23. \( x + \frac{1}{2} = -\frac{1}{5} \)  
\( 10x + 5 = -2 \)  
\( 10x = -7 \)  
\( x = -\frac{7}{10} \)

24. \(-\frac{1}{2} = 3x - \frac{1}{3} \)  
\( -3 = 18x - 2x \)  
\( -3 = 16x \)  
\( x = -\frac{3}{16} = x \)
Write an equation for costs with each calling card. Then "Let x represent..." ending with "Card B" and its equation.

Let x represent the number of minutes and y represent the total cost.

Card A: \( y = 0.05x + 0.50 \)
Card B: \( y = 0.08x + 0.20 \)

Step 2 Solve the system by using a table of values.

\[
\begin{array}{c|c|c}
\text{ } & y = 0.05x + 0.50 & \text{ } \\
\hline
5 & 0.75 & 5 \\
10 & 1.00 & 10 \\
15 & 1.25 & 15 \\
\end{array}
\quad \begin{array}{c|c|c}
\text{ } & y = 0.08x + 0.20 & \text{ } \\
\hline
5 & 0.60 & 5 \\
10 & 1.00 & 10 \\
15 & 1.40 & 15 \\
\end{array}
\]

So the cost is the same for each card at 10 min.

**THINK AND DISCUSS**

1. Graph the two equations on the same grid. If the lines intersect once, there is 1 solution; if they are parallel, there is no solution; if they coincide, there are infinitely many solutions.

2. A solution to a system is represented on a graph by a point of intersection, and parallel lines never intersect.

3.

<table>
<thead>
<tr>
<th>Exactly One Solution</th>
<th>Infinitely Many Solutions</th>
<th>No Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Example</td>
<td>( y = 2x + 1 )</td>
<td>( y = 2x + 1 )</td>
</tr>
<tr>
<td></td>
<td>( y = x + 2 )</td>
<td>( y = 2x + 1 )</td>
</tr>
<tr>
<td></td>
<td>( y = 2x + 1 )</td>
<td>( y = 2x + 4 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Graphs</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="graph.png" alt="Graphs" /></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Slopes</th>
<th>y-intercepts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Different</td>
<td>Either</td>
</tr>
<tr>
<td>Same</td>
<td>Same</td>
</tr>
<tr>
<td>Same</td>
<td>Different</td>
</tr>
</tbody>
</table>

**EXERCISES**

**GUIDED PRACTICE**

1. inconsistent

2. \( 2x - y = 3 \)

\[
\begin{array}{c|c|c|c|c|c|c|c}
2(3) - (3) & 3 & \frac{3}{3} & 3 & \checkmark \\
\end{array}
\]

Because the point is a solution of both equations, \((3, 3)\) is a solution of the system.

3. \( y - 4x = -7 \)

\[
\begin{array}{c|c|c|c|c|c|c|c|c}
-3 & -4(1) & \frac{-7}{1} & \checkmark \\
\end{array}
\]

Because the point is not a solution of both equations, \((1, -3)\) is not a solution of the system.
10. \( y = -7x + 13 \) 
   \[ 4y = -28x - 12 \] 
   \[ y = -7x - 3 \] 
   inconsistent; no solution

11. \( 2x - 3y = -15 \) 
   \[ 3y = 2x + 15 \] 
   \[ y = \frac{2}{3}x + 5 \] 
   consistent, dependent; infinite number of solutions

12. \( 8y - 24x = 64 \) 
   \[ 8y = 24x + 64 \] 
   \[ y = 3x + 8 \] 
   \[ 9y + 45x = 72 \] 
   \[ 4x + 4y = -16 \] 
   \[ 9y = -45x + 72 \] 
   \[ 4y = -4x - 16 \] 
   \[ y = -5x + 8 \] 
   \[ y = -x - 4 \] 
   consistent, independent; one solution

13. \( 2x + 2y = -10 \) 
   \[ 2y = -2x - 10 \] 
   \[ y = -x - 5 \] 
   consistent, dependent; infinite number of solutions

14. Step 1 Write an equation for draining rate for each tank. [then “Let x represent...” ending with the two equations]
   Let \( x \) represent the number of minutes, and let \( y \) represent the depth of the water.
   Tank A: \( y = -1x + 7 \)
   Tank B: \( y = -0.5x + 5 \)

Step 2 Solve the system by using a table of values.

\[
\begin{array}{c|c}
   x & y \\
   \hline
   1 & 6 \\
   2 & 5 \\
   3 & 4 \\
   4 & 3 \\
   5 & 2 \\
\end{array}
\]

\[
\begin{array}{c|c}
   x & y \\
   \hline
   1 & 4.5 \\
   2 & 4 \\
   3 & 3.5 \\
   4 & 3 \\
   5 & 2.5 \\
\end{array}
\]

So the two tanks have the same amount of water at 4 min.

PRACTICE AND PROBLEM SOLVING

15. \( x + y = 0 \) 
   \[ -2 + 2 = 0 \] 
   \[ 0 = 0 \] 
   \[ \sqrt{42} \] 
   \[ \sqrt{2} \] 
   Because the point is a solution of both equations, \((-2, 2)\) is a solution of the system.

16. \( 2y - 6x = 8 \) 
   \[ 2(-5) - 6(-3) = 8 \] 
   \[ \frac{4y = 8x + 4}{8} \] 
   \[ \frac{8(-3) = 4}{-20} \] 
   \[ \frac{-20}{-20} \] 
   Because the point is a solution of both equations, \((-3, -5)\) is a solution of the system.

17. \( \frac{y = 2}{2} \) 
   \[ \frac{y + 8 = 6x}{(2) + 8 = 6(3)} \] 
   \[ \frac{10}{18, x} \] 
   Because the point is not a solution of both equations, \((3, 2)\) is not a solution of the system.

18. \( y = 8x + 2 \) 
   \[ 1 \] 
   \[ 8(6) + 2 \] 
   \[ 50 \] 
   Because the point is not a solution of both equations, \((6, 1)\) is not a solution of the system.

19. \[ 2 + y = x \] 
   \[ x + y = 4 \] 
   \[ y = x - 2 \] 
   \[ y = -x + 4 \] 
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

   The solution to the system is \((3, 1)\).

20. \[ 4y - 2x = 4 \] 
   \[ 10x - 5y = 10 \] 
   \[ y = \frac{1}{2}x + 1 \] 
   \[ y = 2x - 2 \] 
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>4</td>
<td>6</td>
</tr>
</tbody>
</table>

   The solution to the system is \((2, 2)\).

21. \[ 12x + 4y = -4 \] 
   \[ 2x - y = 6 \] 
   \[ y = -3x - 1 \] 
   \[ y = 2x - 6 \] 
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>1</td>
<td>-4</td>
<td>1</td>
<td>-4</td>
</tr>
<tr>
<td>2</td>
<td>-7</td>
<td>2</td>
<td>-2</td>
</tr>
</tbody>
</table>

   The solution to the system is \((1, -4)\).

22. \[ y = 10 - x \] 
   \[ 3x - 3y = 0 \] 
   \[ y = -x + 10 \] 
   \[ y = x \] 
   
<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-5</td>
<td>15</td>
<td>-5</td>
<td>-5</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

   The solution to the system is \((5, 5)\).

23. \[ -27y = -24x + 42 \] 
   \[ y = \frac{8x - 14}{9} \] 
   \[ 9y = 8x - 14 \] 
   \[ y = \frac{8x - 14}{9} \] 
   \[ 4y - 6x = 36 \] 
   \[ 4y = 6x + 36 \] 
   \[ y = \frac{3x + 9}{2} \] 
   consistent, dependent; infinite number of solutions

24. \[ \frac{3}{2}x + 9 = 45 \] 
   \[ y = \frac{3}{2}x + 9 \] 
   \[ 9y = 8x - 14 \] 
   \[ y = \frac{8x - 14}{9} \] 
   \[ 4y = 6x + 36 \] 
   \[ y = \frac{3}{2}x + 9 \] 
   consistent, dependent; infinite number of solutions
25. \(7y + 42x = 56\)  
\[ \begin{align*} 
7y &= -42x + 56 \\
y &= -6x + 8 
\end{align*} \]

26. \(3y = 2x\)  
\[ \begin{align*} 
y &= \frac{2}{3}x
\end{align*} \]

27. Let \(x\) be the number of systems sold, and \(y\) be the total money earned.

Jamal: \(y = 100x + 2400\)  
Wanda: \(y = 120x + 2200\)

<table>
<thead>
<tr>
<th>(x)</th>
<th>(y)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2600</td>
<td>2</td>
<td>2440</td>
</tr>
<tr>
<td>4</td>
<td>2800</td>
<td>4</td>
<td>2680</td>
</tr>
<tr>
<td>6</td>
<td>3000</td>
<td>6</td>
<td>2920</td>
</tr>
<tr>
<td>8</td>
<td>3200</td>
<td>8</td>
<td>3160</td>
</tr>
<tr>
<td>10</td>
<td>3400</td>
<td>10</td>
<td>3400</td>
</tr>
</tbody>
</table>

So they have to sell 10 systems to earn the same amount.

28. \(y - x = 2\)  
\[ \begin{align*} 
\begin{array}{c|c|c|c|c}
       & 2x + y &= 8 \\
\hline
2(2) & 2x   & 8 \\
\hline
2(4) & 2x   & 8 \\
\hline
-2   & 2x   & 8 \\
\hline
-2   & 2x   & 8 \\
\hline
\end{array}
\end{align*} \]

Because the point is not a solution of both equations, \((4, 2)\) is not a solution of the system.

So \(y = -2x + 8\)  
\[ \begin{align*} 
y &= -2(2) + 8 \\
y &= -4 + 8 \\
y &= 4
\end{align*} \]

29. \(3x + y = -1\)  
\[ \begin{align*} 
3(-1) &+ (2) = -1 \\
8(-1) &+ 6 = -2 \\
-1 &- 1 \checkmark \\
-2 &- 2 \checkmark 
\end{align*} \]

Because the point is a solution of both equations, \((-1, 2)\) is a solution of the system.

30. \(x + y = 9\)  
\[ \begin{align*} 
x &= \frac{9}{9} \checkmark \\
4(2) &+ 4 = 7 \\
12 &+ 7 \checkmark 
\end{align*} \]

Because the point is not a solution of both equations, \((7, 2)\) is not a solution of the system.

So \(y = -x + 9\)  
\[ \begin{align*} 
y &= -8 + 9 \\
y &= 1
\end{align*} \]

Because the point is a solution of both equations, \((-1, 2)\) is a solution of the system.

31. \(3x + 4y = -9\)  
\[ \begin{align*} 
3(0) &+ 4(6) = -9 \\
24 &- 9 \checkmark 
\end{align*} \]

\((0, 6)\) is not a solution.

So \(y = -3x - 9\)  
\[ \begin{align*} 
y &= -\frac{3}{4}x - \frac{9}{4} \\
\end{align*} \]

The solution is \((-3, 0)\).

32. Roberto: \(r = 15x + 12\)  
Alexandra: \(a = 18x + 8\)

33. Lynn: \(l = -200x + 10,000\)  
Miguel: \(m = 50x + 5000\)

34. Plan A: \(y = 0.4x + 15\)  
Plan B: \(y = 0.25x + 30\)

c. 2 hours = 120 min

Plan A: \(y = 0.4(120) + 15 = 63\)  
Plan B: \(y = 0.25(120) + 30 = 60\)

He should use plan B. It is $3.00 cheaper than plan A.
35. \[
\begin{align*}
\begin{cases}
y = -x + 6 \\
y = 2x - 3 \\
\end{cases}
\end{align*}
\]
consistent, independent
\[
\begin{align*}
-x + 6 &= 2x - 3 \\
-3x + 6 &= -3 \\
-3x &= -9 \\
3 &= x \\
\end{align*}
\]
The solution of the system is (3, 3).

36. \[
\begin{align*}
\begin{cases}
x = 2 \\
y = 3 \\
\end{cases}
\end{align*}
\]
consistent, independent
inconsistent; no solution
The solution to the system is (2, 3).

37. \[
\begin{align*}
\begin{cases}
y = 3x + 1 \\
y = 3x - 3 \\
\end{cases}
\end{align*}
\]
The solution of the system is \(\left(\frac{8}{3}, \frac{32}{3}\right)\).

38. (2, -3)
39. (-0.25, 4)
40. (6.444, 131)
41. (2.831, -30.403)

42a. Truck:
\[
\begin{align*}
\text{city: } 408 &= 24 \text{ mi/gal} \\
\text{city: } 364 &= 26 \text{ mi/gal} \\
hwy: 476 &= 28 \text{ mi/gal} \\
hwy: 490 &= 35 \text{ mi/gal} \\
\end{align*}
\]

b. Truck:
\[
\begin{align*}
\left(\frac{476}{60}\right) &= 7\frac{14}{15} \text{ h} \\
\left(\frac{490}{60}\right) &= 8\frac{1}{6} \text{ h}
\end{align*}
\]
The truck will have an empty tank after 7\(\frac{14}{15}\) h.
The truck must travel at 58\(\frac{2}{7}\) mi/h.
The car will have an empty tank after 8\(\frac{1}{6}\) h.

43. \[
2y = -x + 4 \\
y = -\frac{1}{2}x + 2
\]
infinite number of solutions \(\rightarrow\) any multiple of the original equation. For example,
\[
2y = -x + 4 \\
4y = -2x + 8 \ldots \text{etc.}
\]
no solution \(\rightarrow\) any equation with the same slope as \(\frac{-1}{2}\) but a different \(y\)-intercept. For example,
\[
y = -\frac{1}{2}x - 6
\]
one solution \(\rightarrow\) any equation with a different slope. For example,
\[
y = -\frac{3}{4}x + 2
\]

44. Possible answer: (3.5, 0.25)

45. Consistent, independent; one solution. The solution is the \(y\)-intercept since it is the same point for each line.

46. Possible answer: One hot-air balloon starts at 120 ft and rises quickly. The other balloon starts at 200 ft and rises slowly. After 4 min, the balloons are at the same height, 280 ft, as represented by the point of intersection.

**TEST PREP**

47. D
48. G
49. B
50. \[
\begin{align*}
x + y &= 8 \\
y &= 4x
\end{align*}
\]
So \(y = x + 8\) and \(y = 4x\).
\[
x + 8 &= 4x \\
8 &= 3x \\
\frac{8}{3} &= x \\
y &= 4\left(\frac{8}{3}\right)
\]
The solution of the system is \(\left(\frac{8}{3}, \frac{32}{3}\right)\).

Three times the value of \(y\) is 32.

**CHALLENGE AND EXTEND**

51. \[
55x + 100 = 20x + 600 \\
x = 20 \left(\frac{100}{7}\right) + 600
\]
55x + 100 = 600
35x = 500
\[x = \frac{500}{35} = \frac{2000}{7} + \frac{4200}{7}\]
\[x = \frac{100}{7} = \frac{6200}{7}\]
The solution of the system is \(\left(\frac{100}{7}, \frac{6200}{7}\right)\).

52. \[
\begin{align*}
5y &= -20x + 135 \\
y &= -4x + 27 \\
y &= -4x + 27
\end{align*}
\]
\[y = -4\left(\frac{5}{46}\right) + 27
\]
So \(y = -4x + 27\) and \(y = 20\)
\[y = \frac{20}{46} + 27 \ldots \text{etc.}
\]
\[y = -46x + 32 = -4x + 27
\]
\[-46x + 32 = 27 \ldots \text{etc.}
\]
\[-46x = -5
\]
\[x = \frac{5}{46}
\]
The solution of the system is \(\left(\frac{5}{46}, \frac{611}{23}\right)\).

53. \[
18y = -9x + 126 \\
14y = -7x + 98 \\
y = -\frac{1}{2}x + 7
\]
infinite number of solutions

54. \[
0.25x - y = 2.25 \\
y = 0.75x + 3.75
\]
So \(y = 0.75x + 3.75\) and \(y = 0.25x - 2.25\).
\[0.25x - 2.25 = 0.75x + 3.75
\]
\[-0.5x = -2.25 = 3.75
\]
\[-0.5x = 6
\]
\[x = -12
\]
\[y = 0.25x - 2.25
\]
\[y = 0.25(-12) - 2.25
\]
\[y = -3 - 2.25
\]
\[y = -5.25
\]
The solution of the system is \((-12, -5.25)\).

55. The solution is meaningless in the real world. Time and cost cannot be negative. The costs for the 2 products will never be equal.
56a. Brad: \( y = -12x + 70 \)
Cliff: \( y = -15x + 100 \)
\(-12x + 70 = -15x + 100\)
\(3x + 70 = 100\)
\(3x = 30\)
\(x = 10\) days

b. Brad: \( y = -12(10) + 70 \)
y = 50 lb
Cliff: \( y = -15(10) + 100 \)
y = 50 lb
No; they will both run out of food before that time.

c. Brad: \( y = -12(4) + 70 + 100 \)
y = 122 lb
Cliff: \( y = -15(4) + 100 + 100 \)
y = 140 lb
On day 10 (6 days later) they have used up:
Brad: \(-12(10) = -72\) lb
\(122 - 72 = 50\) lb
Cliff: \(-15(6) = -90\) lb
\(140 - 90 = 50\) lb
The answer would make sense. Each farmer would have 50 lb of food on day 10.

3-2 USING ALGEBRAIC METHODS TO SOLVE LINEAR SYSTEMS, PAGES 190–197

CHECK IT OUT!

