
10. $(-1, -12)$, $(1, 0)$, and $(2, 9)$

SEE EXAMPLE 3 p. 376

11. **Hobbies** The cost of mounting different-sized photos is shown in the table. Find a quadratic model for the cost given the average side length. (For an 8 in. × 10 in. photo, the average side length is $\frac{8+10}{2} = 9$ in.) Estimate the cost of mounting a 24 in. × 36 in. photo.

Costs of Mounting Photos

Size (in.)	Cost ($)
8 × 10	10
14 × 18	16
16 × 20	19
24 × 30	27
32 × 40	39

PRACTICE AND PROBLEM SOLVING

Determine whether each data set could represent a quadratic function. Explain.

12.

x	0	2	4	6	8
$f(x)$	−1	2	11	26	47

13.

x	0	1	2	3	4
$f(x)$	10	9	6	1	−6

14.

x	1	2	3	4	5
$f(x)$	−3	0	3	6	9

Independent Practice	
For Exercises	See Example
12–14	1
15–18	2
19	3

Extra Practice

Skills Practice p. S13

Application Practice p. S36

Write a quadratic function that fits each set of points.

15. $(-2, 5)$, $(-1, 0)$, and $(1, -2)$

16. $(1, 2)$, $(2, -1)$, and $(5, 2)$

17. $(-4, 12)$, $(-2, 0)$, and $(2, -12)$

18. $(-1, 2.6)$, $(1, 4.2)$, and $(2, 14)$

19. **Gardening** The table shows the amount spent on water gardening in the United States between 1999 and 2003. Find a quadratic model for the annual amount in millions of dollars spent on water gardening based on number of years since 1999. Estimate the amount that people in the United States will spend on water gardening in 2015.

Water Gardening	
Year	Amount Spent (million $)
1999	806
2000	943
2001	1205
2002	1441
2003	1565

Write a function rule for each situation, and identify each relationship as linear, quadratic, or neither.

20. the circumference C of a bicycle wheel, given its radius r

21. the area of a triangle A with a constant height, given its base length b

22. the population of bacteria P in a petri dish doubling every hour t

23. the area of carpet A needed for square rooms of length s

24. **Physics** In the past, different mathematical descriptions of falling objects were proposed.

a. Which rule shows the greatest increase in the distance fallen per second and thus the greatest rate of increase in speed?

b. Identify each rule as linear, quadratic, or neither.

c. Describe the differences in da Vinci's rule, and compare it with the differences in Galileo's.

d. The most accurate rule is sometimes described as the odd-number law. Which rule shows an odd-number pattern of first differences and correctly describes the distance for falling objects?

Relative Distance Fallen (units)			
Time Interval (s)	Aristotle's Rule	da Vinci's Rule	Galileo's Rule
0	0	0	0
1	1	1	1
2	2	3	4
3	3	6	9
4	4	10	16

Find the missing value for each quadratic function.

25.

x	-1	0	1	2	3
$f(x)$	0	1	0	■	-8

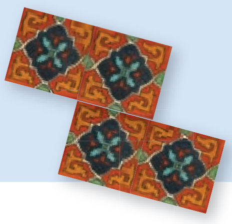
26.

x	-3	-2	-1	0	1
$f(x)$	12	2	■	0	8

27.

x	-2	0	2	4	6
$f(x)$	-2	■	2	7	14

MULTI-STEP TEST PREP



28. This problem will prepare you for the Multi-Step Test Prep on page 390. A home-improvement store sells several sizes of rectangular tiles, as shown in the table.

a. Find a quadratic model for the area of a tile based on its length.

b. The store begins selling a new size of tile with a length of 9 in. Based on your model, estimate the area of a tile of this size.

Length (in.)	Area (in^2)
4	28
6	54
8	88
10	130

29. **Food** The pizza prices for DeAngelo's pizza parlor are shown at right.

a. Find a quadratic model for the price of a pizza based upon the size (diameter).

b. Use the quadratic model to find the price of a pizza with an 18 in. diameter.

c. Graph the quadratic function. Does the function have a minimum or maximum point? What does this point represent?

d. **What if...?** According to the model, how much should a 30 in. pizza cost? How much should an 8 in. pizza cost?

e. Is the quadratic function a good model for the price of DeAngelo's pizza? Explain your reasoning.

Determine whether each data set could represent a quadratic function. If so, find a quadratic function rule.

30.

x	0	1	2	3	4
y	−1	0	−1	−4	−9

31.

x	1	2	3	4	5
y	10	20	40	60	80

32.

x	2	4	6	8	10
y	−1	0	1	3	5

33.

x	−2	−1	0	1	2
y	16	3	0	7	24

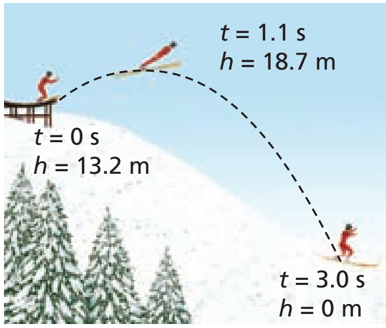
34.

x	0	1	2	3	4
y	9	5	3	1	0

35.

x	−2	−1	0	1	2
y	0	3	9	27	81

36. **Winter Sports** The diagram shows the motion of a skier following a jump. Find a quadratic model of the skier's height h in meters based on time t in seconds. Estimate the skier's height after 2 s.



37. **Data Collection** Use a graphing calculator and a motion detector to measure the height of a basketball over time. Drop the ball from a height of 1 m, and let it bounce several times. Position the motion detector 0.5 m above the release point of the ball.

a. What is the greatest height the ball reaches during its first bounce?

b. Find an appropriate model for the height of the ball as a function of time during its first bounce.

38. **Safety** The light produced by high-pressure sodium vapor streetlamps for different energy usages is shown in the table.

High-Pressure Sodium Vapor Streetlamps					
Energy Use (watts)	35	50	70	100	150
Light Output (lumens)	2250	4000	5800	9500	16,000

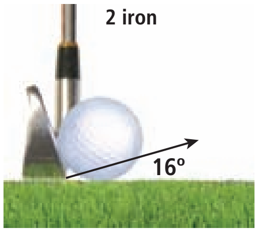
a. Find a quadratic model for the light output with respect to energy use.

b. Find a linear model for the light output with respect to energy use.

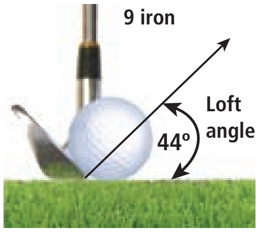
c. Apply each model to estimate the light output in lumens of a 200-watt bulb.

d. Which model gives the better estimate? Explain.

39. **Sports** The table lists the average distance that a normal shot travels for different golf clubs.



Average Distance for Normal Shot								
Club Iron (no.)	2	3	4	5	6	7	8	9
Loft Angle	16°	20°	24°	28°	32°	36°	40°	44°
Distance (yd)	186	176	166	155	143	132	122	112



a. Select three data values (club number, distance), and use a system of equations to find a quadratic model. Check your model by using a quadratic regression.

b. Is there a quadratic relationship between club number and average distance of a normal shot? Explain.

c. Is the relationship between club number and loft angle quadratic or linear? Find a model of this relationship.

40. **Multi-Step** Use the table of alloy-steel chain data.

a. Do each of the last two columns appear to be quadratic functions with respect to the nominal chain size? Explain.

b. Verify your response in part **a** by finding each of the quadratic regression equations. Do the models fit the data well? Explain.

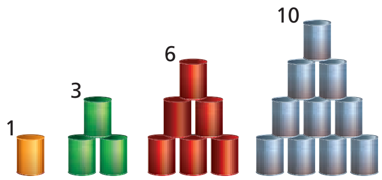
c. Predict the values for the last two columns for a chain with a nominal size of $\frac{5}{8}$ in.

Alloy-Steel Chain Specifications		
Nominal Size (in.)	Maximum Length 100 Links (in.)	Maximum Weight 100 Links (lb)
$\frac{1}{4}$	98	84
$\frac{1}{2}$	156	288
$\frac{3}{4}$	208	655
1	277	1170
$1\frac{1}{4}$	371	1765

Math History

Pythagoras made numerous contributions to mathematics, including the Pythagorean Theorem, which bears his name.

41. **Math History** The Greek mathematician Pythagoras developed a formula for triangular numbers, the first four of which are shown. Write a quadratic function that determines a triangular number t in terms of its place in the sequence n. (*Hint:* The fourth triangular number has $n = 4$.)



