

Polygons and Quadrilaterals

6A Polygons and Parallelograms

- Lab Construct Regular Polygons
- 6-1 Properties and Attributes of Polygons
- Lab Explore Properties of Parallelograms
- 6-2 Properties of Parallelograms
- 6-3 Conditions for Parallelograms

MULTI-STEP TEST PREP


6B Other Special Quadrilaterals

- 6-4 Properties of Special Parallelograms
- Lab Predict Conditions for Special Parallelograms
- 6-5 Conditions for Special Parallelograms
- Lab Explore Isosceles Trapezoids
- 6-6 Properties of Kites and Trapezoids

MULTI-STEP TEST PREP

Divide and Conquer

Some of the trickiest puzzles are based on simple polygonal shapes. You can use polygons to solve and create a variety of puzzles.

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Chapter Project Online

KEYWORD: MG7 ChProj

ARE YOU READY?

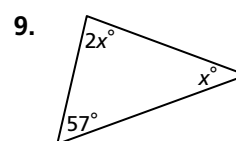
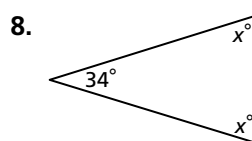
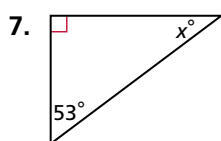
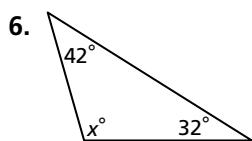
✓ Vocabulary

Match each term on the left with a definition on the right.

- | | |
|------------------------|--|
| 1. exterior angle | A. lines that intersect to form right angles |
| 2. parallel lines | B. lines in the same plane that do not intersect |
| 3. perpendicular lines | C. two angles of a polygon that share a side |
| 4. polygon | D. a closed plane figure formed by three or more segments that intersect only at their endpoints |
| 5. quadrilateral | E. a four-sided polygon |
| | F. an angle formed by one side of a polygon and the extension of a consecutive side |

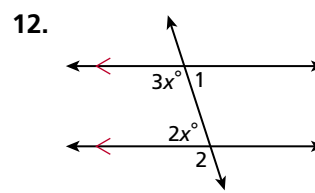
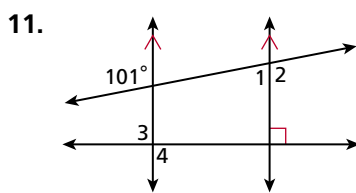
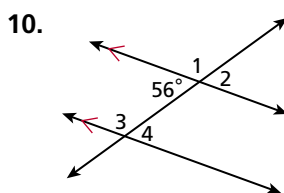
✓ Triangle Sum Theorem

Find the value of x .



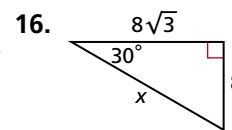
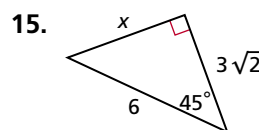
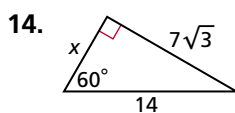
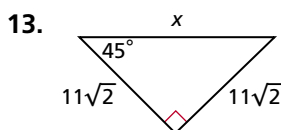
✓ Parallel Lines and Transversals

Find the measure of each numbered angle.



✓ Special Right Triangles

Find the value of x . Give the answer in simplest radical form.



✓ Conditional Statements

Tell whether the given statement is true or false. Write the converse. Tell whether the converse is true or false.

- If two angles form a linear pair, then they are supplementary.
- If two angles are congruent, then they are right angles.
- If a triangle is a scalene triangle, then it is an acute triangle.

Where You've Been

Previously, you

- learned properties of triangles.
- studied properties of parallel and perpendicular lines.
- classified triangles based on their side lengths and angle measures.
- wrote proofs involving congruent triangles.

In This Chapter

You will study

- properties of polygons.
- properties of special quadrilaterals.
- how to show that a polygon is a special quadrilateral.
- how to write proofs involving special quadrilaterals.

Where You're Going

You can use the skills learned in this chapter

- to find areas and volumes in geometry, algebra, and advanced math courses.
- to study motion and mechanics in physics courses.
- to use devices such as cameras and binoculars and to work on hobbies and craft projects outside of school.

Key Vocabulary/Vocabulario


concave	cóncavo
diagonal	diagonal
isosceles trapezoid	trapecio isósceles
kite	cometa
parallelogram	paralelogramo
rectangle	rectángulo
regular polygon	polígono regular
rhombus	rombo
square	cuadrado
trapezoid	trapecio

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. The word **concave** is made up of two parts: *con* and *cave*. Sketch a polygon that looks like it caves in.
2. In Greek, *dia* means “through” or “across,” and *gonia* means “angle” or “corner.” How can you use these meanings to understand the term **diagonal**?
3. If a triangle is *isosceles*, then it has two congruent legs. What do you think is a special property of an **isosceles trapezoid**?
4. A **parallelogram** has four sides. What do you think is a special property of the sides of a parallelogram?
5. One of the meanings of the word *regular* is “orderly.” What do you think the term **regular polygon** means?

Writing Strategy: Write a Convincing Argument


Throughout this book, the  icon identifies exercises that require you to write an explanation or argument to support an idea. Your response to a Write About It exercise shows that you have a solid understanding of the mathematical concept.

To be effective, a written argument should contain

- a clear statement of your mathematical claim.
- evidence or reasoning that supports your claim.

Example

From Lesson 5-4

-  36. **Write About It**
An isosceles triangle has two congruent sides. Does it also have two congruent midsegments? Explain.

Step 1 Make a statement of your mathematical claim.

Draw a sketch to investigate the properties of the midsegments of an isosceles triangle. You will find that the midsegments parallel to the legs of the isosceles triangle are congruent.

Claim: The midsegments parallel to the legs of an isosceles triangle are congruent.



Step 2 Give evidence to support your claim.

Identify any properties or theorems that support your claim. In this case, the Triangle Midsegment Theorem states that the length of a midsegment of a triangle is $\frac{1}{2}$ the length of the parallel side.

To clarify your argument, label your diagram and use it in your response.

Step 3 Write a complete response.

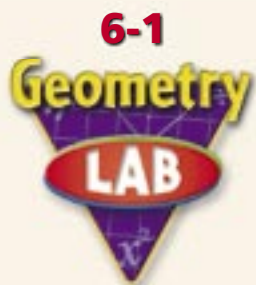
Yes, the two midsegments parallel to the legs of an isosceles triangle are congruent. Suppose $\triangle ABC$ is isosceles with $\overline{AB} \cong \overline{AC}$. \overline{XZ} and \overline{YZ} are midsegments of $\triangle ABC$. By the Triangle Midsegment Theorem, $XZ = \frac{1}{2}AC$ and $YZ = \frac{1}{2}AB$. Since $\overline{AB} \cong \overline{AC}$, $AB = AC$. So $\frac{1}{2}AB = \frac{1}{2}AC$ by the Multiplication Property of Equality. By substitution, $XZ = YZ$, so $\overline{XZ} \cong \overline{YZ}$.



Try This

Write a convincing argument.

1. Compare the circumcenter and the incenter of a triangle.
2. If you know the side lengths of a triangle, how do you determine which angle is the largest?



Construct Regular Polygons

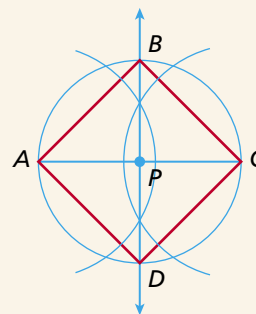
In Chapter 4, you learned that an equilateral triangle is a triangle with three congruent sides. You also learned that an equilateral triangle is equiangular, meaning that all its angles are congruent.

In this lab, you will construct polygons that are both equilateral and equiangular by inscribing them in circles.

Use with Lesson 6-1

Activity 1

- 1 Construct circle P . Draw a diameter \overline{AC} .
- 2 Construct the perpendicular bisector of \overline{AC} . Label the intersections of the bisector and the circle as B and D .
- 3 Draw \overline{AB} , \overline{BC} , \overline{CD} , and \overline{DA} . The polygon $ABCD$ is a *regular quadrilateral*. This means it is a four-sided polygon that has four congruent sides and four congruent angles.

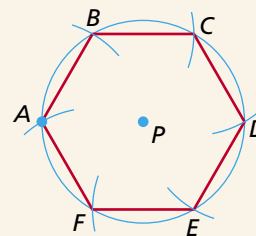


Try This

1. Describe a different method for constructing a regular quadrilateral.
2. The regular quadrilateral in Activity 1 is inscribed in the circle. What is the relationship between the circle and the regular quadrilateral?
3. A *regular octagon* is an eight-sided polygon that has eight congruent sides and eight congruent angles. Use angle bisectors to construct a regular octagon from a regular quadrilateral.

Activity 2

- 1 Construct circle P . Draw a point A on the circle.
- 2 Use the same compass setting. Starting at A , draw arcs to mark off equal parts along the circle. Label the other points where the arcs intersect the circle as B , C , D , E , and F .
- 3 Draw \overline{AB} , \overline{BC} , \overline{CD} , \overline{DE} , \overline{EF} , and \overline{FA} . The polygon $ABCDEF$ is a *regular hexagon*. This means it is a six-sided polygon that has six congruent sides and six congruent angles.

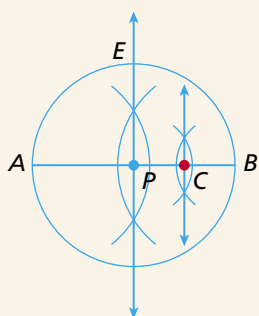


Try This

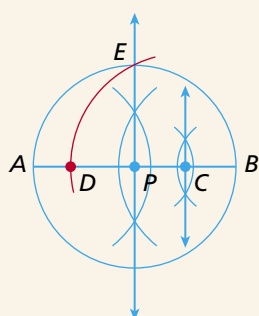
4. Justify the conclusion that $ABCDEF$ is a regular hexagon. (*Hint*: Draw diameters \overline{AD} , \overline{BE} , and \overline{CF} . What types of triangles are formed?)
5. A *regular dodecagon* is a 12-sided polygon that has 12 congruent sides and 12 congruent angles. Use the construction of a regular hexagon to construct a regular dodecagon. Explain your method.

Activity 3

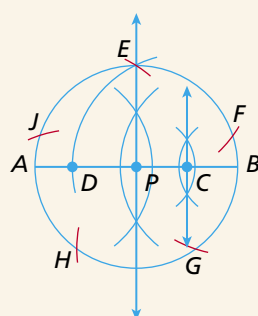
- 1 Construct circle P . Draw a diameter \overline{AB} .
- 2 Construct the perpendicular bisector of \overline{AB} . Label one point where the bisector intersects the circle as point E .
- 3 Construct the midpoint of radius \overline{PB} . Label it as point C .
- 4 Set your compass to the length CE . Place the compass point at C and draw an arc that intersects \overline{AB} . Label the point of intersection D .
- 5 Set the compass to the length ED . Starting at E , draw arcs to mark off equal parts along the circle. Label the other points where the arcs intersect the circle as F , G , H , and J .
- 6 Draw \overline{EF} , \overline{FG} , \overline{GH} , \overline{HJ} , and \overline{JE} . The polygon $EFGHJ$ is a *regular pentagon*. This means it is a five-sided polygon that has five congruent sides and five congruent angles.



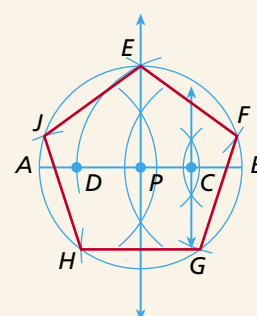
Steps 1–3



Step 4



Step 5



Step 6

Try This

6. A *regular decagon* is a ten-sided polygon that has ten congruent sides and ten congruent angles. Use the construction of a regular pentagon to construct a regular decagon. Explain your method.
7. Measure each angle of the regular polygons in Activities 1–3 and complete the following table.

REGULAR POLYGONS				
Number of Sides	3	4	5	6
Measure of Each Angle	60°			
Sum of Angle Measures	180°			

8. **Make a Conjecture** What is a general rule for finding the sum of the angle measures in a regular polygon with n sides?
9. **Make a Conjecture** What is a general rule for finding the measure of each angle in a regular polygon with n sides?

6-1

Properties and Attributes of Polygons

Objectives

Classify polygons based on their sides and angles.

Find and use the measures of interior and exterior angles of polygons.

Vocabulary

side of a polygon
vertex of a polygon
diagonal
regular polygon
concave
convex

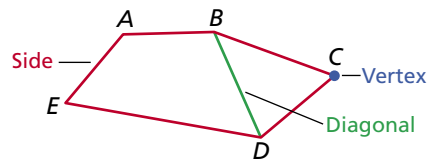
Why learn this?

The opening that lets light into a camera lens is created by an aperture, a set of blades whose edges may form a polygon. (See Example 5.)



In Lesson 2-4, you learned the definition of a polygon. Now you will learn about the parts of a polygon and about ways to classify polygons.

Each segment that forms a polygon is a **side of the polygon**. The common endpoint of two sides is a **vertex of the polygon**. A segment that connects any two nonconsecutive vertices is a **diagonal**.



You can name a polygon by the number of its sides. The table shows the names of some common polygons. Polygon $ABCDE$ is a pentagon.

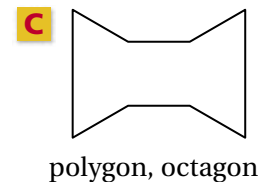
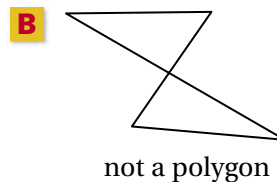
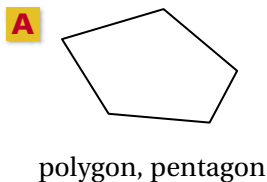
Number of Sides	Name of Polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n -gon

EXAMPLE 1 Identifying Polygons

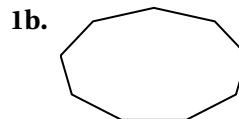
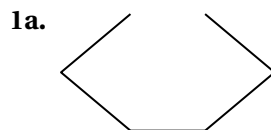
Remember!

A polygon is a closed plane figure formed by three or more segments that intersect only at their endpoints.

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.

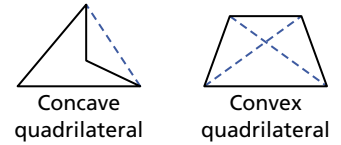


Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



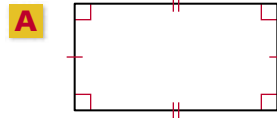
All the sides are congruent in an equilateral polygon. All the angles are congruent in an equiangular polygon. A **regular polygon** is one that is both equilateral and equiangular. If a polygon is not regular, it is called irregular.

A polygon is **concave** if any part of a diagonal contains points in the exterior of the polygon. If no diagonal contains points in the exterior, then the polygon is **convex**. A regular polygon is always convex.

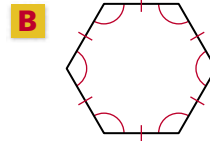


EXAMPLE 2 Classifying Polygons

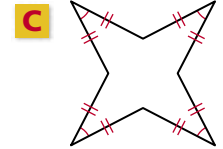
Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.



irregular, convex



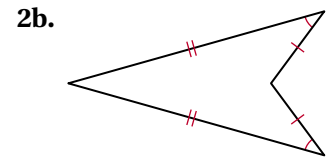
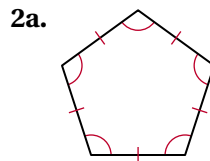
regular, convex



irregular, concave



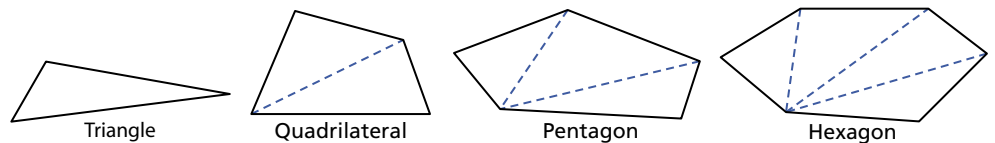
Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.



Remember!

By the Triangle Sum Theorem, the sum of the interior angle measures of a triangle is 180° .

To find the sum of the interior angle measures of a convex polygon, draw all possible diagonals from one vertex of the polygon. This creates a set of triangles. The sum of the angle measures of all the triangles equals the sum of the angle measures of the polygon.



Polygon	Number of Sides	Number of Triangles	Sum of Interior Angle Measures
Triangle	3	1	(1) $180^\circ = 180^\circ$
Quadrilateral	4	2	(2) $180^\circ = 360^\circ$
Pentagon	5	3	(3) $180^\circ = 540^\circ$
Hexagon	6	4	(4) $180^\circ = 720^\circ$
n -gon	n	$n - 2$	$(n - 2)180^\circ$

In each convex polygon, the number of triangles formed is two less than the number of sides n . So the sum of the angle measures of all these triangles is $(n - 2)180^\circ$.



Theorem 6-1-1 Polygon Angle Sum Theorem

The sum of the interior angle measures of a convex polygon with n sides is $(n - 2)180^\circ$.

EXAMPLE 3 Finding Interior Angle Measures and Sums in Polygons

A Find the sum of the interior angle measures of a convex octagon.

$$(n - 2)180^\circ \quad \text{Polygon } \angle \text{ Sum Thm.}$$

$$(8 - 2)180^\circ \quad \text{An octagon has 8 sides, so substitute 8 for } n.$$

$$1080^\circ \quad \text{Simplify.}$$

B Find the measure of each interior angle of a regular nonagon.

Step 1 Find the sum of the interior angle measures.

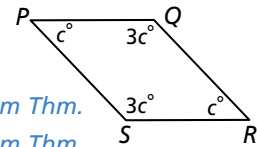
$$(n - 2)180^\circ \quad \text{Polygon } \angle \text{ Sum Thm.}$$

$$(9 - 2)180^\circ = 1260^\circ \quad \text{Substitute 9 for } n \text{ and simplify.}$$

Step 2 Find the measure of one interior angle.

$$\frac{1260^\circ}{9} = 140^\circ \quad \text{The int. } \angle \text{ are } \cong, \text{ so divide by 9.}$$

C Find the measure of each interior angle of quadrilateral PQRS.



$$(4 - 2)180^\circ = 360^\circ \quad \text{Polygon } \angle \text{ Sum Thm.}$$

$$m\angle P + m\angle Q + m\angle R + m\angle S = 360^\circ \quad \text{Polygon } \angle \text{ Sum Thm.}$$

$$c + 3c + c + 3c = 360 \quad \text{Substitute.}$$

$$8c = 360 \quad \text{Combine like terms.}$$

$$c = 45 \quad \text{Divide both sides by 8.}$$

$$m\angle P = m\angle R = 45^\circ$$

$$m\angle Q = m\angle S = 3(45^\circ) = 135^\circ$$

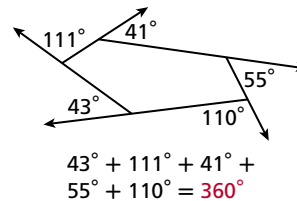
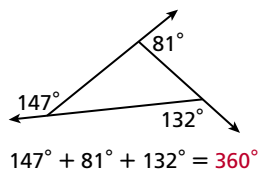
xy Algebra



3a. Find the sum of the interior angle measures of a convex 15-gon.

3b. Find the measure of each interior angle of a regular decagon.

In the polygons below, an exterior angle has been measured at each vertex. Notice that in each case, the sum of the exterior angle measures is 360° .



Remember!

An exterior angle is formed by one side of a polygon and the extension of a consecutive side.

Know it!

Note

Theorem 6-1-2 Polygon Exterior Angle Sum Theorem

The sum of the exterior angle measures, one angle at each vertex, of a convex polygon is 360° .

EXAMPLE 4 Finding Exterior Angle Measures in Polygons

A Find the measure of each exterior angle of a regular hexagon.

A hexagon has 6 sides and 6 vertices.

$$\text{sum of ext. } \angle = 360^\circ$$

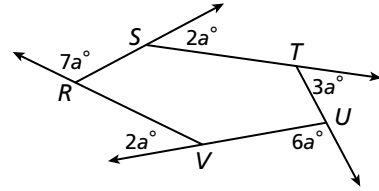
$$\text{measure of one ext. } \angle = \frac{360^\circ}{6} = 60^\circ$$

Polygon Ext. } \angle \text{ Sum Thm.}

A regular hexagon has 6 } \cong \text{ ext. } \angle, \text{ so divide the sum by 6.}

The measure of each exterior angle of a regular hexagon is 60° .

B Find the value of a in polygon $RSTUV$.



$$7a^\circ + 2a^\circ + 3a^\circ + 6a^\circ + 2a^\circ = 360^\circ$$

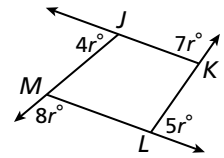
$$20a = 360$$

$$a = 18$$

*Polygon Ext. \angle Sum Thm.
Combine like terms.
Divide both sides by 20.*

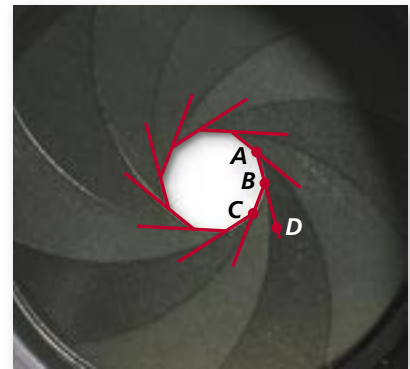


- 4a. Find the measure of each exterior angle of a regular dodecagon.
4b. Find the value of r in polygon $JKLM$.



EXAMPLE 5 Photography Application

The aperture of the camera is formed by ten blades. The blades overlap to form a regular decagon. What is the measure of $\angle CBD$?



$\angle CBD$ is an exterior angle of a regular decagon. By the Polygon Exterior Angle Sum Theorem, the sum of the exterior angle measures is 360° .

$$m\angle CBD = \frac{360^\circ}{10} = 36^\circ$$

A regular decagon has 10 \cong ext. \angle , so divide the sum by 10.



5. **What if...?** Suppose the shutter were formed by 8 blades. What would the measure of each exterior angle be?

THINK AND DISCUSS

1. Draw a concave pentagon and a convex pentagon. Explain the difference between the two figures.
2. Explain why you cannot use the expression $\frac{360^\circ}{n}$ to find the measure of an exterior angle of an irregular n -gon.
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each cell, write the formula for finding the indicated value for a regular convex polygon with n sides.



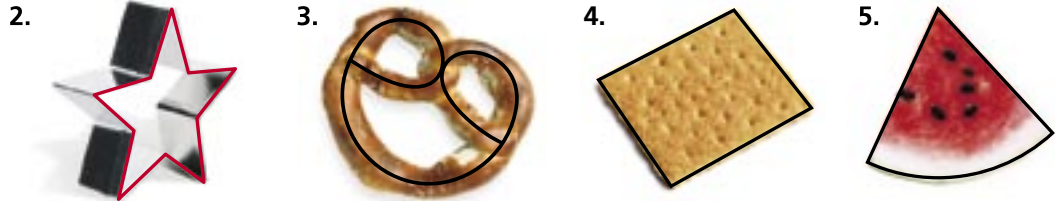
	Interior Angles	Exterior Angles
Sum of Angle Measures		
One Angle Measure		

GUIDED PRACTICE

1. **Vocabulary** Explain why an equilateral polygon is not necessarily a *regular* polygon.

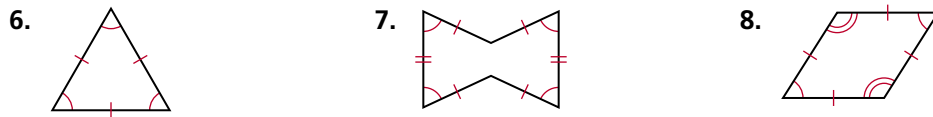
SEE EXAMPLE 1
p. 382

Tell whether each outlined shape is a polygon. If it is a polygon, name it by the number of its sides.



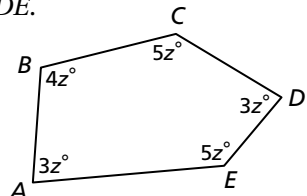
SEE EXAMPLE 2
p. 383

Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.



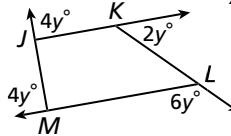
SEE EXAMPLE 3
p. 384

9. Find the measure of each interior angle of pentagon $ABCDE$.
 10. Find the measure of each interior angle of a regular dodecagon.
 11. Find the sum of the interior angle measures of a convex 20-gon.



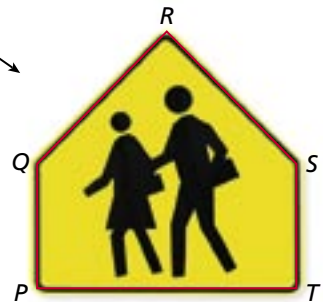
SEE EXAMPLE 4
p. 384

12. Find the value of y in polygon $JKLM$.
 13. Find the measure of each exterior angle of a regular pentagon.



SEE EXAMPLE 5
p. 385

- Safety** Use the photograph of the traffic sign for Exercises 14 and 15.
 14. Name the polygon by the number of its sides.
 15. In the polygon, $\angle P$, $\angle R$, and $\angle T$ are right angles, and $\angle Q \cong \angle S$. What are $m\angle Q$ and $m\angle S$?

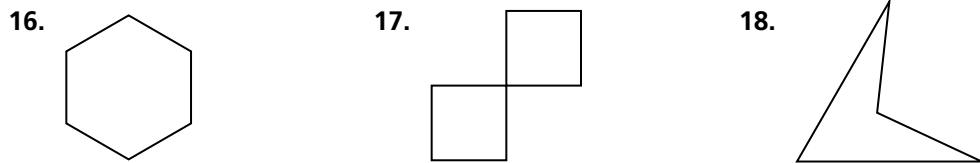


PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
16–18	1
19–21	2
22–24	3
25–26	4
27–28	5

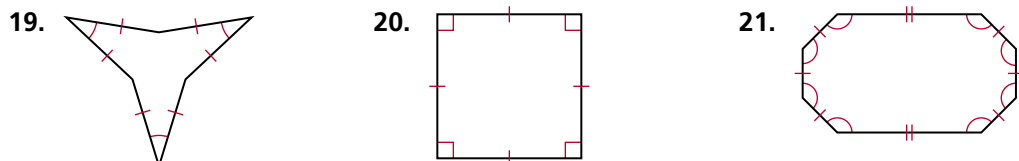
Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



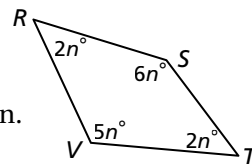
Extra Practice

Skills Practice p. S14
 Application Practice p. S33

Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.