1a. \[
\begin{align*}
x + 2y &= -25 \\
-12x - 7y &= 19
\end{align*}
\]
Step 1 Solve one equation for one variable. The first equation is already solved for \(x\):
\(x = \frac{6}{8}\)
\(x = \frac{3}{4}\)
Step 2 Substitute the \(x\)-value into one of the original equations to solve for \(y\):
\(y = -2\cdot\frac{3}{4} + 2\)
\(y = -4\)
The solution is the ordered pair \((3, -4)\).

b. \[
\begin{align*}
x + 2y &= -9 \\
y &= -2x + 2
\end{align*}
\]
Step 1 Solve one equation for one variable. The first equation is already solved for \(y\):
\(y = -2x + 2\)
Step 2 Substitute the \(x\)-value into the other equation.
\(5x + 6y = -9\)
\(5x + 6(-2x + 2) = -9\)
\(5x - 12x + 12 = -9\)
\(-7x + 12 = -9\)
\(-7x = -21\)
\(x = 3\)

2a. \[
\begin{align*}
4x + 7y &= -25 \\
-12x - 7y &= 19
\end{align*}
\]
Step 1 Find the value of one variable.
\(-8x = -6\)
\(x = \frac{6}{8}\)
\(x = \frac{3}{4}\)
Step 2 Substitute the \(x\)-value into one of the original equations to solve for \(y\):
\(4\cdot\frac{3}{4} + 7y = -25\)
\(3 + 7y = -25\)
\(7y = -28\)
\(y = -4\)
The solution is the ordered pair \(\left(\frac{3}{4}, -4\right)\).

b. \[
\begin{align*}
x + 2y &= -9 \\
y &= -2x + 2
\end{align*}
\]
Step 1 To eliminate \(y\), multiply both sides of the first equation by 5 and both sides of the second equation by 3.
\(5(5x - 3y) = 5(42)\)
\(3(8x + 5y) = 3(28)\)
Add to eliminate \(y\):
\(25x = 210\)
\(24x = 84\)
\(49x = 294\)
\(x = 6\)
Step 2 Substitute the \(x\)-value into one of the original equations to solve for \(y\):
\(5(6) - 3y = 42\)
\(-30 = 3y + 42\)
\(3y = -52\)
\(y = -17\)
The solution is the ordered pair \((6, -17)\).
3a. \(56x + 8y = -32\)  
\[
\begin{align*}
56x + 8y &= -32 \\
8x + y &= -4 \\
7x + y &= -4
\end{align*}
\]
Since \(7x + y = -4\), the system is consistent, dependent, and has an infinite number of solutions.

4. Let \(x\) represent amount of Sumatra beans. Let \(y\) represent amount of Kona beans.
\[
\begin{align*}
x + y &= 50 \\
5x + 13y &= 10(50) \\
5x + 13y &= 500
\end{align*}
\]
5. \(x = 12\), the solution is the ordered pair \((5, 12)\).

**EXERCISES**

**GUIDED PRACTICE**

1. elimination

2. **Step 1** Solve one equation for one variable. The second equation is already solved for \(y\): \(y = x + 7\).
   **Step 2** Substitute the expression into the other equation.
   \[
   \begin{align*}
x + y &= 17 \\
x + (x + 7) &= 17 \\
2x + 7 &= 17 \\
2x &= 10 \\
x &= 5
   \end{align*}
   \]
   **Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
   \[
   \begin{align*}
y &= x + 7 \\
y &= 5 + 7 \\
y &= 12
   \end{align*}
   \]
The solution is the ordered pair \((5, 12)\).

3. **Step 1** Solve one equation for one variable. The first equation is already solved for \(y\): \(y = x - 19\).
   **Step 2** Substitute the expression into the other equation.
   \[
   \begin{align*}
x - y &= 27 \\
x - (x - 19) &= 27 \\
2x - x + 19 &= 27 \\
x &= 8
   \end{align*}
   \]
   **Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
   \[
   \begin{align*}
y &= x - 19 \\
y &= 8 - 19 \\
y &= -11
   \end{align*}
   \]
The solution is the ordered pair \((8, -11)\).

4. **Step 1** Solve one equation for one variable. \(2x - y = 2\)
   \[
   \begin{align*}
y &= 2x - 2
   \end{align*}
   \]
   **Step 2** Substitute the expression into the other equation.
   \[
   \begin{align*}
3x - 2y &= 11 \\
3x - 2(2x - 2) &= 11 \\
3x - 4x + 4 &= 11 \\
-x &= 7 \\
x &= -7
   \end{align*}
   \]
   **Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
   \[
   \begin{align*}
y &= 2x - 2 \\
y &= (2(-7)) - 2 \\
y &= -14 - 2 \\
y &= -16
   \end{align*}
   \]
The solution is the ordered pair \((-7, -16)\).

5. **Step 1** Solve one equation for one variable. \(y = 3x + 5\)
   \[
   \begin{align*}
y &= 3(-3y - 5) + 5 \\
y &= -9y - 15 + 5 \\
10y &= -10 \\
y &= -1
   \end{align*}
   \]
   **Step 3** Substitute the \(y\)-value into one of the original equations to solve for \(x\).
   \[
   \begin{align*}
y &= 3x + 5 \\
-1 &= 3x + 5 \\
6 &= 3x \\
x &= 2
   \end{align*}
   \]
The solution is the ordered pair \((-2, -1)\).
6. Step 1 Find the value
of one variable. Add to eliminate y.
\[2x + y = 12 \quad \text{(1)}\]
\[-5x - y = -33 \quad \text{(2)}\]
\[-3x = -21\]
x = 7
Step 2 Substitute the x-value into one of the original equations to solve for y.
\[2(7) + y = 12\]
y = -2
The solution is the ordered pair (7, -2).

8. Step 1 To eliminate y, multiply both sides of the second equation by 2.
\[2(5x - 3y) = 2(88) \quad \text{(2)}\]
Add to eliminate y.
\[2x + 6y = -8 \quad \text{(1)}\]
\[10x - 6y = 176 \quad \text{(2)}\]
\[12x = 168\]
x = 14
Step 2 Substitute the x-value into one of the original equations to solve for y.
\[2(14) + 6y = -8\]
6y = -36
y = -6
The solution is the ordered pair (14, -6).

10. 5x - y = -3
y = 5x + 3
15x - 3(5x + 3) = -9
15x - 15x - 9 = -9
-9 = -9
consistent, dependent, infinite number of solutions

12. 2x + 3y = -24
3y = -2x - 24
y = -\(\frac{2}{3}x - 8\)
8x + 12 \(\left(\frac{2}{3}x - 8\right)\) = 60
8x + \(\frac{2}{3}x - 8\) = 96
8x - 8x - 96 = 60
-96 = 60
inconsistent, no solution

7. Step 1 Find the value
of one variable. Add to eliminate x.
\[2x - 5y = -5 \quad \text{(1)}\]
\[-2x + 8y = -58 \quad \text{(2)}\]
\[3y = -63\]
y = -21
Step 2 Substitute the y-value into one of the original equations to solve for x.
\[2x - 5(-21) = -5\]
x = -5
The solution is the ordered pair (-55, -21)

9. Step 1 To eliminate x, multiply both sides of the first equation by 4.
\[4 \left(\frac{1}{2}x + y\right) = 4(4) \quad \text{(1)}\]
Add to eliminate x.
\[2x + 4y = 16 \quad \text{(1)}\]
\[-2x - 2y = -6 \quad \text{(2)}\]
\[2y = 10\]
y = 5
Step 2 Substitute the y-value into one of the original equations to solve for x.
\[2x - 2(5) = -6\]
x = -2
The solution is the ordered pair (-2, 5).

11. \[x = 2y - 8\]
\[4(2y - 8) = 8y - 56\]
\[8y \times 2 - 32 = 8y - 56\]
\[-32 = -56 x\]
inconsistent, no solution

13. \[x - \frac{1}{3}y = -2\]
\[-\frac{1}{3}y = -x - 2\]
y = 3x + 6
6x - 2(3x + 6) = -12
6x - 6x - 12 = -12
\[-12 = -12\]
consistent, dependent, infinite number of solutions

14. Let x be the amount of 85% ethanol, and y be the amount of 25% ethanol.
\[x + y = 20\]
\[0.85x + 0.25y = 20(0.5) \quad \text{(2)}\]
x + y = 20
\[0.85x + 0.25y = 10\]
x + y = 20
\[0.85x + 0.25(20 - x) = 10\]
\[0.85x + 5 - 0.25x = 10\]
\[0.6x = 5\]
x = 8 \(\frac{1}{3}\)
y = 20 - \left(8 \frac{1}{3}\right)
y = 11 \(\frac{2}{3}\)

Denise needs 8 \(\frac{1}{3}\) gal of 85% ethanol fuel and 11 \(\frac{2}{3}\) gal of 25% ethanol fuel.

PRACTICE AND PROBLEM SOLVING

15. Step 1 Solve one equation for one variable. The first equation is already solved for x:
\[x = -4y\]
Step 2 Substitute the expression into the other equation.
\[2x + 6y = -3\]
\[2(-4y) + 6y = -3\]
\[-8y + 6y = -3\]
\[-2y = -3\]
y = 3 \(\frac{2}{3}\)
Step 3 Substitute the x-value into one of the original equations to solve for y.
\[x = -4y\]
x = -4 \(\frac{3}{2}\)
y = -6 + 21
y = 15
The solution is the ordered pair \(\left(-6, \frac{3}{2}\right)\).

16. Step 1 Solve one equation for one variable.
\[12x + y = 21\]
y = -12x + 21
Step 2 Substitute the expression into the other equation.
\[18x - 3(-12x + 21) = -36\]
\[18x + 36x - 63 = -36\]
\[54x = 27\]
x = \(\frac{1}{2}\)
Step 3 Substitute the x-value into one of the original equations to solve for y.
y = -12x + 21
y = -12(\(\frac{1}{2}\)) + 21
y = -6 + 21
y = -6 + 21
y = 15
The solution is the ordered pair \(\left(\frac{1}{2}, 15\right)\).
17. **Step 1** Solve one equation for one variable. The first equation is already solved for $y$.

$y = -4x$.

**Step 2** Substitute the expression into the other equation.

$32x + 21y = 29$
$32x + 21(4x) = 29$
$32x + 84x = 29$
$116x = 29$

$x = \frac{29}{116}$

$x = \frac{1}{4}$

**Step 3** Substitute the $x$-value into one of the original equations to solve for $y$.

$y = 4 \left( \frac{1}{4} \right)$

$y = 1$

The solution is the ordered pair $\left( \frac{1}{4}, 1 \right)$.

19. **Step 1** To eliminate $x$, multiply both sides of the second equation by $-1$.

$-1(4x - 5y) = -1(2)$

$-4x + 9y = 26$

**Step 2** Substitute the $y$-value into one of the original equations to solve for $x$.

$4x - 9(6) = 26$
$4x + 54 = 26$
$4x = -28$

$x = -7$

The solution is the ordered pair $(-7, -6)$.

20. **Step 1** To eliminate $x$, multiply both sides of the first equation by 5 and both sides of the second equation by 6.

$5(6x - 3y) = 5(-6)$
$6(-5x + 7y) = 6(41)$

**Step 2** Substitute the $y$-value into one of the original equations to solve for $x$.

$6x - 3(8) = -6$
$6x - 24 = -6$
$6x = 18$

$x = 3$

The solution is the ordered pair $(3, 8)$.

21. **Step 1** To eliminate $y$, multiply both sides of the first equation by 8 and both sides of the second equation by 3.

$8(12x - 3y) = 8(-15)$
$3(8x + 8y) = 3(-58)$

Add to eliminate $y$.

$96x - 24y = -120$  
$24x + 24y = -174$

$x = -2.45$

**Step 2** Substitute the $x$-value into one of the original equations to solve for $y$.

$8(-2.45) + 8y = -58$
$-19.6 + 8y = -58$

$8y = -38.4$

$y = -4.8$

The solution is the ordered pair $(-2.45, -4.8)$.

22. **Step 1** To eliminate $y$, multiply both sides of the first equation by 2.

$-2(3x + y) = -2(7)$

$3x = 12\left( \frac{1}{4}x - \frac{1}{6} \right) + 72$

Add to eliminate $y$.

$-6x - 2y = -14$
$-3x + 2y = 11$

$-9x = 3$

$x = \frac{1}{3}$

**Step 2** Substitute the $x$-value into one of the original equations to solve for $y$.

$3\left( \frac{1}{3} \right) + y = 7$
$1 + y = 7$

$y = 6$

The solution is the ordered pair $\left( \frac{1}{3}, 6 \right)$.

23. $4y = x - 24$

$y = \frac{1}{4}x - 6$

$-2(3x + y) = -2(7)$

$3x = 12\left( \frac{1}{4}x - \frac{1}{6} \right) + 72$

Add to eliminate $y$.

$-6x - 2y = -14$
$-3x + 2y = 11$

$-9x = 3$

$x = \frac{1}{3}$

24. $10x - 2y = 22$

$2y = 10x - 22$
$y = 5x - 11$

$5(5x - 11) - 25x = 65$
$25x - 55 - 25x = 65$

$x = -55 = 65 \times x$  

inconsistent; no solution

$\frac{3}{4}x + 64 - 6x = 64$

$\frac{3}{4}x + 64 - 6x = 64$

$x = 64$

consistent, dependent; infinite number of solutions
26. \(-x + \frac{3}{4}y = 4\)
\[3y = x + 4\]
\[y = \frac{4}{3}x + \frac{16}{3}\]
\[8x - 6\left(\frac{4}{3}x + \frac{16}{3}\right) = -8\]
\[8x - 2\frac{2}{3}x - 22\frac{2}{3} = -8\]
\[8x - 8x - 32 = -8\]
\[-32 = -8x\]

inconsistent; no solution

27. **Step 1** Solve one equation for one variable. The first equation is already solved for \(x\):
\[x = 3y + 3\]

**Step 2** Substitute the expression into the other equation.
\[y + 3x = -21\]
\[y + 3(3y + 3) = -21\]
\[y + 9y + 9 = -21\]
\[9y + 9 = -21\]
\[10y = -30\]
\[y = -3\]

**Step 3** Substitute the \(y\)-value into the original equation to solve for \(x\).
\[x + 800 = 1200\]
\[x = 400\]

printer A produces 400 copies; printer B produces 800 copies

28. **Step 1** Solve one equation for one variable. The first equation is already solved for \(y\):
\[y = -2x + 14\]

**Step 2** Substitute the expression into the other equation.
\[1.5x - 3.5y = 2\]
\[1.5x - 3.5(-2x + 14) = 2\]
\[1.5x + 7x - 49 = 2\]
\[8.5x = 51\]
\[x = 6\]

**Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[y = -2x + 14\]
\[y = 31\]
\[y = 31\]

The solution is the ordered pair \((-6, -3)\).

29. **Step 1** Solve one equation for one variable. The first equation is already solved for \(y\):
\[y = -2x + 14\]

**Step 2** Substitute the expression into the other equation.
\[1.5x - 3.5y = 2\]
\[1.5x - 3.5(-2x + 14) = 2\]
\[1.5x + 7x - 49 = 2\]
\[8.5x = 51\]
\[x = 6\]

**Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[y = -2x + 14\]
\[y = -88\]

The solution is the ordered pair \((6, 2)\).

30. **Step 1** Solve one equation for one variable. The first equation is already solved for \(y\):
\[y = x + 8\]

**Step 2** Substitute the expression into the other equation.
\[\frac{4}{5}y - 3x = \frac{1}{5}\]
\[y = x + 8\]
\[\frac{4}{5}(x + 8) - 3x = \frac{1}{5}\]
\[\frac{4}{5}x + \frac{32}{5} - 3x = \frac{1}{5}\]
\[4x + 32 - 15x = 1\]
\[-11x = -31\]
\[x = \frac{31}{11}\]
\[= 2\frac{9}{11}\]

**Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[y = -2x + 14\]
\[y = 31\]

The solution is the ordered pair \((2\frac{9}{11}, 10\frac{9}{11})\).

31. **Step 1** Solve one equation for one variable. The first equation is already solved for \(y\):
\[y = x + 8\]

**Step 2** Substitute the expression into the other equation.
\[\frac{4}{5}y - 3x = \frac{1}{5}\]
\[y = x + 8\]
\[\frac{4}{5}(x + 8) - 3x = \frac{1}{5}\]
\[\frac{4}{5}x + \frac{32}{5} - 3x = \frac{1}{5}\]
\[4x + 32 - 15x = 1\]
\[-11x = -31\]
\[x = \frac{31}{11}\]
\[= 2\frac{9}{11}\]

**Step 3** Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[y = -2x + 14\]
\[y = 31\]

The solution is the ordered pair \((6, 2)\).

32a. Let \(x\) be the time spent mowing the lawn, and \(y\) be the time spent raking leaves.
\[x + y = 3\]
\[325x + 275y = 885\]

b. \[x + y = 3\]
\[x = 3 - y\]
\[325(3 - y) + 275y = 885\]
\[975 - 325y + 275y = 885\]
\[-50y = -90\]
\[y = 1.8\]
\[x + (1.8) = 3\]
\[x = 1.2\]

He spent 1.2 h mowing the lawn and 1.8 h raking leaves.

33a. 22 ways; 21 dimes, 20 dimes and 2 nickels, ..., 1 dime and 40 nickels, 42 nickels

b. The total number of coins increases because every dime is replaced by 2 nickels.
34. The error is in solution A. The equation \( y = 2 + x \) must be substituted into the other equation, not the equation in which it was solved for \( y \).

35a. 500 \( \times \) 0.533 = 266.5 mi

b. \( d = st \)
\[
0.533 = 14.94s
\]
\[
s = 0.035676 mi/s \times 60 s/min \times 60 min/h
\]
\[
s = 128.43 mi/h
\]
\[
y = 128.43x
\]
c. Lead car: 266.5 = 128.43x
\[
x = 2.0751 h
\]
2nd car: 266.5 = 125x
\[
x = 2.132 h
\]
\[
2.132 - 2.0751 = 0.0569 h
\]
\[
125 mi/h \times 0.0569 h = 7.12 mi
\]
approximately 7.1 mi

36a. Malcolm: \( y = 45x + 300 \)
Owen: \( y = 60x + 325 \)
\[
45x + 300 = 60x + 325
\]
\[
-15x + 300 = 325
\]
\[
-15x = 25
\]
\[
x = -\frac{5}{3} = -1\frac{2}{3}
\]
b. Possible answer: No; the solution is not reasonable since the number of customers cannot be a negative fraction.

37. Let \( x \) be the cost of student tickets, and \( y \) be the cost of adult tickets.
\[
\begin{align*}
16x + 3y &= 110.50 \quad \text{(1)} \\
12x + 4y &= 96 \quad \text{(2)}
\end{align*}
\]
Step 1 To eliminate \( y \), multiply both sides of the first equation by 4 and both sides of the second equation by -3.
\[
4(16x + 3y) = 4(110.50) \quad \text{(1)}
\]
\[
-3(12x + 4y) = -3(96) \quad \text{(2)}
\]
Add to eliminate \( y \).
\[
6x + 12y = 442 \quad \text{(1)}
\]
\[
36x + 12y = 288 \quad \text{(2)}
\]
\[
\begin{align*}
28x &= 154 \\
x &= 5.5
\end{align*}
\]
Step 2 Substitute the \( x \)-value into one of the original equations to solve for \( y \).
\[
16(5.5) + 3y = 110.50
\]
\[
88 + 3y = 110.50
\]
\[
3y = 22.5
\]
\[
y = 7.5
\]
A student ticket costs $5.50, and an adult ticket costs $7.50.