42. **Critical Thinking** Two points define a unique line. How many points define a unique parabola, and what restriction applies to the points?

43. **Critical Thinking** Consider the following data set.

x	10	8	13	9	11	14	6	4	12	7	5
y	9.14	8.14	8.74	8.77	9.29	8.1	6.13	3.1	9.13	7.26	4.74

a. Create a scatter plot of the data.

b. Perform a linear regression on the data.

c. Perform a quadratic regression on the data.

d. Which model best describes the data set? Explain your answer.

44. **Write About It** What does it mean when the coefficient a in a quadratic regression model is zero?

TEST PREP

45. Which of the following would best be modeled by a quadratic function?

Ⓐ Relationship between circumference and diameter

Ⓑ Relationship between area of a square and side length

Ⓒ Relationship between diagonal of a square and side length

Ⓓ Relationship between volume of a cube and side length

46. If $(7, 11)$ and $(3, 11)$ are two points on a parabola, what is the x-value of the vertex of this parabola?

Ⓕ 3 Ⓖ 5 Ⓗ 7 Ⓙ 11

47. If y is a quadratic function of x, which value completes the table?

x	−2	0	2	4	6
y	−8	0	12	28	■

Ⓐ 12 Ⓑ 20 Ⓒ 44 Ⓓ 48

48. The graph of a quadratic function having the form $f(x) = ax^2 + bx + c$ passes through the points $(0, -8)$, $(3, 10)$, and $(6, 34)$. What is the value of the function when $x = -3$?

Ⓕ −32 Ⓖ −26 Ⓗ −20 Ⓙ 10

49. **Extended Response** Write a quadratic function in standard form that fits the data points $(0, -5)$, $(1, -3)$, and $(2, 3)$. Use a system of equations, and show all of your work.

CHALLENGE AND EXTEND

50. Three points defining a quadratic function are $(1, 2)$, $(4, 6)$, and $(7, w)$.

a. If $w = 9$, what is the quadratic function? Does it have a maximum value or a minimum value? What is the vertex?

b. If $w = 11$, what is the quadratic function? Does it have a maximum value or a minimum value? What is the vertex?

c. If $w = 10$, what function best fits the points?

51. Explain how you can determine from three points whether the parabola that fits the points opens upward or downward.

SPIRAL REVIEW

Determine whether each data set could represent a linear function. *(Lesson 2-3)*

52.

x	−2	1	4
$f(x)$	−5	7	1

53.

x	−8	−6	0
$f(x)$	−1	0	3

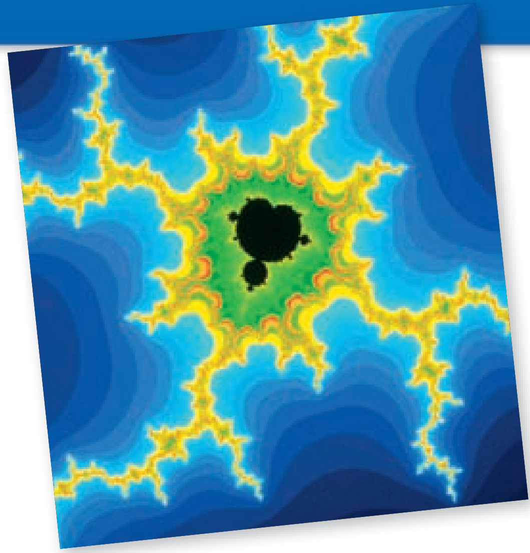
Find the inverse of the matrix, if it is defined. *(Lesson 4-5)*

54. $\begin{bmatrix} \frac{1}{3} & 0 \\ -4 & 1 \end{bmatrix}$ 55. $\begin{bmatrix} 2 & -2 \\ 1 & -1 \end{bmatrix}$ 56. $\begin{bmatrix} -2 & 0 & 1 \\ 0 & 0 & 1 \\ 4 & 2 & 2 \end{bmatrix}$ 57. $\begin{bmatrix} 3 & -4 \\ 0 & -\frac{1}{2} \end{bmatrix}$

Find the zeros of each function by using the Quadratic Formula. *(Lesson 5-6)*

58. $f(x) = 2x^2 - 4x + 1$ 59. $f(x) = x^2 + 9$ 60. $f(x) = -3x^2 + 10x + 12$

5-9 Operations with Complex Numbers



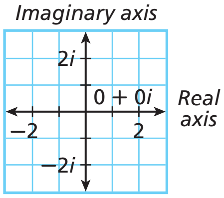
Objective
Perform operations with complex numbers.

Vocabulary
complex plane
absolute value of a complex number

Why learn this?
Complex numbers can be used in formulas to create patterns called fractals. (See Exercise 84.)

Just as you can represent real numbers graphically as points on a number line, you can represent complex numbers in a special coordinate plane.

The **complex plane** is a set of coordinate axes in which the horizontal axis represents real numbers and the vertical axis represents imaginary numbers.



EXAMPLE 1 **Graphing Complex Numbers**

Helpful Hint
The real axis corresponds to the x-axis, and the imaginary axis corresponds to the y-axis. Think of $a + bi$ as $x + yi$.

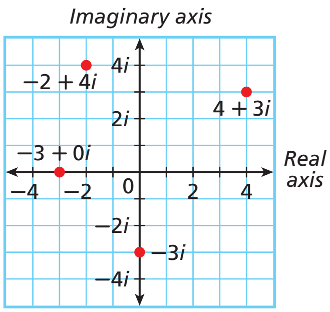
Graph each complex number.

A $-3 + 0i$

B $-3i$

C $4 + 3i$

D $-2 + 4i$

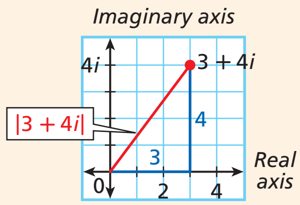


CHECK IT OUT! Graph each complex number.

1a. $3 + 0i$ **1b.** $2i$ **1c.** $-2 - i$ **1d.** $3 + 2i$

Recall that the absolute value of a real number is its distance from 0 on the real axis, which is also a number line. Similarly, the absolute value of an imaginary number is its distance from 0 along the imaginary axis.

Know it! Note **Absolute Value of a Complex Number**

WORDS	ALGEBRA	EXAMPLE
The **absolute value of a complex number** $a + bi$ is the distance from the origin to the point (a, b) in the complex plane, and is denoted $\lvert a + bi \rvert$.	$\lvert a + bi \rvert = \sqrt{a^2 + b^2}$	 $\lvert 3 + 4i \rvert = \sqrt{3^2 + 4^2}$ $= \sqrt{9 + 16}$ $= 5$

EXAMPLE 2 Determining the Absolute Value of Complex Numbers

Find each absolute value.

A $|-9+i|$

$|-9+1i|$

$\sqrt{(-9)^2+1^2}$

$\sqrt{81+1}$

$\sqrt{82}$

B $|6|$

$|6+0i|$

$\sqrt{6^2+0^2}$

$\sqrt{36}$

6

C $|-4i|$

$|0+(-4)i|$

$\sqrt{0^2+(-4)^2}$

$\sqrt{16}$

4

CHECK IT OUT! Find each absolute value.

2a. $|1-2i|$ **2b.** $\left|-\frac{1}{2}\right|$ **2c.** $|23i|$

Adding and subtracting complex numbers is similar to adding and subtracting variable expressions with like terms. Simply combine the real parts, and combine the imaginary parts.

The set of complex numbers has all the properties of the set of real numbers. So you can use the Commutative, Associative, and Distributive Properties to simplify complex number expressions.

EXAMPLE 3 Adding and Subtracting Complex Numbers

Add or subtract. Write the result in the form $a + bi$.

A $(-2+4i)+(3-11i)$

$(-2+3)+(4i-11i)$	*Associative and Commutative Properties*
$1-7i$	*Add real parts and imaginary parts.*

B $(4-i)-(5+8i)$

$(4-i)-5-8i$	*Distributive Property*
$(4-5)+(-i-8i)$	*Associative and Commutative Properties*
$-1-9i$	*Add real parts and imaginary parts.*

C $(6-2i)+(-6+2i)$

$(6-6)+(-2i+2i)$	*Associative and Commutative Properties*
$0+0i$	*Add real parts and imaginary parts.*
0	

D $(10+3i)-(10-4i)$

$(10+3i)-10-(-4i)$	*Distributive Property*
$(10-10)+(3i+4i)$	*Associative and Commutative Properties*
$0+7i$	*Add real parts and imaginary parts.*
$7i$	

Helpful Hint

Complex numbers also have additive inverses. The additive inverse of $a + bi$ is $-(a + bi)$, or $-a - bi$.