22. Find the measure of each interior angle of quadrilateral $RSTV$.
23. Find the measure of each interior angle of a regular 18-gon.
24. Find the sum of the interior angle measures of a convex heptagon.
25. Find the measure of each exterior angle of a regular nonagon.



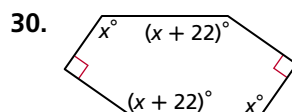
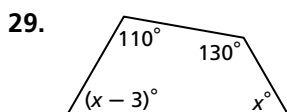
26. A pentagon has exterior angle measures of $5a^\circ$, $4a^\circ$, $10a^\circ$, $3a^\circ$, and $8a^\circ$. Find the value of a .



Crafts The folds on the lid of the gift box form a regular hexagon. Find each measure.

27. $m\angle JKM$
28. $m\angle MKL$

Algebra Find the value of x in each figure.



Find the number of sides a regular polygon must have to meet each condition.

32. Each interior angle measure equals each exterior angle measure.
33. Each interior angle measure is four times the measure of each exterior angle.
34. Each exterior angle measure is one eighth the measure of each interior angle.

Name the convex polygon whose interior angle measures have each given sum.

35. 540° 36. 900° 37. 1800° 38. 2520°

Multi-Step An exterior angle measure of a regular polygon is given. Find the number of its sides and the measure of each interior angle.

39. 120° 40. 72° 41. 36° 42. 24°

43. **/// ERROR ANALYSIS ///** Which conclusion is incorrect? Explain the error.

A The figure is a polygon.

B The figure is not a polygon.

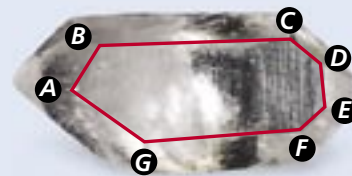


44. **Estimation** Graph the polygon formed by the points $A(-2, -6)$, $B(-4, -1)$, $C(-1, 2)$, $D(4, 0)$, and $E(3, -5)$. Estimate the measure of each interior angle. Make a conjecture about whether the polygon is equiangular. Now measure each interior angle with a protractor. Was your conjecture correct?

MULTI-STEP TEST PREP



45. This problem will prepare you for the Multi-Step Test Prep on page 406. In this quartz crystal, $m\angle A = 95^\circ$, $m\angle B = 125^\circ$, $m\angle E = m\angle D = 130^\circ$, and $\angle C \cong \angle F \cong \angle G$.
- Name polygon $ABCDEFG$ by the number of sides.
 - What is the sum of the interior angle measures of $ABCDEFG$?
 - Find $m\angle F$.



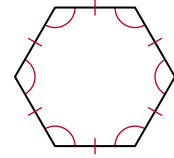
46. The perimeter of a regular polygon is 45 inches. The length of one side is 7.5 inches. Name the polygon by the number of its sides.

Draw an example of each figure.

47. a regular quadrilateral 48. an irregular concave heptagon
49. an irregular convex pentagon 50. an equilateral polygon that is not equiangular

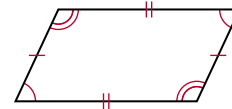


51. **Write About It** Use the terms from the lesson to describe the figure as specifically as possible.
52. **Critical Thinking** What geometric figure does a regular polygon begin to resemble as the number of sides increases?



53. Which terms describe the figure shown?
I. quadrilateral II. concave III. regular

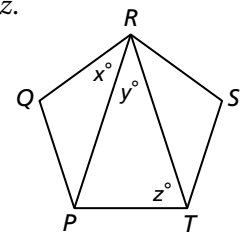
- (A) I only (C) I and II
(B) II only (D) I and III



54. Which statement is NOT true about a regular 16-gon?
(F) It is a convex polygon.
(G) It has 16 congruent sides.
(H) The sum of the interior angle measures is 2880°.
(J) The sum of the exterior angles, one at each vertex, is 360°.
55. In polygon $ABCD$, $m\angle A = 49^\circ$, $m\angle B = 107^\circ$, and $m\angle C = 2m\angle D$. What is $m\angle C$?
(A) 24° (B) 68° (C) 102° (D) 136°

CHALLENGE AND EXTEND

56. The interior angle measures of a convex pentagon are consecutive multiples of 4. Find the measure of each interior angle.
57. Polygon $PQRST$ is a regular pentagon. Find the values of x , y , and z .
58. **Multi-Step** Polygon $ABCDEFGHIJK$ is a regular decagon. Sides \overline{AB} and \overline{DE} are extended so that they meet at point L in the exterior of the polygon. Find $m\angle BLD$.
59. **Critical Thinking** Does the Polygon Angle Sum Theorem work for concave polygons? Draw a sketch to support your answer.



SPIRAL REVIEW

Solve by factoring. (*Previous course*)

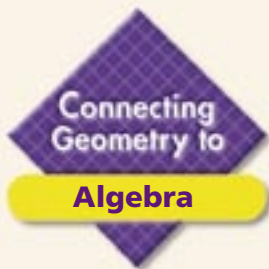
60. $x^2 + 3x - 10 = 0$ 61. $x^2 - x - 12 = 0$ 62. $x^2 - 12x = -35$

The lengths of two sides of a triangle are given. Find the range of possible lengths for the third side. (*Lesson 5-5*)

63. 4, 4 64. 6, 12 65. 3, 7

Find each side length for a 30°-60°-90° triangle. (*Lesson 5-8*)

66. the length of the hypotenuse when the length of the shorter leg is 6
67. the length of the longer leg when the length of the hypotenuse is 10



Relations and Functions

Connecting
Geometry to

Algebra

See Skills Bank
page S61

Many numeric relationships in geometry can be represented by algebraic relations. These relations may or may not be functions, depending on their domain and range.

A *relation* is a set of ordered pairs. All the first coordinates in the set of ordered pairs are the *domain* of the relation. All the second coordinates are the *range* of the relation.

A *function* is a type of relation that pairs each element in the domain with exactly one element in the range.

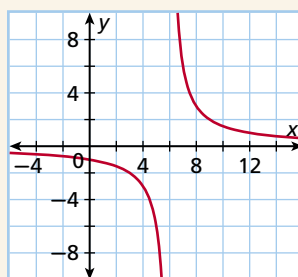
Example

Give the domain and range of the relation $y = \frac{6}{x-6}$. Tell whether the relation is a function.

Step 1 Make a table of values for the relation.

x	-6	0	5	6	7	12
y	-0.5	-1	-6	Undefined	6	1

Step 2 Plot the points and connect them with smooth curves.



Step 3 Identify the domain and range.

Since y is undefined at $x = 6$, the domain of the relation is the set of all real numbers except 6. Since there is no x -value such that $y = 0$, the range of the relation is the set of all real numbers except 0.

Step 4 Determine whether the relation is a function.

From the graph, you can see that only one y -value exists for each x -value, so the relation is a function.

Try This

Give the domain and range of each relation. Tell whether the relation is a function.

1. $y = (x - 2)180$

2. $y = 360$

3. $y = \frac{(x - 2)180}{x}$

4. $y = \frac{360}{x}$

5. $x = 3y - 10$

6. $x^2 + y^2 = 9$

7. $x = -2$

8. $y = x^2 + 4$

9. $-x + 8y = 5$

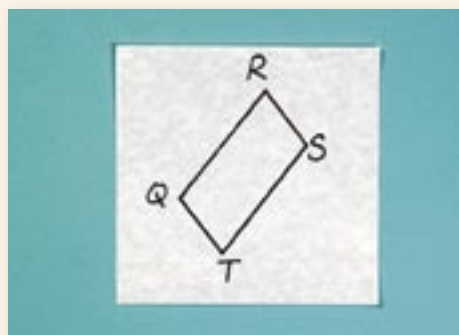
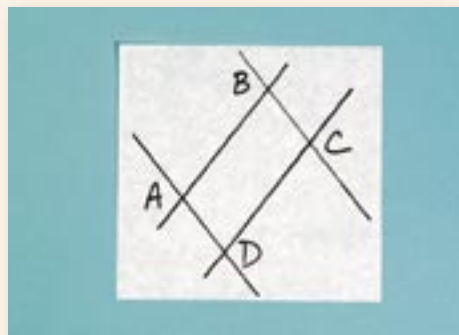
Explore Properties of Parallelograms

In this lab you will investigate the relationships among the angles and sides of a special type of quadrilateral called a *parallelogram*. You will need to apply the Transitive Property of Congruence. That is, if figure $A \cong$ figure B and figure $B \cong$ figure C , then figure $A \cong$ figure C .

Use with Lesson 6-2

Activity

- Use opposite sides of an index card to draw a set of parallel lines on a piece of patty paper. Then use opposite sides of a ruler to draw a second set of parallel lines that intersects the first. Label the points of intersection A , B , C , and D , in that order. Quadrilateral $ABCD$ has two pairs of parallel sides. It is a *parallelogram*.
- Place a second piece of patty paper over the first and trace $ABCD$. Label the points that correspond to A , B , C , and D as Q , R , S , and T , in that order. The parallelograms $ABCD$ and $QRST$ are congruent. Name all the pairs of congruent corresponding sides and angles.
- Lay $ABCD$ over $QRST$ so that \overline{AB} overlays \overline{ST} . What do you notice about their lengths? What does this tell you about \overline{AB} and \overline{CD} ? Now move $ABCD$ so that \overline{DA} overlays \overline{RS} . What do you notice about their lengths? What does this tell you about \overline{DA} and \overline{BC} ?
- Lay $ABCD$ over $QRST$ so that $\angle A$ overlays $\angle S$. What do you notice about their measures? What does this tell you about $\angle A$ and $\angle C$? Now move $ABCD$ so that $\angle B$ overlays $\angle T$. What do you notice about their measures? What does this tell you about $\angle B$ and $\angle D$?
- Arrange the pieces of patty paper so that \overline{RS} overlays \overline{AD} . What do you notice about \overline{QR} and \overline{AB} ? What does this tell you about $\angle A$ and $\angle R$? What can you conclude about $\angle A$ and $\angle B$?
- Draw diagonals \overline{AC} and \overline{BD} . Fold $ABCD$ so that A matches C , making a crease. Unfold the paper and fold it again so that B matches D , making another crease. What do you notice about the creases? What can you conclude about the diagonals?



Try This

- Repeat the above steps with a different parallelogram. Do you get the same results?
- Make a Conjecture** How do you think the sides of a parallelogram are related to each other? the angles? the diagonals? Write your conjectures as conditional statements.

6-2

Properties of Parallelograms

Objectives

Prove and apply properties of parallelograms.
Use properties of parallelograms to solve problems.

Vocabulary
parallelogram

Who uses this?

Race car designers can use a parallelogram-shaped linkage to keep the wheels of the car vertical on uneven surfaces. (See Example 1.)



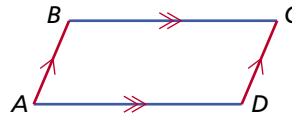
Any polygon with four sides is a quadrilateral. However, some quadrilaterals have special properties. These *special quadrilaterals* are given their own names.

A quadrilateral with two pairs of parallel sides is a **parallelogram**. To write the name of a parallelogram, you use the symbol \square .

Helpful Hint

Opposite sides of a quadrilateral do not share a vertex. Opposite angles do not share a side.

Parallelogram $ABCD$
 $\square ABCD$



$$\overline{AB} \parallel \overline{CD}, \overline{BC} \parallel \overline{DA}$$



Theorem 6-2-1 Properties of Parallelograms

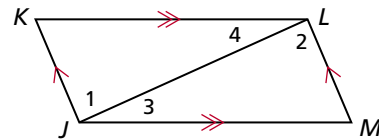
THEOREM	HYPOTHESIS	CONCLUSION
If a quadrilateral is a parallelogram, then its opposite sides are congruent. ($\square \rightarrow$ opp. sides \cong)		$\overline{AB} \cong \overline{CD}$ $\overline{BC} \cong \overline{DA}$

PROOF

Theorem 6-2-1

Given: $JKLM$ is a parallelogram.
Prove: $\overline{JK} \cong \overline{LM}, \overline{KL} \cong \overline{MJ}$

Proof:



Statements	Reasons
1. $JKLM$ is a parallelogram.	1. Given
2. $\overline{JK} \parallel \overline{LM}, \overline{KL} \parallel \overline{MJ}$	2. Def. of \square
3. $\angle 1 \cong \angle 2, \angle 3 \cong \angle 4$	3. Alt. Int. \triangle Thm.
4. $\overline{JL} \cong \overline{JL}$	4. Reflex. Prop. of \cong
5. $\triangle JKL \cong \triangle LMJ$	5. ASA Steps 3, 4
6. $\overline{JK} \cong \overline{LM}, \overline{KL} \cong \overline{MJ}$	6. CPCTC



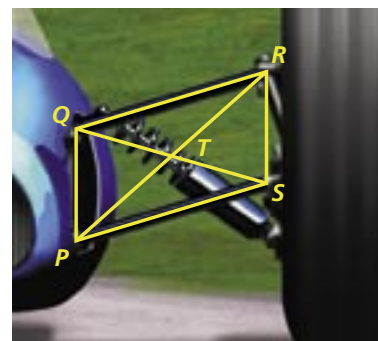
Theorems Properties of Parallelograms

THEOREM	HYPOTHESIS	CONCLUSION
6-2-2 If a quadrilateral is a parallelogram, then its opposite angles are congruent. ($\square \rightarrow \text{opp. } \angle \cong$)		$\angle A \cong \angle C$ $\angle B \cong \angle D$
6-2-3 If a quadrilateral is a parallelogram, then its consecutive angles are supplementary. ($\square \rightarrow \text{cons. } \angle \text{ supp.}$)		$m\angle A + m\angle B = 180^\circ$ $m\angle B + m\angle C = 180^\circ$ $m\angle C + m\angle D = 180^\circ$ $m\angle D + m\angle A = 180^\circ$
6-2-4 If a quadrilateral is a parallelogram, then its diagonals bisect each other. ($\square \rightarrow \text{diags. bisect each other}$)		$\overline{AZ} \cong \overline{CZ}$ $\overline{BZ} \cong \overline{DZ}$

You will prove Theorems 6-2-3 and 6-2-4 in Exercises 45 and 44.

EXAMPLE 1 Racing Application

The diagram shows the parallelogram-shaped linkage that joins the frame of a race car to one wheel of the car. In $\square PQRS$, $QR = 48$ cm, $RT = 30$ cm, and $m\angle QPS = 73^\circ$. Find each measure.



A PS
 $\overline{PS} \cong \overline{QR}$ $\square \rightarrow \text{opp. sides } \cong$
 $PS = QR$ *Def. of \cong segs.*
 $PS = 48$ cm *Substitute 48 for QR.*

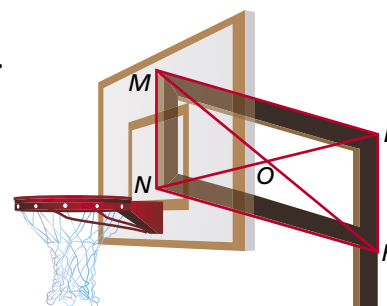
B $m\angle PQR$
 $m\angle PQR + m\angle QPS = 180^\circ$ $\square \rightarrow \text{cons. } \angle \text{ supp.}$
 $m\angle PQR + 73 = 180$ *Substitute 73 for $m\angle QPS$.*
 $m\angle PQR = 107^\circ$ *Subtract 73 from both sides.*

C PT
 $\overline{PT} \cong \overline{RT}$ $\square \rightarrow \text{diags. bisect each other}$
 $PT = RT$ *Def. of \cong segs.*
 $PT = 30$ cm *Substitute 30 for RT.*



In $\square KLMN$, $LM = 28$ in.,
 $LN = 26$ in., and $m\angle LKN = 74^\circ$.
 Find each measure.

- 1a. KN
- 1b. $m\angle NML$
- 1c. LO

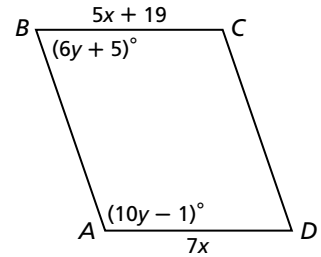


EXAMPLE 2 Using Properties of Parallelograms to Find Measures



$ABCD$ is a parallelogram. Find each measure.

- A** $\overline{AD} \cong \overline{BC}$ $\square \rightarrow \text{opp. sides} \cong$
 $AD = BC$ *Def. of \cong segs.*
 $7x = 5x + 19$ *Substitute the given values.*
 $2x = 19$ *Subtract $5x$ from both sides.*
 $x = 9.5$ *Divide both sides by 2.*
 $AD = 7x = 7(9.5) = 66.5$

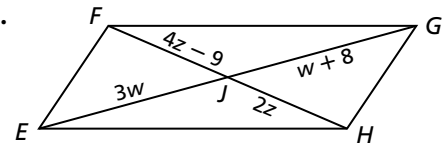


- B** $m\angle B$
 $m\angle A + m\angle B = 180^\circ$ $\square \rightarrow \text{cons. } \angle \text{ supp.}$
 $(10y - 1) + (6y + 5) = 180$ *Substitute the given values.*
 $16y + 4 = 180$ *Combine like terms.*
 $16y = 176$ *Subtract 4 from both sides.*
 $y = 11$ *Divide both sides by 16.*
 $m\angle B = (6y + 5)^\circ = [6(11) + 5]^\circ = 71^\circ$



$EFGH$ is a parallelogram. Find each measure.

- 2a. JG
 2b. FH



EXAMPLE 3 Parallelograms in the Coordinate Plane

Remember!

When you are drawing a figure in the coordinate plane, the name $ABCD$ gives the order of the vertices.

Three vertices of $\square ABCD$ are $A(1, -2)$, $B(-2, 3)$, and $D(5, -1)$. Find the coordinates of vertex C .
 Since $ABCD$ is a parallelogram, both pairs of opposite sides must be parallel.

Step 1 Graph the given points.

Step 2 Find the slope of \overline{AB} by counting the units from A to B .
 The rise from -2 to 3 is 5 .
 The run from 1 to -2 is -3 .

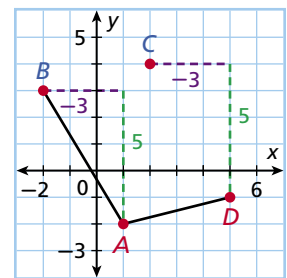
Step 3 Start at D and count the same number of units.
 A rise of 5 from -1 is 4 .
 A run of -3 from 5 is 2 . Label $(2, 4)$ as vertex C .

Step 4 Use the slope formula to verify that $\overline{BC} \parallel \overline{AD}$.

$$\text{slope of } \overline{BC} = \frac{4 - 3}{2 - (-2)} = \frac{1}{4}$$

$$\text{slope of } \overline{AD} = \frac{-1 - (-2)}{5 - 1} = \frac{1}{4}$$

The coordinates of vertex C are $(2, 4)$.



3. Three vertices of $\square PQRS$ are $P(-3, -2)$, $Q(-1, 4)$, and $S(5, 0)$. Find the coordinates of vertex R .

EXAMPLE 4 Using Properties of Parallelograms in a Proof

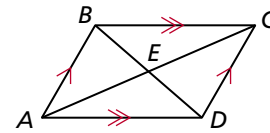
Write a two-column proof.

A Theorem 6-2-2

Given: $ABCD$ is a parallelogram.

Prove: $\angle BAD \cong \angle DCB$, $\angle ABC \cong \angle CDA$

Proof:

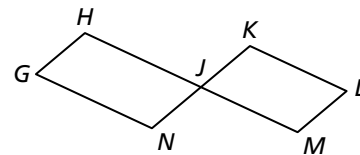


Statements	Reasons
1. $ABCD$ is a parallelogram.	1. Given
2. $\overline{AB} \cong \overline{CD}$, $\overline{DA} \cong \overline{BC}$	2. $\square \rightarrow$ opp. sides \cong
3. $\overline{BD} \cong \overline{BD}$	3. Reflex. Prop. of \cong
4. $\triangle BAD \cong \triangle DCB$	4. SSS Steps 2, 3
5. $\angle BAD \cong \angle DCB$	5. CPCTC
6. $\overline{AC} \cong \overline{AC}$	6. Reflex. Prop. of \cong
7. $\triangle ABC \cong \triangle CDA$	7. SSS Steps 2, 6
8. $\angle ABC \cong \angle CDA$	8. CPCTC

B Given: $GHJN$ and $JKLM$ are parallelograms. H and M are collinear. N and K are collinear.

Prove: $\angle G \cong \angle L$

Proof:



Statements	Reasons
1. $GHJN$ and $JKLM$ are parallelograms.	1. Given
2. $\angle HJN \cong \angle G$, $\angle MJK \cong \angle L$	2. $\square \rightarrow$ opp. $\angle \cong$
3. $\angle HJN \cong \angle MJK$	3. Vert. \angle Thm.
4. $\angle G \cong \angle L$	4. Trans. Prop. of \cong



4. Use the figure in Example 4B to write a two-column proof.

Given: $GHJN$ and $JKLM$ are parallelograms.

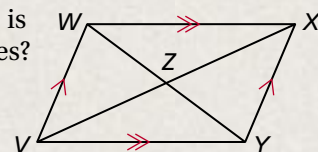
H and M are collinear. N and K are collinear.

Prove: $\angle N \cong \angle K$

THINK AND DISCUSS

1. The measure of one angle of a parallelogram is 71° . What are the measures of the other angles?

2. In $\square VWXY$, $VW = 21$, and $WY = 36$. Find as many other measures as you can. Justify your answers.



3. **GET ORGANIZED** Copy and complete the graphic organizer. In each cell, draw a figure with markings that represents the given property.

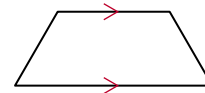


Properties of Parallelograms				
Opp. sides \parallel	Opp. sides \cong	Opp. $\angle \cong$	Cons. \angle supp.	Diags. bisect each other.

GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

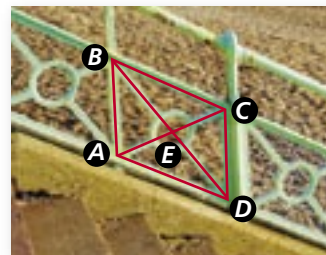
1. Explain why the figure at right is NOT a *parallelogram*.
2. Draw $\square PQRS$. Name the opposite sides and opposite angles.



SEE EXAMPLE 1
 p. 392

Safety The handrail is made from congruent parallelograms. In $\square ABCD$, $AB = 17.5$, $DE = 18$, and $m\angle BCD = 110^\circ$. Find each measure.

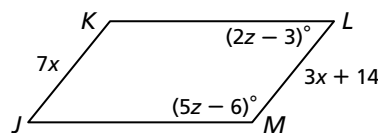
3. BD
4. CD
5. BE
6. $m\angle ABC$
7. $m\angle ADC$
8. $m\angle DAB$



SEE EXAMPLE 2
 p. 393

$JKLM$ is a parallelogram. Find each measure.

9. JK
10. LM
11. $m\angle L$
12. $m\angle M$

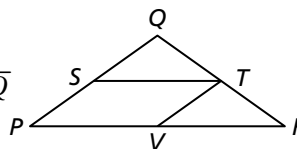


SEE EXAMPLE 3
 p. 393

Multi-Step Three vertices of $\square DFGH$ are $D(-9, 4)$, $F(-1, 5)$, and $G(2, 0)$. Find the coordinates of vertex H .

SEE EXAMPLE 4
 p. 394

14. Write a two-column proof.
 Given: $PSTV$ is a parallelogram. $\overline{PQ} \cong \overline{RQ}$
 Prove: $\angle STV \cong \angle R$



PRACTICE AND PROBLEM SOLVING

Independent Practice

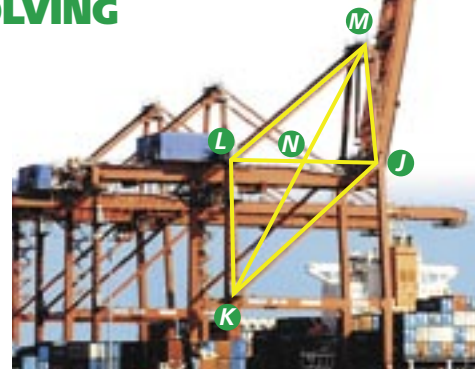
For Exercises	See Example
15–20	1
21–24	2
25	3
26	4

Extra Practice

Skills Practice p. S14
 Application Practice p. S33

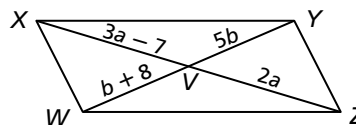
Shipping Cranes can be used to load cargo onto ships. In $\square JKLM$, $JL = 165.8$, $JK = 110$, and $m\angle JML = 50^\circ$. Find the measure of each part of the crane.

15. JN
16. LM
17. LN
18. $m\angle JKL$
19. $m\angle KLM$
20. $m\angle MJK$



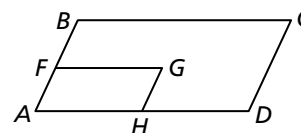
$WXYZ$ is a parallelogram. Find each measure.

21. WV
22. YW
23. XZ
24. ZV



Multi-Step Three vertices of $\square PRTV$ are $P(-4, -4)$, $R(-10, 0)$, and $V(5, -1)$. Find the coordinates of vertex T .

26. Write a two-column proof.
 Given: $ABCD$ and $AFGH$ are parallelograms.
 Prove: $\angle C \cong \angle G$

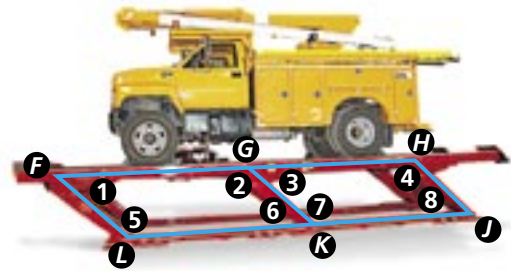


xy Algebra The perimeter of $\square PQRS$ is 84. Find the length of each side of $\square PQRS$ under the given conditions.

27. $PQ = QR$ 28. $QR = 3(RS)$ 29. $RS = SP - 7$ 30. $SP = RS^2$

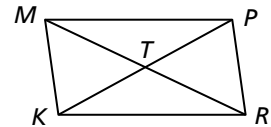
31. **Cars** To repair a large truck, a mechanic might use a *parallelogram lift*. In the lift, $\overline{FG} \cong \overline{GH} \cong \overline{LK} \cong \overline{KJ}$, and $\overline{FL} \cong \overline{GK} \cong \overline{HJ}$.