38a. Step 1 To eliminate \( y \), multiply both sides of the second equation by 2.
\[
2(6x + 3y) = 2(24) \quad \text{(2)}
\]
Add to eliminate \( y \).
\[
3x - 6y = -13 \quad \text{(1)}
\]
\[
12x + 6y = 48 \quad \text{(2)}
\]
\[
15x + = 35 \quad \text{(1)} + 2(\quad \text{(2)}
\]
\[
x = \frac{7}{3} = 2\frac{1}{3}
\]
Step 2 Substitute the \( x \)-value into one of the original equations to solve for \( y \).
\[
3\left(\frac{7}{3}\right) - 6y = -13
\]
\[
7 - 6y = -13
\]
\[
y = 10 \quad \frac{3}{3}
\]
The solution is incorrect. The correct solution is \( \left(\frac{21}{3}, -3\frac{1}{3}\right) \).

b. Solving by graphing is best for equations that intersect at whole-number values or values that are clear on a graph. Solving algebraically is best for solutions that are not easy to read on a graph. When solving algebraically, if a variable is isolated, use substitution. Otherwise use elimination.

39. Possible answer: Solve both equations for \( y \). The slopes and the \( y \)-intercepts should be the same.

TEST PREP

40. D

41. G; Let \( x \) represent the number of small tables, and let \( y \) represent the number of small tables.
\[
\begin{align*}
12x + 8y &= 100 \quad \text{(1)} \\
50x + 25y &= 350 \quad \text{(2)}
\end{align*}
\]
Step 1 To eliminate \( y \), divide both sides of the first equation by -4 and both sides of the second equation by 12.5.
\[
\frac{12x + 8y}{-4} = \frac{100}{-4} \quad \text{(1)}
\]
\[
\frac{50x + 25y}{12.5} = \frac{350}{12.5} \quad \text{(2)}
\]
Add to eliminate \( y \).
\[
-3x - 2y = -25 \quad \text{(1)}
\]
\[
4x + 2y = 28 \quad \text{(2)}
\]
\[
x = \frac{3}{3}
\]
Step 2 Substitute the \( x \)-value into one of the original equations to solve for \( y \).
\[
-3(3) - 2y = -25
\]
\[
-9 - 2y = -25
\]
\[
-2y = -16
\]
\[
y = 8
\]
To seat 100 guests, they need to rent 3 large tables and 8 small tables.
42. A. \[ \begin{align*}
4x + y &= 7 \\
x - y &= -3 \\
x + y &= 7 \\
y &= -x + 7 \\
x - (-x + 7) &= -3 \\
2x - 7 &= -3 \\
2x &= 4 \\
x &= 2 \\
x + y &= 7 \\
(2) + y &= 7 \\
y &= 5 \\
\text{solution: } (2, 5)
\end{align*} \]

43. \[ \begin{align*}
x - 4y &= 6 \\
y &= -4x + 7 \\
x - 4(-4x + 7) &= 6 \\
17x - 28 &= 6 \\
17x &= 34 \\
x &= 2 \\
y &= -4(2) + 7 \\
y &= -1 \\
\text{solution (2, -1)} \\
\text{The system is independent and consistent.}
\end{align*} \]

44. H

CHALLENGE AND EXTEND

45. Possible answer:
\[ y = -\frac{3}{2}x \]

46. Eliminate x by adding:
\[ y - x = 4 \quad \text{①} \\
y + x = 2 \quad \text{②} \\
2y = 6 \quad \text{③} + \text{②} \\
y = 3 \\
(3) - 1 = -2x \\
\begin{align*}
2 &= -2x \\
-1 &= x \\
\end{align*}

solution (-1, 3)

Possible answer: Use the eliminate method on the first and third equations to solve for y. Then substitute the y-value back into another equation to find x.

47a. \[ \begin{align*}
p &= 5 + 2q \\
p &= 5 + 2(100 - 4p) \\
p &= 5 + 200 - 8p \\
p &= 205 \\
p &= 9 \\
p &= 90 \quad \text{(227,8)} \\
p &= 90 \quad \text{9} \\
q &= 100 - 4p \\
q &= 100 - \frac{820}{9} \\
q &= 900 - \frac{820}{9} \\
q &= 80 \quad \text{(291,8)} \\
q &= 80 \quad \text{9} \\
\text{solution: } (227,8) \quad \text{9}
\end{align*} \]

47b. \[ \begin{align*}
p &= 3 + 0.5q \\
p &= 100 - 4p \\
p &= 3 + 0.5(100 - 4p) \\
p &= 3 + 50 - 2p \\
3p &= 53 \\
p &= 53 \quad \text{(3)} \quad \text{(3)} \\
q &= 100 - 4p \\
q &= 100 - \frac{212}{3} \\
q &= 300 - \frac{212}{3} \\
q &= \frac{88}{3} = 29 \frac{1}{3} \\
\text{solution: } \left(17 \frac{2}{3}, 29 \frac{1}{3}\right)
\end{align*} \]

SPIRAL REVIEW

48. \[ \begin{align*}
b^2(2b + 4) + b^5 &= -2b^2 - 4b^2 + b^5 \\
&= -2b^2 - 4b^2 + b^5 \\
&= -2(-1)^3 - 4(-1)^2 + (-1)^5 \\
&= 28b^2 + 1 \\
&= -2(-1) - 4(1) + (-1) \\
&= 2 - 4 - 1 \\
&= -3 \\
&= 252 + 1 \\
&= 253
\end{align*} \]

49. \[ \begin{align*}
c^2 + 1 + (5c)^2 &= 3c^2 + 1 + 25c^2 \\
&= 28c^2 + 1 \\
&= 28(3)^2 + 1 \\
&= 28(9) + 1 \\
&= 252 + 1 \\
&= 253
\end{align*} \]

50. \[ \begin{align*}
\frac{20 - 2x^2}{x} &= \frac{20}{x} - 2x \\
\text{②} &= \frac{20}{x} - 2(-2) \\
&= -10 + 4 \\
&= -6 \\
\text{③} &= \frac{2}{9} \\
\text{④} &= \frac{2}{9y^2} \\
\text{⑤} &= \frac{2}{9(-3)^2} \\
&= \frac{2}{81}
\end{align*} \]

51. \[ \begin{align*}
y^3 &= \frac{2y}{9} \\
2y^2 &= \frac{2}{9} \\
9y^2 &= 2 \\
9(-3)^2 &= \frac{2}{81}
\end{align*} \]

52. \[ \begin{align*}
m &= \frac{4 - 2}{6 - 2} \\
f(x) - 2 &= \frac{1}{2}(x - 2) \\
m &= \frac{2}{4} \\
f(x) - 2 &= \frac{1}{2}x - 1 \\
m &= \frac{1}{2} \\
f(x) &= \frac{1}{2}x + 1
\end{align*} \]

53. \[ \begin{align*}
m &= \frac{-5 - (-2)}{3 - 1} \\
f(x) - (-2) &= -\frac{3}{2}(x - 1) \\
m &= \frac{-3}{2} \\
f(x) + 2 &= -\frac{3}{2}x + \frac{3}{2} \\
m &= \frac{-3}{2} \\
f(x) &= -\frac{3}{2}x - \frac{1}{2}
\end{align*} \]

54. \[ \begin{align*}
f(x) &= -1.5x - 0.5
\end{align*} \]
55. \(3x - y \leq 2(x - 2)\)
   \[-y \leq 2x - 4 - 3x\]
   \[-y \leq -x - 4\]
   \[y \geq x + 4\]

56. \(5x + 4y > 18\)
   \[4y > -5x + 18\]
   \[y > \frac{-5x + 9}{4}\]

3-3 SOLVING SYSTEMS OF LINEAR INEQUALITIES, PAGES 199–204

CHECK IT OUT!

1a. \(x - 3y < 6\)
   \[-3y < -x + 6\]
   \[y > \frac{x}{3} - 2\]
   \[2x + y > 1.5\]
   \[y > -2x + 1.5\]

1b. \(y \leq 4\)
   \[2x + y < 1\]
   \[y < -2x + 1\]

2. \(d + s \leq 40\)
   \[s \leq -d + 40\]
   \[2d + 2.5s \geq 90\]
   \[2.5s \geq -2d + 90\]
   \[s \geq 0.8d + 36\]

3a. \(\) triangle
   \[\) trapezoid

THINK AND DISCUSS

1. Possible answer: The region containing the solutions is the region where the shadings overlap.

EXERCISES

GUIDED PRACTICE

1. Possible answer: Both are composed of straight lines. However, a system of linear equations has a solution at a point \((a, b)\), whereas a system of linear inequalities has a region containing many points as solutions.

2. \[y \geq x + 1\]
   \[y \geq -x + 1\]

3. \[y \geq x + 1\]
   \[y \leq \frac{1}{2}x + 1\]

4. \[y > 7x + 16\]
   \[y \leq -5x - 2\]

5. \[2y \leq -2x + 4\]
   \[y \leq -x + 2\]
   \[y < 3x - 1\]
6. Let \( x \) be the number of adult T-shirts sold, and \( y \) be the number of student T-shirts sold.
\[
\begin{align*}
\text{Adult T-shirts} & \quad \text{Student T-shirts} \\
0 & \quad 0 \\
60 & \quad 60 \\
120 & \quad 120 \\
180 & \quad 180 \\
240 & \quad 240
\end{align*}
\]
\[
\begin{align*}
x + y & \leq 250 \\
y & \leq -x + 250 \\
15x + 10y & \geq 3000 \\
10y & \geq -15x + 3000 \\
y & \geq -1.5x + 300
\end{align*}
\]

7. \( x \geq 0 \) and \( y \geq 0 \) since the number of CDs cannot be negative.

8. \( x \geq 0 \) and \( y \geq 0 \) since the number of CDs cannot be negative.

9. \( x \geq 0 \) and \( y \geq 0 \) since the number of CDs cannot be negative.

10. \( x \geq 0 \) and \( y \geq 0 \) since the number of CDs cannot be negative.

11. \( y < 5x \) \\
\( y < x \)

12. \( 3y \geq 2x - 3 \) \\
\( y \geq \frac{2}{3}x - 1 \)

13. \( x + y > 5 \) \\
\( y > -x + 5 \)

14. \( y > 4 \) \\
\( x + 4y \geq 8 \)

15. \( y \geq \frac{1}{3}x - 4 \) \\
\( y \leq -\frac{1}{3}x - 1 \)

16. \( y \geq 0 \) \\
\( y \geq \frac{1}{3}x - 4 \)

17. \( y \geq 0 \) \\
\( y \geq -\frac{1}{3}x - 1 \)
18. rectangle

19. isosceles right triangle

20. Let \( x \) be the number of receiving yards, and \( y \) be the number of rushing yards.
\[
\begin{align*}
  x + y &< 2370 \\
  y &< -x + 2370 \\
  y &> 1645
\end{align*}
\]

21. Possible answer:
\[
\begin{cases} 
  y \leq 2x \\
  y \geq 2x - 1 \\
  y \leq \frac{1}{2}x \\
  y \geq \frac{1}{2}x + 3
\end{cases}
\]

22. Possible answer:
\[
\begin{cases} 
  y \leq x + 2 \\
  y \geq x - 3 \\
  y \leq -x \\
  y \geq -x - 5
\end{cases}
\]

23. Possible answer:
\[
\begin{cases} 
  y \leq 3 \\
  x \geq 1 \\
  y \geq x - 1
\end{cases}
\]

24. Possible answer:
\[
\begin{cases} 
  y \leq 4 \\
  y \leq \frac{1}{2}x \\
  y \leq -3x + 12
\end{cases}
\]

25a. Possible answer: Let \( x \) be the weight of the driver.
Champ Car: \( y \geq 1565 \)
Formula One: \( y \geq 1322.77 - x \), where \( x \) represents the weight of the driver.

b. Take the average weight of racecar drivers.
Possible answer: \( D : \{ x \in \mathbb{R} \mid 100 \leq x \leq 250 \} \)
\( R : \{ y \in \mathbb{R} \mid 1000 \leq y \leq 2000 \} \)

c. Possible answer: \( D = \{ t \in \mathbb{R} \mid -40 \leq t \leq 40 \} \)
\( R = \{ w \in \mathbb{R} \mid w \geq 0 \} \)

26a. Possible answer: \( D = \{ x \in \mathbb{R} \mid -97 \leq x \leq 97 \} \)
\( R = \{ y \in \mathbb{R} \mid y \leq -97 \} \)

27. Let \( x \) be Brian’s earnings, and \( y \) be Maria’s earnings.
\[
\begin{align*}
  x &\geq 0 \\
  y &\geq 0 \\
  x + y &\leq 114,650 \\
  x + y &\geq 56,801 \\
  y &\geq x + 2000
\end{align*}
\]

28. \( y > \frac{2}{5}x \)
\( y \geq \frac{2}{5}x - 4 \)
Possible answer: \( (3, 1), (5, -1), (10, 5) \)

29. \( y > -7 \)
\( y < 2x + 5 \)
\( y < -3x + 4 \)
Possible answer: \( (0, 0), (-1, 2), \left(1, -\frac{1}{2}\right) \)
30. $y \geq -8$
   $y < -\frac{1}{2}x + 2$
   $x > -6$
   Possible answer: $(-2, -3), (-1, 1), (10, -4)$

31. $y \leq -\frac{1}{6}x + \frac{2}{3}$
   $y < x - 3$
   Possible answer: $(0, -5), (3, -2), (-6, -10)$

32. If the boundary lines are parallel, the possible solutions are
   i) above the upper line
   ii) below the lower line
   iii) between the two lines
   iv) no solution

33. Possible answer: Yes; if the lines are parallel and the regions do not overlap; for example, $\left\{ \begin{array}{l} y > x + 4 \\ y < x - 7 \end{array} \right.$

**TEST PREP**
34. D
35. G
36. C

**CHALLENGE AND EXTEND**
37. $\left\{ \begin{array}{l} x \geq 0 \\ y \geq 0 \end{array} \right.$
   Possible answer: $y \leq x + 1$
   $\left\{ \begin{array}{l} y \geq x - 1 \\ y \leq -x + 4 \end{array} \right.$

38. Yes; $m = -3$ makes the equations parallel and the regions do not overlap. Therefore there is no solution for the system.

39. Let $x$ be the amount in low risk, and $y$ be the amount in high risk.
   $x + y \leq 30,000$
   $y \leq 30,000 - x$
   $0.05x + 0.07y \geq 1900$
   $0.05x + 0.07(30,000 - x) \geq 1900$
   $0.05x + 2100 - 0.07x \geq 1900$
   $-0.02x \geq -200$
   $x \leq 10,000$

   $y = 30,000 - x$
   $y = 30,000 - 10,000$
   $y = 20,000$

   Kira must invest at least $20,000 in the high-risk investment.

**SPIRAL REVIEW**
40. $-7, \frac{1}{7}$
41. $\frac{3}{4}, -\frac{4}{3}$
42. $-2.48, \frac{1}{2.48}$
43. $1, -1$
44. $m = 1 - (-\frac{7}{2})$
45. $y = -3$
46. $m = \frac{0 - (-1)}{0 - 1}$
47. $y - 6 = -\frac{1}{3}(x - 9)$
48. $m = -\frac{1}{4}$
49. $m = -1$
40. $y - 4.5 = -\frac{1}{4}(x + 2)$
41. $y - 2 = -1(x - 3)$
42. $y - 4.5 = -\frac{1}{4}x - 0.5$
43. $y = -\frac{1}{4}x + 4$
44. $y = -\frac{1}{4}x + 0$
45. $y = -x$
46. $y = -\frac{1}{3}x + 3$
47. $y = \frac{1}{3}x + 9$
48. $y = -\frac{1}{4}x + 4$
49. $r = 0.985$

**3-4 LINEAR PROGRAMMING, PAGES 205–211**

**CHECK IT OUT!**
1. [Graph of a linear programming problem with shaded feasible region]
2. Evaluate the vertices of the feasible region.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( P = 25x + 30y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1.5</td>
<td>25(0) + 30(1.5) = 45</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>25(0) + 30(4) = 120</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>25(2) + 30(3) = 140</td>
</tr>
<tr>
<td>3</td>
<td>1.5</td>
<td>25(3) + 30(1.5) = 120</td>
</tr>
</tbody>
</table>

The maximum value is \( P = 140 \).

3. Let \( x \) be the number of bookcase A, and \( y \) be the number of bookcase B.

\[
\begin{align*}
x & \leq 8 \\
y & \leq 12 \\
32x + 16y & \geq 320 \\
x & > 0 \\
y & > 0
\end{align*}
\]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( P = 200x + 125y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>12</td>
<td>200(8) + 125(12) = 3100</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>200(4) + 125(12) = 2300</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>200(8) + 125(4) = 2100</td>
</tr>
</tbody>
</table>

8 of bookcase A and 4 of bookcase B will minimize the cost.

**THINK AND DISCUSS**

1. Possible answer: Most of the questions are based on physical objects, where a negative quantity doesn’t make sense.

2. The region is unbounded.

3. Possible answer: Look for specific units, e.g. dollars, pounds. If the question asks for the maximum profit, look for $ units.

4. Constraints:
\[
\begin{align*}
b & \geq 0 \\
y & \geq 0 \\
1.2b + 2r & \leq 600 \\
2.50b + 2.50r & \leq 1000
\end{align*}
\]

**EXERCISES**

**GUIDED PRACTICE**

1. constraints

5. Find the vertices of the feasible region.
Maximize the objective function \( P = 10x + 16y \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( P = 10x + 16y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>10(0) + 16(0) = 0</td>
</tr>
<tr>
<td>0</td>
<td>3</td>
<td>10(0) + 16(3) = 48</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>10(1) + 16(6) = 106</td>
</tr>
<tr>
<td>7</td>
<td>0</td>
<td>10(7) + 16(0) = 70</td>
</tr>
</tbody>
</table>

\( P = 106 \) is the maximum.