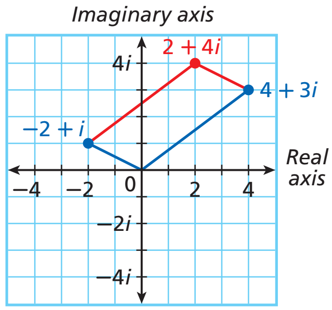
CHECK IT OUT! Add or subtract. Write the result in the form $a + bi$.

3a. $(-3+5i)+(-6i)$ **3b.** $2i-(3+5i)$ **3c.** $(4+3i)+(4-3i)$

You can also add complex numbers by using coordinate geometry.

EXAMPLE 4 Adding Complex Numbers on the Complex Plane

Find $(4 + 3i) + (-2 + i)$ by graphing on the complex plane.



Step 1 Graph $4 + 3i$ and $-2 + i$ on the complex plane. Connect each of these numbers to the origin with a line segment.

Step 2 Draw a parallelogram that has these two line segments as sides. The vertex that is opposite the origin represents the sum of the two complex numbers, $2 + 4i$. Therefore, $(4 + 3i) + (-2 + i) = 2 + 4i$.

Check Add by combining the real parts and combining the imaginary parts.

$$(4 + 3i) + (-2 + i) = [4 + (-2)] + (3i + i) = 2 + 4i$$

CHECK IT OUT! **Find each sum by graphing on the complex plane.**

4a. $(3 + 4i) + (1 - 3i)$ **4b.** $(-4 - i) + (2 - 2i)$

You can multiply complex numbers by using the Distributive Property and treating the imaginary parts as like terms. Simplify by using the fact $i^2 = -1$.

EXAMPLE 5 Multiplying Complex Numbers

Multiply. Write the result in the form $a + bi$.

A $2i(3 - 5i)$

$6i - 10i^2$ *Distribute.*

$6i - 10(-1)$ *Use $i^2 = -1$.*

$10 + 6i$ *Write in a + bi form.*

B $(5 - 6i)(4 - 3i)$

$20 - 15i - 24i + 18i^2$ *Multiply.*

$20 - 39i + 18(-1)$ *Use $i^2 = -1$.*

$2 - 39i$

C $(7 + 2i)(7 - 2i)$

$49 - 14i + 14i - 4i^2$ *Multiply.*

$49 - 4(-1)$ *Use $i^2 = -1$.*

53

D $(6i)(6i)$

$36i^2$

$36(-1)$ *Use $i^2 = -1$.*

-36

CHECK IT OUT! **Multiply. Write the result in the form $a + bi$.**

5a. $2i(3 - 5i)$ **5b.** $(4 - 4i)(6 - i)$ **5c.** $(3 + 2i)(3 - 2i)$

Helpful Hint

Notice the repeating pattern in each row of the table. The pattern allows you to express any power of i as one of four possible values: i, -1, $-i$, or 1.

The imaginary unit i can be raised to higher powers as shown below.

Powers of i		
$i^1 = i$	$i^5 = i^4 \cdot i = 1 \cdot i = i$	$i^9 = i$
$i^2 = -1$	$i^6 = i^4 \cdot i^2 = 1 \cdot (-1) = -1$	$i^{10} = -1$
$i^3 = i^2 \cdot i = -1 \cdot i = -i$	$i^7 = i^4 \cdot i^3 = 1 \cdot (-i) = -i$	$i^{11} = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot (-1) = 1$	$i^8 = i^4 \cdot i^4 = 1 \cdot 1 = 1$	$i^{12} = 1$

EXAMPLE 6 Evaluating Powers of i

A **Simplify $-3i^{12}$.**

$-3i^{12} = -3\left(i^2\right)^6$ *Rewrite i^{12} as a power of i^2.*

$= -3(-1)^6 = -3(1) = -3$ *Simplify.*

B **Simplify i^{25}.**

$i^{25} = i \cdot i^{24}$ *Rewrite as a product of i and an even power of i.*

$= i \cdot \left(i^2\right)^{12}$ *Rewrite i^{24} as a power of i^2.*

$= i \cdot (-1)^{12} = i \cdot 1 = i$ *Simplify.*

CHECK IT OUT! **6a.** Simplify $\frac{1}{2}i^7$. **6b.** Simplify i^{42}.

Remember!

The complex conjugate of a complex number $a + bi$ is $a - bi$. (Lesson 5-5)

Recall that expressions in simplest form cannot have square roots in the denominator (Lesson 1-3). Because the imaginary unit represents a square root, you must rationalize any denominator that contains an imaginary unit. To do this, multiply the numerator and denominator by the complex conjugate of the denominator.

EXAMPLE 7 Dividing Complex Numbers

A **Simplify $\frac{3 + 7i}{8i}$.**

$\frac{3 + 7i}{8i}\left(\frac{-8i}{-8i}\right)$ *Multiply by the conjugate.*

$\frac{-24i - 56i^2}{-64i^2}$ *Distribute.*

$\frac{-24i + 56}{64}$ *Use $i^2 = -1$.*

$\frac{-3i + 7}{8} = \frac{7}{8} - \frac{3}{8}i$ *Simplify.*

B **Simplify $\frac{5 + i}{2 - 4i}$.**

$\frac{5 + i}{2 - 4i}\left(\frac{2 + 4i}{2 + 4i}\right)$

$\frac{10 + 20i + 2i + 4i^2}{4 + 8i - 8i - 16i^2}$

$\frac{10 + 22i - 4}{4 + 16}$

$\frac{6 + 22i}{20} = \frac{3}{10} + \frac{11}{10}i$

CHECK IT OUT! **7a.** Simplify $\frac{3 + 8i}{-i}$. **7b.** Simplify $\frac{3 - i}{2 - i}$.

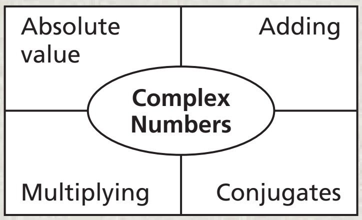
THINK AND DISCUSS

1. Explain when a complex number $a + bi$ and its conjugate are equal.

2. Find the product $(a + bi)(c + di)$, and identify which terms in the product are real and which are imaginary.

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, give an example.



5-9 Exercises

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Homework Help Online
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Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** In the complex number plane, the horizontal axis represents __?__ numbers, and the vertical axis represents __?__ numbers. (*real, irrational,* or *imaginary*)

SEE EXAMPLE 1 p. 382

Graph each complex number.

2. 4
3. $-i$
4. $3 + 2i$
5. $-2 - 3i$

SEE EXAMPLE 2 p. 383

Find each absolute value.

6. $|4 - 5i|$
7. $|-33.3|$
8. $|-9i|$
9. $|5 + 12i|$
10. $|-1 + i|$
11. $|15i|$

SEE EXAMPLE 3 p. 383

Add or subtract. Write the result in the form $a + bi$.

12. $(2 + 5i) + (-2 + 5i)$
13. $(-1 - 8i) + (4 + 3i)$
14. $(1 - 3i) - (7 + i)$
15. $(4 - 8i) + (-13 + 23i)$
16. $(6 + 17i) - (18 - 9i)$
17. $(-30 + i) - (-2 + 20i)$

SEE EXAMPLE 4 p. 384

Find each sum by graphing on the complex plane.

18. $(3 + 4i) + (-2 - 4i)$
19. $(-2 - 5i) + (-1 + 4i)$
20. $(-4 - 4i) + (4 + 2i)$

SEE EXAMPLE 5 p. 384

Multiply. Write the result in the form $a + bi$.

21. $(1 - 2i)(1 + 2i)$
22. $3i(5 + 2i)$
23. $(9 + i)(4 - i)$
24. $(6 + 8i)(5 - 4i)$
25. $(3 + i)^2$
26. $(-4 - 5i)(2 + 10i)$

SEE EXAMPLE 6 p. 385

Simplify.

27. $-i^9$
28. $2i^{15}$
29. i^{30}

SEE EXAMPLE 7 p. 385

30. $\frac{5 - 4i}{i}$
31. $\frac{11 - 5i}{2 - 4i}$
32. $\frac{8 + 2i}{5 + i}$
33. $\frac{17}{4 + i}$
34. $\frac{45 - 3i}{7 - 8i}$
35. $\frac{-3 - 12i}{6i}$

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
36–39	1
40–45	2
46–51	3
52–54	4
55–60	5
61–63	6
64–69	7

Extra Practice

Skills Practice p. S13
Application Practice p. S36

Graph each complex number.