- Which angles are congruent to $\angle 1$? Justify your answer.
- What is the relationship between $\angle 1$ and each of the remaining labeled angles? Justify your answer.

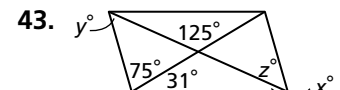
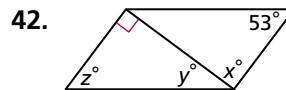
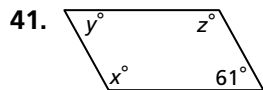


Complete each statement about $\square KMPR$. Justify your answer.

32. $\angle MPR \cong$? 33. $\angle PRK \cong$? 34. $\overline{MT} \cong$?
 35. $\overline{PR} \cong$? 36. $\overline{MP} \parallel$? 37. $\overline{MK} \parallel$?
 38. $\angle MPK \cong$? 39. $\angle MTK \cong$? 40. $m\angle MKR + m\angle PRK =$?

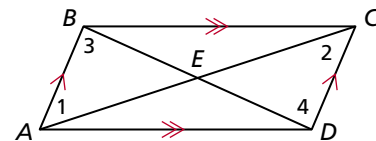


Find the values of x , y , and z in each parallelogram.



44. Complete the paragraph proof of Theorem 6-2-4 by filling in the blanks.

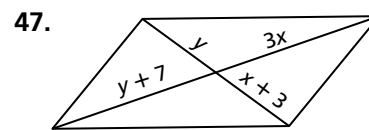
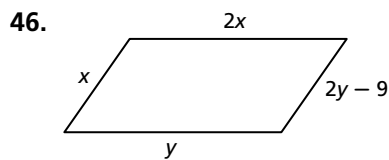
Given: $ABCD$ is a parallelogram.
Prove: \overline{AC} and \overline{BD} bisect each other at E .



Proof: It is given that $ABCD$ is a parallelogram. By the definition of a parallelogram, $\overline{AB} \parallel$ a. ? . By the Alternate Interior Angles Theorem, $\angle 1 \cong$ b. ? , and $\angle 3 \cong$ c. ? . $\overline{AB} \cong \overline{CD}$ because d. ? . This means that $\triangle ABE \cong \triangle CDE$ by e. ? . So by f. ? , $\overline{AE} \cong \overline{CE}$, and $\overline{BE} \cong \overline{DE}$. Therefore \overline{AC} and \overline{BD} bisect each other at E by the definition of g. ? .

45. Write a two-column proof of Theorem 6-2-3: If a quadrilateral is a parallelogram, then its consecutive angles are supplementary.

xy Algebra Find the values of x and y in each parallelogram.



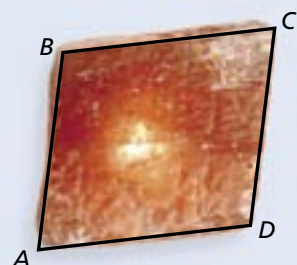
MULTI-STEP TEST PREP



48. This problem will prepare you for the Multi-Step Test Prep on page 406.

In this calcite crystal, the face $ABCD$ is a parallelogram.

- In $\square ABCD$, $m\angle B = (6x + 12)^\circ$, and $m\angle D = (9x - 33)^\circ$. Find $m\angle B$.
- Find $m\angle A$ and $m\angle C$. Which theorem or theorems did you use to find these angle measures?

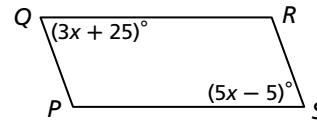


49. **Critical Thinking** Draw any parallelogram. Draw a second parallelogram whose corresponding sides are congruent to the sides of the first parallelogram but whose corresponding angles are not congruent to the angles of the first.
- Is there an SSSS congruence postulate for parallelograms? Explain.
 - Remember the meaning of triangle rigidity. Is a parallelogram rigid? Explain.
50. **Write About It** Explain why every parallelogram is a quadrilateral but every quadrilateral is not necessarily a parallelogram.



51. What is the value of x in $\square PQRS$?

- (A) 15 (C) 30
(B) 20 (D) 70



52. The diagonals of $\square JKLM$ intersect at Z . Which statement is true?

- (F) $JL = KM$ (G) $JL = \frac{1}{2}KM$ (H) $JL = \frac{1}{2}JZ$ (J) $JL = 2JZ$

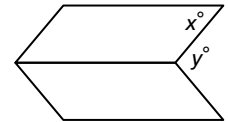
53. **Gridded Response** In $\square ABCD$, $BC = 8.2$, and $CD = 5$. What is the perimeter of $\square ABCD$?

CHALLENGE AND EXTEND

The coordinates of three vertices of a parallelogram are given. Give the coordinates for all possible locations of the fourth vertex.

54. $(0, 5)$, $(4, 0)$, $(8, 5)$ 55. $(-2, 1)$, $(3, -1)$, $(-1, -4)$

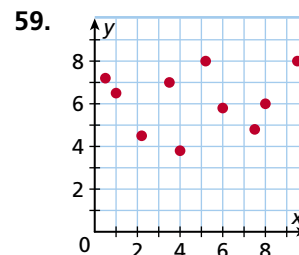
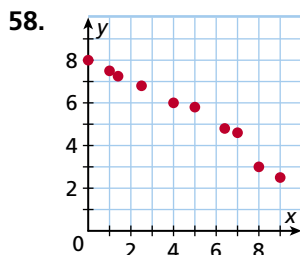
56. The feathers on an arrow form two congruent parallelograms that share a common side. Each parallelogram is the reflection of the other across the line they share. Show that $y = 2x$.



57. Prove that the bisectors of two consecutive angles of a parallelogram are perpendicular.

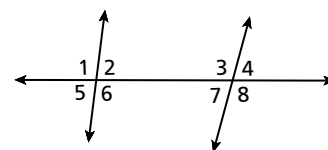
SPIRAL REVIEW

Describe the correlation shown in each scatter plot as positive, negative, or no correlation. (*Previous course*)



Classify each angle pair. (*Lesson 3-1*)

60. $\angle 2$ and $\angle 7$ 61. $\angle 5$ and $\angle 4$
62. $\angle 6$ and $\angle 7$ 63. $\angle 1$ and $\angle 3$



An interior angle measure of a regular polygon is given. Find the number of sides and the measure of each exterior angle. (*Lesson 6-1*)

64. 120° 65. 135° 66. 156°

6-3

Conditions for Parallelograms



Objective

Prove that a given quadrilateral is a parallelogram.

Who uses this?

A bird watcher can use a *parallelogram mount* to adjust the height of a pair of binoculars without changing the viewing angle. (See Example 4.)

You have learned to identify the properties of a parallelogram. Now you will be given the properties of a quadrilateral and will have to tell if the quadrilateral is a parallelogram. To do this, you can use the definition of a parallelogram or the conditions below.



Theorems Conditions for Parallelograms

THEOREM	EXAMPLE
6-3-1 If one pair of opposite sides of a quadrilateral are parallel and congruent, then the quadrilateral is a parallelogram. (quad. with pair of opp. sides \parallel and $\cong \rightarrow \square$)	
6-3-2 If both pairs of opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. sides $\cong \rightarrow \square$)	
6-3-3 If both pairs of opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram. (quad. with opp. $\sphericalangle \cong \rightarrow \square$)	

Remember!

In the converse of a theorem, the hypothesis and conclusion are exchanged.

You will prove Theorems 6-3-2 and 6-3-3 in Exercises 26 and 29.

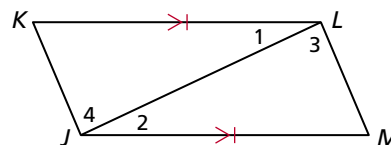
PROOF

Theorem 6-3-1

Given: $\overline{KL} \parallel \overline{MJ}$, $\overline{KL} \cong \overline{MJ}$
 Prove: $JKLM$ is a parallelogram.

Proof:

It is given that $\overline{KL} \cong \overline{MJ}$. Since $\overline{KL} \parallel \overline{MJ}$, $\angle 1 \cong \angle 2$ by the Alternate Interior Angles Theorem. By the Reflexive Property of Congruence, $\overline{JL} \cong \overline{JL}$. So $\triangle JKL \cong \triangle LMJ$ by SAS. By CPCTC, $\angle 3 \cong \angle 4$, and $\overline{JK} \parallel \overline{LM}$ by the Converse of the Alternate Interior Angles Theorem. Since the opposite sides of $JKLM$ are parallel, $JKLM$ is a parallelogram by definition.



The two theorems below can also be used to show that a given quadrilateral is a parallelogram.



Theorems Conditions for Parallelograms

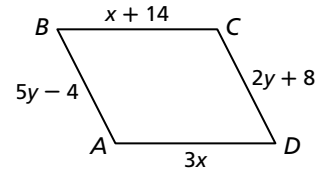
THEOREM	EXAMPLE
<p>6-3-4 If an angle of a quadrilateral is supplementary to both of its consecutive angles, then the quadrilateral is a parallelogram. (quad. with \angle supp. to cons. \angle \rightarrow \square)</p>	
<p>6-3-5 If the diagonals of a quadrilateral bisect each other, then the quadrilateral is a parallelogram. (quad. with diags. bisecting each other \rightarrow \square)</p>	

You will prove Theorems 6-3-4 and 6-3-5 in Exercises 27 and 30.

EXAMPLE 1 Verifying Figures are Parallelograms



A Show that $ABCD$ is a parallelogram for $x = 7$ and $y = 4$.



Step 1 Find BC and DA .

$BC = x + 14$ *Given*

$BC = 7 + 14 = 21$ *Substitute and simplify.*

$DA = 3x$

$DA = 3x = 3(7) = 21$

Step 2 Find AB and CD .

$AB = 5y - 4$ *Given*

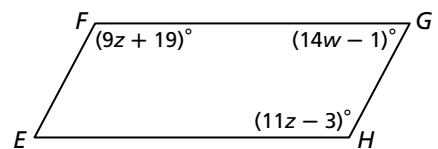
$AB = 5(4) - 4 = 16$ *Substitute and simplify.*

$CD = 2y + 8$

$CD = 2(4) + 8 = 16$

Since $BC = DA$ and $AB = CD$, $ABCD$ is a parallelogram by Theorem 6-3-2.

B Show that $EFGH$ is a parallelogram for $z = 11$ and $w = 4.5$.



$m\angle F = (9z + 19)^\circ$ *Given*

$m\angle F = [9(11) + 19]^\circ = 118^\circ$ *Substitute 11 for z and simplify.*

$m\angle H = (11z - 3)^\circ$ *Given*

$m\angle H = [11(11) - 3]^\circ = 118^\circ$ *Substitute 11 for z and simplify.*

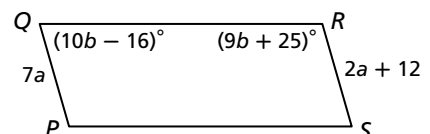
$m\angle G = (14w - 1)^\circ$ *Given*

$m\angle G = [14(4.5) - 1]^\circ = 62^\circ$ *Substitute 4.5 for w and simplify.*

Since $118^\circ + 62^\circ = 180^\circ$, $\angle G$ is supplementary to both $\angle F$ and $\angle H$. $EFGH$ is a parallelogram by Theorem 6-3-4.

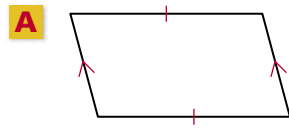


1. Show that $PQRS$ is a parallelogram for $a = 2.4$ and $b = 9$.

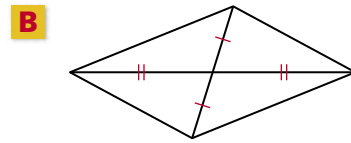


EXAMPLE 2 Applying Conditions for Parallelograms

Determine if each quadrilateral must be a parallelogram. Justify your answer.



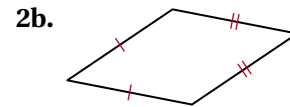
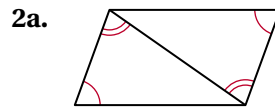
No. One pair of opposite sides are parallel. A different pair of opposite sides are congruent. The conditions for a parallelogram are not met.



Yes. The diagonals bisect each other. By Theorem 6-3-5, the quadrilateral is a parallelogram.



Determine if each quadrilateral must be a parallelogram. Justify your answer.



EXAMPLE 3 Proving Parallelograms in the Coordinate Plane

Show that quadrilateral $ABCD$ is a parallelogram by using the given definition or theorem.

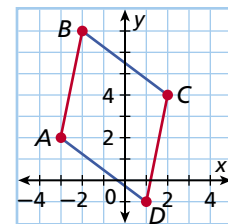
- A** $A(-3, 2)$, $B(-2, 7)$, $C(2, 4)$, $D(1, -1)$; definition of parallelogram
Find the slopes of both pairs of opposite sides.

$$\text{slope of } \overline{AB} = \frac{7 - 2}{-2 - (-3)} = \frac{5}{1} = 5$$

$$\text{slope of } \overline{CD} = \frac{-1 - 4}{1 - 2} = \frac{-5}{-1} = 5$$

$$\text{slope of } \overline{BC} = \frac{4 - 7}{2 - (-2)} = \frac{-3}{4} = -\frac{3}{4}$$

$$\text{slope of } \overline{DA} = \frac{2 - (-1)}{-3 - 1} = \frac{3}{-4} = -\frac{3}{4}$$



Since both pairs of opposite sides are parallel, $ABCD$ is a parallelogram by definition.

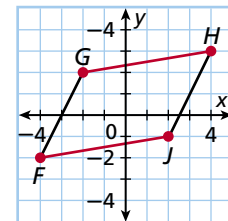
- B** $F(-4, -2)$, $G(-2, 2)$, $H(4, 3)$, $J(2, -1)$; Theorem 6-3-1
Find the slopes and lengths of one pair of opposite sides.

$$\text{slope of } \overline{GH} = \frac{3 - 2}{4 - (-2)} = \frac{1}{6}$$

$$\text{slope of } \overline{JF} = \frac{-2 - (-1)}{-4 - 2} = \frac{-1}{-6} = \frac{1}{6}$$

$$GH = \sqrt{[4 - (-2)]^2 + (3 - 2)^2} = \sqrt{37}$$

$$JF = \sqrt{(-4 - 2)^2 + [-2 - (-1)]^2} = \sqrt{37}$$



\overline{GH} and \overline{JF} have the same slope, so $\overline{GH} \parallel \overline{JF}$.
Since $GH = JF$, $\overline{GH} \cong \overline{JF}$. So by Theorem 6-3-1, $FGJH$ is a parallelogram.

Helpful Hint

To say that a quadrilateral is a parallelogram by definition, you must show that both pairs of opposite sides are parallel.



3. Use the definition of a parallelogram to show that the quadrilateral with vertices $K(-3, 0)$, $L(-5, 7)$, $M(3, 5)$, and $N(5, -2)$ is a parallelogram.

You have learned several ways to determine whether a quadrilateral is a parallelogram. You can use the given information about a figure to decide which condition is best to apply.

Helpful Hint

To show that a quadrilateral is a parallelogram, you only have to show that it satisfies one of these sets of conditions.

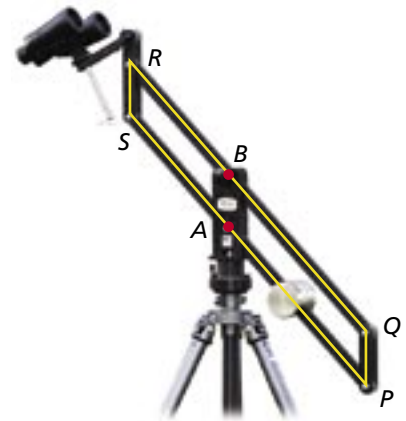
Conditions for Parallelograms

Both pairs of opposite sides are parallel. (definition)
One pair of opposite sides are parallel and congruent. (Theorem 6-3-1)
Both pairs of opposite sides are congruent. (Theorem 6-3-2)
Both pairs of opposite angles are congruent. (Theorem 6-3-3)
One angle is supplementary to both of its consecutive angles. (Theorem 6-3-4)
The diagonals bisect each other. (Theorem 6-3-5)

EXAMPLE 4 Bird-Watching Application

In the parallelogram mount, there are bolts at P , Q , R , and S such that $PQ = RS$ and $QR = SP$. The frame $PQRS$ moves when you raise or lower the binoculars. Why is $PQRS$ always a parallelogram?

When you move the binoculars, the angle measures change, but PQ , QR , RS , and SP stay the same. So it is always true that $PQ = RS$ and $QR = SP$. Since both pairs of opposite sides of the quadrilateral are congruent, $PQRS$ is always a parallelogram.



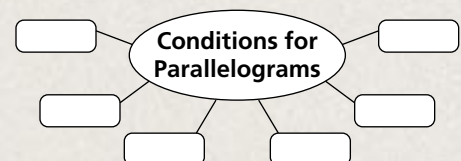
4. The frame is attached to the tripod at points A and B such that $AB = RS$ and $BR = SA$. So $ABRS$ is also a parallelogram. How does this ensure that the angle of the binoculars stays the same?

THINK AND DISCUSS

1. What do all the theorems in this lesson have in common?
2. How are the theorems in this lesson different from the theorems in Lesson 6-2?



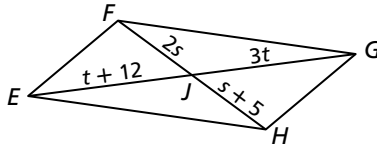
3. **GET ORGANIZED** Copy and complete the graphic organizer. In each box, write one of the six conditions for a parallelogram. Then sketch a parallelogram and label it to show how it meets the condition.



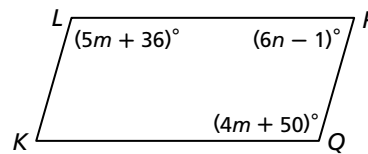
GUIDED PRACTICE

SEE EXAMPLE 1
p. 399

1. Show that $EFGH$ is a parallelogram for $s = 5$ and $t = 6$.



2. Show that $KLPQ$ is a parallelogram for $m = 14$ and $n = 12.5$.



SEE EXAMPLE 2
p. 400

Determine if each quadrilateral must be a parallelogram. Justify your answer.

3. 4. 5.

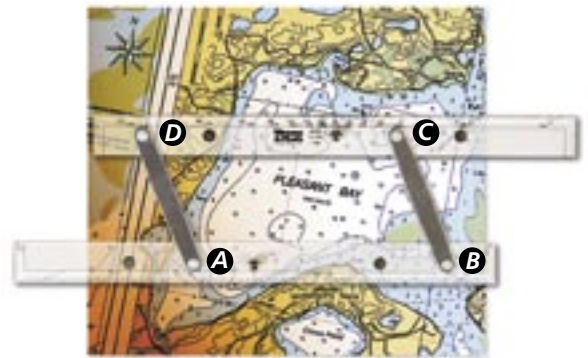
SEE EXAMPLE 3
p. 400

Show that the quadrilateral with the given vertices is a parallelogram.

6. $W(-5, -2), X(-3, 3), Y(3, 5), Z(1, 0)$
7. $R(-1, -5), S(-2, -1), T(4, -1), U(5, -5)$

SEE EXAMPLE 4
p. 401

8. **Navigation** A parallel rule can be used to plot a course on a navigation chart. The tool is made of two rulers connected at hinges to two congruent crossbars \overline{AD} and \overline{BC} . You place the edge of one ruler on your desired course and then move the second ruler over the compass rose on the chart to read the bearing for your course. If $\overline{AD} \parallel \overline{BC}$, why is \overline{AB} always parallel to \overline{CD} ?

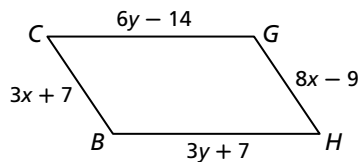


PRACTICE AND PROBLEM SOLVING

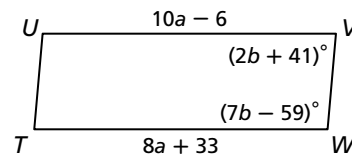
Independent Practice

For Exercises	See Example
9–10	1
11–13	2
14–15	3
16	4

9. Show that $BCGH$ is a parallelogram for $x = 3.2$ and $y = 7$.



10. Show that $TUVW$ is a parallelogram for $a = 19.5$ and $b = 22$.



Extra Practice

Skills Practice p. S14
Application Practice p. S33

Determine if each quadrilateral must be a parallelogram. Justify your answer.

11. 12. 13.

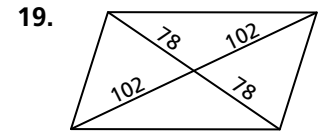
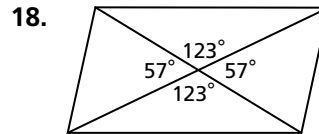
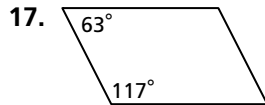
Show that the quadrilateral with the given vertices is a parallelogram.

14. $J(-1, 0), K(-3, 7), L(2, 6), M(4, -1)$
15. $P(-8, -4), Q(-5, 1), R(1, -5), S(-2, -10)$

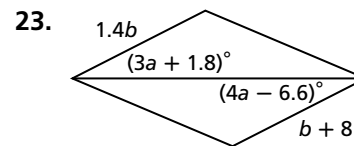
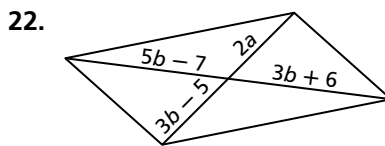
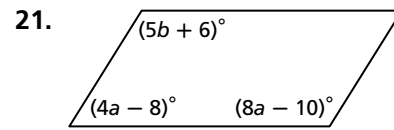
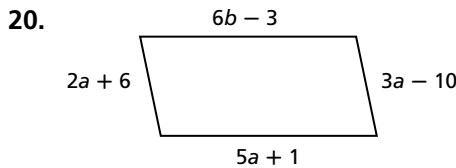
16. **Design** The toolbox has cantilever trays that pull away from the box so that you can reach the items beneath them. Two congruent brackets connect each tray to the box. Given that $AD = BC$, how do the brackets \overline{AB} and \overline{CD} keep the tray horizontal?



Determine if each quadrilateral must be a parallelogram. Justify your answer.



Algebra Find the values of a and b that would make the quadrilateral a parallelogram.



24. **Critical Thinking** Draw a quadrilateral that has congruent diagonals but is not a parallelogram. What can you conclude about using congruent diagonals as a condition for a parallelogram?

25. **Social Studies** The angles at the corners of the flag of the Republic of the Congo are right angles. The red and green triangles are congruent isosceles right triangles. Why is the shape of the yellow stripe a parallelogram?

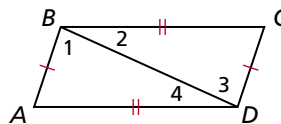


26. Complete the two-column proof of Theorem 6-3-2 by filling in the blanks.

Given: $\overline{AB} \cong \overline{CD}$,
 $\overline{BC} \cong \overline{DA}$

Prove: $ABCD$ is a parallelogram.

Proof:



Statements	Reasons
1. $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{DA}$	1. Given
2. $\overline{BD} \cong \overline{BD}$	2. a. <u> </u> ?
3. $\triangle DAB \cong$ b. <u> </u> ?	3. c. <u> </u> ?
4. $\angle 1 \cong$ d. <u> </u> ? , $\angle 4 \cong$ e. <u> </u> ?	4. CPCTC
5. $\overline{AB} \parallel \overline{CD}$, $\overline{BC} \parallel \overline{DA}$	5. f. <u> </u> ?
6. $ABCD$ is a parallelogram.	6. g. <u> </u> ?



Measurement



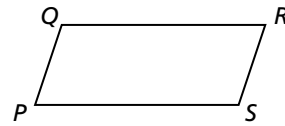
Ancient balance scales had one beam that moved on a single hinge. The stress on the hinge often made the scale imprecise.

27. Complete the paragraph proof of Theorem 6-3-4 by filling in the blanks.

Given: $\angle P$ is supplementary to $\angle Q$.

$\angle P$ is supplementary to $\angle S$.

Prove: $PQRS$ is a parallelogram.

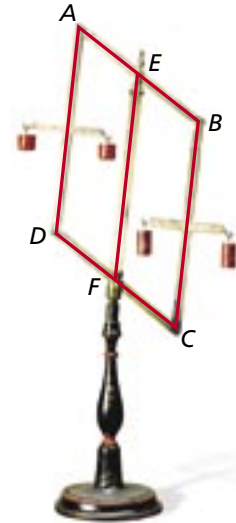


Proof:

It is given that $\angle P$ is supplementary to **a.** $\underline{\hspace{1cm}}$ and **b.** $\underline{\hspace{1cm}}$.

By the Converse of the Same-Side Interior Angles Theorem, $\overline{QR} \parallel$ **c.** $\underline{\hspace{1cm}}$ and $\overline{PQ} \parallel$ **d.** $\underline{\hspace{1cm}}$. So $PQRS$ is a parallelogram by the definition of **e.** $\underline{\hspace{1cm}}$.

28. **Measurement** In the eighteenth century, Gilles Personne de Roberval designed a scale with two beams and two hinges. In $\square ABCD$, E is the midpoint of \overline{AB} , and F is the midpoint of \overline{CD} . Write a paragraph proof that $Aefd$ and $Ebcf$ are parallelograms.

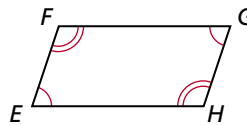


Prove each theorem.

29. Theorem 6-3-3

Given: $\angle E \cong \angle G$, $\angle F \cong \angle H$

Prove: $EFGH$ is a parallelogram.



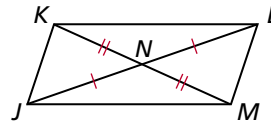
Plan: Show that the sum of the interior angles of $EFGH$ is 360° . Then apply properties of equality to show that $m\angle E + m\angle F = 180^\circ$ and $m\angle E + m\angle H = 180^\circ$. Then you can conclude that $\overline{EF} \parallel \overline{GH}$ and $\overline{FG} \parallel \overline{HE}$.

30. Theorem 6-3-5

Given: \overline{JL} and \overline{KM} bisect each other.

Prove: $JKLM$ is a parallelogram.

Plan: Show that $\triangle JNK \cong \triangle LNM$ and $\triangle KNL \cong \triangle MNJ$. Then use the fact that the corresponding angles are congruent to show $\overline{JK} \parallel \overline{LM}$ and $\overline{KL} \parallel \overline{MJ}$.



31. Prove that the figure formed by two midsegments of a triangle and their corresponding bases is a parallelogram.



32. **Write About It** Use the theorems from Lessons 6-2 and 6-3 to write three biconditional statements about parallelograms.