6. Find the vertices of the feasible region.
Minimize the objective function: \( P = 3x + 5y \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( P = 3x + 5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>3(0) + 5(-1) = -5</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>3(0) + 5(1) = 5</td>
</tr>
<tr>
<td>24</td>
<td>0</td>
<td>3(24) + 5(0) = 72</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>3(4) + 5(5) = 37</td>
</tr>
</tbody>
</table>

\( P = -5 \) is the minimum.

7. Find the vertices of the feasible region. Maximize the objective function \( P = 2.4x + 1.5y \).

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>( P = 2.4x + 1.5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3</td>
<td>2.4(-2) + 1.5(-3) = -9.3</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2.4(1) + 1.5(1) = 3.9</td>
</tr>
<tr>
<td>-2</td>
<td>1</td>
<td>2.4(-2) + 1.5(1) = -3.3</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>2.4(2) + 1.5(-1) = 3.3</td>
</tr>
</tbody>
</table>

\( P = 3.9 \) is the maximum.
8. Let \( x \) be the number of cleanings, and \( y \) be the number of cavities filled.

\[
\begin{align*}
0 & \leq y \leq 4 \\
0.5x + y & \leq 7
\end{align*}
\]

Maximize the objective function \( P = 40x + 95y \).

\[
\begin{array}{c|cc|c}
 x & y & P = 40x + 95y \\
0 & 0 & 40(0) + 95(0) = 0 \\
0 & 4 & 40(0) + 95(4) = 380 \\
6 & 4 & 40(6) + 95(4) = 620 \\
14 & 0 & 40(14) + 95(0) = 560 \\
\end{array}
\]

Dr. Lee should book 6 cleanings and 4 fillings to maximize his earnings.

10. Let \( x \) be the number of journey bags, and \( y \) be the number of trek bags.

\[
\begin{align*}
x & \geq 0 \\
y & \geq 0
\end{align*}
\]

Maximize the objective function \( P = x + 3y \).

\[
\begin{array}{c|cc|c}
x & y & P = x + 3y \\
0 & −3 & (0) + 3(−3) = −9 \\
0 & 4 & (0) + 3(4) = 12 \\
5 & −1 & (5) + 3(−1) = 2 \\
5 & −2 & (5) + 3(−2) = −1 \\
\end{array}
\]

\( P = 12 \) is the maximum.

11. Let \( x \) be the number of radio commercials, and \( y \) be the number of prime-time TV commercials.

\[
\begin{align*}
0.5x + 1500y & \leq 60,000 \\
y & > 0
\end{align*}
\]

Maximize the objective function \( P = 20x + 30y \).

\[
\begin{array}{c|cc|c}
x & y & P = 20x + 30y \\
30 & 0 & 20(30) + 30(0) = 600 \\
60 & 0 & 20(60) + 30(0) = 1200 \\
30 & 32 & 20(30) + 30(32) = 1560 \\
60 & 24 & 20(60) + 30(24) = 1920 \\
\end{array}
\]

There should be 60 radio and 24 prime-time TV commercials.

12. Find the vertices of the feasible region.

Maximize the objective function \( P = −21x + 11y \).

\[
\begin{array}{c|cc|c}
x & y & P = −21x + 11y \\
0 & 0 & −21(0) + 11(0) = 0 \\
1 & 0 & −21(1) + 11(0) = −21 \\
0 & 5 & −21(0) + 11(5) = 55 \\
3 & 8 & −21(3) + 11(8) = 25 \\
\end{array}
\]

\( P = 55 \) is the maximum.

13. Find the vertices of the feasible region.

Minimize the objective function \( P = −2x − 4y \).

\[
\begin{array}{c|cc|c}
x & y & P = −2x − 4y \\
−7 & 0 & −2(−7) − 4(0) = 14 \\
0 & 9 & −2(0) − 4(9) = −36 \\
−8 & 9 & −2(−8) − 4(9) = −20 \\
−3.5 & 0 & −2(−3.5) − 4(0) = 7 \\
\end{array}
\]

\( P = −36 \) is the minimum.

14. Find the vertices of the feasible region.

Maximize the objective function \( P = x + 3y \).

\[
\begin{array}{c|cc|c}
x & y & P = x + 3y \\
0 & −3 & (0) + 3(−3) = −9 \\
0 & 4 & (0) + 3(4) = 12 \\
5 & −1 & (5) + 3(−1) = 2 \\
5 & −2 & (5) + 3(−2) = −1 \\
\end{array}
\]

15. Let \( x \) be the number of radio commercials, and \( y \) be the number of prime-time TV commercials.

\[
\begin{align*}
30 & \leq x \leq 60 \\
y & > 0
\end{align*}
\]

Maximize the objective function \( P = 20x + 30y \).

\[
\begin{array}{c|cc|c}
x & y & P = 20x + 30y \\
30 & 0 & 20(30) + 30(0) = 600 \\
60 & 0 & 20(60) + 30(0) = 1200 \\
30 & 32 & 20(30) + 30(32) = 1560 \\
60 & 24 & 20(60) + 30(24) = 1920 \\
\end{array}
\]

16a. Let \( x \) be the number of fans in the upper deck, and \( y \) be the number of fans in the lower deck.

\[
\begin{align*}
0 & \leq y \leq 60,000 \\
0 & \leq x \leq 120,000 \\
x + y & \leq 160,000
\end{align*}
\]

Maximize the objective function \( P = 25x + 45y \).

\[
\begin{array}{c|cc|c}
x & y & P = 25x + 45y \\
0 & 0 & 0 \\
0 & 60,000 & 2,700,000 \\
100,000 & 60,000 & 5,200,000 \\
120,000 & 40,000 & 4,800,000 \\
120,000 & 0 & 3,000,000 \\
\end{array}
\]

100,000 upper deck and 60,000 lower deck tickets will maximize profit.

b. Possible answer: The system of inequalities does not change, but the objective function changes. The new solution is 120,000 upper deck and 40,000 lower deck tickets to maximize profits.

17. Let \( x \) be the number of journey bags, and \( y \) be the number of trek bags.

\[
\begin{align*}
0 & \leq x \leq 4 \\
0 & \leq y \leq 15 \\
40x + 80y & \geq 400
\end{align*}
\]

Minimize the objective function \( P = 4x + 6y \).

\[
\begin{array}{c|cc|c}
x & y & P = 4x + 6y \\
0 & 5 & 4(0) + 6(5) = 30 \\
0 & 15 & 4(0) + 6(15) = 90 \\
4 & 3 & 4(4) + 6(3) = 32 \\
4 & 15 & 4(4) + 6(15) = 106 \\
\end{array}
\]

The minimum number of hours required is 32 h.
18. parallelogram
\[ \begin{align*}
-1 & \leq x \leq 2 \\
x - 4 & \leq y \leq x + 2 
\end{align*} \]

19. right triangle
\[ \begin{align*}
y & \leq x + 2 \\
y & \geq 2x \\
y & \geq -\frac{1}{2}x 
\end{align*} \]

20. trapezoid
\[ \begin{align*}
1 & \leq y \leq 6 \\
y & \leq -2x + 8 \\
y & \leq 2x + 10 
\end{align*} \]

21. Let \( x \) be the number of gas stops, and \( y \) be the number of flat tire stops.
\[ \begin{align*}
0 & \leq x \leq 14 \\
0 & \leq y \\
0.25x + 0.75y & < 8 
\end{align*} \]
Maximize the objective function \( P = x + y \): 

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( P = x + y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>(0) + (0) = 0</td>
</tr>
<tr>
<td>0</td>
<td>10</td>
<td>(0) + (10) = 10</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>(2) + (10) = 12</td>
</tr>
<tr>
<td>14</td>
<td>6</td>
<td>(14) + (6) = 20</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>(14) + (0) = 14</td>
</tr>
</tbody>
</table>

20 stops is the maximum.

22. Possible answer: If the feasible region is unbounded, the objective function may not have a maximum value, or a minimum value, and such a linear programming problem has no solution.

23. Let \( x \) be the number of Soy Joy smoothies, and \( y \) be the number of Vitamin Boost smoothies.
\[ \begin{align*}
x & \geq 0 \\
y & \geq 0 \\
2x + y & \leq 100 \\
x + 3y & \leq 100 
\end{align*} \]
Maximize the objective function \( P = 2.75x + 3.25y \):

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( P = 2.75x + 3.25y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>2.75(0) + 3.25(0) = 0</td>
</tr>
<tr>
<td>0</td>
<td>33</td>
<td>2.75(0) + 3.25(33) = 107.25</td>
</tr>
<tr>
<td>50</td>
<td>0</td>
<td>2.75(50) + 3.25(0) = 137.5</td>
</tr>
<tr>
<td>40</td>
<td>20</td>
<td>2.75(40) + 3.25(20) = 175</td>
</tr>
</tbody>
</table>

The store should make 40 Soy Joy and 20 Vitamin Boost smoothies.

24. Possible answer: The objective function could calculate the profit/loss and the loss could be negative values.

25. Possible answer: Substitute the values from every vertex of the feasible region into the objective function one pair at a time. Depending on the problem statement, look for the greatest value of the objective function to identify the maximum or the least value to identify the minimum.

26. Possible answer: Once you have identified the feasible region, find the intersection points of the lines that form that region. The coordinates of the intersections are the coordinates of the vertices.

**TEST PREP**

27. D

28. A:
\[ P(0,0) = -4(0) + (0) - 1 = -1 \]

29. G

**CHALLENGE AND EXTEND**

b. (350, 400) represents 350 of bacteria type A and 400 of bacteria type B. (400, 350) represents 400 of bacteria type A and 350 of bacteria type B.

C. Only the point (350, 400) satisfies the constraint, as the minimum value for type B must be at least 400.

**SPIRAL REVIEW**

31. \( f(7) = 1 = \frac{1}{2} \frac{1}{7} - 3 = \frac{1}{11} \)
\[ f(-\frac{1}{2}) = \frac{1}{2} = \frac{1}{1} \]
\[ = -1 - 3 = -1 \frac{1}{4} \]

32. \( f(7) = 0.5(7) = 3.5 \)
\[ f(-\frac{1}{2}) = 0.5(-\frac{1}{2}) = -\frac{1}{4} \]

33. \( f(7) = \frac{7^2 - 1}{7 - 1} = \frac{48}{6} = 8 \)
\[ f(-\frac{1}{2}) = \frac{(-\frac{1}{2})^2 - 1}{-\frac{1}{2} - 1} = \frac{1}{4} - 1 = \frac{1}{2} \]

34. \( f(x) = |x| \)
\[ g(x) = |x - 6| - 3 \]

35. \( f(x) = |x| \)
\[ g(x) = |x - \frac{1}{3} + \frac{4}{3} | \]

36. \( f(x) = |x| \)
\[ g(x) = |x + 2.5| + 0.75 \]
1. \[
\begin{align*}
2x + y &= -5 \\
x + 2y &= 2 \\
y &= -2x - 5
\end{align*}
\]
\[
\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
2 & -9 \\
0 & -5 \\
-2 & -1 \\
-4 & 3 \\
\end{array}
\]
The solution to the system is \((-4, 3).\)

2. \[
\begin{align*}
x + y &= -1 \\
x - 2y &= -4 \\
y &= -x - 1
\end{align*}
\]
\[
\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
2 & -3 \\
0 & -1 \\
-2 & 1 \\
-4 & 3 \\
\end{array}
\]
The solution to the system is \((-2, 1).\)

3. \[
\begin{align*}
x &= y - 2 \\
3x - y &= 2 \\
y &= x + 2
\end{align*}
\]
\[
\begin{array}{c|c}
\text{x} & \text{y} \\
\hline
-1 & 1 \\
0 & 2 \\
1 & 3 \\
2 & 4 \\
\end{array}
\]
The solution to the system is \((2, 4).\)

4. \[
\begin{align*}
y &= \frac{2}{3}x - 4 \\
y &= \frac{2}{3}x - \frac{4}{3}
\end{align*}
\]
inconsistent; no solution

5. \[
\begin{align*}
y &= \frac{5}{6}x - \frac{7}{3} \\
y &= -\frac{1}{3}x + 5
\end{align*}
\]
independent; one solution

6. \[
\begin{align*}
y &= \frac{1}{2}x + 5 \\
y &= \frac{1}{2}x + 5
\end{align*}
\]
dependent; infinitely many solutions

7. Step 1: Solve one equation for one variable. The first equation is already solved for \(y\): \(y = x + 3.\)
Step 2: Substitute the expression into the other equation.
\[
\begin{align*}
2x + 4y &= 24 \\
2x + 4(x + 3) &= 24 \\
6x + 12 &= 24 \\
6x &= 12 \\
x &= 2
\end{align*}
\]
The solution is the ordered pair \((4, 5).\)

8. Step 1: Solve one equation for one variable. The first equation is already solved for \(x\): \(x = 5.\)
Step 2: Substitute the expression into the other equation.
\[
\begin{align*}
2x + 3y &= 19 \\
2(5) + 3y &= 19 \\
10 + 3y &= 19 \\
3y &= 9 \\
y &= 3
\end{align*}
\]
The solution is the ordered pair \((4, 3).\)

9. \[
\begin{align*}
x - y &= 5 \\
x &= y + 5 \\
3x - 2y &= 14 \\
3(y + 5) - 2y &= 14 \\
3y + 15 - 2y &= 14 \\
y + 15 &= 14 \\
y &= -1
\end{align*}
\]
\[
\begin{align*}
x &= y + 5 \\
x &= (-1) + 5 \\
x &= 4 \\
solution (4, -1)
\end{align*}
\]

10. Step 1: Find the value of one variable. Add to eliminate \(y\).
\[
\begin{align*}
x + 2y &= 15 \\
x - 2y &= -9 \\
2x &= 6 \\
x &= 3
\end{align*}
\]
Step 2: Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[
\begin{align*}
x + 2y &= 15 \\
(3) + 2y &= 15 \\
2y &= 12 \\
y &= 6
\end{align*}
\]
The solution is the ordered pair \((3, 6).\)

11. Step 1: To eliminate \(y\), multiply both sides of the second equation by \(-1\).
\[-1(8x - 4y) = -1(12) \quad 2\]
Add to eliminate \(y\).
\[
\begin{align*}
5x - 4y &= 0 \\
-8x + 4y &= -12 \\
-3x &= -12 \\
x &= 4
\end{align*}
\]
Step 2: Substitute the \(x\)-value into one of the original equations to solve for \(y\).
\[
\begin{align*}
5x - 4y &= 0 \\
5(4) - 4y &= 0 \\
20 - 4y &= 0 \\
-4y &= -20 \\
y &= 5
\end{align*}
\]
The solution is the ordered pair \((4, 5).\)

12. Step 1: To eliminate \(x\), multiply both sides of the second equation by \(-2\).
\[-2(2x + 6y) = -2(-4) \quad 2\]
Add to eliminate \(x\).
\[
\begin{align*}
4x + 2y &= 12 \\
-4x - 12y &= -8 \\
-10y &= 20 \\
y &= -2
\end{align*}
\]
Step 2: Substitute the \(y\)-value into one of the original equations to solve for \(x\).
\[
\begin{align*}
4x + 2y &= 12 \\
4x + 2(-2) &= 12 \\
4x &= 16 \\
x &= 4
\end{align*}
\]
The solution is the ordered pair \((4, -2).\)
13. The smallest possible value is $P = 4$ when $x = 1$ and $y = 0$.

14. The biggest possible value is $P = 34$ when $x = 1$ and $y = 6$.

15. $x + y < 7$

16. $60x + 50y < 350$

17. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$P = 4x + 5y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>4(1) + 5(0) = 4</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>4(8) + 5(0) = 32</td>
</tr>
<tr>
<td>10</td>
<td>7/3</td>
<td>4(10)/3 + 5(7)/3 = 25</td>
</tr>
</tbody>
</table>

18. 

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$P = 4x + 5y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>0</td>
<td>4(-2) + 5(0) = -8</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>4(2) + 5(0) = 8</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
<td>4(2) + 5(3) = 23</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>4(1) + 5(6) = 34</td>
</tr>
</tbody>
</table>

19. Let $x$ be the number of special services, and $y$ be the number of haircuts in a day.

\[
\begin{align*}
0 & \leq x \leq 4 \\
y & \geq 0 \\
x + 0.5y & \leq 8
\end{align*}
\]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
<th>$P = 45x + 20y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>45(0) + 20(0) = 0</td>
</tr>
<tr>
<td>0</td>
<td>16</td>
<td>45(0) + 20(16) = 320</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>45(4) + 20(0) = 180</td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>45(4) + 20(8) = 340</td>
</tr>
</tbody>
</table>

To produce the maximum income, the salon should schedule 4 special services and 8 haircuts per day; $340.

3-5 LINEAR EQUATIONS IN THREE DIMENSIONS, PAGES 214–218

CHECK IT OUT!

1. a.–c.

2. $x$-intercept: $x = 4(0) + 2(0) = 4$

   $x = 4$

   $y$-intercept: $0 - 4y + 2(0) = 4$

   $-4y = 4$

   $y = -1$

   $z$-intercept: $0 - 4(0) + 2(z) = 4$

   $2z = 4$

   $z = 2$

3a. $3.5x + 1.5y + 0.75z = 61.50$

   b. $3.5(6) + 1.5y + 0.75(24) = 61.50$

   $21 + 1.5y + 18 = 61.50$

   $1.5y = 22.50$

   $y = 15$

THINK AND DISCUSS

1. Possible answer: (10, 4, 3)

2. Passes through (0, 0, 1) and (0, 1, 0) and contains lines parallel to the $x$-axis.
**EXERCISES**

**GUIDED PRACTICE**

1. Possible answer: The 3-dimensional coordinate system has 3 axes instead of 2 and uses 3 coordinates instead of 2 to denote a point.