36. -3
37. $-2.5i$
38. $1 + i$
39. $4 - 3i$

Find each absolute value.

40. $|2 + 3i|$
41. $|-18|$
42. $\left|\frac{4}{5}i\right|$
43. $|6 - 8i|$
44. $|-0.5i|$
45. $|10 - 4i|$

Add or subtract. Write the result in the form $a + bi$.

46. $(8 - 9i) - (-2 - i)$
47. $4i - (11 - 3i)$
48. $(4 - 2i) + (-9 - 5i)$
49. $(13 + 6i) + (15 + 35i)$
50. $(3 - i) - (-3 + i)$
51. $-16 + (12 + 9i)$

Find each sum by graphing on the complex plane.

52. $(4 + i) + (-3i)$
53. $(5 + 4i) + (-1 + 2i)$
54. $(-3 - 3i) + (4 - 3i)$

Multiply. Write the result in the form $a + bi$.

55. $-12i(-1 + 4i)$ **56.** $(3 - 5i)(2 + 9i)$ **57.** $(7 + 2i)(7 - 2i)$

58. $(5 + 6i)^2$ **59.** $(7 - 5i)(-3 + 9i)$ **60.** $-4(8 + 12i)$

Simplify.

61. i^{27} **62.** $-i^{11}$ **63.** $5i^{10}$

64. $\frac{2 - 3i}{i}$ **65.** $\frac{5 - 2i}{3 + i}$ **66.** $\frac{3}{-1 - 5i}$

67. $\frac{19 + 9i}{5 + i}$ **68.** $\frac{8 + 4i}{7 + i}$ **69.** $\frac{6 + 3i}{2 - 2i}$

Write the complex number represented by each point on the graph.

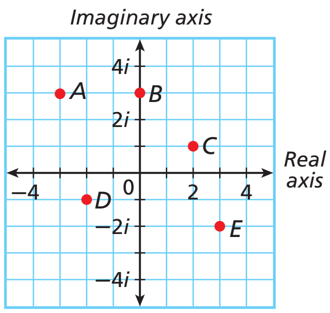
70. A

71. B

72. C

73. D

74. E



Find the absolute value of each complex number.

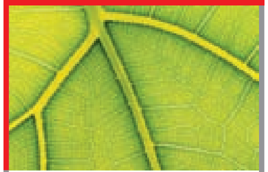
75. $3 - i$ **76.** $7i$ **77.** $-2 - 6i$

78. $-1 - 8i$ **79.** 0 **80.** $5 + 4i$

81. $\frac{3}{2} - \frac{1}{2}i$ **82.** $5 - i\sqrt{3}$ **83.** $2\sqrt{2} - i\sqrt{3}$

LINK

Fractals



Fractals are self-similar, which means that smaller parts of a fractal are similar to the fractal as a whole. Many objects in nature, such as the veins of leaves and snow crystals, also exhibit self-similarity. As a result, scientists can use fractals to model these objects.

84. Fractals Fractals are patterns produced using complex numbers and the repetition of a mathematical formula. Substitute the first number into the formula. Then take the result, put it back into the formula, and so on. Each complex number produced by the formula can be used to assign a color to a pixel on a computer screen. The result is an image such as the one at right. Many common fractals are based on the Julia Set, whose formula is $Z_{n+1} = (Z_n)^2 + c$, where c is a constant.

a. Find Z_2 using $Z_2 = (Z_1)^2 + 0.25$. Let $Z_1 = 0.5 + 0.6i$.

b. Find Z_3 using $Z_3 = (Z_2)^2 + 0.25$. Use Z_2 that you obtained in part **a.**

c. Find Z_4 using $Z_4 = (Z_3)^2 + 0.25$. Use Z_3 that you obtained in part **b.**

Simplify. Write the result in the form $a + bi$.

85. $(3.5 + 5.2i) + (6 - 2.3i)$ **86.** $6i - (4 + 5i)$ **87.** $(-2.3 + i) - (7.4 - 0.3i)$

88. $(-8 - 11i) + (-1 + i)$ **89.** $i(4 + i)$ **90.** $(6 - 5i)^2$

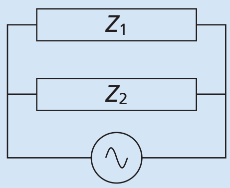
91. $(-2 - 3i)^2$ **92.** $(5 + 7i)(5 - 7i)$ **93.** $(2 - i)(2 + i)(2 - i)$

94. $3 - i^{11}$ **95.** $i^{52} - i^{48}$ **96.** $i^{35} - i^{24} + i^{18}$

97. $\frac{12 + i}{i}$ **98.** $\frac{18 - 3i}{i}$ **99.** $\frac{4 + 2i}{6 + i}$

100. $\frac{1 + i}{-2 + 4i}$ **101.** $\frac{4}{2 - 3i}$ **102.** $\frac{6}{\sqrt{2} - i}$

Multi-Step *Impedance* **is a measure of the opposition of a circuit to an electric current. Electrical engineers find it convenient to model impedance Z with complex numbers. In a parallel AC circuit with two impedances Z_1 and Z_2, the *equivalent* or total impedance in ohms can be determined by using the formula $Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$.**



12 V
Parallel AC circuit

103. Find the equivalent impedance Z_{eq} for $Z_1 = 3 + 2i$ and $Z_2 = 1 - 2i$ arranged in a parallel AC circuit.

104. Find the equivalent impedance Z_{eq} for $Z_1 = 2 + 2i$ and $Z_2 = 4 - i$ arranged in a parallel AC circuit.

Tell whether each statement is sometimes, always, or never true. If the statement is sometimes true, give an example and a counterexample. If the statement is never true, give a counterexample.

105. The sum of any complex number $a + bi$ and its conjugate is a real number.

106. The difference between any complex number $a + bi$ $(b \neq 0)$ and its conjugate is a real number.

107. The product of any complex number $a + bi$ $(a \neq 0)$ and its conjugate is a positive real number.

108. The product of any two imaginary numbers bi $(b \neq 0)$ and di $(d \neq 0)$ is a positive real number.

109. **ERROR ANALYSIS** Two attempts to simplify $\frac{3}{2+i}$ are shown. Which is incorrect? Explain the error.

A

$$\frac{3}{2+i} = \frac{3}{2+i}\left(\frac{2+i}{2+i}\right)$$
$$= \frac{6+3i}{4+i^2}$$
$$= \frac{6+3i}{3} = 2 + i$$

B

$$\frac{3}{2+i} = \frac{3}{2+i}\left(\frac{2-i}{2-i}\right)$$
$$= \frac{6-3i}{4-i^2}$$
$$= \frac{6-3i}{5}$$

110. **Critical Thinking** Why are the absolute value of a complex number and the absolute value of its conjugate equal? Use a graph to justify your answer.

111. **Write About It** Discuss how the difference of two squares, $a^2 - b^2 = (a + b)(a - b)$, relates to the product of a complex number and its conjugate.

MULTI-STEP TEST PREP

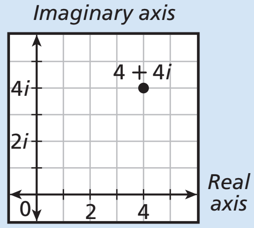
112. This problem will prepare you for the Multi-Step Test Prep on page 390.

You have seen how to graph sums of complex numbers on the complex plane.

a. Find three pairs of complex numbers whose sum is $4 + 4i$.

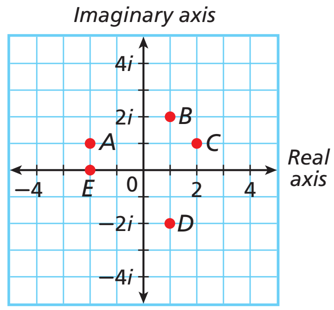
b. Graph each of the sums on the same complex plane.

c. Describe the results of your graph.



TEST PREP

Use the graph for Exercises 113–114.



113. Which point on the graph represents $1 - 2i$?

Ⓐ A Ⓒ C
Ⓑ B Ⓓ D

114. What is the value of the complex number represented in the graph by E?

Ⓕ -2 Ⓗ $-2i$
Ⓖ 2 Ⓙ $2i$

115. Which expression is equivalent to $(2 - 5i) - (2 + 5i)$?

Ⓐ $10i$ Ⓑ $4 + 10i$ Ⓒ $-10i$ Ⓓ $4 - 10i$

116. Which expression is equivalent to $(-5 + 3i)^2$?

Ⓕ $16 - 15i$ Ⓖ $16 - 30i$ Ⓗ $34 - 15i$ Ⓙ $34 - 30i$

CHALLENGE AND EXTEND

117. Consider the powers of i.

a. Complete the table, and look for a pattern.