33. **Construction** Explain how you can construct a parallelogram based on the conditions of Theorem 6-3-1. Use your method to construct a parallelogram.

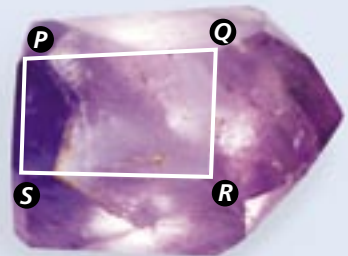
MULTI-STEP TEST PREP



34. This problem will prepare you for the Multi-Step Test Prep on page 406.

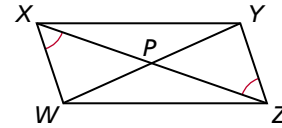
A geologist made the following observations while examining this amethyst crystal. Tell whether each set of observations allows the geologist to conclude that $PQRS$ is a parallelogram. If so, explain why.

- a. $\overline{PQ} \cong \overline{SR}$, and $\overline{PS} \parallel \overline{QR}$.
b. $\angle S$ and $\angle R$ are supplementary, and $\overline{PS} \cong \overline{QR}$.
c. $\angle S \cong \angle Q$, and $\overline{PQ} \parallel \overline{SR}$.



35. What additional information would allow you to conclude that $WXYZ$ is a parallelogram?

- (A) $\overline{XY} \cong \overline{ZW}$ (C) $\overline{WY} \cong \overline{WZ}$
 (B) $\overline{WX} \cong \overline{YZ}$ (D) $\angle XWY \cong \angle ZYW$



36. Which could be the coordinates of the fourth vertex of $\square ABCD$ with $A(-1, -1)$, $B(1, 3)$, and $C(6, 1)$?

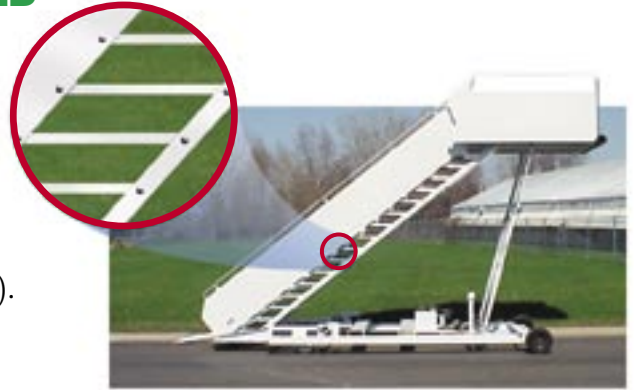
- (F) $D(8, 5)$ (G) $D(4, -3)$ (H) $D(13, 3)$ (J) $D(3, 7)$

37. **Short Response** The vertices of quadrilateral $RSTV$ are $R(-5, 0)$, $S(-1, 3)$, $T(5, 1)$, and $V(2, -2)$. Is $RSTV$ a parallelogram? Justify your answer.

CHALLENGE AND EXTEND



38. **Write About It** As the upper platform of the movable staircase is raised and lowered, the height of each step changes. How does the upper platform remain parallel to the ground?

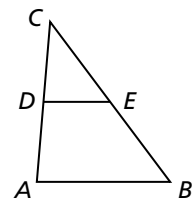


39. **Multi-Step** The diagonals of a parallelogram intersect at $(-2, 1.5)$. Two vertices are located at $(-7, 2)$ and $(2, 6.5)$. Find the coordinates of the other two vertices.

40. **Given:** D is the midpoint of \overline{AC} , and E is the midpoint of \overline{BC} .

Prove: $\overline{DE} \parallel \overline{AB}$, $DE = \frac{1}{2}AB$

(Hint: Extend \overline{DE} to form \overline{DF} so that $\overline{EF} \cong \overline{DE}$. Then show that $DFBA$ is a parallelogram.)



SPIRAL REVIEW

Complete a table of values for each function. Use the domain $\{-5, -2, 0, 0.5\}$.

(Previous course)

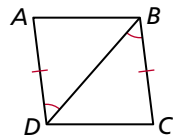
41. $f(x) = 7x - 3$

42. $f(x) = \frac{x+2}{2}$

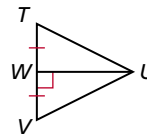
43. $f(x) = 3x^2 + 2$

Use SAS to explain why each pair of triangles are congruent. (Lesson 4-4)

44. $\triangle ABD \cong \triangle CDB$



45. $\triangle TUW \cong \triangle VUW$



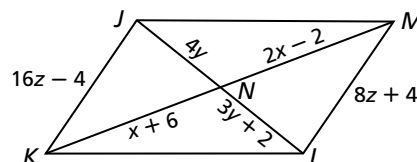
For $\square JKLM$, find each measure. (Lesson 6-2)

46. NM

47. LM

48. JL

49. JK



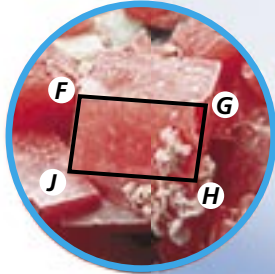
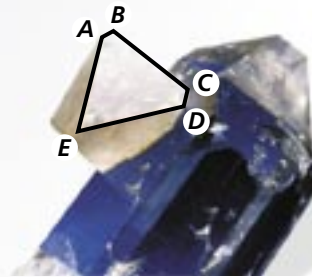
MULTI-STEP TEST PREP

Polygons and Parallelograms

Crystal Clear A crystal is a mineral formation that has polygonal faces. Geologists classify crystals based on the types of polygons that the faces form.

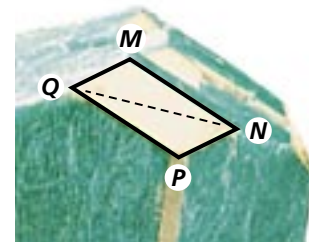


1. What type of polygon is $ABCDE$ in the fluorite crystal? Given that $\overline{AE} \parallel \overline{CD}$, $m\angle B = 120^\circ$, $m\angle E = 65^\circ$, and $\angle C \cong \angle D$, find $m\angle A$.



2. The pink crystals are called rhodochrosite. The face $FGHJ$ is a parallelogram. Given that $m\angle F = (9x - 13)^\circ$ and $m\angle J = (7x + 1)^\circ$, find $m\angle G$. Explain how you found this angle measure.

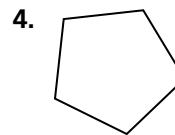
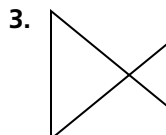
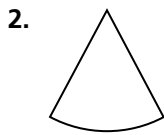
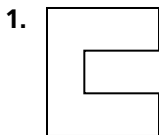
3. While studying the amazonite crystal, a geologist found that $\overline{MN} \cong \overline{QP}$ and $\angle NQP \cong \angle QNM$. Can the geologist conclude that $MNPQ$ is a parallelogram? Why or why not? Justify your answer.



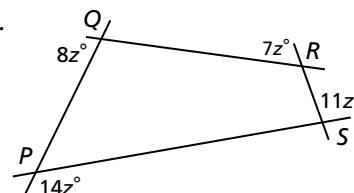
Quiz for Lessons 6-1 Through 6-3

6-1 Properties and Attributes of Polygons

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



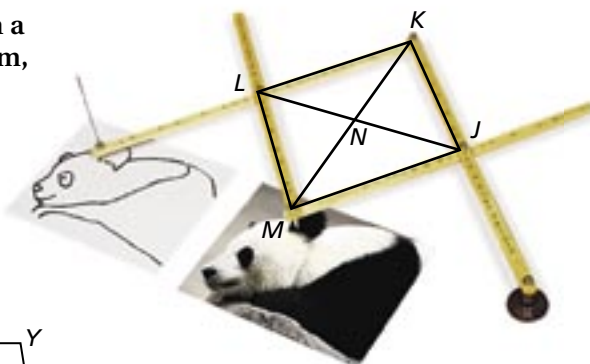
- Find the sum of the interior angle measures of a convex 16-gon.
- The surface of a trampoline is in the shape of a regular hexagon. Find the measure of each interior angle of the trampoline.
- A park in the shape of quadrilateral $PQRS$ is bordered by four sidewalks. Find the measure of each exterior angle of the park.
- Find the measure of each exterior angle of a regular decagon.



6-2 Properties of Parallelograms

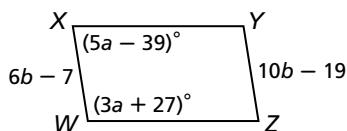
A pantograph is used to copy drawings. Its legs form a parallelogram. In $\square JKLM$, $LM = 17$ cm, $KN = 13.5$ cm, and $m\angle KJM = 102^\circ$. Find each measure.

- KM
- KJ
- MN
- $m\angle JKL$
- $m\angle JML$
- $m\angle KLM$
- Three vertices of $\square ABCD$ are $A(-3, 1)$, $B(5, 7)$, and $C(6, 2)$. Find the coordinates of vertex D .



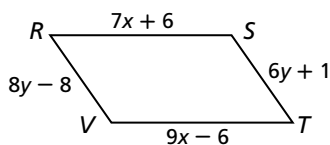
$WXYZ$ is a parallelogram. Find each measure.

- WX
- YZ
- $m\angle X$
- $m\angle W$

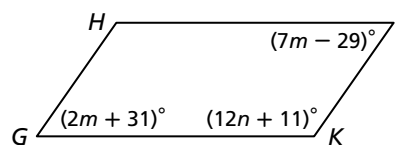


6-3 Conditions for Parallelograms

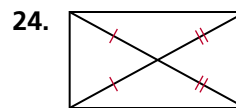
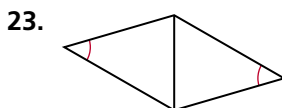
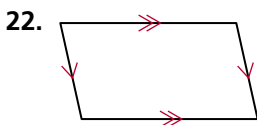
- Show that $RSTV$ is a parallelogram for $x = 6$ and $y = 4.5$.



- Show that $GHJK$ is a parallelogram for $m = 12$ and $n = 9.5$.



Determine if each quadrilateral must be a parallelogram. Justify your answer.



- Show that a quadrilateral with vertices $C(-9, 4)$, $D(-4, 8)$, $E(2, 6)$, and $F(-3, 2)$ is a parallelogram.

6-4

Properties of Special Parallelograms



Objectives

Prove and apply properties of rectangles, rhombuses, and squares.

Use properties of rectangles, rhombuses, and squares to solve problems.

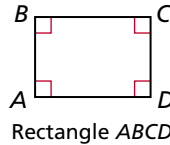
Vocabulary

rectangle
rhombus
square

Who uses this?

Artists who work with stained glass can use properties of rectangles to cut materials to the correct sizes.

A second type of special quadrilateral is a **rectangle**. A **rectangle** is a quadrilateral with four right angles.



Theorems Properties of Rectangles

THEOREM	HYPOTHESIS	CONCLUSION
6-4-1 If a quadrilateral is a rectangle, then it is a parallelogram. (rect. \rightarrow \square)		ABCD is a parallelogram.
6-4-2 If a parallelogram is a rectangle, then its diagonals are congruent. (rect. \rightarrow diags. \cong)		$\overline{AC} \cong \overline{BD}$

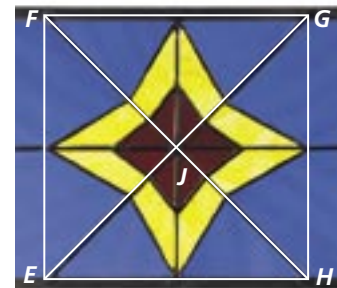
You will prove Theorems 6-4-1 and 6-4-2 in Exercises 38 and 35.

Since a rectangle is a parallelogram by Theorem 6-4-1, a rectangle “inherits” all the properties of parallelograms that you learned in Lesson 6-2.

EXAMPLE 1 Craft Application

An artist connects stained glass pieces with lead strips. In this rectangular window, the strips are cut so that $FG = 24$ in. and $FH = 34$ in. Find JG .

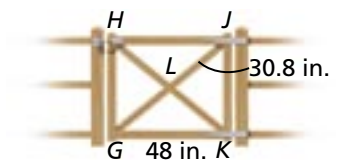
$$\begin{aligned} \overline{EG} &\cong \overline{FH} && \text{Rect.} \rightarrow \text{diags.} \cong \\ EG &= FH = 34 && \text{Def. of} \cong \text{ segs.} \\ JG &= \frac{1}{2}EG && \square \rightarrow \text{diags. bisect} \\ &&& \text{each other} \\ JG &= \frac{1}{2}(34) = 17 \text{ in.} && \text{Substitute and simplify.} \end{aligned}$$



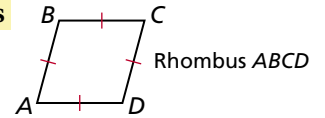
Carpentry The rectangular gate has diagonal braces. Find each length.

1a. HJ

1b. HK



A *rhombus* is another special quadrilateral. A **rhombus** is a quadrilateral with four congruent sides.



Theorems Properties of Rhombuses

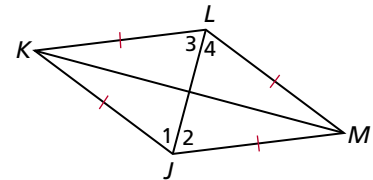
THEOREM	HYPOTHESIS	CONCLUSION
6-4-3 If a quadrilateral is a rhombus, then it is a parallelogram. (rhombus \rightarrow \square)		ABCD is a parallelogram.
6-4-4 If a parallelogram is a rhombus, then its diagonals are perpendicular. (rhombus \rightarrow diags. \perp)		$\overline{AC} \perp \overline{BD}$
6-4-5 If a parallelogram is a rhombus, then each diagonal bisects a pair of opposite angles. (rhombus \rightarrow each diag. bisects opp. \angle s)		$\angle 1 \cong \angle 2$ $\angle 3 \cong \angle 4$ $\angle 5 \cong \angle 6$ $\angle 7 \cong \angle 8$

You will prove Theorems 6-4-3 and 6-4-4 in Exercises 34 and 37.

PROOF

Theorem 6-4-5

Given: JKLM is a rhombus.
Prove: \overline{JL} bisects $\angle KJM$ and $\angle KLM$.
 \overline{KM} bisects $\angle JKL$ and $\angle JML$.



Proof:

Since JKLM is a rhombus, $\overline{JK} \cong \overline{JM}$, and $\overline{KL} \cong \overline{ML}$ by the definition of a rhombus. By the Reflexive Property of Congruence, $\overline{JL} \cong \overline{JL}$. Thus $\triangle JKL \cong \triangle JML$ by SSS. Then $\angle 1 \cong \angle 2$, and $\angle 3 \cong \angle 4$ by CPCTC. So \overline{JL} bisects $\angle KJM$ and $\angle KLM$ by the definition of an angle bisector. By similar reasoning, \overline{KM} bisects $\angle JKL$ and $\angle JML$.

Like a rectangle, a rhombus is a parallelogram. So you can apply the properties of parallelograms to rhombuses.

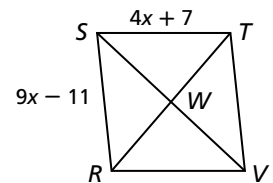
EXAMPLE 2 Using Properties of Rhombuses to Find Measures



RSTV is a rhombus. Find each measure.

A VT

- $ST = SR$ *Def. of rhombus*
- $4x + 7 = 9x - 11$ *Substitute the given values.*
- $18 = 5x$ *Subtract 4x from both sides and add 11 to both sides.*
- $3.6 = x$ *Divide both sides by 5.*
- $VT = ST$ *Def. of rhombus*
- $VT = 4x + 7$ *Substitute 4x + 7 for ST.*
- $VT = 4(3.6) + 7 = 21.4$ *Substitute 3.6 for x and simplify.*



$RSTV$ is a rhombus. Find each measure.

B $m\angle WSR$

$$m\angle SWT = 90^\circ \quad \text{Rhombus} \rightarrow \text{diags. } \perp$$

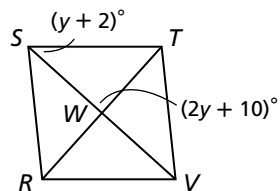
$$2y + 10 = 90 \quad \text{Substitute } 2y + 10 \text{ for } m\angle SWT.$$

$$y = 40 \quad \text{Subtract 10 from both sides and divide both sides by 2.}$$

$$m\angle WSR = m\angle TSW \quad \text{Rhombus} \rightarrow \text{each diag. bisects opp. } \sphericalangle$$

$$m\angle WSR = (y + 2)^\circ \quad \text{Substitute } y + 2 \text{ for } m\angle TSW.$$

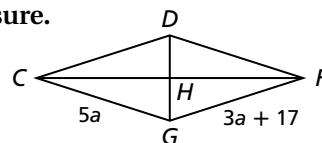
$$m\angle WSR = (40 + 2)^\circ = 42^\circ \quad \text{Substitute 40 for } y \text{ and simplify.}$$



$CDFG$ is a rhombus. Find each measure.

2a. CD

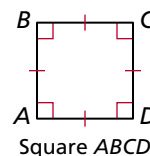
2b. $m\angle GCH$ if $m\angle GCD = (b + 3)^\circ$
and $m\angle CDF = (6b - 40)^\circ$



Helpful Hint

Rectangles, rhombuses, and squares are sometimes referred to as *special parallelograms*.

A **square** is a quadrilateral with four right angles and four congruent sides. In the exercises, you will show that a square is a parallelogram, a rectangle, and a rhombus. So a square has the properties of all three.



EXAMPLE 3 Verifying Properties of Squares

Show that the diagonals of square $ABCD$ are congruent perpendicular bisectors of each other.

Step 1 Show that \overline{AC} and \overline{BD} are congruent.

$$AC = \sqrt{[2 - (-1)]^2 + (7 - 0)^2} = \sqrt{58}$$

$$BD = \sqrt{[4 - (-3)]^2 + (2 - 5)^2} = \sqrt{58}$$

$$\text{Since } AC = BD, \overline{AC} \cong \overline{BD}.$$

Step 2 Show that \overline{AC} and \overline{BD} are perpendicular.

$$\text{slope of } \overline{AC} = \frac{7 - 0}{2 - (-1)} = \frac{7}{3}$$

$$\text{slope of } \overline{BD} = \frac{2 - 5}{4 - (-3)} = \frac{-3}{7} = -\frac{3}{7}$$

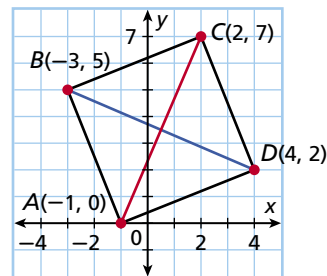
$$\text{Since } \left(\frac{7}{3}\right)\left(-\frac{3}{7}\right) = -1, \overline{AC} \perp \overline{BD}.$$

Step 3 Show that \overline{AC} and \overline{BD} bisect each other.

$$\text{mdpt. of } \overline{AC}: \left(\frac{-1 + 2}{2}, \frac{0 + 7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

$$\text{mdpt. of } \overline{BD}: \left(\frac{-3 + 4}{2}, \frac{5 + 2}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$$

Since \overline{AC} and \overline{BD} have the same midpoint, they bisect each other. The diagonals are congruent perpendicular bisectors of each other.



3. The vertices of square $STVW$ are $S(-5, -4)$, $T(0, 2)$, $V(6, -3)$, and $W(1, -9)$. Show that the diagonals of square $STVW$ are congruent perpendicular bisectors of each other.

Student to Student

Special Parallelograms

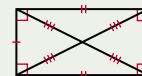


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To remember the properties of rectangles, rhombuses, and squares, I start with a **square**, which has all the properties of the others.



To get a **rectangle** that is not a square, I stretch the square in one direction. Its diagonals are still congruent, but they are no longer perpendicular.

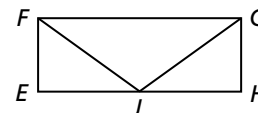


To get a **rhombus** that is not a square, I go back to the square and slide the top in one direction. Its diagonals are still perpendicular and bisect the opposite angles, but they aren't congruent.



EXAMPLE 4 Using Properties of Special Parallelograms in Proofs

Given: $EFGH$ is a rectangle. J is the midpoint of \overline{EH} .
Prove: $\triangle FJG$ is isosceles.

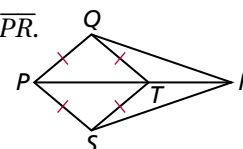


Proof:

Statements	Reasons
1. $EFGH$ is a rectangle. J is the midpoint of \overline{EH} .	1. Given
2. $\angle E$ and $\angle H$ are right angles.	2. Def. of rect.
3. $\angle E \cong \angle H$	3. Rt. $\angle \cong$ Thm.
4. $EFGH$ is a parallelogram.	4. Rect. \rightarrow \square
5. $\overline{EF} \cong \overline{HG}$	5. $\square \rightarrow$ opp. sides \cong
6. $\overline{EJ} \cong \overline{HJ}$	6. Def. of mdpt.
7. $\triangle FJE \cong \triangle GJH$	7. SAS Steps 3, 5, 6
8. $\overline{FJ} \cong \overline{GJ}$	8. CPCTC
9. $\triangle FJG$ is isosceles.	9. Def. of isosc. \triangle

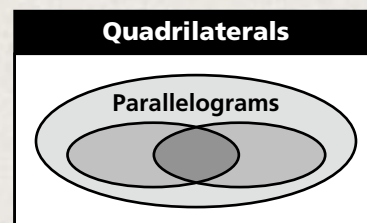


4. Given: $PQTS$ is a rhombus with diagonal \overline{PR} .
Prove: $\overline{RQ} \cong \overline{RS}$

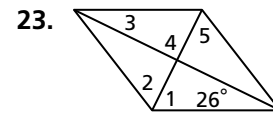
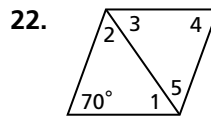
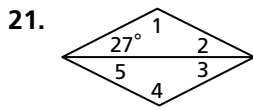


THINK AND DISCUSS

- Which theorem means “The diagonals of a rectangle are congruent”? Why do you think the theorem is written as a conditional?
- What properties of a rhombus are the same as the properties of all parallelograms? What special properties does a rhombus have?
- GET ORGANIZED** Copy and complete the graphic organizer. Write the missing terms in the three unlabeled sections. Then write a definition of each term.



Find the measures of the numbered angles in each rhombus.



Tell whether each statement is sometimes, always, or never true.

(Hint: Refer to your graphic organizer for this lesson.)

24. A rectangle is a parallelogram. 25. A rhombus is a square.
 26. A parallelogram is a rhombus. 27. A rhombus is a rectangle.
 28. A square is a rhombus. 29. A rectangle is a quadrilateral.
 30. A square is a rectangle. 31. A rectangle is a square.
 32. **Critical Thinking** A triangle is equilateral if and only if the triangle is equiangular. Can you make a similar statement about a quadrilateral? Explain your answer.



History



Pompeii was located in what is today southern Italy. In C.E. 79, Mount Vesuvius erupted and buried Pompeii in volcanic ash. The ruins have been excavated and provide a glimpse into life in ancient Rome.

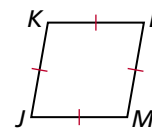
33. **History** There are five shapes of clay tiles in this tile mosaic from the ruins of Pompeii.

- Make a sketch of each shape of tile and tell whether the shape is a polygon.
- Name each polygon by its number of sides. Does each shape appear to be regular or irregular?
- Do any of the shapes appear to be special parallelograms? If so, identify them by name.
- Find the measure of each interior angle of the center polygon.



34. **ERROR ANALYSIS** Find and correct the error in this proof of Theorem 6-4-3.

Given: $JKLM$ is a rhombus.
 Prove: $JKLM$ is a parallelogram.

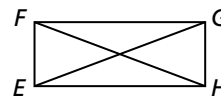


Proof:

It is given that $JKLM$ is a rhombus. So by the definition of a rhombus, $\overline{JK} \cong \overline{LM}$, and $\overline{KL} \cong \overline{MJ}$. Theorem 6-2-1 states that if a quadrilateral is a parallelogram, then its opposite sides are congruent. So $JKLM$ is a parallelogram by Theorem 6-2-1.

35. Complete the two-column proof of Theorem 6-4-2 by filling in the blanks.

Given: $EFGH$ is a rectangle.
 Prove: $\overline{FH} \cong \overline{GE}$



Proof:

Statements	Reasons
1. $EFGH$ is a rectangle.	1. Given
2. $EFGH$ is a parallelogram.	2. a. <u> ?</u>
3. $\overline{EF} \cong$ b. <u> ?</u>	3. $\square \rightarrow$ opp. sides \cong
4. $\overline{EH} \cong \overline{EH}$	4. c. <u> ?</u>
5. $\angle FEH$ and $\angle GHE$ are right angles.	5. d. <u> ?</u>
6. $\angle FEH \cong$ e. <u> ?</u>	6. Rt. $\angle \cong$ Thm.
7. $\triangle FEH \cong \triangle GHE$	7. f. <u> ?</u>
8. $\overline{FH} \cong \overline{GE}$	8. g. <u> ?</u>



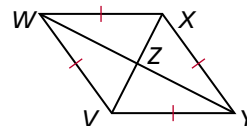
36. This problem will prepare you for the Multi-Step Test Prep on page 436. The organizers of a fair plan to fence off a plot of land given by the coordinates $A(2, 4)$, $B(4, 2)$, $C(-1, -3)$, and $D(-3, -1)$.
- Find the slope of each side of quadrilateral $ABCD$.
 - What type of quadrilateral is formed by the fences? Justify your answer.
 - The organizers plan to build a straight path connecting A and C and another path connecting B and D . Explain why these two paths will have the same length.

37. Use this plan to write a proof of Theorem 6-4-4.

Given: $VWXY$ is a rhombus.

Prove: $\overline{VX} \perp \overline{WY}$

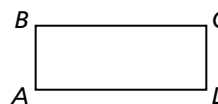
Plan: Use the definition of a rhombus and the properties of parallelograms to show that $\triangle WZX \cong \triangle YZX$. Then use CPCTC to show that $\angle WZX$ and $\angle YZX$ are right angles.



38. Write a paragraph proof of Theorem 6-4-1.

Given: $ABCD$ is a rectangle.

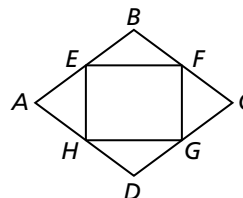
Prove: $ABCD$ is a parallelogram.



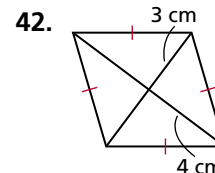
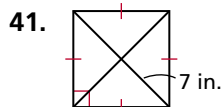
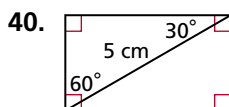
39. Write a two-column proof.