2. 
   - x-int.: $x + 0 + 0 = 3$
   - $x = 3$
   - y-int.: $0 + y + 0 = 3$
   - $y = 3$
   - z-int.: $0 + 0 + z = 3$
   - $z = 3$

3. 
   - $x = 1.5$ and $y = 3$

4. 
   - $x = 1.5$ and $y = 3$

5. 
   - $x = 1.5$ and $y = 3$

6. 
   - $x = 1.5$ and $y = 3$

7. 
   - $x = 1.5$ and $y = 3$

8. 
   - x-int.: $1.5x + 3(0) - 2(0) = -6$  
     $1.5x = -6$
     $x = -4$
   - y-int.: $1.5(0) + 3y - 2(0) = -6$  
     $3y = -6$
     $y = -2$
   - z-int.: $1.5(0) + 3(0) - 2z = -6$  
     $-2z = -6$
     $z = 3$

9a. $225x + 150y + 300z = 3000$
   b. i. $225(8) + 150(6) + 300z = 3000$
       $300z = 300$
       $z = 1$
   ii. $225(x) + 150(1) + 300(5) = 3000$
       $225x = 1350$
       $x = 6$
   iii. $225(4) + 150y + 300(4) = 3000$
       $150y = 900$
       $y = 6$
   iv. $225(10) + 150(5) + 300z = 3000$
       $300z = 0$
       $z = 0$

c. 20 (all dishwashers)

**PRACTICE AND PROBLEM SOLVING**

10. 
    - $(2, -4, 3)$
    - $x = 2$

11. 
    - $(-1, 1, 4)$
    - $x = 1$

12. 
    - $(3, 0, 0)$
    - $y = 3$

13. 
    - $(0, -2, 0)$
    - $z = 0$

14. 
    - $(5, 0, 2)$
    - $x = 5$

15. 
    - $(-3, -3, -3)$
    - $y = -3$

16. 
    - $(0, -3, 2)$
    - $z = 2$

17. 
    - $(-4, -1, 1)$
    - $z = 1$
18. x-int.: $x + (0) - (0) = -1$
\[ x = -1 \]

y-int.: $(0) + y - (0) = -1$
\[ y = -1 \]

z-int.: $(0) + (0) - z = -1$
\[ z = 1 \]

19. x-int.: $2x - (0) + 2(0) = 4$
\[ 2x = 4 \]
\[ x = 2 \]

y-int.: $2(0) - y + 2(0) = 4$
\[ -y = 4 \]
\[ y = -4 \]

z-int.: $2(0) - (0) + 2z = 4$
\[ 2z = 4 \]
\[ z = 2 \]

20. x-int.: $2x + \frac{1}{2}(0) + (0) = -2$
\[ 2x = -2 \]
\[ x = -1 \]

y-int.: $2(0) + \frac{1}{2}y + (0) = -2$
\[ \frac{1}{2}y = -2 \]
\[ y = -4 \]

z-int.: $2(0) + \frac{1}{2}(0) + z = -2$
\[ z = -2 \]

21. x-int.: $5x + (0) - (0) = -5$
\[ 5x = -5 \]
\[ x = -1 \]

y-int.: $5(0) + y - (0) = -5$
\[ y = -5 \]

z-int.: $5(0) + (0) - z = -5$
\[ -z = -5 \]
\[ z = 5 \]

22. x-int.: $8x + 6(0) + 4(0) = 24$
\[ 8x = 24 \]
\[ x = 3 \]

y-int.: $8(0) + 6y + 4(0) = 24$
\[ 6y = 24 \]
\[ y = 4 \]

z-int.: $8(0) + 6(0) + 4z = 24$
\[ 4z = 24 \]
\[ z = 6 \]

23. x-int.: $3x - 3(0) + 2.5(0) = 7.5$
\[ 3x = 7.5 \]
\[ x = 2.5 \]

y-int.: $3(0) - 3y + 2.5(0) = 7.5$
\[ -3y = 7.5 \]
\[ y = -2.5 \]

z-int.: $3(0) - 3(0) + 2.5z = 7.5$
\[ 2.5z = 7.5 \]
\[ z = 3 \]

24a. $10x + 15y + 2.5z = 80$

25. Let $x$ be the number of free throws, $y$ be the number of 2-pt. field goals, and $z$ be the number of 3-pt. field goals.

\[ x + 2y + 3z = 60 \]
\[ 20 + 2y + 3z = 60 \]
\[ 2y + 3z = 40 \]

Possible answer:

<table>
<thead>
<tr>
<th>Three-pointers</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Two-pointers</td>
<td>17</td>
<td>14</td>
<td>11</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

26. $(-3, -3, 3), (-3, 0, 3), (0, -3, 3), (0, 0, 3), (-3, -3, 0), (0, 0, 0), (0, -3, 0)$

27. $(-2, -2, 2), (-2, 2, 2), (2, -2, 2), (2, 2, 2), (-2, -2, -2), (-2, -2, -2)$

28. $6h + 4t + 2c = 8000$
\[ 6h + 4(400) + 2c = 8000 \]
\[ 6h + 2c = 6400 \]

$6400$ remaining
\[ 2000 \times 400 = 1600 \text{ ft}^2 \text{ remaining} \]
\[ 800 \text{ ft}^2 \text{ hardwood and } 800 \text{ ft}^2 \text{ carpet} \]
\[ 6(800) = $4800 for hardwood \]
\[ 2(800) = $1600 for carpet \]

Total is $6400$.

Yes, it is possible to finish the rest of the flooring half in hardwood and half in carpet.

29. Possible answer: No, it only represents 1 dimension. For 2 dimensions you must move up and down as well as forward and backward or left and right.

30. Possible answer: Draw 2 perpendicular lines just like a 2D grid. Label the horizontal “$y$” and the vertical “$z$”. Draw a line which would look like $y = x$ in 2D, through this graph. Label it “$x$”.

31a. $(7 \pm 4, 12 \pm 4, 10)$

Possible answers: $(3, 8, 10)$ and $(11, 16, 10)$.

b. $(7, 12, 10 - 1.5) = (7, 12, 8.5)$

c. $(7, 12, 8.5 + 4) = (7, 12, 12.5)$

32. Solution B is incorrect. To find the $x$-intercept, the $z$-value must equal 0.

**TEST PREP**

33. A

34. C; intercepts are at $(2, 0, 0), (0, 1, 0)$, and $(0, 0, 3)$
35. H 36. \( \frac{3}{4} \); \( 5(0) - 2(0) - 4z = -3 \)

\[ -4z = -3 \]

\[ z = \frac{3}{4} \]

**CHALLENGE AND EXTEND**

37. [Diagram of a square pyramid]

38. [Diagram of a triangular prism]

40. [Diagram of a sphere]

41. Possible answer:

\[ x + 2y - 4z = 4 \]

**SPIRAL REVIEW**

43. square pyramid

45. sphere

47. **Step 1** Solve one equation for one variable. The first equation is already solved for \( x; x = 5y \). **Step 2** Substitute the expression into the other equation.

\[ \frac{2}{5} x + 7y = 18 \]

\[ \frac{2}{5}(5y) + 7y = 18 \]

\[ 2y + 7y = 18 \]

\[ 9y = 18 \]

\[ y = 2 \]

**Step 3** Substitute the \( y \)-value into one of the original equations to solve for \( x \).

\[ x = 5y \]

\[ x = 5(2) \]

\[ x = 10 \]

The solution is the ordered pair \( (10, 2) \).

48. **Step 1** Solve one equation for one variable.

\[ 6x - y = 5 \]

\[ y = -6x + 5 \]

**Step 2** Substitute the expression into the other equation.

\[ 4y - 3x = 1 \]

\[ 4(6x - 5) - 3x = 1 \]

\[ 24x - 20 - 3x = 1 \]

\[ 21x = 21 \]

\[ x = 1 \]

**Step 3** Substitute the \( x \)-value into one of the original equations to solve for \( y \).

\[ y = -6x + 5 \]

\[ y = -6(1) + 5 \]

\[ y = 1 \]

The solution is the ordered pair \( (1, 1) \).

49. **Step 1** Solve one equation for one variable.

\[ x + 3y = 6 \]

\[ x = -3y + 6 \]

**Step 2** Substitute the expression into the other equation.

\[ 2x - 3y = 9 \]

\[ 2(-3y + 6) - 3y = 9 \]

\[ -6y + 12 - 3y = 9 \]

\[ -9y = -3 \]

\[ y = \frac{1}{3} \]

**Step 3** Substitute the \( y \)-value into one of the original equations to solve for \( x \).

\[ x = -3y + 6 \]

\[ x = -3\left(\frac{1}{3}\right) + 6 \]

\[ x = -1 + 6 \]

\[ x = 5 \]

The solution is the ordered pair \( \left(5, \frac{1}{3}\right) \).}

**3-6 SOLVING LINEAR SYSTEMS IN THREE VARIABLES, PAGES 220–226**

**CHECK IT OUT!**

\[ \begin{align*}
-x + y + 2z &= 7 \\
2x + 3y + z &= 1 \\
-3x - 4y + z &= 4 \\
6x + 8y - 2z &= -8 \\
5x + 9y &= -1 \\
5x + 7y &= -3 \\
5x + 9y &= -1
\end{align*} \]

**Step 1** Eliminate one variable. Multiply equation \( 3 \) by \(-1\) and eliminate \( z \) from equations \( 2 \) and \( 3 \) by adding.

\[ \begin{align*}
2x + 3y + z &= 1 \\
3x + 4y + z &= -4 \\
5x + 7y &= -3
\end{align*} \]

Multiply equation \( 3 \) by \(-2\) and eliminate \( z \) from equations \( 1 \) and \( 3 \) by adding.

\[ \begin{align*}
x + y + 2z &= 7 \\
6x + 8y - 2z &= -8 \\
5x + 9y &= -1 \\
5x + 7y &= -3 \\
5x + 9y &= -1
\end{align*} \]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation \( 5 \) by \(-1\) and eliminate \( x \) from equations \( 3 \) and \( 5 \) by adding.

\[ \begin{align*}
5x + 7y &= -3 \\
-5x - 9y &= 1 \\
\hline
2y &= -2 \\
y &= 1
\end{align*} \]

**Step 3** Use one of the equations in your 2-by-2 system to solve for \( x \).

\[ \begin{align*}
5x + 7y &= -3 \\
5x + 7(1) &= -3 \\
5x &= -10 \\
x &= -2
\end{align*} \]

**Step 4** Substitute for \( x \) and \( y \) in one of the original equations to solve for \( z \).

\[ \begin{align*}
-x + y + 2z &= 7 \\
2 + 1 + 2z &= 7 \\
2z &= 4 \\
z &= 2
\end{align*} \]

The solution is \((-2, 1, 2)\).
2. **Step 1** Let $x$ represent the number of points for a first-place vote, $y$ for a second-place vote, and $z$ for a third-place vote.

- $3x + y + 4z = 15$ Jada’s points
- $2x + 4y = 14$ Maria’s points
- $2x + 2y + 3z = 13$ Al’s points

**Step 2** Solve for $x$ in equation (2).

\[2x + 4y = 14\]

- $2x = -4y + 14$
- $x = -2y + 7$

**Step 3** Substitute for $x$ in equations (1) and (3).

\[
\begin{align*}
3(-2y + 7) + y + 4z &= 15 \quad \text{(1)} \\
2(-2y + 7) + 2y + 3z &= 13 \quad \text{(3)} \\
-5y + 4z &= -6 \quad \text{(4)} \\
-2y + 3z &= -1 \quad \text{(5)}
\end{align*}
\]

**Step 4** Multiply equation (5) by $-1$ and solve for $z$ by adding equations (4) and (5).

\[
\begin{align*}
-5y + 4z &= -6 \quad \text{(4)} \\
2y - 3z &= 1 \quad \text{(5)} \\
-3y &= z - 5
\end{align*}
\]

- $z = 3y - 5$

**Step 5** Substitute for $z$ in equation (4).

\[\begin{align*}
-5y + 4(3y - 5) &= -6 \\
7y &= 14 \\
y &= 2
\end{align*}\]

**Step 6** Substitute for $y$ to solve for $x$ and then for $z$.

\[
\begin{align*}
2x + 4y &= 14 \quad \text{(2)} \\
2x + 4(2) &= 14 \\
2x &= 6 \\
x &= 3
\end{align*}\]

\[
\begin{align*}
-9x + 3y &= 7 \\
-9x + 3(2) &= 7 \\
-9x &= 1 \\
x &= 1
\end{align*}\]

The solution to the system is $(3, 2, 1)$.

3a. \[
\begin{align*}
3x - y + 2z &= 4 \quad \text{(1)} \\
2x - y + 3z &= 7 \\
-9x + 3y - 6z &= -12 \quad \text{(3)}
\end{align*}\]

Multiply equation (2) by $-1$, and eliminate $y$ from equations (1) and (3) by adding.

\[
\begin{align*}
3x - y + 2z &= 4 \quad \text{(1)} \\
-2x + y - 3z &= -2 \quad \text{(2)} \\
x - z &= -3 \quad \text{(4)}
\end{align*}\]

Multiply equation (2) by 3, and eliminate $y$ from equations (1) and (3) by adding.

\[
\begin{align*}
6x - 3y + 9z &= 21 \quad \text{(2)} \\
-9x + 3y - 6z &= -12 \quad \text{(3)} \\
-3x + 3z &= 9 \quad \text{(5)}
\end{align*}\]

\[
\begin{align*}
x - z &= -3 \quad \text{(4)} \\
-3x + 3z &= 9 \quad \text{(5)}
\end{align*}\]

Multiply equation (3) by 3, and eliminate $x$ from equations (4) and (5) by adding.

\[
\begin{align*}
3x - 3z &= -9 \quad \text{(4)} \\
-3x + 3z &= 9 \quad \text{(5)}
\end{align*}\]

- $0 = 0$

consistent and dependent; infinite number of solutions

b. \[
\begin{align*}
x - y + 3z &= 6 \\
2x - 4y + 6z &= 10 \\
y - z &= -2
\end{align*}\]

Multiply equation (1) by $-2$, and eliminate $z$ from equations (1) and (2) by adding.

\[
\begin{align*}
-2x + 2y - 6z &= -12 \quad \text{(1)} \\
2x - 4y + 6z &= 10 \quad \text{(2)} \\
-2y &= -2
\end{align*}\]

- $y = 1$

Eliminate $y$ from equations (1) and (3) by adding.

\[
\begin{align*}
x - y + 3z &= 6 \quad \text{(1)} \\
y - z &= -2 \quad \text{(3)} \\
x + 2z &= 4 \quad \text{(5)}
\end{align*}\]

\[
\begin{align*}
y - 1 \quad \text{(4)} \\
x + 2z &= 4 \quad \text{(5)}
\end{align*}\]

inconsistent, no solution

**THINK AND DISCUSS**

1. Possible answer: Inconsistent systems have two vertical, non-intersecting planes each cut by a third vertical plane. Dependent systems have two identical planes cut by a third plane.

2. 

**EXERCISES**

**GUIDED PRACTICE**

\[
\begin{align*}
\begin{align*}
-2x + y + 3z &= 20 \quad \text{(1)} \\
-3x + 2y + z &= 21 \quad \text{(2)} \\
3x - 2y + 3z &= -9 \quad \text{(3)}
\end{align*}
\end{align*}\]

**Step 1** Eliminate one variable. Multiply equation (1) by $-2$, and eliminate $y$ from equations (1) and (2) by adding.

\[
\begin{align*}
4x - 2y - 6z &= -40 \quad \text{(1)} \\
-3x + 2y + z &= 21 \quad \text{(2)} \\
x - 5z &= -19 \quad \text{(4)}
\end{align*}\]

Multiply equation (1) by 2 and eliminate $y$ from equations (1) and (3) by adding.

\[
\begin{align*}
-4x + 2y + 6z &= 40 \quad \text{(1)} \\
3x - 2y + 3z &= -9 \quad \text{(3)} \\
x + 9z &= 31 \quad \text{(5)}
\end{align*}\]

Multiply equation (1) by 2 and eliminate $z$ from equations (1) and (3) by adding.

\[
\begin{align*}
-x - 5z &= -19 \quad \text{(3)} \\
-x + 9z &= 31 \quad \text{(5)}
\end{align*}\]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Eliminate $x$ from equations (4) and (5) by adding.

\[
\begin{align*}
x - 5z &= -19 \quad \text{(3)} \\
-x + 9z &= 31 \quad \text{(5)}
\end{align*}\]

\[
\begin{align*}
4z &= 12 \\
z &= 3
\end{align*}\]
Step 3 Use one of the equations in your 2-by-2 system to solve for $x$.

\[ x - 5z = -19 \]
\[ x - 5(3) = -19 \]
\[ x = -4 \]

Step 4 Substitute for $x$ and $z$ in one of the original equations to solve for $y$.

\[ 2x + y + 3z = 20 \]
\[ 2(-4) + y + 3(3) = 20 \]
\[ y = 3 \]

The solution is $(-4, 3, 3)$.

---

2.

Step 1 Eliminate one variable. Multiply equation 2 by $-1$ and eliminate $x$ from equations 1 and 2 by adding.

\[ x + 2y + 3z = 9 \] \[ x + 3y + 2z = 5 \]
\[ -x - 3y - 2z = -5 \] \[ -y + z = 4 \]

Multiply equation 3 by $-1$ and eliminate $x$ from equations 2 and 3 by adding.

\[ x + 3y + 2z = 5 \]
\[ -x - 4y + z = 5 \]
\[ -y + z = 4 \]

Then solve for $y$ and $z$ in one of the original equations.

\[ -y + z = 4 \]
\[ -y + 3z = 10 \]

Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation 5 by $-1$ and eliminate $y$ from equations 4 and 5 by adding.

\[ -y + z = 4 \]
\[ -y + 3z = 10 \]

Then solve for $z$.

\[ z = 3 \]

Step 3 Use one of the equations in your 2-by-2 system to solve for $y$.

\[ -y + z = 4 \]
\[ -y + (3) = 4 \]
\[ y = -1 \]

Step 4 Substitute for $y$ and $z$ in one of the original equations to solve for $x$.

\[ x + 2y + 3z = 9 \]
\[ x + 2(-1) + 3(3) = 9 \]
\[ x = 2 \]

The solution is $(2, -1, 3)$.

---

3.