$i^1 = \blacksquare$	$i^0 = \blacksquare$	$i^{-1} = \blacksquare$	$i^{-2} = \blacksquare$	$i^{-3} = \blacksquare$	$i^{-4} = \blacksquare$	$i^{-5} = \blacksquare$

b. Explain the pattern that you observed for i raised to negative powers. What are the only possible values of i raised to a negative integer power?

c. Simplify i^{-12}, i^{-37}, and i^{-90}.

Find the general form of the result for each complex operation.

118. $(a + bi)(c + di)$

119. $\dfrac{a + bi}{c + di}$

SPIRAL REVIEW

120. Money The table shows the amount that James spent for lunches each week over an eight-week period. Make a scatter plot of the data. Sketch a line of best fit, and find its equation. *(Lesson 2-7)*

Lunches Purchased	5	7	3	5	6	2	4	5
Weekly Cost ($)	10	13	8	9	8	5	10	11

Solve each inequality by using algebra. *(Lesson 5-7)*

121. $0 \geq 3x^2 - 6x$

122. $10 < x^2 - 4x - 11$

123. $-6 \geq 2x^2 + 7x - 21$

124. $3 - x^2 < 7 - 5x$

Determine whether each data set could represent a quadratic function. Explain. *(Lesson 5-8)*

125.

x	-2	-1	0	1
y	5	-1	-3	-1

126.

x	0	2	4	6
y	18	10	2	-6

CHAPTER 5

SECTION 5B

MULTI-STEP TEST PREP

Applying Quadratic Functions

Tilted Tiles Mitch and Jacob are making mosaics in an art class. To make one mosaic, Mitch first divides a wall into a grid made up of squares with a side length of 20 cm. Then Jacob glues a tile on each square, making sure that each corner of the tile touches a side of the grid square.

They measure the side length of each tile as well as the distance x from the upper right corner of the grid square to a corner of the tile. They find that for each tile there are two possible values of x, as shown.

1. Complete the table by finding the area of each tile and the ratio y of the area of each tile to the area of the grid square.
2. Make a scatterplot of the ordered pairs (x, y). Find and graph a quadratic model for the data. Is the model a reasonable representation of the data? Explain.
3. Describe the domain for the problem situation. Explain why the domain of the problem situation is different from the domain of the model.
4. Use your model to determine the value of y when $x = 3.8$. Explain the meaning of your answer in the context of the problem.
5. For what values of x does a tile cover at least 75% of the grid square? Round to the nearest tenth.

Side Length of Tile (cm)	x (cm)	Area of Tile (cm^2)	y
15	6.4		
15	13.6		
15.5	5.5		
15.5	14.5		
16	4.7		
16	15.3		
17	3.3		
17	16.7		
18	2.1		
18	17.9		
19	1.1		
19	18.9		
20	0		

READY TO GO ON?

Quiz for Lessons 5-7 Through 5-9

5-7 Solving Quadratic Inequalities

Graph each inequality.

1. $y > -x^2 + 6x$
2. $y \le -x^2 - x + 2$

Solve each inequality by using tables or graphs.

3. $x^2 - 4x + 1 > 6$
4. $2x^2 + 2x - 10 \le 2$

Solve each inequality by using algebra.

5. $x^2 + 4x - 7 \ge 5$
6. $x^2 - 8x < 0$
7. The function $p(r) = -1000r^2 + 6400r - 4400$ models the monthly profit p of a small DVD-rental store, where r is the rental price of a DVD. For what range of rental prices does the store earn a monthly profit of at least \$5000?

5-8 Curve Fitting with Quadratic Models

Determine whether each data set could represent a quadratic function. Explain.

8.

x	5	6	7	8	9
y	13	11	7	1	–7

9.

x	–4	–2	0	2	4
y	10	8	4	8	10

Write a quadratic function that fits each set of points.

10. $(0, 4)$, $(2, 0)$, and $(3, 1)$
11. $(1, 3)$, $(2, 5)$, and $(4, 3)$

For Exercises 12–14, use the table of maximum load allowances for various heights of spruce columns.

Maximum Load Allowance No. 1 Common Spruce	
Height of Column (ft)	Maximum Load (lb)
4	7280
5	7100
6	6650
7	5960

12. Find a quadratic regression equation to model the maximum load given the height.
13. Use your model to predict the maximum load allowed for a 6.5 ft spruce column.
14. Use your model to predict the maximum load allowed for an 8 ft spruce column.

5-9 Operations with Complex Numbers

Find each absolute value.

15. $|-6i|$
16. $|3 + 4i|$
17. $|2 - i|$

Perform each indicated operation, and write the result in the form $a + bi$.

18. $(3 - 5i) - (6 - i)$
19. $(-6 + 4i) + (7 - 2i)$
20. $3i(4 + i)$
21. $(3 + i)(5 - i)$
22. $(1 - 4i)(1 + 4i)$
23. $3i^{15}$
24. $\dfrac{2 - 7i}{-i}$
25. $\dfrac{3 - i}{4 - 2i}$

CHAPTER 5

Study Guide: Review

Vocabulary

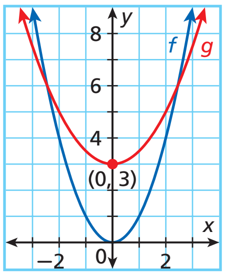
Complete the sentences below with vocabulary words from the list above.

1. The number $5i$ can be classified as both a(n) ___?___ and a ___?___.
2. The value of the input x that makes the output $f(x)$ equal zero is called the ___?___.
3. The ___?___ is the point at which the parabola intersects the axis of symmetry.
4. The type and number of solutions to a quadratic equation can be determined by finding the ___?___.
5. When a parabola opens upward, the y-value of the vertex is the ___?___ of a quadratic function.

5-1 Using Transformations to Graph Quadratic Functions *(pp. 315–322)*

EXAMPLES

■ **Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph $g(x) = \frac{1}{2}x^2 + 3$.**



$g(x) = \frac{1}{2}x^2 + 3$ is f vertically compressed by a factor of $\frac{1}{2}$ and translated 3 units up.

■ **Use the description to write a quadratic function in vertex form. The function $f(x) = x^2$ is translated 1 unit right to create g.**

translation 1 unit right: $h = 1$

$g(x) = a(x - h)^2 + k \rightarrow g(x) = (x - 1)^2$

EXERCISES

Graph each function by using a table.

6. $f(x) = -x^2 - 2x$
7. $f(x) = \frac{1}{2}x^2 + 3x - 4$

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

8. $g(x) = 4(x - 2)^2$
9. $g(x) = -2(x + 1)^2$
10. $g(x) = \frac{1}{3}x^2 - 3$
11. $g(x) = -(x + 2)^2 + 6$

Use the description to write each quadratic function in vertex form.

12. $f(x) = x^2$ is reflected across the x-axis and translated 3 units down to create g.
13. $f(x) = x^2$ is vertically stretched by a factor of 2 and translated 4 units right to create g.
14. $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{4}$ and translated 1 unit left to create g.

5-2 Properties of Quadratic Functions in Standard Form *(pp. 323–330)*

EXAMPLE

- **For $f(x) = -x^2 + 2x + 3$, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.**

a. Because $a < 0$, the parabola opens downward.

b. axis of symmetry:

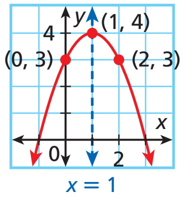
$$x = -\frac{b}{2a} = -\frac{2}{2(-1)} = 1$$

c. $f(1) = -1^2 + 2(1) + 3 = 4$

The vertex is $(1, 4)$.

d. Because $c = 3$, the y-intercept is 3.

e.



EXERCISES

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

15. $f(x) = x^2 - 4x + 3$

16. $g(x) = x^2 + 2x + 3$

17. $h(x) = x^2 - 3x$

18. $j(x) = \frac{1}{2}x^2 - 2x + 4$

Find the minimum or maximum value of each function.