Given: $ABCD$ is a rhombus. E , F , G , and H are the midpoints of the sides.

Prove: $EFGH$ is a parallelogram.



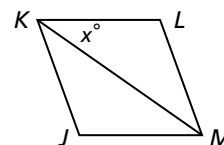
Multi-Step Find the perimeter and area of each figure. Round to the nearest hundredth, if necessary.



43. **Write About It** Explain why each of these conditional statements is true.
- If a quadrilateral is a square, then it is a parallelogram.
 - If a quadrilateral is a square, then it is a rectangle.
 - If a quadrilateral is a square, then it is a rhombus.
44. **Write About It** List the properties that a square “inherits” because it is (1) a parallelogram, (2) a rectangle, and (3) a rhombus.

45. Which expression represents the measure of $\angle J$ in rhombus $JKLM$?

- (A) x° (C) $(180 - x)^\circ$
 (B) $2x^\circ$ (D) $(180 - 2x)^\circ$



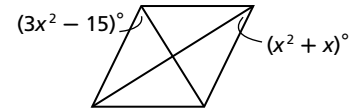
46. **Short Response** The diagonals of rectangle $QRST$ intersect at point P . If $QR = 1.8$ cm, $QP = 1.5$ cm, and $QT = 2.4$ cm, find the perimeter of $\triangle RST$. Explain how you found your answer.

47. Which statement is NOT true of a rectangle?
- (F) Both pairs of opposite sides are congruent and parallel.
 - (G) Both pairs of opposite angles are congruent and supplementary.
 - (H) All pairs of consecutive sides are congruent and perpendicular.
 - (J) All pairs of consecutive angles are congruent and supplementary.

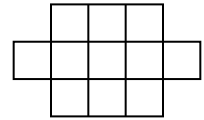
CHALLENGE AND EXTEND



48. **Algebra** Find the value of x in the rhombus.



49. Prove that the segment joining the midpoints of two consecutive sides of a rhombus is perpendicular to one diagonal and parallel to the other.
50. Extend the definition of a triangle midsegment to write a definition for the midsegment of a rectangle. Prove that a midsegment of a rectangle divides the rectangle into two congruent rectangles.
51. The figure is formed by joining eleven congruent squares. How many rectangles are in the figure?



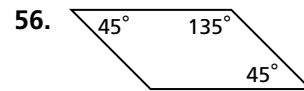
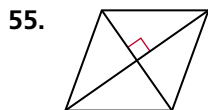
SPIRAL REVIEW

52. The cost c of a taxi ride is given by $c = 2 + 1.8(m - 1)$, where m is the length of the trip in miles. Mr. Hatch takes a 6-mile taxi ride. How much change should he get if he pays with a \$20 bill and leaves a 10% tip? (*Previous course*)

Determine if each conditional is true. If false, give a counterexample. (*Lesson 2-2*)

53. If a number is divisible by -3 , then it is divisible by 3.
54. If the diameter of a circle is doubled, then the area of the circle will double.

Determine if each quadrilateral must be a parallelogram. Justify your answer. (*Lesson 6-3*)



Construction Rhombus

<p>1</p>	<p>2</p>	<p>3</p>	<p>4</p>
----------	----------	----------	----------

Draw \overline{PS} . Set the compass to the length of \overline{PS} . Place the compass point at P and draw an arc above \overline{PS} . Label a point Q on the arc.

Place the compass point at Q and draw an arc to the right of Q .

Place the compass point at S and draw an arc that intersects the arc drawn from Q . Label the point of intersection R .

Draw \overline{PQ} , \overline{QR} , and \overline{RS} .

Predict Conditions for Special Parallelograms

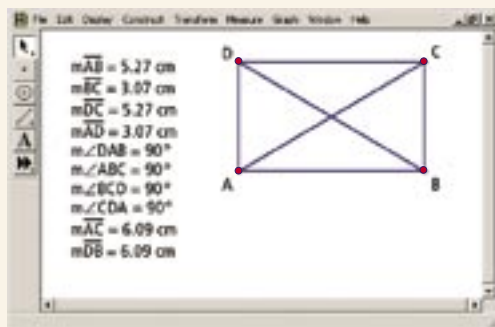
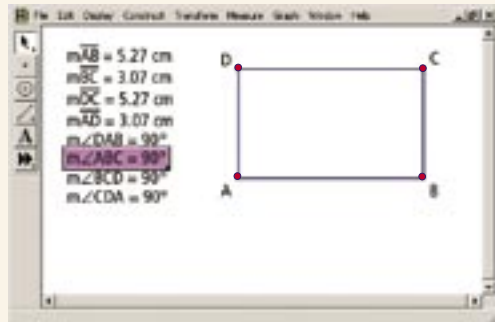
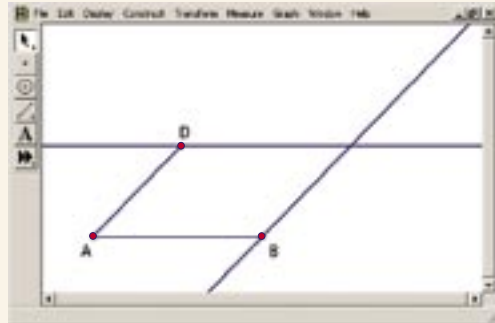
In this lab, you will use geometry software to predict the conditions that are sufficient to prove that a parallelogram is a rectangle, rhombus, or square.

Use with Lesson 6-5



Activity 1

- 1 Construct \overline{AB} and \overline{AD} with a common endpoint A . Construct a line through D parallel to \overline{AB} . Construct a line through B parallel to \overline{AD} .
- 2 Construct point C at the intersection of the two lines. Hide the lines and construct \overline{BC} and \overline{CD} to complete the parallelogram.
- 3 Measure the four sides and angles of the parallelogram.
- 4 Move A so that $m\angle ABC = 90^\circ$. What type of special parallelogram results?
- 5 Move A so that $m\angle ABC \neq 90^\circ$.
- 6 Construct \overline{AC} and \overline{BD} and measure their lengths. Move A so that $AC = BD$. What type of special parallelogram results?



Try This

1. How does the method of constructing $ABCD$ in Steps 1 and 2 guarantee that the quadrilateral is a parallelogram?
2. **Make a Conjecture** What are two conditions for a rectangle? Write your conjectures as conditional statements.

Activity 2

1 Use the parallelogram you constructed in Activity 1. Move A so that $AB = BC$. What type of special parallelogram results?

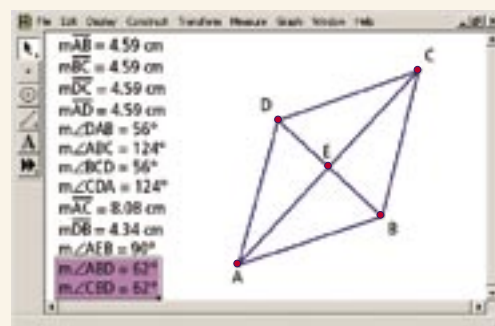
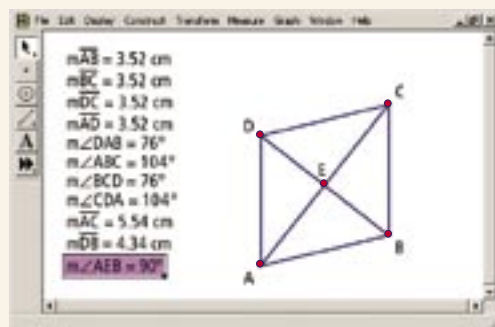
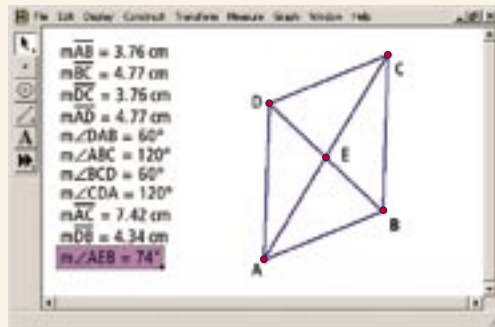
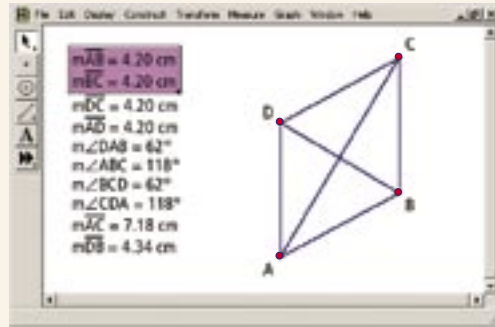
2 Move A so that $AB \neq BC$.

3 Label the intersection of the diagonals as E . Measure $\angle AEB$.

4 Move A so that $m\angle AEB = 90^\circ$. What type of special parallelogram results?

5 Move A so that $m\angle AEB \neq 90^\circ$.

6 Measure $\angle ABD$ and $\angle CBD$. Move A so that $m\angle ABD = m\angle CBD$. What type of special parallelogram results?



Try This

- Make a Conjecture** What are three conditions for a rhombus? Write your conjectures as conditional statements.
- Make a Conjecture** A square is both a rectangle and a rhombus. What conditions do you think must hold for a parallelogram to be a square?

6-5

Conditions for Special Parallelograms

Objective

Prove that a given quadrilateral is a rectangle, rhombus, or square.

Who uses this?

Building contractors and carpenters can use the conditions for rectangles to make sure the frame for a house has the correct shape.



When you are given a parallelogram with certain properties, you can use the theorems below to determine whether the parallelogram is a rectangle.



Theorems Conditions for Rectangles

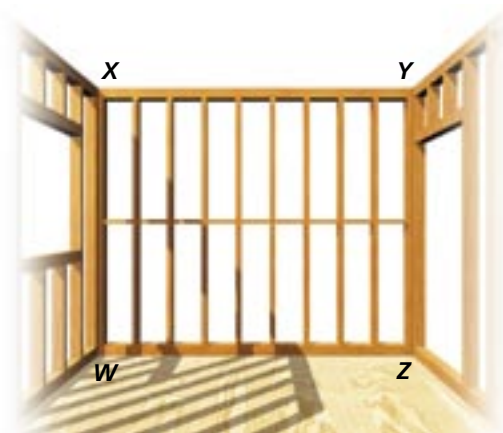
THEOREM	EXAMPLE
<p>6-5-1 If one angle of a parallelogram is a right angle, then the parallelogram is a rectangle. (□ with one rt. $\angle \rightarrow$ rect.)</p>	
<p>6-5-2 If the diagonals of a parallelogram are congruent, then the parallelogram is a rectangle. (□ with diags. $\cong \rightarrow$ rect.)</p>	

You will prove Theorems 6-5-1 and 6-5-2 in Exercises 31 and 28.

EXAMPLE 1 Carpentry Application

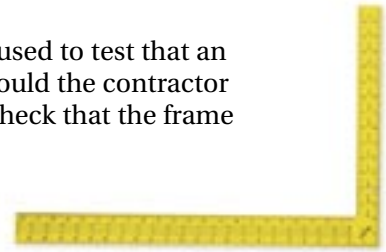
A contractor built a wood frame for the side of a house so that $\overline{XY} \cong \overline{WZ}$ and $\overline{XW} \cong \overline{YZ}$. Using a tape measure, the contractor found that $XZ = WY$. Why must the frame be a rectangle?

Both pairs of opposite sides of $WXYZ$ are congruent, so $WXYZ$ is a parallelogram. Since $XZ = WY$, the diagonals of $\square WXYZ$ are congruent. Therefore the frame is a rectangle by Theorem 6-5-2.





1. A carpenter's square can be used to test that an angle is a right angle. How could the contractor use a carpenter's square to check that the frame is a rectangle?



Below are some conditions you can use to determine whether a parallelogram is a rhombus.

Know it!
Note

Theorems Conditions for Rhombuses

THEOREM	EXAMPLE
6-5-3 If one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. (\square with one pair cons. sides $\cong \rightarrow$ rhombus)	
6-5-4 If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. (\square with diags. $\perp \rightarrow$ rhombus)	
6-5-5 If one diagonal of a parallelogram bisects a pair of opposite angles, then the parallelogram is a rhombus. (\square with diag. bisecting opp. $\sphericalangle \rightarrow$ rhombus)	

Caution!

In order to apply Theorems 6-5-1 through 6-5-5, the quadrilateral must be a parallelogram.

You will prove Theorems 6-5-3 and 6-5-4 in Exercises 32 and 30.

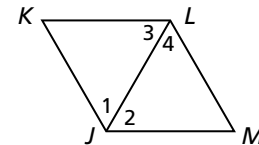
PROOF

Theorem 6-5-5

Given: $JKLM$ is a parallelogram.
 \overline{JL} bisects $\angle KJM$ and $\angle KLM$.

Prove: $JKLM$ is a rhombus.

Proof:



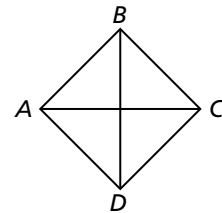
Statements	Reasons
1. $JKLM$ is a parallelogram. \overline{JL} bisects $\angle KJM$ and $\angle KLM$.	1. Given
2. $\angle 1 \cong \angle 2$, $\angle 3 \cong \angle 4$	2. Def. of \angle bisector
3. $\overline{JL} \cong \overline{JL}$	3. Reflex. Prop. of \cong
4. $\triangle JKL \cong \triangle JML$	4. ASA Steps 2, 3
5. $\overline{JK} \cong \overline{JM}$	5. CPCTC
6. $JKLM$ is a rhombus.	6. \square with one pair cons. sides $\cong \rightarrow$ rhombus

To prove that a given quadrilateral is a square, it is sufficient to show that the figure is both a rectangle and a rhombus. You will explain why this is true in Exercise 43.

EXAMPLE 2

Applying Conditions for Special Parallelograms

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



A Given: $\overline{AB} \cong \overline{CD}$, $\overline{BC} \cong \overline{AD}$,
 $\overline{AD} \perp \overline{DC}$, $\overline{AC} \perp \overline{BD}$

Conclusion: $ABCD$ is a square.

Step 1 Determine if $ABCD$ is a parallelogram.

$$\overline{AB} \cong \overline{CD}, \overline{BC} \cong \overline{AD} \quad \text{Given}$$

$ABCD$ is a parallelogram. *Quad. with opp. sides $\cong \rightarrow \square$*

Step 2 Determine if $ABCD$ is a rectangle.

$$\overline{AD} \perp \overline{DC}, \text{ so } \angle ADC \text{ is a right angle.} \quad \text{Def. of } \perp$$

$ABCD$ is a rectangle. *\square with one rt. $\angle \rightarrow \text{rect.}$*

Step 3 Determine if $ABCD$ is a rhombus.

$$\overline{AC} \perp \overline{BD} \quad \text{Given}$$

$ABCD$ is a rhombus. *\square with diags. $\perp \rightarrow \text{rhombus}$*

Step 4 Determine if $ABCD$ is a square.

Since $ABCD$ is a rectangle and a rhombus, it has four right angles and four congruent sides. So $ABCD$ is a square by definition.

The conclusion is valid.

B Given: $\overline{AB} \cong \overline{BC}$

Conclusion: $ABCD$ is a rhombus.

The conclusion is not valid. By Theorem 6-5-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a rhombus. To apply this theorem, you must first know that $ABCD$ is a parallelogram.

Remember!

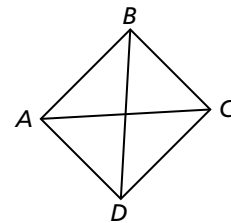
You can also prove that a given quadrilateral is a rectangle, rhombus, or square by using the definitions of the special quadrilaterals.



2. Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

Given: $\angle ABC$ is a right angle.

Conclusion: $ABCD$ is a rectangle.



EXAMPLE 3

Identifying Special Parallelograms in the Coordinate Plane

Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

A $A(0, 2)$, $B(3, 6)$, $C(8, 6)$, $D(5, 2)$

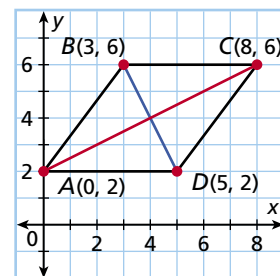
Step 1 Graph $\square ABCD$.

Step 2 Determine if $ABCD$ is a rectangle.

$$\begin{aligned} AC &= \sqrt{(8 - 0)^2 + (6 - 2)^2} \\ &= \sqrt{80} = 4\sqrt{5} \end{aligned}$$

$$\begin{aligned} BD &= \sqrt{(5 - 3)^2 + (2 - 6)^2} \\ &= \sqrt{20} = 2\sqrt{5} \end{aligned}$$

Since $4\sqrt{5} \neq 2\sqrt{5}$, $ABCD$ is not a rectangle. Thus $ABCD$ is not a square.



Step 3 Determine if $ABCD$ is a rhombus.

$$\text{slope of } \overline{AC} = \frac{6-2}{8-0} = \frac{1}{2} \quad \text{slope of } \overline{BD} = \frac{2-6}{5-3} = -2$$

Since $\left(\frac{1}{2}\right)(-2) = -1$, $\overline{AC} \perp \overline{BD}$. $ABCD$ is a rhombus.

B $E(-4, -1), F(-3, 2), G(3, 0), H(2, -3)$

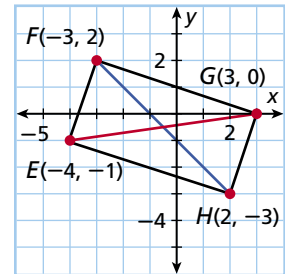
Step 1 Graph $\square EFGH$.

Step 2 Determine if $EFGH$ is a rectangle.

$$\begin{aligned} EG &= \sqrt{[3 - (-4)]^2 + [0 - (-1)]^2} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

$$\begin{aligned} FH &= \sqrt{[2 - (-3)]^2 + (-3 - 2)^2} \\ &= \sqrt{50} = 5\sqrt{2} \end{aligned}$$

Since $5\sqrt{2} = 5\sqrt{2}$, the diagonals are congruent.
 $EFGH$ is a rectangle.



Step 3 Determine if $EFGH$ is a rhombus.

$$\text{slope of } \overline{EG} = \frac{0 - (-1)}{3 - (-4)} = \frac{1}{7}$$

$$\text{slope of } \overline{FH} = \frac{-3 - 2}{2 - (-3)} = \frac{-5}{5} = -1$$

Since $\left(\frac{1}{7}\right)(-1) \neq -1$, $\overline{EG} \not\perp \overline{FH}$.

So $EFGH$ is not a rhombus and cannot be a square.



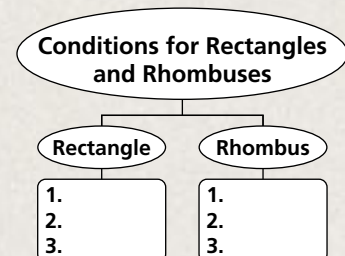
Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

3a. $K(-5, -1), L(-2, 4), M(3, 1), N(0, -4)$

3b. $P(-4, 6), Q(2, 5), R(3, -1), S(-3, 0)$

THINK AND DISCUSS

- What special parallelogram is formed when the diagonals of a parallelogram are congruent? when the diagonals are perpendicular? when the diagonals are both congruent and perpendicular?
- Draw a figure that shows why this statement is not necessarily true: If one angle of a quadrilateral is a right angle, then the quadrilateral is a rectangle.
- A rectangle can also be defined as a parallelogram with a right angle. Explain why this definition is accurate.
- GET ORGANIZED** Copy and complete the graphic organizer. In each box, write at least three conditions for the given parallelogram.

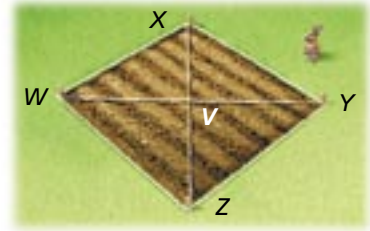


GUIDED PRACTICE

SEE EXAMPLE 1

p. 418

1. **Gardening** A city garden club is planting a square garden. They drive pegs into the ground at each corner and tie strings between each pair. The pegs are spaced so that $\overline{WX} \cong \overline{XY} \cong \overline{YZ} \cong \overline{ZW}$. How can the garden club use the diagonal strings to verify that the garden is a square?

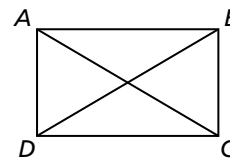


SEE EXAMPLE 2

p. 420

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

2. Given: $\overline{AC} \cong \overline{BD}$
Conclusion: $ABCD$ is a rectangle.
3. Given: $\overline{AB} \parallel \overline{CD}$, $\overline{AB} \cong \overline{CD}$, $\overline{AB} \perp \overline{BC}$
Conclusion: $ABCD$ is a rectangle.



SEE EXAMPLE 3

p. 420

Multi-Step Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

4. $P(-5, 2)$, $Q(4, 5)$, $R(6, -1)$, $S(-3, -4)$
5. $W(-6, 0)$, $X(1, 4)$, $Y(2, -4)$, $Z(-5, -8)$

PRACTICE AND PROBLEM SOLVING

Independent Practice

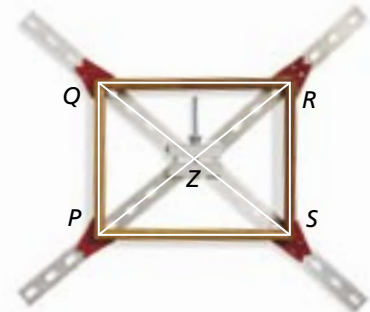
For Exercises	See Example
6	1
7-8	2
9-10	3

Extra Practice

Skills Practice p. S15

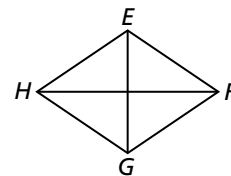
Application Practice p. S33

6. **Crafts** A framer uses a clamp to hold together the pieces of a picture frame. The pieces are cut so that $\overline{PQ} \cong \overline{RS}$ and $\overline{QR} \cong \overline{SP}$. The clamp is adjusted so that \overline{PZ} , \overline{QZ} , \overline{RZ} , and \overline{SZ} are all equal. Why must the frame be a rectangle?



Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

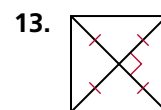
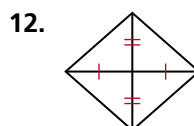
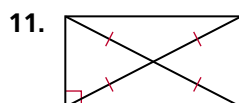
7. Given: \overline{EG} and \overline{FH} bisect each other. $\overline{EG} \perp \overline{FH}$
Conclusion: $EFGH$ is a rhombus.
8. Given: \overline{FH} bisects $\angle EFG$ and $\angle EHG$.
Conclusion: $EFGH$ is a rhombus.



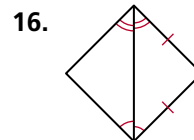
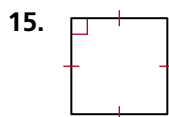
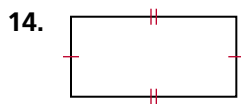
Multi-Step Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

9. $A(-10, 4)$, $B(-2, 10)$, $C(4, 2)$, $D(-4, -4)$
10. $J(-9, -7)$, $K(-4, -2)$, $L(3, -3)$, $M(-2, -8)$

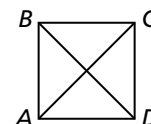
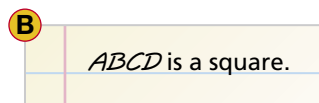
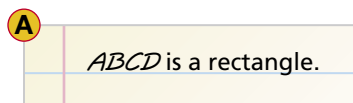
Tell whether each quadrilateral is a parallelogram, rectangle, rhombus, or square. Give all the names that apply.



Tell whether each quadrilateral is a parallelogram, rectangle, rhombus, or square. Give all the names that apply.

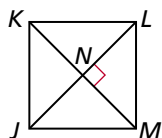


17. **/// ERROR ANALYSIS ///** In $\square ABCD$, $\overline{AC} \cong \overline{BD}$. Which conclusion is incorrect? Explain the error.

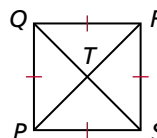


Give one characteristic of the diagonals of each figure that would make the conclusion valid.

18. Conclusion: $JKLM$ is a rhombus.



19. Conclusion: $PQRS$ is a square.



The coordinates of three vertices of $\square ABCD$ are given. Find the coordinates of D so that the given type of figure is formed.

20. $A(4, -2)$, $B(-5, -2)$, $C(4, 4)$; rectangle

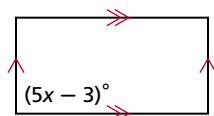
21. $A(-5, 5)$, $B(0, 0)$, $C(7, 1)$; rhombus

22. $A(0, 2)$, $B(4, -2)$, $C(0, -6)$; square

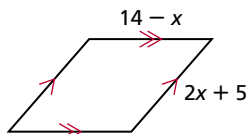
23. $A(2, 1)$, $B(-1, 5)$, $C(-5, 2)$; square

Find the value of x that makes each parallelogram the given type.

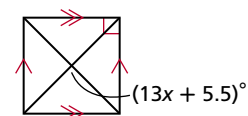
24. rectangle



25. rhombus



26. square



27. **Critical Thinking** The diagonals of a quadrilateral are perpendicular bisectors of each other. What is the best name for this quadrilateral? Explain your answer.

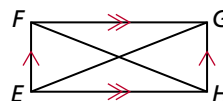
28. Complete the two-column proof of Theorem 6-5-2 by filling in the blanks.

Given: $EFGH$ is a parallelogram.

$$\overline{EG} \cong \overline{HF}$$

Prove: $EFGH$ is a rectangle.

Proof:



Statements	Reasons
1. $EFGH$ is a parallelogram. $\overline{EG} \cong \overline{HF}$	1. Given
2. $\overline{EF} \cong \overline{HG}$	2. a. <u> </u> ?
3. b. <u> </u> ?	3. Reflex. Prop. of \cong
4. $\triangle EFH \cong \triangle HGE$	4. c. <u> </u> ?
5. $\angle FEH \cong$ d. <u> </u> ?	5. e. <u> </u> ?
6. $\angle FEH$ and $\angle GHE$ are supplementary.	6. f. <u> </u> ?
7. g. <u> </u> ?	7. $\cong \triangle$ supp. \rightarrow rt. \triangle
8. $EFGH$ is a rectangle.	8. h. <u> </u> ?



29. This problem will prepare you for the Multi-Step Test Prep on page 436.

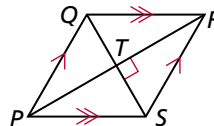
A state fair takes place on a plot of land given by the coordinates $A(-2, 3)$, $B(1, 2)$, $C(2, -1)$, and $D(-1, 0)$.