Step 1 Eliminate one variable. Multiply equation 3 by $-1$ and eliminate $x$ from equations 1 and 2 by adding.

\[ x + 2y + z = 8 \]
\[ 2x + y - z = 4 \]
\[ x + y + 3z = 7 \]

Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation 4 by $-6$ and eliminate $y$ from equations 4 and 5 by adding.

\[ 5y + 8z = 155 \]
\[ 30y + 33z = 780 \]

Step 3 Use one of the equations in your 2-by-2 system to solve for $y$.

\[ 5y + 8z = 155 \]
\[ 5y + 5(10) = 155 \]
\[ 5y = 75 \]
\[ y = 15 \]

Step 4 Substitute for $y$ and $z$ in one of the original equations to solve for $x$.

\[ x + 2y + 3z = 9 \]
\[ x + 2(15) + 3(10) = 92 \]
\[ 4x = 32 \]
\[ x = 8 \]

Step 5 Inconsistent; no solution

---

4.

Step 1 Eliminate one variable. Multiply equation 2 by $-1$ and eliminate $x$ from equations 1 and 2 by adding.

\[ 5x + 10y + 12z = 310 \]
\[ 5x + 5y + 4z = 155 \]
\[ 4x + 2y + 3z = 92 \]

Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation 4 by $-6$ and eliminate $y$ from equations 4 and 5 by adding.

\[ 5y + 8z = 155 \]
\[ 30y + 33z = 780 \]

Step 3 Use one of the equations in your 2-by-2 system to solve for $y$.

\[ 2x + 4y - 2z = 4 \]
\[ -x - 2y + z = 4 \]
\[ 3x + 6y - 3z = 10 \]

Step 4 Substitute for $y$ and $z$ in one of the original equations to solve for $x$.

\[ x + 2y + z = 8 \]
\[ x + 2(3) + (1) = 8 \]
\[ x = 1 \]

Step 5 Inconsistent; no solution
Multiply equation 3 by -1 and eliminate y from equations 1 and 2 by adding.

\[
\begin{align*}
2x + 4y & - 5z = -10 \\
-x - 2y + 8z & = 16 \\
-2x + 4y + 2z & = 4
\end{align*}
\]

Multiply equation 2 by 1 and eliminate y from equations 2 and 3 by adding.

\[
\begin{align*}
2x + 4y & - 5z = -10 \\
-x - 2y + 8z & = 16 \\
-2x - 4y & + 2z = -4
\end{align*}
\]

Multiply equation 2 by 2 and eliminate y from equations 2 and 3 by adding.

\[
\begin{align*}
4x + 7z & = -14 \\
-4x + 18z & = 36
\end{align*}
\]

Eliminate x from equations 4 and 5 by adding.

\[
\begin{align*}
4x + 7z & = -14 \\
-4x + 18z & = 36
\end{align*}
\]

\[
\begin{align*}
11z & = 22 \\
z & = 2
\end{align*}
\]

consistent; one solution

\[
\begin{align*}
-2x + 3y + z & = 15 \\
x + 3y - z & = -1 \\
-5x - 6y + 4z & = -16
\end{align*}
\]

Multiply equation 2 by -1 and eliminate y from equations 1 and 2 by adding.

\[
\begin{align*}
-2x + 3y + z & = 15 \\
-x - 3y + z & = 1 \\
-3x + 2z & = 16
\end{align*}
\]

Multiply equation 2 by 2 and eliminate y from equations 2 and 3 by adding.

\[
\begin{align*}
2x + 6y - 2z & = -2 \\
-5x - 6y + 4z & = -16 \\
-3x + 2z & = -18
\end{align*}
\]

\[
\begin{align*}
-3x + 2z & = 16 \\
-3x + 2z & = -18
\end{align*}
\]

Multiply equation by -1 and eliminate x by adding.

\[
\begin{align*}
-3x + 2z & = 16 \\
3x - 2z & = 18
\end{align*}
\]

\[
0 = 34
\]

inconsistent; no solution

**PRACTICE AND PROBLEM SOLVING**

\[
\begin{align*}
2x - y - 3z & = 1 \\
4x + 3y + 2z & = -4 \\
-3x + 2y + 5z & = -3
\end{align*}
\]

**Step 1** Eliminate one variable. Multiply equation 1 by 2 and eliminate y from equations 3 and 5 by adding.

\[
\begin{align*}
4x - 2y - 6z & = 2 \\
-3x + 2y + 5z & = -3
\end{align*}
\]

Multiply equation 1 by 3 and eliminate y from equations 1 and 2 by adding.

\[
\begin{align*}
6x - 3y - 9z & = 3 \\
4x + 3y + 2z & = -4
\end{align*}
\]

\[
\begin{align*}
10x - 7z & = -1
\end{align*}
\]

\[
\begin{align*}
x - z & = -1 \\
10x - 7z & = -1
\end{align*}
\]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation 4 by -10 and eliminate x from 4 and 5 by adding.

\[
\begin{align*}
-10x + 10z & = 10 \\
10x - 7z & = -1
\end{align*}
\]

\[
\begin{align*}
3z & = 9 \\
z & = 3
\end{align*}
\]

**Step 3** Use one of the equations in your 2-by-2 system to solve for x.

\[
\begin{align*}
x - z & = -1 \\
x - (3) & = -1 \\
x & = 2
\end{align*}
\]

**Step 4** Substitute for x and z in one of the original equations to solve for y.

\[
\begin{align*}
2x - y - 3z & = 1 \\
2(2) - y - 3(3) & = 1 \\
y & = -6
\end{align*}
\]

The solution is \( (2, -6, 3) \).
9. \[
\begin{align*}
5x - 6y + 2z &= 21 \\
2x + 3y - 3z &= -9 \\
-3x + 9y - 4z &= -24
\end{align*}
\]

**Step 1** Eliminate one variable. Multiply equation 1 by 2, and eliminate \( z \) from equations 1 and 3 by adding.

\[
\begin{align*}
10x - 12y + 4z &= 42 \\
-3x + 9y - 4z &= -24
\end{align*}
\]

Multiply equation 1 by 3 and equation 2 by 2.

\[
\begin{align*}
7x - 3y &= 18
\end{align*}
\]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation 4 by \(-4\), and eliminate \( y \) from equations 4 and 5 by adding.

\[
\begin{align*}
-28x + 12y &= -72 \\
19x - 12y &= 45
\end{align*}
\]

\[9x = -27\]

\[x = 3\]

**Step 3** Use one of the equations in your 2-by-2 system to solve for \( y \).

\[
\begin{align*}
7x - 3y &= 18 \\
7(3) - 3y &= 18 \\
-3y &= -3
\end{align*}
\]

\[y = 1\]

**Step 4** Substitute for \( x \) and \( y \) in one of the original equations to solve for \( z \).

\[
\begin{align*}
2x + 3y - 3z &= -9 \\
2(3) + 3(1) - 3z &= -9 \\
-3z &= -18 \\
z &= 6
\end{align*}
\]

The solution is \((3, 1, 6)\).

10. \[
\begin{align*}
4x + 7y - z &= 42 \\
-2x + 2y + 3z &= -26 \\
2x - 3y + 5z &= 10
\end{align*}
\]

**Step 1** Eliminate one variable. Eliminate \( x \) from equations 2 and 3 by adding.

\[
\begin{align*}
-2x + 2y + 3z &= -26 \\
2x - 3y + 5z &= 10
\end{align*}
\]

Multiply equation 3 by \(-2\) and eliminate \( x \) from equations 1 and 3 by adding.

\[
\begin{align*}
4x + 7y - z &= 42 \\
-4x + 6y - 10z &= -20
\end{align*}
\]

\[13y - 11z = 22\]

\[-y + 8z = -16\]

\[13y - 11z = 22\]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation 4 by 13, and eliminate \( y \) from equations 4 and 5 by adding.

\[
\begin{align*}
-13y + 104z &= -208 \\
13y - 11z &= 22
\end{align*}
\]

\[93z = -186\]

\[z = -2\]

**Step 3** Use one of the equations in your 2-by-2 system to solve for \( y \).

\[-y + 8z = -16\]

\[-y + 8(-2) = -16\]

\[y = 0\]

**Step 4** Substitute for \( y \) and \( z \) in one of the original equations to solve for \( x \).

\[
\begin{align*}
2x - 3y + 5z &= 10 \\
2x - 3(0) + 5(-2) &= 10 \\
2x &= 20
\end{align*}
\]

\[x = 10\]

The solution is \((10, 0, -2)\).
11. \[
\begin{align*}
8x + 9y + 10z &= 9.2 \\
9x + 7y + 8z &= 8.1 \\
6x + 10y + 8z &= 7.8
\end{align*}
\]
**Step 1** Eliminate one variable. Multiply equation 3 by \(-1\) and eliminate \(z\) from equations 2 and 3 by adding.
\[
\begin{align*}
9x + 7y + 8z &= 8.1 \\
-6x - 10y - 8z &= -7.8 \\
3x - 3y &= 0.3
\end{align*}
\]
Multiply equation 1 by 4 and equation 3 by \(-5\).
\[
\begin{align*}
3x - 3y &= 0.3 \\
2x - 14y &= -2.2
\end{align*}
\]
**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation 2 by 2, and equation 3 by \(-3\). Eliminate \(x\) by adding.
\[
\begin{align*}
6x - 6y &= 0.6 \\
-6x + 42y &= 6.6 \\
36y &= 7.2 \\
y &= 0.2
\end{align*}
\]
**Step 3** Use one of the equations in your 2-by-2 system to solve for \(y\).
\[
\begin{align*}
3x - 3y &= 0.3 \\
3x - 3(0.2) &= 0.3 \\
x &= 0.9 \\
x &= 0.3
\end{align*}
\]
**Step 4** Substitute for \(x\) and \(y\) in one of the original equations to solve for \(z\).
\[
\begin{align*}
8x + 9y + 10z &= 9.2 \\
8(0.3) + 9(0.2) + 10z &= 9.2 \\
10z &= 5 \\
z &= 0.5
\end{align*}
\]
talent: 30%; presentation: 20%; star quality: 50%

12. \[
\begin{align*}
4x - 3y + z &= -9 \\
-3x + 2y - z &= 6 \\
-x + 3y + 2z &= 9
\end{align*}
\]
Eliminate \(z\) from equations 1 and 2 by adding.
\[
\begin{align*}
4x - 3y + z &= -9 \\
-3x + 2y - z &= 6 \\
x - y &= -3
\end{align*}
\]
Multiply equation 1 by \(-2\) and eliminate \(z\) from equations 1 and 3 by adding.
\[
\begin{align*}
-8x + 6y - 2z &= 18 \\
-x + 3y + 2z &= 9 \\
-9x + 9y &= 27
\end{align*}
\]
Multiply equation 3 by 9 and eliminate \(x\) from equations 4 and 5 by adding.
\[
\begin{align*}
x - y &= -3 \\
-9x + 9y &= 27
\end{align*}
\]
Multiply equation 4 by 9 and eliminate \(x\) from equations 4 and 5 by adding.
\[
\begin{align*}
x - 9y &= -27 \\
-9x + 9y &= 27
\end{align*}
\]
consistent; infinite number of solutions

13. \[
\begin{align*}
3x + 3y + 3z &= 4 \\
2x - y - 5z &= 2 \\
5x + 2y - 2z &= 8
\end{align*}
\]
Multiply equation 2 by 3 and eliminate \(y\) from equations 1 and 2 by adding.
\[
\begin{align*}
3x + 3y + 3z &= 4 \\
6x - 3y - 15z &= 6 \\
9x - 12z &= 10
\end{align*}
\]
Multiply equation 2 by 2 and eliminate \(y\) from equations 2 and 3 by adding.
\[
\begin{align*}
4x - 2y - 10z &= 4 \\
5x + 2y - 2z &= 8 \\
9x - 12z &= 12
\end{align*}
\]
\[
\begin{align*}
9x - 12z &= 10 \\
9x - 12z &= 12
\end{align*}
\]
inconsistent; no solution

14. \[
\begin{align*}
2x - 2y - 2z &= -16 \\
2x + y + 4z &= -6
\end{align*}
\]
Eliminate \(y\) from equations 1 and 2 by adding.
\[
\begin{align*}
x + y + z &= 8 \\
2x + y + 4z &= -6 \\
2x + 5z &= 2
\end{align*}
\]
Multiply equation 3 by \(-2\) and eliminate \(y\) from equations 2 and 3 by adding.
\[
\begin{align*}
x + 5z &= 2 \\
-2x - 10z &= -4
\end{align*}
\]
Multiply equation 4 by 2 and eliminate \(x\) from equations 4 and 5 by adding.
\[
\begin{align*}
x + 5z &= 2 \\
-2x - 10z &= -4
\end{align*}
\]
consistent; infinite number of solutions
15. \[
\begin{align*}
\angle A &= 2\angle B + 2\angle C && (1) \\
\angle B &= 3\angle C && (2) \\
\angle A + \angle B + \angle C &= 180 && (3)
\end{align*}
\]

**Step 1** Solve for \(\angle B\). The second equation is already solved for \(\angle B\): \(\angle B = 3\angle C\). 

**Step 2** Substitute for \(\angle B\) in equations (1) and (3). 
\[
\begin{align*}
\angle A &= 2(3\angle C) + 2\angle C && (1) \\
\angle A + 3\angle C + \angle C &= 180 && (3)
\end{align*}
\]

**Step 3** Solve equation (3) for \(\angle A\). 
\[
\begin{align*}
\angle A &= 3\angle C && (4) \\
\angle A + 4\angle C &= 180 && (5)
\end{align*}
\]

**Step 4** Substitute for \(\angle A\) in equation (4). 
\[
\begin{align*}
\angle A &= 8\angle C && (4) \\
(-4\angle C + 180) &= 8\angle C \\
180 &= 12\angle C \\
15 &= \angle C
\end{align*}
\]

**Step 5** Substitute for \(\angle C\) to solve for \(\angle B\) and then for \(\angle A\). 
\[
\begin{align*}
\angle B &= 3\angle C && (2) \\
\angle A &= 2\angle B \quad 2\angle C && (1) \\
\angle B &= 45 \\
\angle A &= 120, \angle B = 45^\circ, \angle C = 15^\circ
\end{align*}
\]

16. **Step 1** Let \(x\) be the number of 3-pt. baskets, \(y\) be the number of 2-pt. baskets, and \(z\) be the number of 1-pt. free throws. 
\[
\begin{align*}
y &= z + 2144 && (1) \\
z &= x + 1558 && (2) \\
3x + 2y + z &= 13,726 && (3)
\end{align*}
\]

**Step 2** Solve for \(x\) in equation (2). 
\[
\begin{align*}
z &= x + 1558 && (2) \\
x &= z - 1558
\end{align*}
\]

Equation (1) is already solved for \(y\): \(y = z + 2144\). 

**Step 3** Substitute for \(x\) and \(y\) in equation (3). 
\[
\begin{align*}
3(z - 1558) + 2(z + 2144) + z &= 13,726 \\
6z &= 14,112 \\
z &= 2352
\end{align*}
\]

**Step 4** Solve equation (1) for \(y\). 
\[
\begin{align*}
y &= z + 2144 && (1) \\
y &= (2353) + 2144 \\
y &= 4496
\end{align*}
\]

**Step 5** Substitute for \(z\) in equation (2) to solve for \(x\). 
\[
\begin{align*}
x &= z + 1558 && (2) \\
x &= 2352 + x + 1558 \\
x &= 794 \\
\end{align*}
\]

Dampier made 794 3-point baskets, 4496 2-point baskets, and 2352 free throws. 

17a. Possible answer: You find an infinite number of solutions. 

b. Possible answer: A single solution in 3 dimensions is a point with 3 coordinates, so you need 3 equations to identify the point. Each equation represents a piece of information, so you must have 3. 

18. Possible answer: The type of solution will depend on the third equation. The third equation could represent a plane that contains the line, intersects the line in a single point, or does not intersect the line at all. There could be 1, 0, or infinitely many solutions. 
\[
\begin{align*}
x + y + z &= 53 && (1) \\
3x - 2y + z &= 69 && (2) \\
-x + 2y - z &= -59 && (3)
\end{align*}
\]

**Step 1** Eliminate one variable. Multiply equation (1) by \(-1\), and eliminate \(z\) from equations (1) and (2) by adding. 
\[
\begin{align*}
-x - y - z &= -53 \quad (1) \\
3x - 2y + z &= 69 \quad (2)
\end{align*}
\]

\[
\begin{align*}
2x - 3y &= 16 \quad (3)
\end{align*}
\]

Eliminate \(z\) from equations (1) and (2) by adding. 
\[
\begin{align*}
x + y + z &= 53 \quad (1) \\
-x + 2y - z &= -59 \quad (2)
\end{align*}
\]

\[
\begin{align*}
3y &= -6 \\
y &= -2 \quad (5)
\end{align*}
\]

\[
\begin{align*}
3x - 3y &= 16 \quad (4) \\
y &= -2 \quad (5)
\end{align*}
\]

**Step 2** Use one of the equations in the 2-by-2 system to solve for \(x\). 
\[
\begin{align*}
2x - 3y &= 16 \quad (4) \\
2x - 3(-2) &= 16 \\
2x &= 10 \\
x &= 5
\end{align*}
\]

**Step 3** Substitute for \(x\) and \(y\) in one of the original equations to solve for \(z\). 
\[
\begin{align*}
x + y + z &= 53 \quad (1) \\
(5) + (-2) + z &= 53 \\
z &= 50
\end{align*}
\]

The solution is \((5,-2, 50)\). 

b. 50 ft 

c. \((5, -2, 0)\) 

**TEST PREP**

20. B: \(2(0) + (2) + 3(-1) = -1\) ✓ 
\[
4(0) + 2(2) + 3(-1) = 1 \checkmark
\]
\[
(1) - 2(2) + 4(-1) = -6 \checkmark
\]

21. J; 

**Step 1** Let \(a\) represent Ann’s age, \(b\) represent Betty’s age, and \(c\) represent Charlotte’s age. 
\[
\begin{align*}
a &= 2b && (1) \\
b &= c - 12 && (2) \\
c + 5 &= 2(b + 5) && (3)
\end{align*}
\]

**Step 2** Solve for \(c\) in equation (2). 
\[
\begin{align*}
b &= c - 12 && (2) \quad \Rightarrow c = b + 12
\end{align*}
\]

**Step 3** Substitute for \(c\) in equation (3). 
\[
\begin{align*}
(b + 12) + 5 &= 2b + 10 \quad (3) \\
7 &= b
\end{align*}
\]

**Step 4** Substitute for \(b\) to solve for \(a\) and then for \(c\). 
\[
\begin{align*}
a &= 2b \quad (1) \\
c + 5 &= 2(b + 5) \quad (3) \\
a &= 2(7) \\
c + 5 &= 2(7 + 5) \\
a &= 14 \\
c &= 19
\end{align*}
\]
22. \( x + 4y = 6 \)  \\
\( 2x + 3z = 12 \)  \\
\( 4y + z = 10 \)

From ① \( x = 6 - 4y \)

From ③ \( z = 10 - 4y \)

Substitute these into ②.

\[
\begin{align*}
2(6 - 4y) + 3(10 - 4y) &= 12 \\
12 - 8y + 30 - 12y &= 12 \\
-20y &= -30 \\
y &= 3/2
\end{align*}
\]

Substitute \( y = \frac{3}{2} \) into ①.

\[
\begin{align*}
x + 4\left(\frac{3}{2}\right) &= 6 \\
x + 6 &= 6 \\
x &= 0
\end{align*}
\]

**CHALLENGE AND EXTEND**

\[
\begin{align*}
w + 2x + 2y + z &= -2  \\
w + 3x - 2y - z &= -6 \quad  \\
-2w - x + 3y + 3z &= 6 \quad  \\
w + 4x + y - 2z &= -14
\end{align*}
\]

**Step 1** Eliminate \( z \) from equations ① and ② by adding.

\[
\begin{align*}
w + 2x + 2y + z &= -2  \\
w + 3x - 2y - z &= -6
\end{align*}
\]

Multiply equation ② by 2 and equation ④ by -1.