19. $f(x) = x^2 + 2x + 6$

20. $g(x) = 6x - 2x^2$

21. $f(x) = x^2 - 5x + 1$

22. $g(x) = -2x^2 - 8x + 10$

23. $f(x) = -x^2 - 4x + 8$

24. $g(x) = 3x^2 + 7$

5-3 Solving Quadratic Equations by Graphing and Factoring *(pp. 333–340)*

EXAMPLES

- **Find the roots of $x^2 + x = 30$ by factoring.**

$x^2 + x - 30 = 0$	*Rewrite in standard form.*
$(x - 5)(x + 6) = 0$	*Factor.*
$x - 5 = 0$ or $x + 6 = 0$	*Zero Product Property.*
$x = 5$ or $x = -6$	*Solve each equation.*

- **Write a quadratic function with zeros 8 and −8.**

$x = 8$ or $x = -8$	*Write zeros as solutions.*
$x - 8 = 0$ or $x + 8 = 0$	*Set equations equal to 0.*
$(x - 8)(x + 8) = 0$	*Converse Zero Product Property*
$f(x) = x^2 - 64$	*Replace 0 with f(x).*

EXERCISES

Find the roots of each equation by factoring.

25. $x^2 - 7x - 8 = 0$

26. $x^2 - 5x + 6 = 0$

27. $x^2 = 144$

28. $x^2 - 21x = 0$

29. $4x^2 - 16x + 16 = 0$

30. $2x^2 + 8x + 6 = 0$

31. $x^2 + 14x = 32$

32. $9x^2 + 6x + 1 = 0$

Write a quadratic function in standard form for each given set of zeros.

33. 2 and −3

34. 1 and −1

35. 4 and 5

36. −2 and −3

37. −5 and −5

38. 9 and 0

5-4 Completing the Square *(pp. 342–349)*

EXAMPLE

- **Solve $x^2 - 8x = 12$ by completing the square.**

$x^2 - 8x + \square = 12 + \square$	*Set up equation.*
$x^2 - 8x + 16 = 12 + 16$	*Add $\left(\frac{b}{2}\right)^2$.*
$(x - 4)^2 = 28$	*Factor.*
$x - 4 = \pm\sqrt{28}$	*Take square roots.*
$x = 4 \pm 2\sqrt{7}$	*Solve for x.*

EXERCISES

Solve each equation by completing the square.

39. $x^2 - 16x + 48 = 0$

40. $x^2 + 20x + 84 = 0$

41. $x^2 - 6x = 16$

42. $x^2 - 14x = 13$

Write each function in vertex form, and identify its vertex.

43. $f(x) = x^2 - 4x + 9$

44. $g(x) = x^2 + 2x - 7$

5-5 Complex Numbers and Roots *(pp. 350–355)*

EXAMPLE

- Solve $x^2 - 22x + 133 = 0$.

$$x^2 - 22x + \blacksquare = -133 + \blacksquare \quad \textit{Rewrite.}$$

$$x^2 - 22x + 121 = -133 + 121 \quad \textit{Add } \left(\tfrac{b}{2}\right)^2.$$

$$(x - 11)^2 = -12 \quad \textit{Factor.}$$

$$x - 11 = \pm\sqrt{-12} \quad \textit{Take square roots.}$$

$$x = 11 \pm 2i\sqrt{3} \quad \textit{Solve.}$$

EXERCISES

Solve each equation.

45. $x^2 = -81$

46. $6x^2 + 150 = 0$

47. $x^2 + 6x + 10 = 0$

48. $x^2 + 12x + 45 = 0$

49. $x^2 - 14x + 75 = 0$

50. $x^2 - 22x + 133 = 0$

Find each complex conjugate.

51. $5i - 4$

52. $3 + i\sqrt{5}$

5-6 The Quadratic Formula *(pp. 356–363)*

EXAMPLES

- Find the zeros of $f(x) = 3x^2 - 5x + 3$ by using the Quadratic Formula.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \textit{Quadratic Formula}$$

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(3)}}{2(3)} \quad \textit{Substitute.}$$

$$= \frac{5 \pm \sqrt{-11}}{6} = \frac{5}{6} \pm i\frac{\sqrt{11}}{6} \quad \textit{Simplify.}$$

- Find the type and number of solutions for $x^2 + 9x + 20 = 0$.

$$b^2 - 4ac = 9^2 - 4(1)(20)$$
$$= 81 - 80 = 1$$

There are two distinct real roots because the discriminant is positive.

EXERCISES

Find the zeros of each function by using the Quadratic Formula.

53. $f(x) = x^2 - 3x - 8$

54. $h(x) = (x - 5)^2 + 12$

55. $f(x) = 2x^2 - 10x + 18$

56. $g(x) = x^2 + 3x + 3$

57. $h(x) = x^2 - 5x + 10$

Find the type and number of solutions for each equation.

58. $2x^2 - 16x + 32 = 0$

59. $x^2 - 6x = -5$

60. $x^2 + 3x + 8 = 0$

61. $x^2 - 246x = -144$

62. $x^2 + 5x = -12$

63. $3x^2 - 5x + 3 = 0$

5-7 Solving Quadratic Inequalities *(pp. 366–373)*

EXAMPLE

- Solve $x^2 - 4x - 9 \geq 3$ by using algebra.

Write and solve the related equation.

$$x^2 - 4x - 12 = 0 \quad \textit{Write in standard form.}$$

$$(x + 2)(x - 6) = 0 \quad \textit{Factor.}$$

$$x = -2 \text{ or } x = 6 \quad \textit{Solve.}$$

The critical values are -2 and 6. These values divide the number line into three intervals: $x \leq -2$, $-2 \leq x \leq 6$, and $x \geq 6$.

Testing an x-value in each interval gives the solution of $x \leq -2$ or $x \geq 6$.

EXERCISES

Graph each inequality.

64. $y > x^2 + 3x + 4$

65. $y \leq 2x^2 - x - 5$

Solve each inequality by using tables or graphs.

66. $x^2 + 2x - 4 \geq -1$

67. $-x^2 - 5x > 4$

Solve each inequality by using algebra.

68. $-x^2 + 6x < 5$

69. $3x^2 - 25 \leq 2$

70. $x^2 - 3 < 0$

71. $3x^2 + 4x - 3 \leq 1$

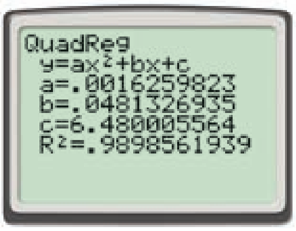
5-8 Curve Fitting with Quadratic Models *(pp. 374–381)*

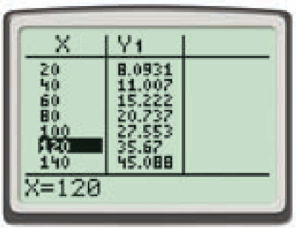
EXAMPLE

- **Find a quadratic model for the wattage of fluorescent bulbs *F* given the comparable incandescent bulb wattage *I*. Use the model to estimate the wattage of a fluorescent bulb that produces the same amount of light as a 120-watt incandescent bulb.**

Wattage Comparison					
Incandescent (watts)	40	60	75	90	100
Fluorescent (watts)	11	15	20	23	28

Enter the data into two lists in a graphing calculator. Use the quadratic regression feature.





The model is $F(I) \approx 0.0016I^2 + 0.0481I + 6.48$. A 36-watt fluorescent bulb produces about the same amount of light as a 120-watt incandescent bulb.

EXERCISES

Write a quadratic function that fits each set of points.

72. $(-1, 8)$, $(0, 6)$, and $(1, 2)$

73. $(0, 0)$, $(1, -1)$, and $(2, -6)$

Construction **For Exercises 74–77, use the table of copper wire gauges.**

Common U.S. Copper Wire Gauges		
Gauge	Diameter (in.)	Resistance per 1000 ft (ohms)
24	0.0201	25.67
22	0.0254	16.14
20	0.0320	10.15
18	0.0403	6.385

74. Find a quadratic regression equation to model the diameter given the wire gauge.

75. Use your model to predict the diameter for a 12-gauge copper wire.

76. Find a quadratic regression equation to model the resistance given the wire gauge.

77. Use your model to predict the resistance for a 26-gauge copper wire.

5-9 Operations with Complex Numbers *(pp. 382–389)*

EXAMPLES

Perform each indicated operation, and write the result in the form $a + bi$.