- Show that the opposite sides of quadrilateral $ABCD$ are parallel.
- A straight path connects A and C , and another path connects B and D . Use slopes to prove that these two paths are perpendicular.
- What can you conclude about $ABCD$? Explain your answer.

30. Complete the paragraph proof of Theorem 6-5-4 by filling in the blanks.

Given: $PQRS$ is a parallelogram. $\overline{PR} \perp \overline{QS}$

Prove: $PQRS$ is a rhombus.



Proof:

It is given that $PQRS$ is a parallelogram. The diagonals of a parallelogram bisect each other, so $\overline{PT} \cong$ **a.** $\underline{\hspace{1cm}}$ $?$. By the Reflexive Property of Congruence, $\overline{QT} \cong$ **b.** $\underline{\hspace{1cm}}$ $?$. It is given that $\overline{PR} \perp \overline{QS}$, so $\angle QTP$ and $\angle QTR$ are right angles by the definition of **c.** $\underline{\hspace{1cm}}$ $?$. Then $\angle QTP \cong \angle QTR$ by the **d.** $\underline{\hspace{1cm}}$ $?$.

So $\triangle QTP \cong \triangle QTR$ by **e.** $\underline{\hspace{1cm}}$ $?$, and $\overline{QP} \cong$ **f.** $\underline{\hspace{1cm}}$ $?$, by CPCTC.

By Theorem 6-5-3, if one pair of consecutive sides of a parallelogram are congruent, then the parallelogram is a **g.** $\underline{\hspace{1cm}}$ $?$. Therefore $PQRS$ is rhombus.

31. Write a two-column proof of Theorem 6-5-1.

Given: $ABCD$ is a parallelogram. $\angle A$ is a right angle.

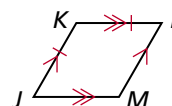
Prove: $ABCD$ is a rectangle.



32. Write a paragraph proof of Theorem 6-5-3.

Given: $JKLM$ is a parallelogram. $\overline{JK} \cong \overline{KL}$

Prove: $JKLM$ is a rhombus.



33. **Algebra** Four lines are represented by the equations below.

$$l: y = -x + 1 \quad m: y = -x + 7 \quad n: y = 2x + 1 \quad p: y = 2x + 7$$

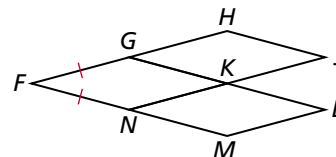
- Graph the four lines in the coordinate plane.
- Classify the quadrilateral formed by the lines.
- What if...?** Suppose the slopes of lines n and p change to 1. Reclassify the quadrilateral.

34. Write a two-column proof.

Given: $FHJN$ and $GLMF$ are parallelograms.

$$\overline{FG} \cong \overline{FN}$$

Prove: $FGKN$ is a rhombus.



35. **Write About It** Use Theorems 6-4-2 and 6-5-2 to write a biconditional statement about rectangles. Use Theorems 6-4-4 and 6-5-4 to write a biconditional statement about rhombuses. Can you combine Theorems 6-4-5 and 6-5-5 to write a biconditional statement? Explain your answer.



Construction Use the diagonals to construct each figure. Then use the theorems from this lesson to explain why your method works.

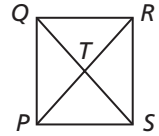
36. rectangle

37. rhombus

38. square

39. In $\square PQRS$, \overline{PR} and \overline{QS} intersect at T . What additional information is needed to conclude that $PQRS$ is a rectangle?

- (A) $\overline{PT} \cong \overline{QT}$ (C) $\overline{PT} \perp \overline{QT}$
 (B) $\overline{PT} \cong \overline{RT}$ (D) \overline{PT} bisects $\angle QPS$.

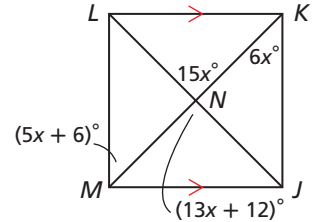


40. Which of the following is the best name for figure $WXYZ$ with vertices $W(-3, 1)$, $X(1, 5)$, $Y(8, -2)$, and $Z(4, -6)$?

- (F) Parallelogram (G) Rectangle (H) Rhombus (J) Square

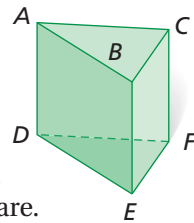
41. **Extended Response**

- a. Write and solve an equation to find the value of x .
 b. Is $JKLM$ a parallelogram? Explain.
 c. Is $JKLM$ a rectangle? Explain.
 d. Is $JKLM$ a rhombus? Explain.



CHALLENGE AND EXTEND

42. Given: $\overline{AC} \cong \overline{DF}$, $\overline{AB} \cong \overline{DE}$, $\overline{AB} \perp \overline{BC}$, $\overline{DE} \perp \overline{EF}$,
 $\overline{BE} \perp \overline{EF}$, $\overline{BC} \parallel \overline{EF}$
 Prove: $EBCF$ is a rectangle.



43. **Critical Thinking** Consider the following statement: If a quadrilateral is a rectangle and a rhombus, then it is a square.

- a. Explain why the statement is true.
 b. If a quadrilateral is a rectangle, is it necessary to show that all four sides are congruent in order to conclude that it is a square? Explain.
 c. If a quadrilateral is a rhombus, is it necessary to show that all four angles are right angles in order to conclude that it is a square? Explain.

44. **Cars** As you turn the crank of a car jack, the platform that supports the car rises. Use the diagonals of the parallelogram to explain whether the jack forms a rectangle, rhombus, or square.

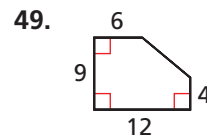
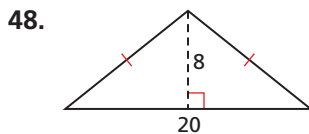


SPIRAL REVIEW

Sketch the graph of each function. State whether the function is linear or nonlinear. (Previous course)

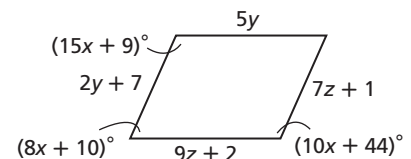
45. $y = -3x + 1$ 46. $y = x^2 - 4$ 47. $y = 3$

Find the perimeter of each figure. Round to the nearest tenth. (Lesson 5-7)



Find the value of each variable that would make the quadrilateral a parallelogram. (Lesson 6-3)

50. x 51. y 52. z





Explore Isosceles Trapezoids

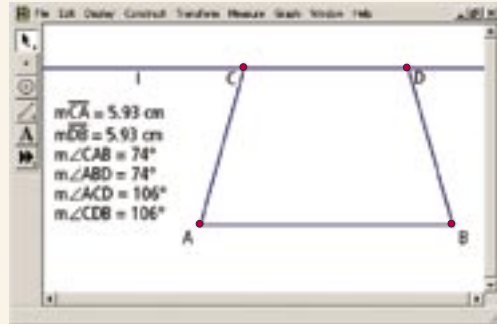
In this lab you will investigate the properties and conditions of an *isosceles trapezoid*. A *trapezoid* is a quadrilateral with one pair of parallel sides, called *bases*. The sides that are not parallel are called *legs*. In an isosceles trapezoid, the legs are congruent.

Use with Lesson 6-6



Activity 1

- 1 Draw \overline{AB} and a point C not on \overline{AB} . Construct a parallel line ℓ through C .
- 2 Draw point D on line ℓ . Construct \overline{AC} and \overline{BD} .
- 3 Measure AC , BD , $\angle CAB$, $\angle ABD$, $\angle ACD$, and $\angle CDB$.
- 4 Move D until $AC = BD$. What do you notice about $m\angle CAB$ and $m\angle ABD$? What do you notice about $m\angle ACD$ and $m\angle CDB$?
- 5 Move D so that $AC \neq BD$. Now move D so that $m\angle CAB = m\angle ABD$. What do you notice about AC and BD ?

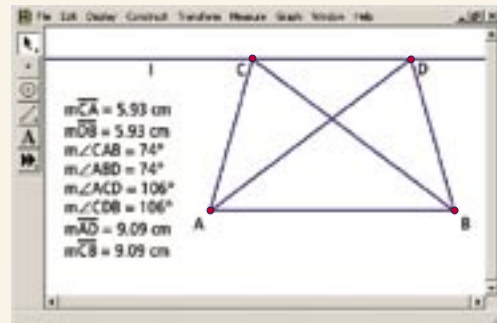


Try This

1. **Make a Conjecture** What is true about the base angles of an isosceles trapezoid? Write your conjecture as a conditional statement.
2. **Make a Conjecture** How can the base angles of a trapezoid be used to determine if the trapezoid is isosceles? Write your conjecture as a conditional statement.

Activity 2

- 1 Construct \overline{AD} and \overline{CB} .
- 2 Measure AD and CB .
- 3 Move D until $AC = BD$. What do you notice about AD and CB ?
- 4 Move D so that $AC \neq BD$. Now move D so that $AD = BC$. What do you notice about AC and BD ?



Try This

3. **Make a Conjecture** What is true about the diagonals of an isosceles trapezoid? Write your conjecture as a conditional statement.
4. **Make a Conjecture** How can the diagonals of a trapezoid be used to determine if the trapezoid is isosceles? Write your conjecture as a conditional statement.

6-6

Properties of Kites and Trapezoids

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Objectives

Use properties of kites to solve problems.

Use properties of trapezoids to solve problems.

Vocabulary

kite

trapezoid

base of a trapezoid

leg of a trapezoid

base angle of a trapezoid

isosceles trapezoid

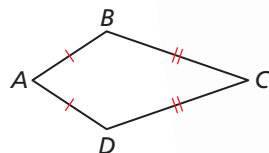
midsegment of a

trapezoid

Why learn this?

The design of a simple kite flown at the beach shares the properties of the geometric figure called a kite.

A **kite** is a quadrilateral with exactly two pairs of congruent consecutive sides.



Kite ABCD



Theorems Properties of Kites

THEOREM	HYPOTHESIS	CONCLUSION
6-6-1 If a quadrilateral is a kite, then its diagonals are perpendicular. (kite \rightarrow diags. \perp)		$\overline{AC} \perp \overline{BD}$
6-6-2 If a quadrilateral is a kite, then exactly one pair of opposite angles are congruent. (kite \rightarrow one pair opp. $\sphericalangle \cong$)		$\angle B \cong \angle D$ $\angle A \not\cong \angle C$

You will prove Theorem 6-6-1 in Exercise 39.

PROOF

Theorem 6-6-2

Given: JKLM is a kite with $\overline{JK} \cong \overline{JM}$ and $\overline{KL} \cong \overline{ML}$.

Prove: $\angle K \cong \angle M$, $\angle KJM \not\cong \angle KLM$

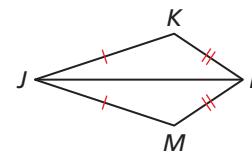
Proof:

Step 1 Prove $\angle K \cong \angle M$.

It is given that $\overline{JK} \cong \overline{JM}$ and $\overline{KL} \cong \overline{ML}$. By the Reflexive Property of Congruence, $\overline{JL} \cong \overline{JL}$. This means that $\triangle JKL \cong \triangle JML$ by SSS. So $\angle K \cong \angle M$ by CPCTC.

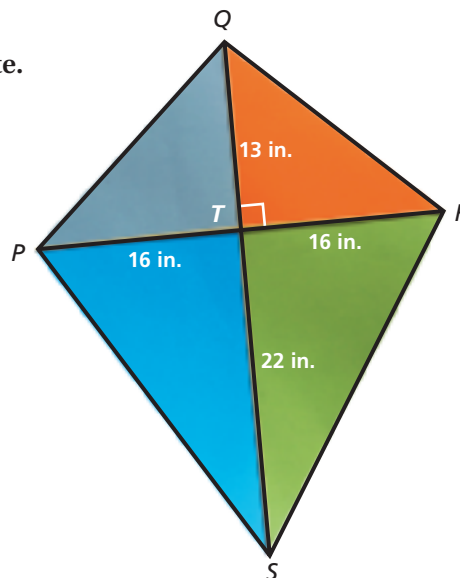
Step 2 Prove $\angle KJM \not\cong \angle KLM$.

If $\angle KJM \cong \angle KLM$, then both pairs of opposite angles of JKLM are congruent. This would mean that JKLM is a parallelogram. But this contradicts the given fact that JKLM is a kite. Therefore $\angle KJM \not\cong \angle KLM$.



EXAMPLE 1**Problem-Solving Application**

Alicia is using a pattern to make a kite. She has made the frame of the kite by placing wooden sticks along the diagonals. She also has cut four triangular pieces of fabric and has attached them to the frame. To finish the kite, Alicia must cover the outer edges with a cloth binding. There are 2 yards of binding in one package. What is the total amount of binding needed to cover the edges of the kite? How many packages of binding must Alicia buy?

**1 Understand the Problem**

The answer has two parts.

- the total length of binding Alicia needs
- the number of packages of binding Alicia must buy

2 Make a Plan

The diagonals of a kite are perpendicular, so the four triangles are right triangles. Use the Pythagorean Theorem and the properties of kites to find the unknown side lengths. Add these lengths to find the perimeter of the kite.

3 Solve

$$PQ = \sqrt{16^2 + 13^2} \quad \text{Pyth. Thm.}$$

$$= \sqrt{425} = 5\sqrt{17} \text{ in.}$$

$$RQ = PQ = 5\sqrt{17} \text{ in.} \quad \overline{PQ} \cong \overline{RQ}$$

$$PS = \sqrt{16^2 + 22^2} \quad \text{Pyth. Thm.}$$

$$= \sqrt{740} = 2\sqrt{185} \text{ in.}$$

$$RS = PS = 2\sqrt{185} \text{ in.} \quad \overline{RS} \cong \overline{PS}$$

$$\text{perimeter of } PQRS = 5\sqrt{17} + 5\sqrt{17} + 2\sqrt{185} + 2\sqrt{185} \approx 95.6 \text{ in.}$$

Alicia needs approximately 95.6 inches of binding.

One package of binding contains 2 yards, or 72 inches.

$$\frac{95.6}{72} \approx 1.3 \text{ packages of binding}$$

In order to have enough, Alicia must buy 2 packages of binding.

4 Look Back

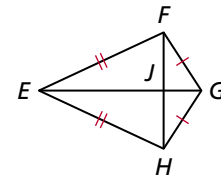
To estimate the perimeter, change the side lengths into decimals and round. $5\sqrt{17} \approx 21$, and $2\sqrt{185} \approx 27$. The perimeter of the kite is approximately $2(21) + 2(27) = 96$. So 95.6 is a reasonable answer.



- 1. What if...?** Daryl is going to make a kite by doubling all the measures in the kite above. What is the total amount of binding needed to cover the edges of his kite? How many packages of binding must Daryl buy?

EXAMPLE 2 Using Properties of Kites

In kite $EFGH$, $m\angle FEJ = 25^\circ$, and $m\angle FGJ = 57^\circ$. Find each measure.



A $m\angle GFJ$

$$\begin{aligned} m\angle FJG &= 90^\circ && \text{Kite} \rightarrow \text{diags. } \perp \\ m\angle GFJ + m\angle FGJ &= 90 && \text{Acute } \triangle \text{ of rt. } \triangle \text{ are comp.} \\ m\angle GFJ + 57 &= 90 && \text{Substitute } 57 \text{ for } m\angle FGJ. \\ m\angle GFJ &= 33^\circ && \text{Subtract } 57 \text{ from both sides.} \end{aligned}$$

B $m\angle JFE$

$\triangle FJE$ is also a right triangle, so $m\angle JFE + m\angle FEJ = 90^\circ$. By substituting 25° for $m\angle FEJ$, you find that $m\angle JFE = 65^\circ$.

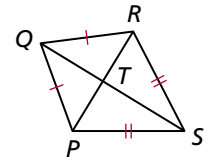
C $m\angle GHE$

$$\begin{aligned} \angle GHE &\cong \angle GFE && \text{Kite} \rightarrow \text{one pair opp. } \triangle \cong \\ m\angle GHE &= m\angle GFE && \text{Def. of } \cong \triangle \\ m\angle GFE &= m\angle GFJ + m\angle JFE && \angle \text{ Add. Post.} \\ m\angle GHE &= 33^\circ + 65^\circ = 98^\circ && \text{Substitute.} \end{aligned}$$

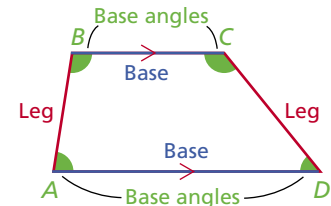


In kite $PQRS$, $m\angle PQR = 78^\circ$, and $m\angle TRS = 59^\circ$. Find each measure.

2a. $m\angle QRT$ 2b. $m\angle QPS$ 2c. $m\angle PSR$



A **trapezoid** is a quadrilateral with exactly one pair of parallel sides. Each of the parallel sides is called a **base**. The nonparallel sides are called **legs**. **Base angles** of a trapezoid are two consecutive angles whose common side is a base.



If the legs of a trapezoid are congruent, the trapezoid is an **isosceles trapezoid**. The following theorems state the properties of an isosceles trapezoid.



Theorems Isosceles Trapezoids

THEOREM	DIAGRAM	EXAMPLE
6-6-3 If a quadrilateral is an isosceles trapezoid, then each pair of base angles are congruent. (isosc. trap. \rightarrow base $\triangle \cong$)		$\angle A \cong \angle D$ $\angle B \cong \angle C$
6-6-4 If a trapezoid has one pair of congruent base angles, then the trapezoid is isosceles. (trap. with pair base $\triangle \cong \rightarrow$ isosc. trap.)		$ABCD$ is isosceles.
6-6-5 A trapezoid is isosceles if and only if its diagonals are congruent. (isosc. trap. \leftrightarrow diags. \cong)		$\overline{AC} \cong \overline{BD} \leftrightarrow$ $ABCD$ is isosceles.

Remember!

Theorem 6-6-5 is a biconditional statement. So it is true both "forward" and "backward."

EXAMPLE 3 Using Properties of Isosceles Trapezoids

A Find $m\angle Y$.

$$m\angle W + m\angle X = 180^\circ$$

$$117 + m\angle X = 180$$

$$m\angle X = 63^\circ$$

$$\angle Y \cong \angle X$$

$$m\angle Y = m\angle X$$

$$m\angle Y = 63^\circ$$

Same-Side Int. \triangle Thm.

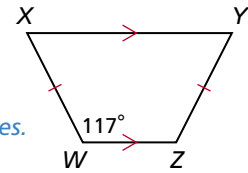
Substitute 117 for $m\angle W$.

Subtract 117 from both sides.

Isosc. trap. \rightarrow base \triangle \cong

Def. of $\cong \triangle$

Substitute 63 for $m\angle X$.



B $RT = 24.1$, and $QP = 9.6$. Find PS .

$$\overline{QS} \cong \overline{RT}$$

$$QS = RT$$

$$QS = 24.1$$

$$QP + PS = QS$$

$$9.6 + PS = 24.1$$

$$PS = 14.5$$

Isosc. trap. \rightarrow diags. \cong

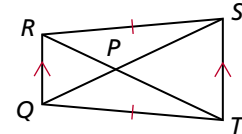
Def. of \cong segs.

Substitute 24.1 for RT .

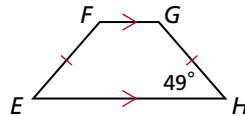
Seg. Add. Post.

Substitute 9.6 for QP and 24.1 for QS .

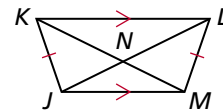
Subtract 9.6 from both sides.



3a. Find $m\angle F$.



3b. $JN = 10.6$, and $NL = 14.8$. Find KM .



EXAMPLE 4 Applying Conditions for Isosceles Trapezoids



A Find the value of y so that $EFGH$ is isosceles.

$$\angle E \cong \angle H$$

Trapez. with pair base \triangle \cong

\rightarrow isosc. trap.

$$m\angle E = m\angle H$$

Def. of $\cong \triangle$

$$2y^2 - 25 = y^2 + 24$$

Substitute $2y^2 - 25$ for $m\angle E$

and $y^2 + 24$ for $m\angle H$.

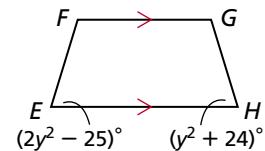
$$y^2 = 49$$

Subtract y^2 from both sides and

add 25 to both sides.

$$y = 7 \text{ or } y = -7$$

Find the square root of both sides.



B $JL = 5z + 3$, and $KM = 9z - 12$. Find the value of z so that $JKLM$ is isosceles.

$$\overline{JL} \cong \overline{KM}$$

Diags. $\cong \rightarrow$ isosc. trap.

$$JL = KM$$

Def. of \cong segs.

$$5z + 3 = 9z - 12$$

Substitute $5z + 3$ for JL and

$9z - 12$ for KM .

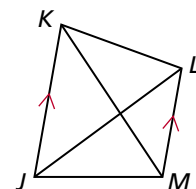
$$15 = 4z$$

Subtract $5z$ from both sides and

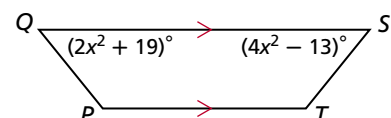
add 12 to both sides.

$$3.75 = z$$

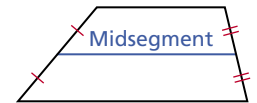
Divide both sides by 4.



4. Find the value of x so that $PQST$ is isosceles.

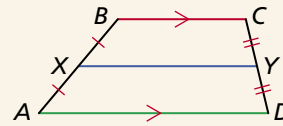


The **midsegment of a trapezoid** is the segment whose endpoints are the midpoints of the legs. In Lesson 5-1, you studied the Triangle Midsegment Theorem. The Trapezoid Midsegment Theorem is similar to it.



Theorem 6-6-6 Trapezoid Midsegment Theorem

The midsegment of a trapezoid is parallel to each base, and its length is one half the sum of the lengths of the bases.



$$\overline{XY} \parallel \overline{BC}, \overline{XY} \parallel \overline{AD}$$

$$XY = \frac{1}{2}(BC + AD)$$

You will prove the Trapezoid Midsegment Theorem in Exercise 46.

EXAMPLE 5 Finding Lengths Using Midsegments

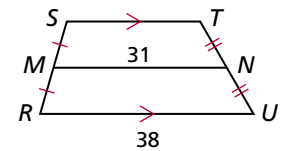
Find ST .

$$MN = \frac{1}{2}(ST + RU) \quad \text{Trap. Midsegment Thm.}$$

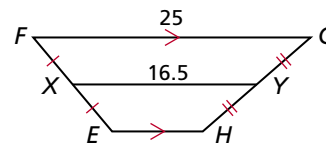
$$31 = \frac{1}{2}(ST + 38) \quad \text{Substitute the given values.}$$

$$62 = ST + 38 \quad \text{Multiply both sides by 2.}$$

$$24 = ST \quad \text{Subtract 38 from both sides.}$$

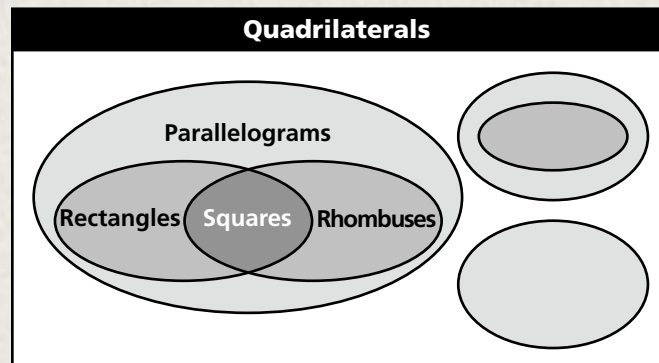


5. Find EH .



THINK AND DISCUSS

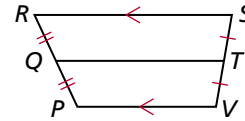
- Is it possible for the legs of a trapezoid to be parallel? Explain.
- How is the midsegment of a trapezoid similar to a midsegment of a triangle? How is it different?
- GET ORGANIZED** Copy and complete the graphic organizer. Write the missing terms in the unlabeled sections. Then write a definition of each term. (*Hint:* This completes the Venn diagram you started in Lesson 6-4.)



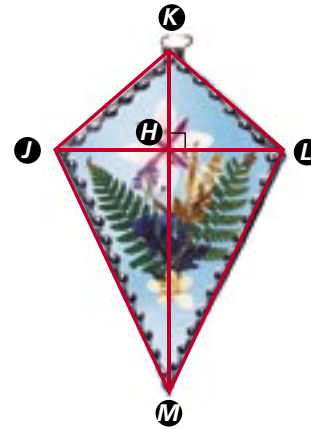
GUIDED PRACTICE

Vocabulary Apply the vocabulary from this lesson to answer each question.

- In trapezoid $PRSV$, name the *bases*, the *legs*, and the *midsegment*.
- Both a parallelogram and a *kite* have two pairs of congruent sides. How are the congruent sides of a kite different from the congruent sides of a parallelogram?



- Crafts** The edges of the kite-shaped glass in the sun catcher are sealed with lead strips. JH , KH , and LH are 2.75 inches, and MH is 5.5 inches. How much lead is needed to seal the edges of the sun catcher? If the craftsperson has two 3-foot lengths of lead, how many sun catchers can be sealed?



SEE EXAMPLE 1

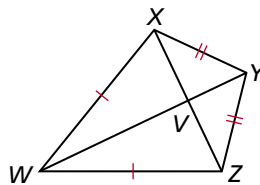
p. 428

SEE EXAMPLE 2

p. 429

In kite $WXYZ$, $m\angle WXY = 104^\circ$, and $m\angle VYZ = 49^\circ$. Find each measure.

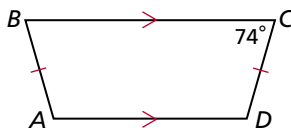
- $m\angle VZY$
- $m\angle VXW$
- $m\angle XWZ$



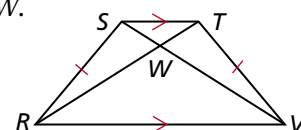
SEE EXAMPLE 3

p. 430

- Find $m\angle A$.



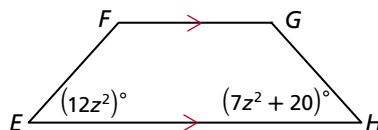
- $RW = 17.7$, and $SV = 23.3$. Find TW .