Eliminate \( z \) by adding.

\[
\begin{align*}
w + 6x - 4y - 2z &= -12  \\
-w - 4x - y + 2z &= 14
\end{align*}
\]

Multiply equation ④ by 3 and eliminate \( z \) from equations ② and ④ by adding.

\[
\begin{align*}
w + 2x &= -8  \\
w + 8x - 3y &= -12
\end{align*}
\]

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation ⑦ by 5 and equation ⑥ by -3. Eliminate \( y \) by adding.

\[
\begin{align*}
w + 40x - 15y &= -60  \\
-3w - 6x + 15y &= -6
\end{align*}
\]

Multiply equation ⑥ by -1 and eliminate \( w \) from equations ⑤ and ⑥ by adding.

\[
\begin{align*}
w + 5x &= -8  \\
-2w - 34x &= 66
\end{align*}
\]

\[
\begin{align*}
w + 5x &= -8  \\
-29x &= 58  \\
x &= -2
\end{align*}
\]
28. \(4x - 3y = -6\)
   \[-3y = -4x - 6\]
   \(y = \frac{4}{3}x + 2\)

29. \(3y - 2x = -12\)
   \(3y = 2x - 12\)
   \(y = \frac{2}{3}x - 4\)

30. \(2x + 5y = 15\)
   \(5y = -2x + 15\)
   \(y = -\frac{2}{5}x + 3\)

**READY TO GO ON? PAGE 229**

1–3.

4.

5.

6.

7. Let \(x\) be the number of teeth cleanings, \(y\) be the number of one-surface fillings, and \(z\) be the number of initial visits.
\[50x + 100y + 75z = 3500\]

8. | Day       | Cleaning | Filling | Initial Visit |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>20</td>
<td>22</td>
<td>4</td>
</tr>
<tr>
<td>Tuesday</td>
<td>25</td>
<td>18</td>
<td>6</td>
</tr>
<tr>
<td>Wednesday</td>
<td>16</td>
<td>24</td>
<td>4</td>
</tr>
<tr>
<td>Thursday</td>
<td>25</td>
<td>21</td>
<td>2</td>
</tr>
</tbody>
</table>

9. \(\begin{cases} x + y + z = 0 & \text{①} \\
                   2x + y - 2z = -8 & \text{②} \\
                   -x + 4z = 10 & \text{③} \end{cases}\)

   **Step 1** Eliminate one variable. Eliminate \(x\) from ① and ③ by adding.
   \[x + y + z = 0 \quad \text{①} \]
   \[-x + 4z = 10 \quad \text{③} \]
   \[y + 5z = 10 \quad \text{④} \]
   Multiply equation ① by 2 and equation ② by \(-1\). Eliminate \(x\) by adding.
   \[2x + 2y + 2z = 0 \quad \text{①} \]
   \[-2x - y + 2z = 8 \quad \text{②} \]
   \[y + 4z = 8 \quad \text{⑤} \]
   \[\begin{align*}
   y + 5z &= 10 \quad \text{④} \\
   y + 4z &= 8 \quad \text{⑤} \\
   \hline
   z &= 2
   \end{align*}\]

   **Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation ① by \(-1\), and eliminate \(y\) from equations ④ and ⑤ by adding.
   \[y + 5z = 10 \quad \text{④} \]
   \[-y - 4z = -8 \quad \text{⑤} \]
   \[z = 2
   \]

   **Step 3** Use one of the equations in your 2-by-2 system to solve for \(y\).
   \[y + 5z = 10 \quad \text{④} \]
   \[y + 5(2) = 10 \]
   \[y = 0
   \]

   **Step 4** Substitute for \(y\) and \(z\) in one of the original equations to solve for \(x\).
   \(x + y + z = 0 \quad \text{①} \)
   \(x + (0) + (2) = 0 \)
   \[x = -2
   \]
   The solution is \((-2, 0, 2)\).

10. \(\begin{cases} x + 2y + z = 7 & \text{①} \\
                   x - 2y - 4z = -3 & \text{②} \end{cases}\)

   **Step 1** Eliminate one variable. Eliminate \(y\) from equations ① and ② by adding.
   \[x + 2y + z = 7 \quad \text{①} \]
   \[x - 2y - 4z = 0 \quad \text{②} \]
   \[2x - 3z = 7 \quad \text{④} \]
   Multiply equation ③ by 2 and eliminate \(y\) from equations ① and ③ by adding.
   \[x + 2y + z = 7 \quad \text{①} \]
   \[4x - 2y - 8z = -6 \quad \text{③} \]
   \[5x + 9z = 1 \quad \text{⑤} \]
   \[\begin{align*}
   2x - 3z &= 7 \quad \text{④} \\
   5x + 9z &= 1 \quad \text{⑤} \\
   \hline
   x &= 2
   \end{align*}\]

   **Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation ④ by 3, and eliminate \(z\) from equations ④ and ⑤ by adding.
   \[2x - 9z = 21 \quad \text{④} \]
   \[5x + 9z = 1 \quad \text{⑤} \]
   \[11x = 22 \]
   \[x = 2
   \]

   **Step 3** Use one of the equations in your 2-by-2 system to solve for \(z\).
   \[2x - 3z = 7 \quad \text{④} \]
   \[2(2) - 3z = 7 \]
   \[-3z = 3 \]
   \[z = -1\]
11.

Step 1 Eliminate one variable. Eliminate $y$ from equations ① and ② by adding.

\[
\begin{align*}
2x + 2y + z &= 10 \quad ① \\
x - 2y + 3z &= 13 \quad ②
\end{align*}
\]

Multiply equation ② by $-1$ and equation ③ by 2.

\[
\begin{align*}
x + 4z &= 23 \quad ③ \\
x + 3z &= 11 \quad ④
\end{align*}
\]

Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation ⑤ by $-3$, and eliminate $x$ from equations ③ and ④ by adding.

\[
\begin{align*}
3x + 4z &= 23 \quad ③ \\
-3x - 9z &= -33 \quad ⑤
\end{align*}
\]

Then solve for $z$ by adding.

\[
\begin{align*}
0z &= -10 \\
z &= 2
\end{align*}
\]

Step 3 Use one of the equations in your 2-by-2 system to solve for $x$.

\[
\begin{align*}
x + 3z &= 11 \quad ⑥ \\
x + 3(2) &= 11 \\
x &= 5
\end{align*}
\]

Step 4 Substitute for $x$ and $z$ in one of the original equations to solve for $y$.

\[
\begin{align*}
x - y + 3z &= 12 \quad ③ \\
(5) - y + 3(2) &= 12 \\
y &= -1
\end{align*}
\]

The solution is $(5, -1, 2)$.

12. Let $x$ be the price of Type A, $y$ be the price of Type B, and $z$ be the price of Type C.

\[
\begin{align*}
6x + 8y + 14z &= 65 \\
10x + 10y + 15z &= 80 \\
12x + 6y + 9z &= 60
\end{align*}
\]

13. The equations can be simplified to

\[
\begin{align*}
6x + 8y + 14z &= 65 \quad ① \\
2x + 2y + 3z &= 16 \quad ② \\
4x + 2y + 3z &= 20 \quad ③
\end{align*}
\]

Step 1 Eliminate one variable. Multiply equation ② by $-4$, and eliminate $y$ from equations ① and ② by adding.

\[
\begin{align*}
6x + 8y + 14z &= 65 \quad ① \\
-8x - 8y - 12z &= -64 \quad ③ \\
-2x + 2z &= 1 \quad ④
\end{align*}
\]

Multiply equation ③ by $-4$ and eliminate $y$ from equations ① and ④ by adding.

\[
\begin{align*}
6x + 8y + 14z &= 65 \quad ① \\
-16x - 8y - 12z &= -80 \quad ③ \\
-10x + 2z &= -15 \quad ④
\end{align*}
\]

Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation ④ by $-1$, and eliminate $z$ from equations ⑥ and ⑦ by adding.

\[
\begin{align*}
-2x + 2z &= 1 \quad ④ \\
10x - 2z &= 15 \quad ⑤ \\
8x &= 16 \\
x &= 2
\end{align*}
\]

Step 3 Use one of the equations in your 2-by-2 system to solve for $z$.

\[
\begin{align*}
-10x + 2z &= -15 \quad ⑤ \\
-10(2) + 2z &= -15 \\
2z &= 5 \\
z &= 2.5
\end{align*}
\]

Step 4 Substitute for $x$ and $z$ in one of the original equations to solve for $y$.

\[
\begin{align*}
2x + 2y + 3z &= 16 \quad ⑥ \\
2(2) + 2y + 3(2.5) &= 16 \\
2y &= 4.5 \\
y &= 2.25
\end{align*}
\]

Type A costs $2, Type B costs $2.25, and Type C costs $2.50.

14. Let $x$ be the price of Type A, $y$ be the price of Type B, and $z$ be the price of Type C.

\[
\begin{align*}
2x - 2y + 3z &= -2 \quad ① \\
4y + 6z &= 1 \quad ② \\
4x - 4y + 6z &= 5 \quad ③
\end{align*}
\]

Multiply equation ① by $-2$ and eliminate $y$ from equations ① and ② by adding.

\[
\begin{align*}
-4x + 4y - 6z &= 4 \quad ① \\
4x - 4y + 6z &= 5 \quad ③ \\
0 &= 9
\end{align*}
\]

inconsistent; no solution
15. \(4x - y + z = 5\)  
\(3x + y + 2z = 5\)  
\(2x - 5z = -8\)

Multiply equation 1 by 5 and eliminate z from equations 1 and 2 by adding.

\[
20x - 5y + 5z = 25 \quad 1
\]
\[
2x - 5z = -8 \quad 3
\]
\[
22x - 5y = 17 \quad 4
\]

Multiply equation 2 by 5 and equation 3 by 2.

Eliminate z by adding.

\[
15x + 5y + 10z = 25 \quad 2
\]
\[
4x - 10z = -16 \quad 3
\]
\[
19x + 5y = 9 \quad 5
\]

Eliminate y from equations 4 and 5 by adding.

\[
22x - 5y = 17 \quad 4
\]
\[
19x + 5y = 9 \quad 5
\]

41x = 26
\[
x = \frac{26}{41}
\]
consistent; one solution

16. \(x - 3y + z = -8\)  
\(-x + 3y - z = 8\)

Multiply equation 3 by 2 and eliminate x from equations 1 and 3.

\[
2x + y - 3z = 4 \quad 1
\]
\[
-2x + 6y - 2z = 16 \quad 3
\]
\[
7y - 5z = 20 \quad 4
\]
\[
2x + y - 3z = 4
\]
\[
-2x + 6y - 2z = 16
\]
\[
7y - 5z = 20
\]
\[
7y - 5z = 20 \quad 4
\]
\[
7y - 5z = 20 \quad 5
\]
dependent; infinitely many solutions

**STUDY GUIDE: REVIEW, PAGES 232–235**

**VOCABULARY**

1. dependent  
2. elimination  
3. system of linear inequalities, feasible region  
4. three-dimensional coordinate system, ordered triple  
5. consistent

**LESSON 3-1**

6. \[\begin{align*}
  y &= 2x \\
  3x - y &= 5 \\
  y &= 2x
\end{align*}\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  0 & 0 \\
  5 & 10
\end{array}
\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  0 & -5 \\
  5 & 10
\end{array}
\]

The solution to the system is \((5, 10)\).

7. \[
\begin{align*}
  x + y &= 6 \\
  x - y &= 2 \\
  y &= -x + 6
\end{align*}\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  -4 & 10 \\
  0 & 6 \\
  4 & 2
\end{array}
\]

The solution to the system is \((4, 2)\).

8. \[
\begin{align*}
  x - 6y &= 2 \\
  2x - 5y &= -3 \\
  y &= \frac{1}{6}x - \frac{1}{3}
\end{align*}\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  -4 & -1 \\
  0 & \frac{1}{3} \\
  \frac{2}{5} & \frac{3}{5}
\end{array}
\]

The solution to the system is \((-4, -1)\).

9. \[
\begin{align*}
  x - 3y &= 6 \\
  3x - y &= 2 \\
  y &= \frac{1}{3}x - 2
\end{align*}\]

\[
\begin{array}{c|c}
  x & y \\
  \hline
  -3 & -3 \\
  0 & -2
\end{array}
\]

The solution to the system is \((0, -2)\).

10. \[
\begin{align*}
  y &= x - 7 \\
  x + 9y &= 16 \\
  9y &= -x + 16 \\
  y &= -\frac{1}{9}x + \frac{16}{9}
\end{align*}\]

independent; no solution

11. \[
\begin{align*}
  \frac{1}{2}x + 2y &= 3 \\
  2y &= -\frac{1}{2}x + 3 \\
  y &= -\frac{1}{4}x + \frac{3}{2}
\end{align*}\]

The solution to the system is \((3, 4)\).

12. \[
\begin{align*}
  5x - 10y &= 8 \\
  10y &= 5x - 8 \\
  y &= \frac{1}{2}x - \frac{4}{5}
\end{align*}\]

inconsistent; no solution

13. \[
\begin{align*}
  4x - 3y &= 21 \\
  3y &= 4x - 21 \\
  y &= \frac{4}{3}x - 7
\end{align*}\]

inconsistent; no solution

116 Holt McDougal Algebra 2
14. **Step 1** Write an equation for costs for each locksmith for a house call and re-keying locks. Let \( x \) represent Locksmith A and \( y \) represent Locksmith B.

- **Locksmith A:** \( y = 15x + 25 \)
- **Locksmith B:** \( y = 20x + 10 \)

**Step 2** Solve the system by using a table of values.

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & \text{x} & \text{y} \\
\hline
1 & 40 & 1 & 30 \\
2 & 55 & 2 & 50 \\
3 & 70 & 3 & 70 \\
\end{array}
\]

The total costs will be the same for 3 locks.

**LEsson 3-2**

15. \[
\begin{align*}
\text{y} &= 3x \\
2x - 3y &= -7 \\
\end{align*}
\]

**Step 1** Solve one equation for one variable.

The first equation is already solved for \( y \): \( y = 3x \).

**Step 2** Substitute the expression into the other equation.

\[
\begin{align*}
2x + 3y &= -7 \\
2x - 3(3x) &= -7 \\
2x - 9x &= -7 \\
-7x &= -7 \\
x &= 1 \\
\end{align*}
\]

**Step 3** Substitute the \( x \)-value into one of the original equations to solve for \( y \).

\[
\begin{align*}
y &= 3x \\
y &= 3(1) \\
y &= 3 \\
\end{align*}
\]

The solution is the ordered pair \((1, 3)\).

16. \[
\begin{align*}
\text{y} &= x - 1 \\
4x - y &= 19 \\
\end{align*}
\]

**Step 1** Solve one equation for one variable.

The first equation is already solved for \( y \): \( y = x - 1 \).

**Step 2** Substitute the expression into the other equation.

\[
\begin{align*}
4x - y &= 19 \\
4x - (x - 1) &= 19 \\
3x + 1 &= 19 \\
x &= 6 \\
\end{align*}
\]

**Step 3** Substitute the \( x \)-value into one of the original equations to solve for \( y \).

\[
\begin{align*}
y &= x - 1 \\
y &= (6) - 1 \\
y &= 5 \\
\end{align*}
\]

The solution is the ordered pair \((6, 5)\).

17. \[
\begin{align*}
4x - y &= 0 \\
6x - 3y &= 12 \\
\end{align*}
\]

**Step 1** Solve one equation for one variable.

\[
\begin{align*}
4x - y &= 0 \\
y &= 4x \\
\end{align*}
\]

**Step 2** Substitute the expression into the other equation.

\[
\begin{align*}
6x - 3y &= 12 \\
6x - 3(4x) &= 12 \\
6x - 12x &= 12 \\
-6x &= 12 \\
x &= -2 \\
\end{align*}
\]

**Step 3** Substitute the \( x \)-value into one of the original equations to solve for \( y \).

\[
\begin{align*}
4(x) + 5y &= 41 \\
16 + 5y &= 41 \\
5y &= 25 \\
y &= 5 \\
\end{align*}
\]

The solution is the ordered pair \((4, 5)\).

18. \[
\begin{align*}
5x &= -10y \\
8x - 4y &= 40 \\
\end{align*}
\]

**Step 1** Solve one equation for one variable.

\[
\begin{align*}
5x &= -10y \\
x &= -2y \\
\end{align*}
\]

**Step 2** Substitute the expression into the other equation.

\[
\begin{align*}
8x - 4y &= 40 \\
8(-2y) - 4y &= 40 \\
-16y + 4y &= 40 \\
-20y &= 40 \\
y &= -2 \\
\end{align*}
\]

**Step 3** Substitute the \( y \)-value into one of the original equations to solve for \( x \).

\[
\begin{align*}
5x &= -10y \\
5x &= -10(-2) \\
x &= 4 \\
\end{align*}
\]

The solution is the ordered pair \((4, -2)\).
21. \[ \begin{align*} 2x - y &= 8 \\ x + 2y &= 9 \end{align*} \]

**Step 1** To eliminate \( x \), multiply both sides of the second equation by \(-2\).

\(-2(x + 2y) = -2(9) \)

Add to eliminate \( x \).