- $|-2 + 4i|$

  $\sqrt{(-2)^2 + 4^2} = \sqrt{4 + 16} = \sqrt{20} = 2\sqrt{5}$

- $(3 + 2i)(4 - 5i)$

  $12 - 15i + 8i - 10i^2$

  $12 - 7i - 10(-1) = 22 - 7i$

- $\dfrac{-5 + 3i}{1 - 2i}$

  $$\frac{-5 + 3i}{1 - 2i}\left(\frac{1 + 2i}{1 + 2i}\right) = \frac{-5 - 7i + 6i^2}{1 - 4i^2}$$

  $$= \frac{-11 - 7i}{1 + 4} = -\frac{11}{5} - \frac{7}{5}i$$

EXERCISES

Perform each indicated operation, and write the result in the form $a + bi$.

78. $|-3i|$

79. $|4 - 2i|$

80. $|12 - 16i|$

81. $|7i|$

82. $(1 + 5i) + (6 - i)$

83. $(9 + 4i) - (3 + 2i)$

84. $(5 - i) - (11 - i)$

85. $-5i(3 - 4i)$

86. $(5 - 2i)(6 + 8i)$

87. $(3 + 2i)(3 - 2i)$

88. $(4 + i)(1 - 5i)$

89. $(-7 + 4i)(3 + 9i)$

90. i^{32}

91. $-5i^{21}$

92. $\dfrac{2 + 9i}{-2i}$

93. $\dfrac{5 + 2i}{3 - 4i}$

94. $\dfrac{8 - 4i}{1 + i}$

95. $\dfrac{-12 + 26i}{2 + 4i}$

CHAPTER 5

CHAPTER TEST

Using the graph of $f(x) = x^2$ as a guide, describe the transformations, and then graph each function.

1. $g(x) = (x+1)^2 - 2$

2. $h(x) = -\frac{1}{2}x^2 + 2$

3. Use the following description to write a quadratic function in vertex form: $f(x) = x^2$ is vertically compressed by a factor of $\frac{1}{2}$ and translated 6 units right to create g.

For each function, (a) determine whether the graph opens upward or downward, (b) find the axis of symmetry, (c) find the vertex, (d) find the y-intercept, and (e) graph the function.

4. $f(x) = -x^2 + 4x + 1$

5. $g(x) = x^2 - 2x + 3$

6. The area A of a rectangle with a perimeter of 32 cm is modeled by the function $A(x) = -x^2 + 16x$, where x is the width of the rectangle in centimeters. What is the maximum area of the rectangle?

Find the roots of each equation by using factoring.

7. $x^2 - 2x + 1 = 0$

8. $x^2 + 10x = -21$

Solve each equation.

9. $x^2 + 4x = 12$

10. $x^2 - 12x = 25$

11. $x^2 + 25 = 0$

12. $x^2 + 12x = -40$

Write each function in vertex form, and identify its vertex.

13. $f(x) = x^2 - 4x + 9$

14. $g(x) = x^2 - 18x + 92$

Find the zeros of each function by using the Quadratic Formula.

15. $f(x) = (x-1)^2 + 7$

16. $g(x) = 2x^2 - x + 5$

17. The height h in feet of a person on a waterslide is modeled by the function $h(t) = -0.025t^2 - 0.5t + 50$, where t is the time in seconds. At the bottom of the slide, the person lands in a swimming pool. To the nearest tenth of a second, how long does the ride last?

18. Graph the inequality $y < x^2 - 3x - 4$.

Solve each inequality.

19. $-x^2 + 3x + 5 \geq 7$

20. $x^2 - 4x + 1 > 1$

For Exercises 21 and 22, use the table showing the average cost of LCD televisions at one store.

Costs of LCD Televisions

Size (in.)	15	17	23	30
Cost ($)	550	700	1500	2500

21. Find a quadratic model for the cost of a television given its size.

22. Use the model to estimate the cost of a 42 in. LCD television.

Perform the indicated operation, and write the result in the form $a + bi$.

23. $(12 - i) - (5 + 2i)$

24. $(6 - 2i)(2 - 2i)$

25. $-2i^{18}$

26. $\frac{1 - 8i}{4i}$

COLLEGE ENTRANCE EXAM PRACTICE

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The SAT Mathematics Subject Tests assess knowledge from course work rather than ability to learn. The Level 1 test is meant to be taken by students who have completed two years of algebra and one year of geometry, and it tests more elementary topics than the Level 2 test.

HOT TIP! You will need to use a calculator for some of the problems on the SAT Mathematics Subject Tests. Before test day, make sure that you are familiar with the features of the calculator that you will be using.

You may want to time yourself as you take this practice test. It should take you about 8 minutes to complete.

---

1. For what value of c will $3x^2 - 2x + c = 0$ have exactly one distinct real root?

(A) $-\frac{2}{3}$

(B) $-\frac{1}{3}$

(C) 0

(D) $\frac{1}{3}$

(E) $\frac{2}{3}$

---

2. If m and n are real numbers, $i^2 = -1$, and $(m - n) - 4i = 7 + ni$, what is the value of m?

(A) -4

(B) -3

(C) 1

(D) 3

(E) 4

---

3. If $x^2 - 5x + 6 = (x - h)^2 + k$, what is the value of k?

(A) $-\frac{25}{4}$

(B) $-\frac{5}{2}$

(C) $-\frac{1}{4}$

(D) 0

(E) 6

---

4. What is the solution set of $y^2 - 2y \leq 3y + 14$?

(A) $y \geq -2$

(B) $y \leq 7$

(C) $y \leq -2$ or $y \geq 7$

(D) $-7 \leq y \leq 2$

(E) $-2 \leq y \leq 7$

---

5. Which of the following is a factor of $(a - 1)^2 - b^2$?

(A) $a + b - 1$

(B) $a - b$

(C) $a - 1$

(D) $a - b + 1$

(E) $1 - b$

---

6. If $z = 5 - 4i$ and $i^2 = -1$, what is $|z|$?

(A) 1

(B) 3

(C) 9

(D) $\sqrt{41}$

(E) $\sqrt{42}$

TEST TACKLER

Standardized Test Strategies

Multiple Choice: Work Backward

When taking a multiple-choice test, you can sometimes work backward to determine which answer is correct. Because this method can be time consuming, it is best used only when you cannot solve a problem in any other way.

EXAMPLE 1

Which expression is equivalent to $2x^2 - 3x - 14$?

(A) $(2x + 7)(x + 2)$ (C) $(2x - 7)(x + 2)$

(B) $(2x - 7)(x - 2)$ (D) $(2x + 7)(x - 2)$

If you have trouble factoring the quadratic expression given in the question, you can multiply the binomials in the answer choices to find the product that is the same as $2x^2 - 3x - 14$.

Try Choice A: $(2x + 7)(x + 2) = 2x^2 + 11x + 14$

Try Choice B: $(2x - 7)(x - 2) = 2x^2 - 11x + 14$

Try Choice C: $(2x - 7)(x + 2) = 2x^2 - 3x - 14$

Choice C is the answer.

***Note:** Trying choice D can help you check your work.*

EXAMPLE 2

What is the solution set of $x^2 - 36 < 0$?

(F) $x < -6$ or $x > 6$ (H) $-36 < x < 36$

(G) $-6 < x < 6$ (J) $x < -36$ or $x > 36$

If you have trouble determining the solution set, substitute values of x into the inequality. Based on whether the values make the inequality true or false, you may be able to eliminate one or more of the answer choices.

Substitute 0 for x: $x^2 - 36 < 0 \rightarrow (0)^2 - 36 \overset{?}{<} 0 \rightarrow -36 < 0$ ✔

When $x = 0$, the inequality is true. Therefore, the solution set must include $x = 0$. Because choices F and J do not include $x = 0$, they can be eliminated.

Substitute 10 for x: $x^2 - 36 < 0 \rightarrow (10)^2 - 36 \overset{?}{<} 0 \rightarrow 64 \overset{?}{<} 0$ ✘

When $x = 10$, the inequality is false. Therefore, the solution set does not include $x = 10$. Because choice H includes $x = 10$, it can be eliminated.

The only remaining choice is choice G. Therefore, choice G must be correct.

HOT TIP! You can also work backward to check whether the answer you found by another method is correct or reasonable.

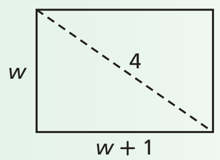
Read each test item, and answer the questions that follow.

Item A
What are the zeros of the function $g(x) = 6x^2 - 8x - 4$, rounded to the nearest hundredth?

Ⓐ -10.32 and 2.32
Ⓑ -1.72 and 0.39
Ⓒ 1.72 and -0.39
Ⓓ 10.32 and -2.32

1. Rachel cannot remember how to determine the zeros of a quadratic function, so she plans to pick one of the answer choices at random. What could Rachel do to make a more educated guess?
2. Describe how to find the correct answer by working backward.