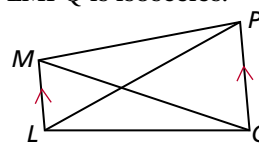
SEE EXAMPLE 4

p. 430

- Find the value of z so that $EFGH$ is isosceles.



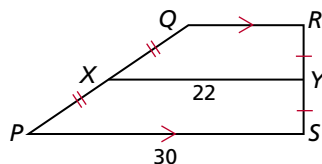
- $MQ = 7y - 6$, and $LP = 4y + 11$. Find the value of y so that $LMPQ$ is isosceles.



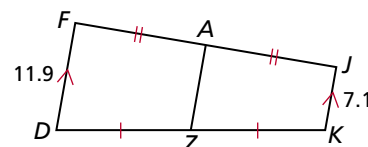
SEE EXAMPLE 5

p. 431

- Find QR .



- Find AZ .



PRACTICE AND PROBLEM SOLVING

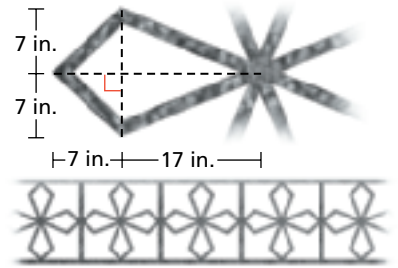
Independent Practice

For Exercises	See Example
13	1
14–16	2
17–18	3
19–20	4
21–22	5

Extra Practice

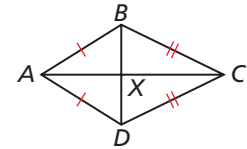
Skills Practice p. S15
Application Practice p. S33

- 13. Design** Each square section in the iron railing contains four small kites. The figure shows the dimensions of one kite. What length of iron is needed to outline one small kite? How much iron is needed to outline one complete section, including the square?

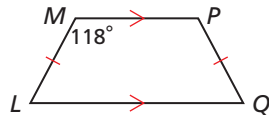


In kite $ABCD$, $m\angle DAX = 32^\circ$, and $m\angle XDC = 64^\circ$. Find each measure.

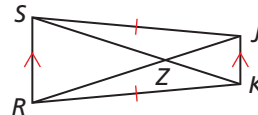
14. $m\angle XDA$ 15. $m\angle ABC$ 16. $m\angle BCD$



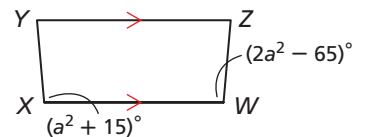
17. Find $m\angle Q$.



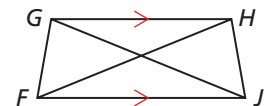
18. $SZ = 62.6$, and $KZ = 34$. Find RJ .



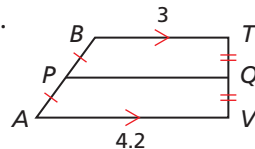
- 19. Algebra** Find the value of a so that $XYZW$ is isosceles. Give your answer as a simplified radical.



- 20. Algebra** $GJ = 4x - 1$, and $FH = 9x - 15$. Find the value of x so that $FGHJ$ is isosceles.



21. Find PQ .



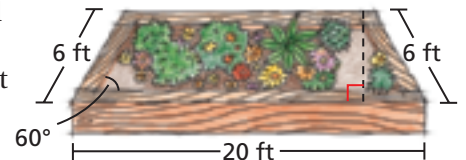
22. Find KR .



Tell whether each statement is sometimes, always, or never true.

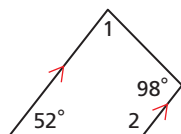
23. The opposite angles of a trapezoid are supplementary.
24. The opposite angles of a kite are supplementary.
25. A pair of consecutive angles in a kite are supplementary.

26. **Estimation** Hal is building a trapezoid-shaped frame for a flower bed. The lumber costs \$1.29 per foot. Based on Hal's sketch, estimate the cost of the lumber. (*Hint*: Find the angle measures in the triangle formed by the dashed line.)

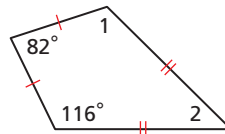


Find the measure of each numbered angle.

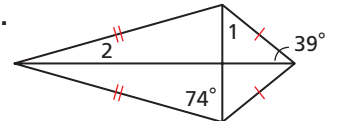
- 27.



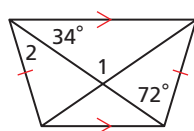
- 28.



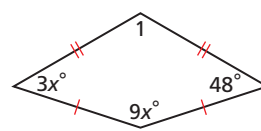
- 29.



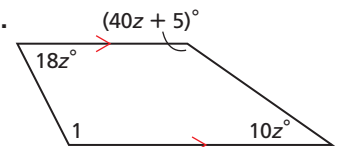
- 30.



- 31.



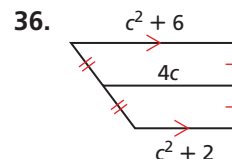
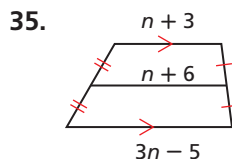
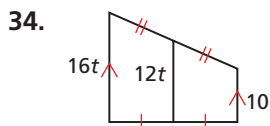
- 32.





33. This problem will prepare you for the Multi-Step Test Prep on page 436. The boundary of a fairground is a quadrilateral with vertices at $E(-1, 3)$, $F(3, 4)$, $G(2, 0)$, and $H(-3, -2)$.
- Use the Distance Formula to show that $EFGH$ is a kite.
 - The organizers need to know the angle measure at each vertex. Given that $m\angle H = 46^\circ$ and $m\angle F = 62^\circ$, find $m\angle E$ and $m\angle G$.

Algebra Find the length of the midsegment of each trapezoid.



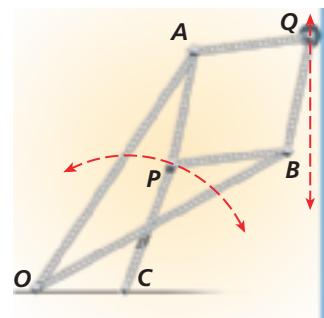
Mechanics



The Peaucellier cell, invented in 1864, converts circular motion into linear motion. This type of linkage was supposedly used in the fans that ventilated the Houses of Parliament in London prior to the invention of electric fans.

The Granger Collection, New York

37. **Mechanics** A Peaucellier cell is made of seven rods connected by joints at the labeled points. $AQBP$ is a rhombus, and $\overline{OA} \cong \overline{OB}$. As P moves along a circular path, Q moves along a linear path. In the position shown, $m\angle AQB = 72^\circ$, and $m\angle AOB = 28^\circ$. What are $m\angle PAQ$, $m\angle OAQ$, and $m\angle OBP$?
38. Prove that one diagonal of a kite bisects a pair of opposite angles and the other diagonal.
39. Prove Theorem 6-6-1: If a quadrilateral is a kite, then its diagonals are perpendicular.

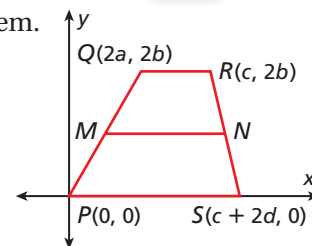


Multi-Step Give the best name for a quadrilateral with the given vertices.

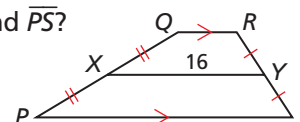
40. $(-4, -1), (-4, 6), (2, 6), (2, -4)$ 41. $(-5, 2), (-5, 6), (-1, 6), (2, -1)$
42. $(-2, -2), (1, 7), (4, 4), (1, -5)$ 43. $(-4, -3), (0, 3), (4, 3), (8, -3)$
44. **Carpentry** The window frame is a regular octagon. It is made from eight pieces of wood shaped like congruent isosceles trapezoids. What are $m\angle A$, $m\angle B$, $m\angle C$, and $m\angle D$?



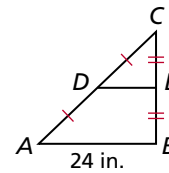
45. **Write About It** Compare an isosceles trapezoid to a trapezoid that is not isosceles. What properties do the figures have in common? What properties does one have that the other does not?
46. Use coordinates to verify the Trapezoid Midsegment Theorem.
- M is the midpoint of \overline{QP} . What are its coordinates?
 - N is the midpoint of \overline{RS} . What are its coordinates?
 - Find the slopes of \overline{QR} , \overline{PS} , and \overline{MN} . What can you conclude?
 - Find QR , PS , and MN . Show that $MN = \frac{1}{2}(PS + QR)$.



47. In trapezoid $PQRS$, what could be the lengths of \overline{QR} and \overline{PS} ?
- (A) 6 and 10 (C) 8 and 32
 (B) 6 and 26 (D) 10 and 24

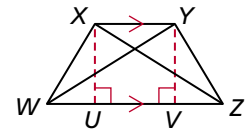


48. Which statement is never true for a kite?
- (F) The diagonals are perpendicular.
 - (G) One pair of opposite angles are congruent.
 - (H) One pair of opposite sides are parallel.
 - (J) Two pairs of consecutive sides are congruent.
49. **Gridded Response** What is the length of the midsegment of trapezoid $ADEB$ in inches?



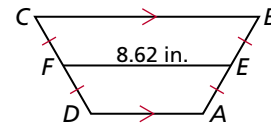
CHALLENGE AND EXTEND

50. Write a two-column proof. (*Hint*: If there is a line and a point not on the line, then there is exactly one line through the point perpendicular to the given line. Use this fact to draw auxiliary lines \overline{UX} and \overline{VY} so that $\overline{UX} \perp \overline{WZ}$ and $\overline{VY} \perp \overline{WZ}$.)



Given: $WXYZ$ is a trapezoid with $\overline{XZ} \cong \overline{YW}$.
Prove: $WXYZ$ is an isosceles trapezoid.

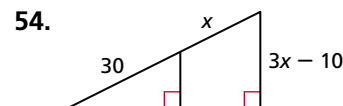
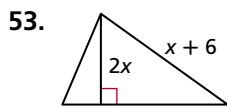
51. The perimeter of isosceles trapezoid $ABCD$ is 27.4 inches. If $BC = 2(AB)$, find AD , AB , BC , and CD .



SPIRAL REVIEW

52. An empty pool is being filled with water. After 10 hours, 20% of the pool is full. If the pool is filled at a constant rate, what fraction of the pool will be full after 25 hours? (*Previous course*)

Write and solve an inequality for x . (*Lesson 3-4*)



Tell whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply. (*Lesson 6-5*)

55. $(-3, 1)$, $(-1, 3)$, $(1, 1)$, and $(-1, -1)$ 56. $(1, 1)$, $(4, 5)$, $(4, 0)$, and $(1, -4)$



Construction Kite

<p>1</p>	<p>2</p>	<p>3</p>	<p>4</p>
Draw a segment \overline{AC} .	Construct line l as the perpendicular bisector of \overline{AC} . Label the intersection as X .	Draw a point B on l above \overline{AC} . Draw \overline{AB} and \overline{CB} .	Draw a point D on l below \overline{AC} so that $DX \neq BX$. Draw \overline{AD} and \overline{CD} .

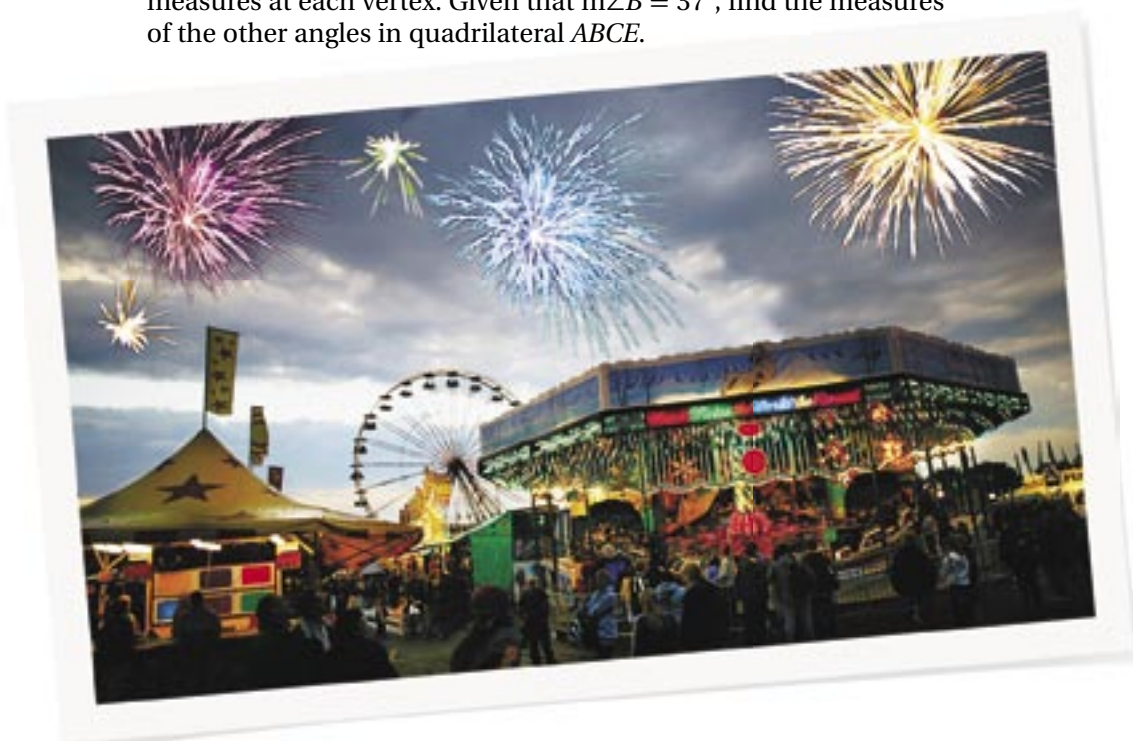
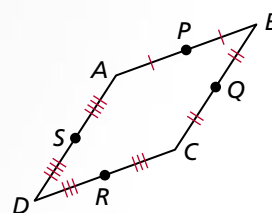
1. **Critical Thinking** How would you modify the construction above so that $ABCD$ is a concave kite?



Other Special Quadrilaterals

A Fair Arrangement The organizers of a county fair are using a coordinate plane to plan the layout of the fairground. The fence that surrounds the fairground will have vertices at $A(-1, 4)$, $B(7, 8)$, $C(3, 0)$, and $D(-5, -4)$.

- The organizers consider creating two straight paths through the fairground: one from point A to point C and another from point B to point D . Use a theorem from Lesson 6-4 to prove that these paths would be perpendicular.
- The organizers instead decide to put an entry gate at the midpoint of each side of the fence, as shown. They plan to create straight paths that connect the gates. Show that the paths \overline{PQ} , \overline{QR} , \overline{RS} , and \overline{SP} form a parallelogram.
- Use the paths \overline{PR} and \overline{SQ} to tell whether $\square PQRS$ is a rhombus, rectangle, or square.
- One section of the fair will contain all the rides and games. The organizers will fence off this area within the fairground by using the existing fences along \overline{AB} and \overline{BC} and adding fences along \overline{AE} and \overline{CE} , where E has coordinates $(-1, 0)$. What type of quadrilateral will be formed by these four fences?
- To construct the fences, the organizers need to know the angle measures at each vertex. Given that $m\angle B = 37^\circ$, find the measures of the other angles in quadrilateral $ABCE$.

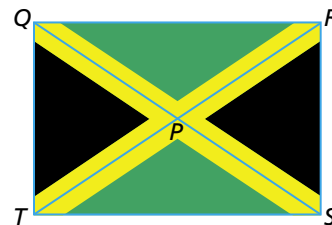


Quiz for Lessons 6-4 Through 6-6

6-4 Properties of Special Parallelograms

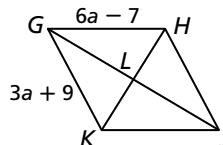
The flag of Jamaica is a rectangle with stripes along the diagonals. In rectangle $QRST$, $QS = 80.5$, and $RS = 36$. Find each length.

- SP
- QT
- TR
- TP

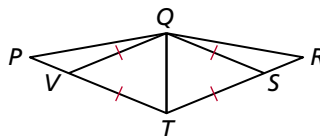


$GHIK$ is a rhombus. Find each measure.

- HJ
- $m\angle HJG$ and $m\angle GHJ$ if $m\angle JLH = (4b - 6)^\circ$ and $m\angle JKH = (2b + 11)^\circ$



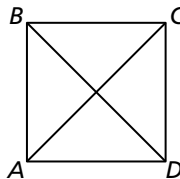
- Given: $QSTV$ is a rhombus. $\overline{PT} \cong \overline{RT}$
Prove: $\overline{PQ} \cong \overline{RQ}$



6-5 Conditions for Special Parallelograms

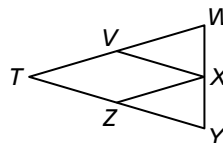
Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

- Given: $\overline{AC} \perp \overline{BD}$
Conclusion: $ABCD$ is a rhombus.
- Given: $\overline{AB} \cong \overline{CD}$, $\overline{AC} \cong \overline{BD}$, $\overline{AB} \parallel \overline{CD}$
Conclusion: $ABCD$ is a rectangle.



Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

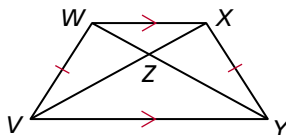
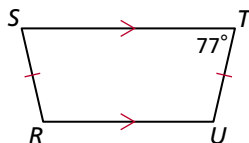
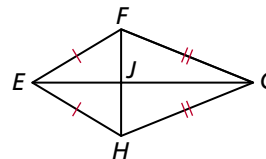
- $W(-2, 2)$, $X(1, 5)$, $Y(7, -1)$, $Z(4, -4)$
- Given: \overline{VX} and \overline{VZ} are midsegments of $\triangle TWY$. $\overline{TW} \cong \overline{TY}$
Prove: $TVXZ$ is a rhombus.



6-6 Properties of Kites and Trapezoids

In kite $EFGH$, $m\angle FHG = 68^\circ$, and $m\angle FEH = 62^\circ$. Find each measure.

- $m\angle FEJ$
- $m\angle FGJ$
- Find $m\angle R$.
- $m\angle EHG$
- $m\angle EHG$
- $YZ = 34.2$, and $VX = 53.4$. Find WZ .



- A dulcimer is a trapezoid-shaped stringed instrument. The bases are 43 in. and 23 in. long. If a string is attached at the midpoint of each leg of the trapezoid, how long is the string?





For a complete list of the postulates and theorems in this chapter, see p. S82.

Vocabulary

base of a trapezoid	429	kite	427	rhombus	409
base angle of a trapezoid	429	leg of a trapezoid	429	side of a polygon	382
concave	383	midsegment of a trapezoid . .	431	square	410
convex	383	parallelogram	391	trapezoid	429
diagonal	382	rectangle	408	vertex of a polygon	382
isosceles trapezoid	429	regular polygon	382		

Complete the sentences below with vocabulary words from the list above.

- The common endpoint of two sides of a polygon is a(n) ? .
- A polygon is ? if no diagonal contains points in the exterior.
- A(n) ? is a quadrilateral with four congruent sides.
- Each of the parallel sides of a trapezoid is called a(n) ? .

6-1 Properties and Attributes of Polygons (pp. 382–388)

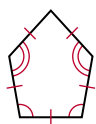
EXAMPLES

- Tell whether the figure is a polygon. If it is a polygon, name it by the number of its sides.



The figure is a closed plane figure made of segments that intersect only at their endpoints, so it is a polygon. It has six sides, so it is a hexagon.

- Tell whether the polygon is regular or irregular. Tell whether it is concave or convex.



The polygon is equilateral, but it is not equiangular. So it is not regular. No diagonal contains points in the exterior, so it is convex.

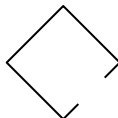
Find each measure.

- the sum of the interior angle measures of a convex 11-gon
 $(n - 2)180^\circ$ *Polygon \angle Sum Thm.*
 $(11 - 2)180^\circ = 1620^\circ$ *Substitute 11 for n.*
- the measure of each exterior angle of a regular pentagon
 sum of ext. $\angle = 360^\circ$ *Polygon Ext. \angle SumThm.*
 measure of one ext. $\angle = \frac{360^\circ}{5} = 72^\circ$

EXERCISES

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.

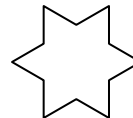
5.



6.

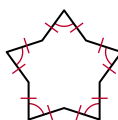


7.

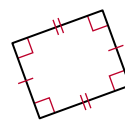


Tell whether each polygon is regular or irregular. Tell whether it is concave or convex.

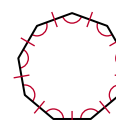
8.



9.

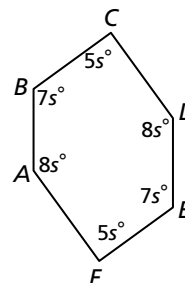


10.



Find each measure.

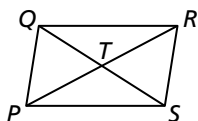
- the sum of the interior angle measures of a convex dodecagon
- the measure of each interior angle of a regular 20-gon
- the measure of each exterior angle of a regular quadrilateral
- the measure of each interior angle of hexagon *ABCDEF*



6-2 Properties of Parallelograms (pp. 391–397)

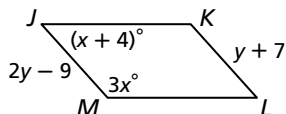
EXAMPLES

- In $\square PQRS$, $m\angle RSP = 99^\circ$, $PQ = 19.8$, and $RT = 12.3$. Find PT .



$$\begin{aligned} \overline{PT} &\cong \overline{RT} && \square \rightarrow \text{diags. bisect each other} \\ PT &= RT && \text{Def. of } \cong \text{ segs.} \\ PT &= 12.3 && \text{Substitute 12.3 for RT.} \end{aligned}$$

- $JKLM$ is a parallelogram. Find each measure.

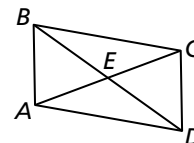


- LK
- $$\begin{aligned} \overline{JM} &\cong \overline{LK} && \square \rightarrow \text{opp. sides } \cong \\ JM &= LK && \text{Def. of } \cong \text{ segs.} \\ 2y - 9 &= y + 7 && \text{Substitute the given values.} \\ y &= 16 && \text{Solve for } y. \\ LK &= 16 + 7 = 23 \end{aligned}$$

- $m\angle M$
- $$\begin{aligned} m\angle J + m\angle M &= 180^\circ && \square \rightarrow \text{cons. } \angle \text{ supp.} \\ (x + 4) + 3x &= 180 && \text{Substitute the given values.} \\ x &= 44 && \text{Solve for } x. \\ m\angle M &= 3(44) = 132^\circ \end{aligned}$$

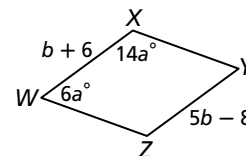
EXERCISES

- In $\square ABCD$, $m\angle ABC = 79^\circ$, $BC = 62.4$, and $BD = 75$. Find each measure.



15. BE 16. AD
17. ED 18. $m\angle CDA$
19. $m\angle BCD$ 20. $m\angle DAB$

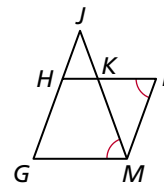
- $WXYZ$ is a parallelogram. Find each measure.



21. WX 22. YZ
23. $m\angle W$ 24. $m\angle X$
25. $m\angle Y$ 26. $m\angle Z$
27. Three vertices of $\square RSTV$ are $R(-8, 1)$, $S(2, 3)$, and $V(-4, -7)$. Find the coordinates of vertex T .

28. Write a two-column proof.
Given: $GHLM$ is a parallelogram.
 $\angle L \cong \angle JMG$

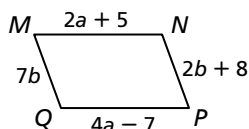
Prove: $\triangle GJM$ is isosceles.



6-3 Conditions for Parallelograms (pp. 398–405)

EXAMPLES

- Show that $MNPQ$ is a parallelogram for $a = 6$ and $b = 1.6$.



$$\begin{aligned} MN &= 2a + 5 && QP &= 4a - 7 \\ MN &= 2(6) + 5 = 17 && QP &= 4(6) - 7 = 17 \\ MQ &= 7b && NP &= 2b + 8 \\ MQ &= 7(1.6) = 11.2 && NP &= 2(1.6) + 8 = 11.2 \end{aligned}$$

Since its opposite sides are congruent, $MNPQ$ is a parallelogram.

- Determine if the quadrilateral must be a parallelogram. Justify your answer.

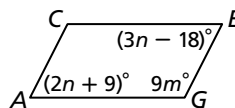


No. One pair of opposite angles are congruent, and one pair of consecutive sides are congruent. None of the conditions for a parallelogram are met.

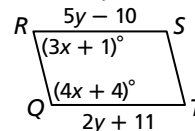
EXERCISES

- Show that the quadrilateral is a parallelogram for the given values of the variables.

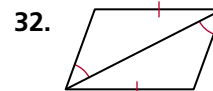
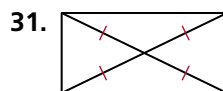
29. $m = 13$, $n = 27$



30. $x = 25$, $y = 7$



- Determine if the quadrilateral must be a parallelogram. Justify your answer.

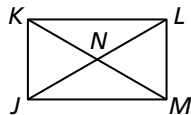


33. Show that the quadrilateral with vertices $B(-4, 3)$, $D(6, 5)$, $F(7, -1)$, and $H(-3, -3)$ is a parallelogram.

6-4 Properties of Special Parallelograms (pp. 408–415)

EXAMPLES

In rectangle $JKLM$, $KM = 52.8$, and $JM = 45.6$. Find each length.



■ KL

$JKLM$ is a \square .
 $KL = JM = 45.6$

*Rect. $\rightarrow \square$
 $\square \rightarrow$ opp. sides \cong*

■ NL

$JL = KM = 52.8$

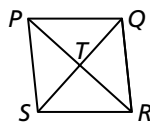
Rect. \rightarrow diags. \cong

$NL = \frac{1}{2}JL = 26.4$

$\square \rightarrow$ diags. bisect each other

■ $PQRS$ is a rhombus.

Find $m\angle QPR$, given that $m\angle QTR = (6y + 6)^\circ$ and $m\angle SPR = 3y^\circ$.



$m\angle QTR = 90^\circ$

Rhombus \rightarrow diags. \perp

$6y + 6 = 90$

Substitute the given value.

$y = 14$

Solve for y .

$m\angle QPR = m\angle SPR$

Rhombus \rightarrow each

$m\angle QPR = 3(14)^\circ = 42^\circ$

diag. bisects opp. \angle .

■ The vertices of square $ABCD$ are $A(5, 0)$, $B(2, 4)$, $C(-2, 1)$, and $D(1, -3)$. Show that the diagonals of square $ABCD$ are congruent perpendicular bisectors of each other.

$AC = BD = 5\sqrt{2}$

Diags. are \cong .

slope of $\overline{AC} = -\frac{1}{7}$

Product of slopes is -1 ,

slope of $\overline{BD} = 7$

so diags. are \perp .

mdpt. of \overline{AC}

$=$ mdpt. of $\overline{BD} = \left(\frac{3}{2}, \frac{1}{2}\right)$

Diags. bisect each other.

EXERCISES

In rectangle $ABCD$, $CD = 18$, and $CE = 19.8$.

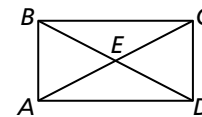
Find each length.

34. AB

35. AC

36. BD

37. BE



In rhombus $WXYZ$, $WX = 7a + 1$,

$WZ = 9a - 6$, and $VZ = 3a$.

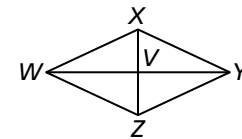
Find each measure.

38. WZ

39. XV

40. XY

41. XZ



In rhombus $RSTV$, $m\angle TZV = (8n + 18)^\circ$,

and $m\angle SRV = (9n + 1)^\circ$.

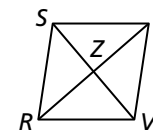
Find each measure.

42. $m\angle TRS$

43. $m\angle RSV$

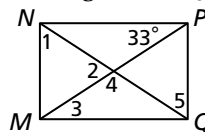
44. $m\angle STV$

45. $m\angle TVR$

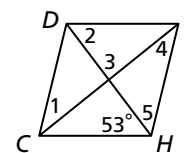


Find the measures of the numbered angles in each figure.

46. rectangle $MNPQ$



47. rhombus $CDGH$



Show that the diagonals of the square with the given vertices are congruent perpendicular bisectors of each other.

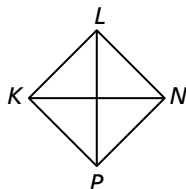
48. $R(-5, 0)$, $S(-1, -2)$, $T(-3, -6)$, and $U(-7, -4)$

49. $E(2, 1)$, $F(5, 1)$, $G(5, -2)$, and $H(2, -2)$

6-5 Conditions for Special Parallelograms (pp. 418–425)

EXAMPLES

■ Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



Given: $\overline{LP} \perp \overline{KN}$

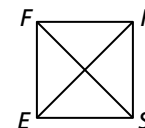
Conclusion: $KLNP$ is a rhombus.

The conclusion is not valid.

If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus. To apply this theorem, you must first know that $KLNP$ is a parallelogram.

EXERCISES

Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.



50. Given: $\overline{ER} \perp \overline{FS}$, $\overline{ER} \cong \overline{FS}$

Conclusion: $EFRS$ is a square.

51. Given: \overline{ER} and \overline{FS} bisect each other.

$\overline{ER} \cong \overline{FS}$

Conclusion: $EFRS$ is a rectangle.

52. Given: $\overline{EF} \parallel \overline{RS}$, $\overline{FR} \parallel \overline{ES}$, $\overline{EF} \cong \overline{RS}$

Conclusion: $EFRS$ is a rhombus.

- Use the diagonals to tell whether a parallelogram with vertices $P(-5, 3)$, $Q(0, 1)$, $R(2, -4)$, and $S(-3, -2)$ is a rectangle, rhombus, or square. Give all the names that apply.

$$PR = \sqrt{98} = 7\sqrt{2} \quad \text{Distance Formula}$$

$$QS = \sqrt{18} = 3\sqrt{2} \quad \text{Distance Formula}$$

Since $PR \neq QS$, $PQRS$ is not a rectangle and not a square.

$$\text{slope of } \overline{PR} = \frac{7}{-7} = -1 \quad \text{Slope Formula}$$

$$\text{slope of } \overline{QS} = \frac{3}{3} = 1 \quad \text{Slope Formula}$$

Since the product of the slopes is -1 , the diagonals are perpendicular. $PQRS$ is a rhombus.

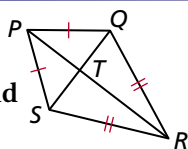
Use the diagonals to tell whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

53. $B(-3, 0)$, $F(-2, 7)$, $J(5, 8)$, $N(4, 1)$
 54. $D(-4, -3)$, $H(5, 6)$, $L(8, 3)$, $P(-1, -6)$
 55. $Q(-8, -2)$, $T(-6, 8)$, $W(4, 6)$, $Z(2, -4)$

6-6 Properties of Kites and Trapezoids (pp. 427–435)

EXAMPLES

- In kite $PQRS$, $m\angle SRT = 24^\circ$, and $m\angle TSP = 53^\circ$. Find $m\angle SPT$.

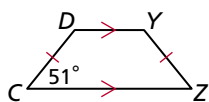


$\triangle PTS$ is a right triangle. *Kite \rightarrow diags. \perp*
 $m\angle SPT + m\angle TSP = 90^\circ$ *Acute \triangle of rt. \triangle are comp.*

$$m\angle SPT + 53 = 90 \quad \text{Substitute } 53 \text{ for } m\angle TSP.$$

$$m\angle SPT = 37^\circ \quad \text{Subtract } 53 \text{ from both sides.}$$

- Find $m\angle D$.

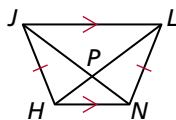


$$m\angle C + m\angle D = 180^\circ \quad \text{Same-Side Int. \angle Thm.}$$

$$51 + m\angle D = 180 \quad \text{Substitute } 51 \text{ for } m\angle C.$$

$$m\angle D = 129^\circ \quad \text{Subtract.}$$

- In trapezoid $HJLN$, $JP = 32.5$, and $HL = 50$. Find PN .



$$\overline{JN} \cong \overline{HL} \quad \text{Isosc. trap. \rightarrow diags. \cong }$$

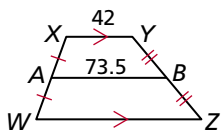
$$JN = HL = 50 \quad \text{Def. of } \cong \text{ segs.}$$

$$JP + PN = JN \quad \text{Seg. Add. Post.}$$

$$32.5 + PN = 50 \quad \text{Substitute.}$$

$$PN = 17.5 \quad \text{Subtract } 32.5 \text{ from both sides.}$$

- Find WZ .



$$AB = \frac{1}{2}(XY + WZ) \quad \text{Trap. Midsegment Thm.}$$

$$73.5 = \frac{1}{2}(42 + WZ) \quad \text{Substitute.}$$

$$147 = 42 + WZ \quad \text{Multiply both sides by } 2.$$

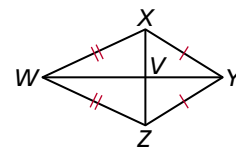
$$105 = WZ \quad \text{Solve for } WZ.$$

EXERCISES

In kite $WXYZ$, $m\angle VXY = 58^\circ$, and $m\angle ZWX = 50^\circ$.

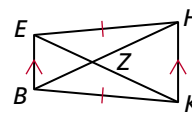
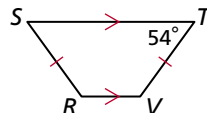
Find each measure.

56. $m\angle XYZ$ 57. $m\angle ZWV$
 58. $m\angle VZW$ 59. $m\angle WZY$

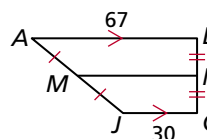


Find each measure.

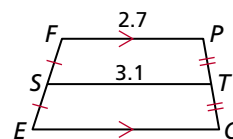
60. $m\angle R$ and $m\angle S$ 61. BZ if $ZH = 70$ and $EK = 121.6$



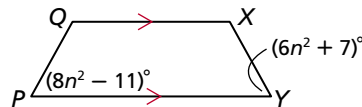
62. MN



63. EQ



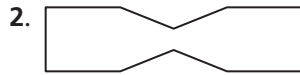
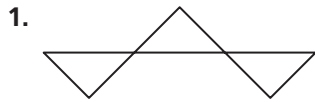
64. Find the value of n so that $PQXY$ is isosceles.



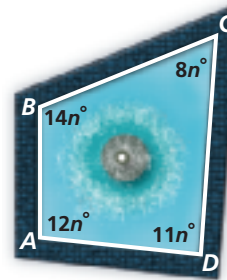
Give the best name for a quadrilateral whose vertices have the given coordinates.

65. $(-4, 5)$, $(-1, 8)$, $(5, 5)$, $(-1, 2)$
 66. $(1, 4)$, $(5, 4)$, $(5, -4)$, $(1, -1)$
 67. $(-6, -1)$, $(-4, 2)$, $(0, 2)$, $(2, -1)$

Tell whether each figure is a polygon. If it is a polygon, name it by the number of its sides.



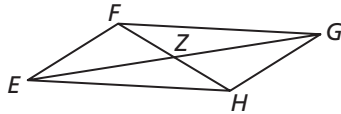
3. The base of a fountain is in the shape of a quadrilateral, as shown. Find the measure of each interior angle of the fountain.



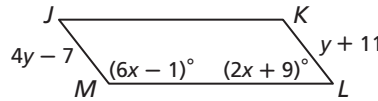
4. Find the sum of the interior angle measures of a convex nonagon.

5. Find the measure of each exterior angle of a regular 15-gon.

6. In $\square EFGH$, $EH = 28$, $HZ = 9$, and $m\angle EHG = 145^\circ$. Find FH and $m\angle FEH$.

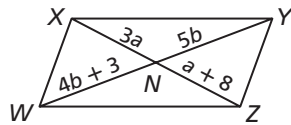


7. $JKLM$ is a parallelogram. Find KL and $m\angle L$.

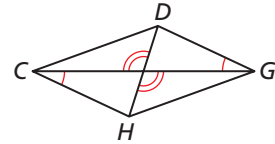


8. Three vertices of $\square PQRS$ are $P(-2, -3)$, $R(7, 5)$, and $S(6, 1)$. Find the coordinates of Q .

9. Show that $WXYZ$ is a parallelogram for $a = 4$ and $b = 3$.

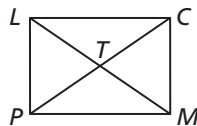


10. Determine if $CDGH$ must be a parallelogram. Justify your answer.

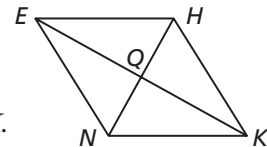


11. Show that a quadrilateral with vertices $K(-7, -3)$, $L(2, 0)$, $S(5, -4)$, and $T(-4, -7)$ is a parallelogram.

12. In rectangle $PLCM$, $LC = 19$, and $LM = 23$. Find PT and PM .



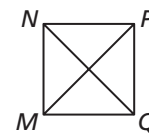
13. In rhombus $EHKN$, $m\angle NQK = (7z + 6)^\circ$, and $m\angle ENQ = (5z + 1)^\circ$. Find $m\angle HEQ$ and $m\angle EHK$.



Determine if the conclusion is valid. If not, tell what additional information is needed to make it valid.

14. Given: $\overline{NP} \cong \overline{PQ} \cong \overline{QM} \cong \overline{MN}$
Conclusion: $MNPQ$ is a square.

15. Given: $\overline{NP} \cong \overline{MQ}$, $\overline{NM} \cong \overline{PQ}$, $\overline{NQ} \cong \overline{MP}$
Conclusion: $MNPQ$ is a rectangle.

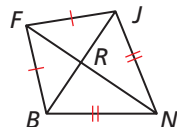


Use the diagonals to determine whether a parallelogram with the given vertices is a rectangle, rhombus, or square. Give all the names that apply.

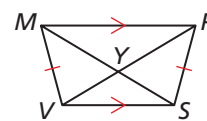
16. $A(-5, 7)$, $C(3, 6)$, $E(7, -1)$, $G(-1, 0)$

17. $P(4, 1)$, $Q(3, 4)$, $R(-3, 2)$, $S(-2, -1)$

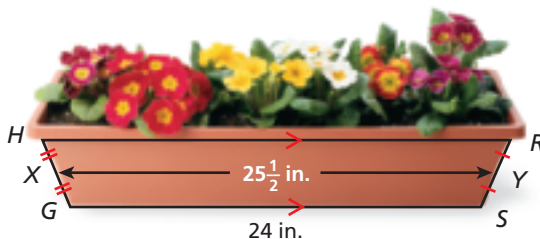
18. $m\angle JFR = 43^\circ$, and $m\angle JNB = 68^\circ$. Find $m\angle FBN$.



19. $PV = 61.1$, and $YS = 24.7$. Find MY .



20. Find HR .



COLLEGE ENTRANCE EXAM PRACTICE



FOCUS ON SAT

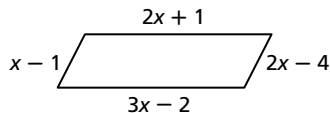
The scores for each SAT section range from 200 to 800. Your score is calculated by subtracting a fraction for each incorrect multiple-choice answer from the total number of correct answers. No points are deducted for incorrect grid-in answers or items you left blank.



If you have time, go back through each section of the test and check as many of your answers as possible. Try to use a different method of solving the problem than you used the first time.

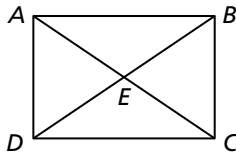
You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

1. Given the quadrilateral below, what value of x would allow you to conclude that the figure is a parallelogram?



- (A) -2
- (B) 0
- (C) 1
- (D) 2
- (E) 3

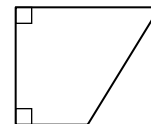
2. In the figure below, if $ABCD$ is a rectangle, what type of triangle must $\triangle ABE$ be?



- (A) Equilateral
- (B) Right
- (C) Equiangular
- (D) Isosceles
- (E) Scalene

3. Which of the following terms best describes the figure below?

- (A) Rhombus
- (B) Trapezoid
- (C) Quadrilateral
- (D) Square
- (E) Parallelogram



4. Three vertices of $\square MNPQ$ are $M(3, 1)$, $N(0, 6)$, and $P(4, 7)$. Which of the following could be the coordinates of vertex Q ?

- (A) $(7, 0)$
- (B) $(-1, 1)$
- (C) $(7, 2)$
- (D) $(11, 3)$
- (E) $(9, 4)$

5. If $ABCDE$ is a regular pentagon, what is the measure of $\angle C$?

- (A) 45°
- (B) 60°
- (C) 90°
- (D) 108°
- (E) 120°



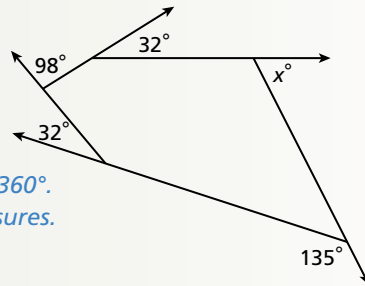
Multiple Choice: Eliminate Answer Choices

For some multiple-choice test items, you can eliminate one or more of the answer choices without having to do many calculations. Use estimation or logic to help you decide which answer choices can be eliminated.

EXAMPLE 1

What is the value of x in the figure?

- (A) 3° (C) 83°
 (B) 63° (D) 153°



The sum of the exterior angle measures of a convex polygon is 360° .
 By rounding, you can estimate the sum of the given angle measures.

$$100^\circ + 30^\circ + 140^\circ + 30^\circ = 300^\circ$$

If $x = 153^\circ$, the sum of the angle measures would be far greater than 360° .
 So eliminate D.

If $x = 3^\circ$, the sum would be far less than 360° . So eliminate A.

From your estimate, it seems likely that the correct choice is B, 63° .
 Confirm that this is correct by doing the actual calculation.

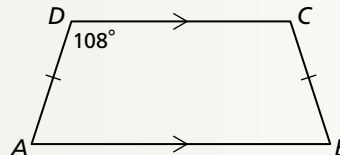
$$98^\circ + 32^\circ + 63^\circ + 135^\circ + 32^\circ = 360^\circ$$

The correct answer is B, 63° .

EXAMPLE 2

What is $m\angle B$ in the isosceles trapezoid?

- (F) 216° (H) 72°
 (G) 108° (J) 58°



Base angles of an isosceles trapezoid are congruent.
 Since $\angle D$ and $\angle B$ are not a pair of base angles, their measures are not equal. Eliminate G, 108° .

$\angle D$ and $\angle C$ are base angles, so $m\angle C = 108^\circ$. $\angle B$ and $\angle C$ are same-side interior angles formed by parallel lines. So they are supplementary angles. Therefore the measure of angle B cannot be greater than 180° . You can eliminate F.

$$m\angle B = 180^\circ - 108^\circ = 72^\circ$$

The correct answer is H, 72° .

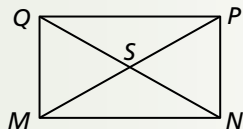


Try to eliminate unreasonable answer choices. Some choices may be too large or too small or may have incorrect units.

Read each test item and answer the questions that follow.

Item A

The diagonals of rectangle $MNPQ$ intersect at S . If $MN = 4.1$ meters, $MS = 2.35$ meters, and $MQ = 2.3$ meters, what is the area of $\triangle MPQ$ to the nearest tenth?

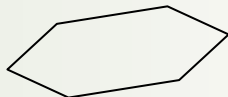


- (A) 4.7 square meters
- (B) 5.4 meters
- (C) 9.4 square meters
- (D) 12.8 meters

- Are there any answer choices you can eliminate immediately? If so, which choices and why?
- Describe how to use estimation to eliminate at least one more answer choice.

Item B

What is the sum of the interior angles of a convex hexagon?

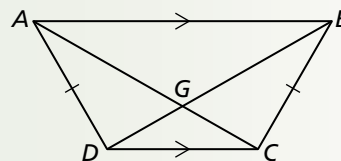


- (F) 180°
- (G) 500°
- (H) 720°
- (J) 1080°

- Can any of the answer choices be eliminated immediately? If so, which choices and why?
- How can you use the fact that 500 is not a multiple of 180 to eliminate choice G?
- A student answered this problem with J. Explain the mistake the student made.

Item C

In isosceles trapezoid $ABCD$, $AC = 18.2$, and $DG = 6.3$. What is GB ?

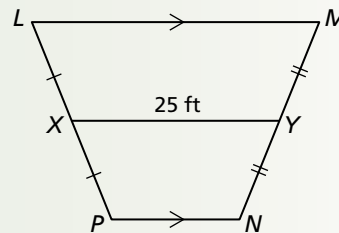


- (A) 24.5
- (B) 11.9
- (C) 6.3
- (D) 2.9

- Will the measure of \overline{GB} be more than, less than, or equal to the measure of \overline{AC} ? What answer choices can you eliminate and why?
- Explain how to use estimation to answer this problem.

Item D

In trapezoid $LMNP$, $\overline{XY} = 25$ feet. What are two possible lengths for \overline{LM} and \overline{PN} ?



- (F) 18 feet and 32 feet
- (G) 49 feet and 2 feet
- (H) 10 feet and 15 feet
- (J) 7 inches and 43 inches

- Which answer choice can you eliminate immediately? Why?
- A student used logic to eliminate choice H. Do you agree with the student's decision? Explain.
- A student used estimation and answered this problem with G. Explain the mistake the student made.

CUMULATIVE ASSESSMENT, CHAPTERS 1–6
Multiple Choice

1. The exterior angles of a triangle have measures of $(x + 10)^\circ$, $(2x + 20)^\circ$, and $3x^\circ$. What is the measure of the smallest interior angle of the triangle?

(A) 15° (C) 55°
 (B) 35° (D) 65°

2. If a plant is a monocot, then its leaves have parallel veins. If a plant is an orchid, then it is a monocot. A Mexican vanilla plant is an orchid. Based on this information, which conclusion is NOT valid?

(F) The leaves of a Mexican vanilla plant have parallel veins.
 (G) A Mexican vanilla plant is a monocot.
 (H) All orchids have leaves with parallel veins.
 (J) All monocots are orchids.

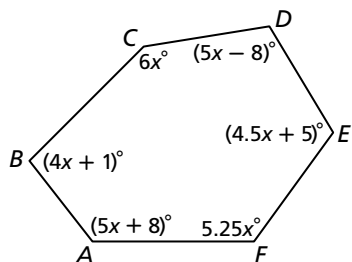
3. If $\triangle ABC \cong \triangle PQR$ and $\triangle RPQ \cong \triangle XYZ$, which of the following angles is congruent to $\angle CAB$?

(A) $\angle QRP$ (C) $\angle YXZ$
 (B) $\angle XZY$ (D) $\angle XYZ$

4. Which line coincides with the line $2y + 3x = 4$?

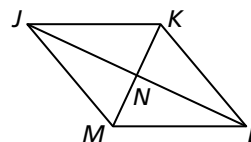
(F) $3y + 2x = 4$
 (G) $y = \frac{2}{3}x + 2$
 (H) a line through $(-1, 1)$ and $(2, 3)$
 (J) a line through $(0, 2)$ and $(4, -4)$

5. What is the value of x in polygon $ABCDEF$?



(A) 12 (C) 24
 (B) 18 (D) 36

Use the figure below for Items 6 and 7.



6. If $\overline{JK} \parallel \overline{ML}$, what additional information do you need to prove that quadrilateral $JKLM$ is a parallelogram?

(F) $\overline{JM} \cong \overline{KL}$
 (G) $\overline{MN} \cong \overline{LN}$
 (H) $\angle MLK$ and $\angle LKJ$ are right angles.
 (J) $\angle JML$ and $\angle KLM$ are supplementary.

7. Given that $JKLM$ is a parallelogram and that $m\angle KLN = 25^\circ$, $m\angle JMN = 65^\circ$, and $m\angle JML = 130^\circ$, which term best describes quadrilateral $JKLM$?

(A) Rectangle
 (B) Rhombus
 (C) Square
 (D) Trapezoid

8. For two lines and a transversal, $\angle 1$ and $\angle 2$ are same-side interior angles, $\angle 2$ and $\angle 3$ are vertical angles, and $\angle 3$ and $\angle 4$ are alternate exterior angles. Which classification best describes the angle pair $\angle 2$ and $\angle 4$?

(F) Adjacent angles
 (G) Alternate interior angles
 (H) Corresponding angles
 (J) Vertical angles

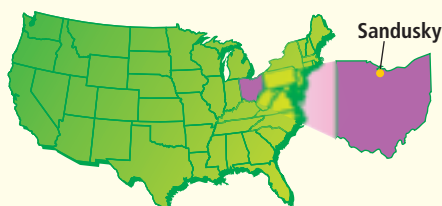
9. For $\triangle ABC$ and $\triangle DEF$, $\angle A \cong \angle F$, and $\overline{AC} \cong \overline{EF}$. Which of the following would allow you to conclude that these triangles are congruent by AAS?

(A) $\angle ABC \cong \angle EDF$
 (B) $\angle ACB \cong \angle EDF$
 (C) $\angle BAC \cong \angle FDE$
 (D) $\angle CBA \cong \angle FED$



Problem Solving on Location

OHIO



★ Handmade Tiles

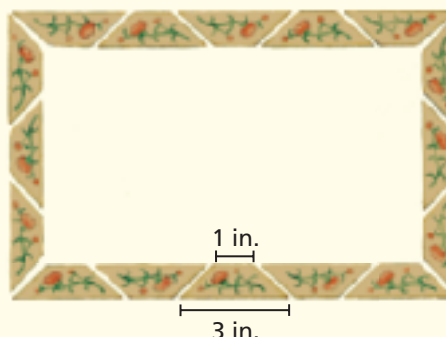
During the nineteenth century, an important industry developed in east central Ohio thanks to an “earthy” discovery—clay! The region’s rich soil and easy access to river transportation helped establish Ohio as the pottery and ceramic capital of the United States. Today the majority of the earthenware clay used in handmade tiles is still mined in Ohio.

Choose one or more strategies to solve each problem.

1. In tile making, soft clay is pressed into long rectangular wooden trays. After the clay has dried, tiles are cut from the rectangular slab. A tile manufacturer wants to make parallelogram-shaped tiles with the dimensions shown. What is the maximum number of such tiles that can be cut from a 12 in. by 40 in. slab of clay?



2. An interior designer is buying tiles that are in the shape of isosceles trapezoids. Each tile has bases that are 1 in. and 3 in. long, and the tiles can be arranged as shown to form a rectangle. How many tiles should the designer buy in order to frame a 25 in. by 49 in. window?
3. A tile manufacturer wants to make a tile in the shape of a rhombus where one diagonal is twice the length of the other diagonal. What should the lengths of the diagonals be in order to make a tile with sides 7 cm long? Round to the nearest hundredth.





Problem Solving Strategies

- Draw a Diagram
- Make a Model
- Guess and Test
- Work Backward
- Find a Pattern
- Make a Table
- Solve a Simpler Problem
- Use Logical Reasoning
- Use a Venn Diagram
- Make an Organized List

★ The Millennium Force Roller Coaster

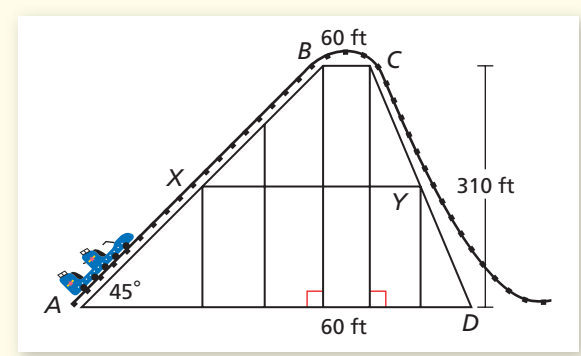
When it opened in May 2000, the Millennium Force roller coaster broke all previous records and became the tallest and fastest roller coaster in the world. One of 16 roller coasters at Cedar Point in Sandusky, Ohio, the Millennium Force takes riders on a wild journey that features 1.25 miles of track, a top speed of 93 miles per hour, and a breathtaking 310-foot drop!



Choose one or more strategies to solve each problem.

1. The first hill of the Millennium Force is 310 ft tall. The ascent to the top of the hill is at a 45° angle. What is the length of the ascent to the nearest tenth of a foot?
2. The Millennium Force was the first coaster in which an elevator lift system was used to pull the trains to the top of the first hill. The system pulls the trains at a speed of 20 ft/s. How long does it take a train to reach the top of the hill?

The figure shows the support structure for the first hill of the Millennium Force. For 3 and 4, use the figure.



3. The length of the first descent \overline{CD} is 314.8 ft. To the nearest foot, what is the total horizontal distance \overline{AD} that the train covers as it goes over the first hill?
4. Engineers designed the support beam \overline{XY} so that X is the midpoint of the ascent \overline{AB} and Y is the midpoint of the descent \overline{CD} . What is the length of the beam to the nearest foot?