Item B
A portable television has a screen with a diagonal of 4 inches. The length of the screen is 1 inch greater than its width. What are the dimensions of the screen to the nearest hundredth?



Ⓕ 1.28 inches by 2.28 inches
Ⓖ 1.28 inches by 3.28 inches
Ⓗ 2.28 inches by 2.28 inches
Ⓙ 2.28 inches by 3.28 inches

3. Can any of the answer choices be eliminated immediately? If so, which choices and why?
4. Describe how you can determine the correct answer by using the Pythagorean Theorem and working backward.

Item C
Which of the following is a solution of $(x + 4)^2 = 25$?

Ⓐ $x = -9$
Ⓑ $x = -1$
Ⓒ $x = 0$
Ⓓ $x = 9$

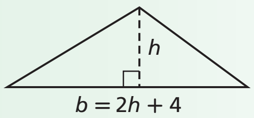
5. Explain how to use substitution to determine the correct answer.
6. Check whether choice A is correct by working backward. Explain your findings. What should you do next?

Item D
The height h of a golf ball in feet t seconds after it is hit into the air is modeled by $h(t) = -16t^2 + 64t$. How long is the ball in the air?

Ⓕ 2 seconds
Ⓖ 4 seconds
Ⓗ 12 seconds
Ⓙ 16 seconds

7. The measurements given in the answer choices represent possible values of which variable in the function?
8. Describe how you can work backward to determine that choice F is not correct.

Item E
The base of a triangle is 4 in. longer than twice its height. If the triangle has an area of 24 in^2, what is its height?



Ⓐ 2 in.
Ⓑ 4 in.
Ⓒ 6 in.
Ⓓ 8 in.

9. What equation do you need to solve to find the value of h?
10. Try choice A by working backward. Explain your findings. What should you do next?

CHAPTER 5

STANDARDIZED TEST PREP

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CUMULATIVE ASSESSMENT, CHAPTERS 1–5

Multiple Choice

1. $M = \begin{bmatrix} 6 & -2 \\ 3 & 7 \end{bmatrix}$ $\quad N = \begin{bmatrix} -1 & 8 & 2 \\ 0 & 1 & 6 \end{bmatrix}$

What is the matrix product $2MN$?

(A) $\begin{bmatrix} -24 & 184 & 0 \\ -12 & 124 & 192 \end{bmatrix}$

(B) $\begin{bmatrix} -12 & 92 & 0 \\ -6 & 62 & 96 \end{bmatrix}$

(C) $\begin{bmatrix} -24 & -12 \\ 184 & 124 \\ 0 & 192 \end{bmatrix}$

(D) $\begin{bmatrix} -12 & -6 \\ 92 & 62 \\ 0 & 96 \end{bmatrix}$

2. Which of these functions does NOT have zeros at -1 and 4?

(F) $f(x) = x^2 - 3x - 4$

(G) $f(x) = 2x^2 + 6x - 8$

(H) $f(x) = -x^2 + 3x + 4$

(J) $f(x) = 2x^2 - 6x - 8$

3. Dawn and Julia are running on a jogging trail. Dawn starts running 5 minutes after Julia does. If Julia runs at an average speed of 8 ft/s and Dawn runs at an average speed of 9 ft/s, how many minutes after Dawn starts running will she catch up with Julia?

(A) 5 minutes

(B) 27 minutes

(C) 40 minutes

(D) 45 minutes

4. Which equation has intercepts at (20, 0, 0), (0, 40, 0), and (0, 0, 5)?

(F) $20x + 40y + 5z = 0$

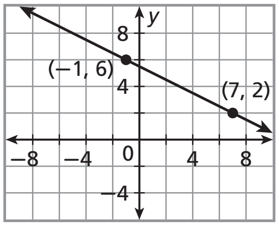
(G) $20x + 40y + 5z = 1$

(H) $4x + 8y + z = 5$

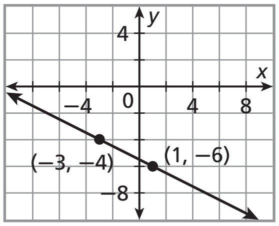
(J) $2x + y + 8z = 40$

5. Which graph represents the function $f(x) = -\frac{1}{2}(x - 3) - 4$?

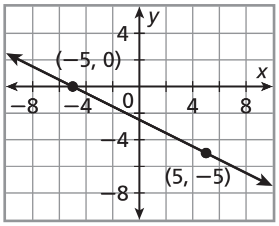
(A)



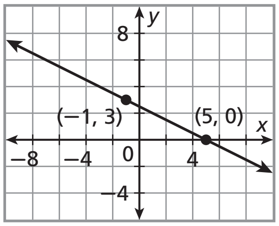
(B)



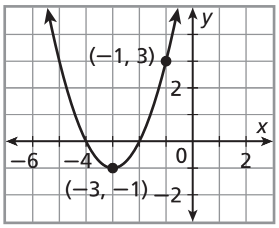
(C)



(D)



6. What is the equation of the function graphed below?



(F) $y = (x - 3)^2 - 1$

(G) $y = (x + 3)^2 - 1$

(H) $y = (x - 1)^2 - 3$

(J) $y = (x + 1)^2 - 3$

7. If the relationship between x and y is quadratic, which value of y completes the table?

x	-3	-1	1	3	5
y	21	7	■	27	61

(A) 3 (C) 9

(B) 7 (D) 17

8. Which is equivalent to the expression $\dfrac{5(6-8i)}{2-i}$?

(F) $-20+10i$ (H) $15-40i$

(G) $15-8i$ (J) $20-10i$

9. What is the inverse of the following matrix?

$$\begin{bmatrix} -2 & -4 \\ 4 & 2 \end{bmatrix}$$

(A) $\begin{bmatrix} -\frac{1}{6} & -\frac{1}{3} \\ \frac{1}{3} & \frac{1}{6} \end{bmatrix}$ (C) $\begin{bmatrix} \frac{1}{6} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{6} \end{bmatrix}$

(B) $\begin{bmatrix} -\frac{1}{2} & -\frac{1}{4} \\ \frac{1}{4} & \frac{1}{2} \end{bmatrix}$ (D) $\begin{bmatrix} 2 & 4 \\ -4 & -2 \end{bmatrix}$

HOT TIP!

In nearly all standardized tests, you cannot enter a negative value as the answer to a gridded-response question. If you get a negative value as an answer to one of these questions, you have probably made a mistake in your calculations.

Gridded Response

10. What value of x makes the equation $x^2+64=16x$ true?

11. The table shows the fees that are charged at an airport parking lot for various lengths of time. What is the slope of the linear function that models the parking fee f in dollars for h number of hours?

Time (h)	1	3	5	7
Parking Fee ($)	3.35	5.05	6.75	8.45

12. What is the x-value of the vertex of $f(x)=2x^2-15x+5$?

13. What is the value of c given that the following system is dependent?

$$\begin{cases} 2y-x+10=0 \\ 3x-6y-c=16 \end{cases}$$

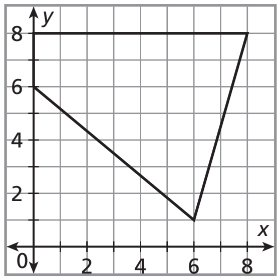
Short Response

14. $\begin{cases} -4x+8y-2z=8 \\ 4x-4y+2z=-5 \\ x+4y-2z=15 \end{cases}$

a. Write the augmented matrix that could be used to solve the system of equations given above.

b. Find the solution of the system, and explain how you determined your answer.

15. The graph below shows a feasible region for a set of constraints.



a. Write the constraints for the feasible region.

b. Maximize the objective function $P=3x-4y$ under these constraints.

16. Consider the function $f(x)=x^2-2x-48$.

a. Determine the roots of the function. Show your work.

b. The function f is translated to produce the function g. The vertex of g is the point $(3, 30)$. Write the function rule for g in vertex form, and explain how you determined your answer.

Extended Response

17. A small alteration store charges $15.00 per hour plus a $12.50 consulting fee for alterations. A competing store charges $20.00 per hour but does not charge a consulting fee.

a. For each store, write a linear function c that can be used to find the total cost of an alteration that takes h hours.

b. For which values of h is the small alteration store less expensive than the competing store? Explain how you determined your answer.

c. The small store wants to adjust its pricing so that it is less expensive than the competing store for any alteration job that takes an hour or more. By how much should the small store lower its consulting fee in order to make this adjustment?