\((-2x - 4y) = -18 \leftarrow \text{original equation} \)

\(-5y = -10 \)

\( y = 2 \)

**Step 2** Substitute the \( y \)-value into one of the original equations to solve for \( x \).

\( x + 2(2) = 9 \)

\( x = 5 \)

The solution is the ordered pair \((5, 2)\).

---

22. \[ \begin{align*} 9x - 5y &= 13 \\ 4x - 6y &= 2 \end{align*} \]

**Step 1** To eliminate \( y \), multiply both sides of the first equation by \( 6 \) and both sides of the second equation by \(-5\).

\( 6(9x - 5y) = 6(13) \quad \text{(1)} \)

\(-5(4x - 6y) = -5(2) \quad \text{(2)} \)

Add to eliminate \( y \).

\( 54x - 30y = 78 \quad \text{(1)} \)

\(-20x + 30y = -10 \quad \text{(2)} \)

\(-34x = -68 \)

\( x = 2 \)

**Step 2** Substitute the \( x \)-value into one of the original equations to solve for \( y \).

\( 4(2) - 6y = 2 \)

\(-6y = -6 \)

\( y = 1 \)

The solution is the ordered pair \((2, 1)\).

---

23. Let \( x \) represent the amount of pine needles.

Let \( y \) represent the amount of lavender.

\[ \begin{align*} 1.5x + 4y &= 200 \\ x + y &= 80 \end{align*} \]

\( x = 80 - y \)

Substitute \( x \) into \( \text{(1)} \).

\( 1.5(80 - y) + 4y = 200 \)

\( 120 - 1.5y + 4y = 200 \)

\( 2.5y = 80 \)

\( y = 32 \)

\( x + (32) = 80 \)

\( x = 48 \)

The potpourri will contain 48 oz pine needles and 32 oz lavender.

---

24. \[ \begin{align*} 2x - y &= 8 \\ x + 2y &= 9 \end{align*} \]

25. \[ \begin{align*} 9x - 5y &= 13 \\ 4x - 6y &= 2 \end{align*} \]

26. right triangle

27. trapezoid

---

28. \[ \begin{align*} x + y &\leq 120 \\ 8x + 11.5y &< 1200 \end{align*} \]

---

29. \[ \begin{align*} y &\leq 8 \\ y &\geq 4 \end{align*} \]

30. \[ \begin{align*} x &\leq 6 \\ y &\geq 3 \end{align*} \]

31. \[ \begin{align*} x &\geq 0 \\ y &\geq 0 \\ y &\leq 3x + 1 \\ y &\leq -\frac{3}{4} + 6 \end{align*} \]

Maximize the objective function \( P = 6x + 10y \).

\[
\begin{array}{c|c|c}
\hline
x & y & P = 6x + 10y \\
\hline
0 & 0 & 6(0) + 10(0) = 0 \\
0 & 1 & 6(0) + 10(1) = 10 \\
\frac{4}{3} & 5 & 6\frac{4}{3} + 10(5) = 58 \\
8 & 0 & 6(8) + 10(0) = 48 \\
\hline
\end{array}
\]

The maximum is 58.

32. \[ \begin{align*} x &< 3 \\ y &\geq 0 \\ y &< 2x + 1 \\ y &\leq -x + 4 \end{align*} \]

Minimize the objective function \( P = 14x + 9y \).

\[
\begin{array}{c|c|c}
\hline
x & y & P = 14x + 9y \\
\hline
0 & \frac{-1}{2} & 14(0) + 9\left(-\frac{1}{2}\right) = -4.5 \\
1 & 3 & 14(1) + 9(3) = 41 \\
3 & 1 & 14(3) + 9(1) = 51 \\
3 & 0 & 14(3) + 9(0) = 42 \\
\hline
\end{array}
\]

The minimum is \(-4.5\).
35. \[
x \geq 0 \\
y \geq 0 \\
6x + 4y \leq 720 \\
x \geq 2y
\]

36. \[
P = 8x + 9y \\
x \geq 0 \\
y \geq 0 \\
6x + 4y \leq 720 \\
x \geq 2y
\]

Maximize the objective function \( P = 8x + 9y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( P = 8x + 9y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>8(0) + 9(0) = 0</td>
</tr>
<tr>
<td>90</td>
<td>45</td>
<td>8(90) + 9(45) = 1125</td>
</tr>
<tr>
<td>120</td>
<td>0</td>
<td>8(120) + 9(0) = 960</td>
</tr>
</tbody>
</table>

The maximum profit for 1 day is $1125.00.

37.

\[
\begin{align*}
10 & \leq x \leq 25 \\
5 & \leq y \leq 10
\end{align*}
\]

Let \( x \) be the number of cell phones with contracts, and \( y \) be the number of cell phones without contracts.

Maximize the objective function \( P = 35x + 5y \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( P = 35x + 5y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>35(10) + 5(5) = 375</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>35(10) + 5(10) = 400</td>
</tr>
<tr>
<td>20</td>
<td>10</td>
<td>35(20) + 5(10) = 750</td>
</tr>
<tr>
<td>25</td>
<td>5</td>
<td>35(25) + 5(5) = 900</td>
</tr>
</tbody>
</table>

25 phones with contracts and 5 phones with no contracts will maximize profits.

38.

\[
\begin{align*}
10 & \leq x \leq 25 \\
5 & \leq y \leq 10
\end{align*}
\]

Let \( d \) be the number of drinks, \( p \) be the number of pizzas, and \( c \) be the number of quarts of ice cream.

\[
2d + 9p + 4c = 35
\]

39–42.

43.

44.

45.

46.

47. Let \( d \) be the number of drinks, \( p \) be the number of pizzas, and \( c \) be the number of quarts of ice cream.

\[
2d + 9p + 4c = 35
\]

**LESSON 3-6**

48. \[
\begin{align*}
x + 3y + 2z &= 13 \quad \text{①} \\
2x + 2y - z &= 3 \quad \text{②} \\
x - 2y + 3z &= 6 \quad \text{③}
\end{align*}
\]

**Step 1** Eliminate one variable. Multiply equation ③ by \(-1\), and eliminate \( x \) from equations ① and ③ by adding.

\[
\begin{align*}
x + 3y + 2z &= 13 \quad \text{①} \\
-x + 2y - 3z &= -6 \quad \text{③}
\end{align*}
\]

Multiply equation ① by 2 and equation ② by \(-1\).

**Step 2** Eliminate another variable. Then solve for the remaining variable. Multiply equation ④ by 5, and eliminate \( z \) from equations ④ and ⑤ by adding.

\[
\begin{align*}
5y - z &= 7 \quad \text{④} \\
4y + 5z &= 23 \quad \text{⑤}
\end{align*}
\]

**Step 3** Use one of the equations in your 2-by-2 system to solve for \( z \).

\[
\begin{align*}
5y - z &= 7 \quad \text{④} \\
5(2) - z &= 7 \\
z &= 3
\end{align*}
\]

**Step 4** Substitute for \( y \) and \( z \) in one of the original equations to solve for \( x \).

\[
\begin{align*}
x - 2y + 3z &= 6 \quad \text{③} \\
x - 2(2) + 3(3) &= 6 \\
x &= 1
\end{align*}
\]

The solution is \((1, 2, 3)\).
49. \[
\begin{align*}
\begin{cases}
x + y + z &= 2 \quad & (1) \\
3x + 2y - z &= -1 \quad & (2) \\
3x - y &= 4 \quad & (3)
\end{cases}
\end{align*}
\]
Step 1 Eliminate one variable. Multiply equation (3) by \(-3\) and eliminate \(x\) from equations (1) and (2) by adding:
\[
\begin{align*}
-3x - 3y - 3z &= -6 \quad & (1) \\
3x + 2y - z &= -1 \quad & (2) \\
\hline
-5y - 4z &= -7 \quad & (3)
\end{align*}
\]
Multiply equation (3) by \(-1\) and eliminate \(x\) from equations (2) and (3) by adding:
\[
\begin{align*}
-3x - 2y + z &= 1 \quad & (2) \\
3x - y &= 4 \quad & (3) \\
\hline
-3y + z &= 5 \quad & (5)
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
-5y - 4z &= -7 \quad & (3) \\
-3y + z &= 5 \quad & (5)
\end{cases}
\end{align*}
\]
Step 2 Eliminate another variable. Then solve for the remaining variable. Multiply equation (5) by 4 and eliminate \(z\) from equations (4) and (5) by adding:
\[
\begin{align*}
-12y + 4z &= 20 \quad & (5) \\
y - 4z &= -7 \quad & (4) \\
\hline
-13y &= 13 \\
y &= -1
\end{align*}
\]
Step 3 Use one of the equations in your 2-by-2 system to solve for \(z\).
\[
\begin{align*}
-y - 4z &= -7 \quad & (4) \\
-( -1) - 4z &= -7 \\
-4z &= -8 \\
z &= 2
\end{align*}
\]
Step 4 Substitute for \(y\) and \(z\) in one of the original equations to solve for \(x\).
\[
\begin{align*}
x + y + z &= 2 \quad & (1) \\
x + ( -1) + (2) &= 2 \\
x &= 1
\end{align*}
\]
The solution is \((1, -1, 2)\).

50. \[
\begin{align*}
\begin{cases}
x + y + z &= -2 \quad & (1) \\
-x + 2y - 5z &= 4 \quad & (2) \\
3x + 3y + 3z &= 5 \quad & (3)
\end{cases}
\end{align*}
\]
Eliminate \(x\) from equations (1) and (2) by adding.
\[
\begin{align*}
x + y + z &= -2 \quad & (1) \\
-x + 2y - 5z &= 4 \quad & (2) \\
\hline
3y - 4z &= 2 \quad & (4)
\end{align*}
\]
Multiply equation (2) by 3, and eliminate \(x\) from equations (2) and (3).
\[
\begin{align*}
-3x + 6y - 15z &= 12 \quad & (2) \\
3x + 3y + 3z &= 5 \quad & (3) \\
\hline
9y - 12z &= 17 \quad & (5)
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
3y - 4z &= 2 \quad & (4) \\
9y - 12z &= 17 \quad & (5)
\end{cases}
\end{align*}
\]
Multiply equation (4) by \(-3\), and eliminate \(y\) from equations (4) and (5) by adding.
\[
\begin{align*}
-9y + 12z &= -6 \quad & (4) \\
9y - 12z &= 17 \quad & (5) \\
\hline
0 &= 11
\end{align*}
\]
Inconsistent; no solution

51. \[
\begin{align*}
\begin{cases}
-x - y + 2z &= -3 \quad & (1) \\
4x + 4y - 8z &= 12 \quad & (2) \\
2x + y - 3z &= -2 \quad & (3)
\end{cases}
\end{align*}
\]
Multiply equation (1) by 2 and eliminate \(x\) from equations (1) and (3) by adding.
\[
\begin{align*}
-2x - 2y + 4z &= -6 \quad & (1) \\
2x + y - 3z &= -2 \quad & (3) \\
\hline
y + z &= 8 \quad & (4)
\end{align*}
\]
Multiply equation (4) by \(-2\) and eliminate \(x\) from equations (2) and (4) by adding.
\[
\begin{align*}
4x + 4y - 8z &= 12 \quad & (2) \\
4x - 2y + 6z &= 4 \quad & (4) \\
\hline
2y - 2z &= 16 \quad & (5)
\end{align*}
\]
\[
\begin{align*}
\begin{cases}
y + z &= -8 \quad & (4) \\
2y - 2z &= 16 \quad & (5)
\end{cases}
\end{align*}
\]
Multiply equation (4) by 2 and eliminate \(z\) from equations (4) and (5).
\[
\begin{align*}
2y + 2z &= -16 \quad & (4) \\
2y - 2z &= 16 \quad & (5) \\
\hline
0 &= 0
\end{align*}
\]
dependent, infinitely many solutions

CHAPTER TEST, PAGE 236

1. \[
\begin{align*}
\begin{cases}
x - y &= -4 \quad & (1) \\
3x - 6y &= -12 \quad & (2)
\end{cases}
\end{align*}
\]
\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & 4 \\
-4 & 0 \\
\hline
\end{array}
\]
\[
y = \frac{1}{2}x + 2
\]
The solution to the system is \((-4, 0)\).

2. \[
\begin{align*}
\begin{cases}
y &= x - 1 \quad & (1) \\
x + 4y &= 6 \quad & (2)
\end{cases}
\end{align*}
\]
\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & -1 \\
2 & 1 \\
\hline
\end{array}
\]
\[
y = -\frac{1}{4}x + \frac{3}{2}
\]
The solution to the system is \((2, 1)\).

3. \[
\begin{align*}
\begin{cases}
x - y &= 3 \quad & (1) \\
2x + 3y &= 6 \quad & (2)
\end{cases}
\end{align*}
\]
\[
\begin{array}{|c|c|}
\hline
x & y \\
\hline
0 & -3 \\
3 & 0 \\
\hline
\end{array}
\]
\[
y = -\frac{2}{3}x + 2
\]
The solution to the system is \((3, 0)\).
Step 1: Equation for one variable.

Example:

\[ \begin{align*}
6y &= 9x \\
y &= \frac{3}{2}x
\end{align*} \]

8x + 4y = 20
4y = -8x + 20
\[ y = -2x + 5 \]

independent; one solution

6. \[ \begin{align*}
3x - 9y &= 21 \\
y &= 3x - 21
\end{align*} \]

\[ y = \frac{1}{3}x - \frac{7}{3} \]

6 = x - 3y
3y = x - 6
\[ y = \frac{1}{3}x - 2 \]
inconsistent; no solution

Step 2: Substitute the expression into the other equation.

\[ \begin{align*}
x + 5y &= 20 \\
x + 5(x - 2) &= 20 \\
6x - 10 &= 20 \\
6x &= 30 \\
x &= 5
\end{align*} \]

Step 3: Substitute the x-value into one of the original equations to solve for y.

\[ \begin{align*}
y &= x - 2 \\
y &= (5) - 2 \\
y &= 3
\end{align*} \]
The solution is the ordered pair \((5, 3)\).

8. \[ \begin{align*}
5x - y &= 33 \\
7x + y &= 51
\end{align*} \]

Step 1: Solve one equation for one variable.

\[ 5x - y = 33 \]
\[ y = 5x - 33 \]

Step 2: Substitute the expression into the other equation.

\[ 7x + y = 51 \]
\[ 7x + (5x - 33) = 51 \]
\[ 12x - 33 = 51 \]
\[ 12x = 84 \]
\[ x = 7 \]

Step 3: Substitute the x-value into one of the original equations to solve for y.

\[ 5x - y = 33 \]
\[ 5(7) - y = 33 \]
\[ y = 2 \]
The solution is the ordered pair \((7, 2)\).

Step 1: Solve one equation for one variable.

\[ x + y = 5 \]
\[ 2x + 5y = 16 \]

Step 2: Substitute the expression into the other equation.

\[ 2x + 5y = 16 \]
\[ 2x + 5(5x - 2) = 16 \]
\[ 10x + 9(3) = 16 \]
\[ 10x = 3 \]

Step 3: Substitute the x-value into one of the original equations to solve for y.

\[ x + y = 5 \]
\[ (3) + y = 5 \]
\[ y = 2 \]
The solution is the ordered pair \((3, 2)\).

9. \[ \begin{align*}
x + y &= 5 \\
2x + 5y &= 16
\end{align*} \]

Step 1: Solve one equation for one variable.

\[ x + y = 5 \]
\[ y = -x + 5 \]

Step 2: Substitute the expression into the other equation.

\[ 2x + 5y = 16 \]
\[ 2x + 5(-x + 5) = 16 \]
\[ -3x + 25 = 16 \]
\[ -3x = -9 \]
\[ x = 3 \]

Step 3: Substitute the x-value into one of the original equations to solve for y.

\[ x + y = 5 \]
\[ (3) + y = 5 \]
\[ y = 2 \]
The solution is the ordered pair \((3, 2)\).

10. \[ \begin{align*}
x + y &\leq 250 \\
0.09x + 0.24y &< 45
\end{align*} \]

Maximize the objective function \[ P = 5x + 9y. \]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( P = 5x + 9y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>5(0) + 9(0) = 0</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
<td>5(0) + 9(1) = 9</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>5(1) + 9(3) = 32</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>5(2) + 9(0) = 10</td>
</tr>
</tbody>
</table>

The minimum is 0.

14–16.

17. Let \( x \) be the number of repairs, \( y \) be the number of installations, and \( z \) be the number of emergencies. \[ 50x + 150y + 200z = 1000 \]

18. Mon.: \[ 50(2) + 150(2) + 200z = 1000 \]

Tues.: \[ 50x + 150(3) + 200(2) = 1000 \]

Wed.: \[ 50(1) + 150y + 200(4) = 1000 \]

Thurs.: \[ 50(4) + 150(4) + 200z = 1000 \]

<table>
<thead>
<tr>
<th>Day</th>
<th>Faucet</th>
<th>Sink</th>
<th>Emergency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monday</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Tuesday</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>Wednesday</td>
<td>1</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Thursday</td>
<td>4</td>
<td>4</td>
<td>1</td>
</tr>
</tbody>
</table>
19. \[
\begin{align*}
x - y + z &= -2 \quad \text{(1)} \\
4x - y + 2z &= -3 \quad \text{(2)} \\
2x - 3y + 2z &= -7 \quad \text{(3)}
\end{align*}
\]

**Step 1** Eliminate one variable. Multiply equation (3) by \(-1\) and eliminate \(z\) from equations (2) and (3) by adding.
\[-1(2x - 3y + 2z = -7) \quad \text{(3)} \]
Add to eliminate \(z\).
\[
\begin{align*}
4x - y + 2z &= -3 \quad \text{(2)} \\
-2x + 3y - 2z &= 7 \quad \text{(3)} \\
2x + 2y &= 4 \quad \text{(4)}
\end{align*}
\]
Multiply equation (1) by 2 and equation (2) by \(-1\) and eliminate \(z\) by adding.
\[
\begin{align*}
2x - 2y + 2z &= -4 \quad \text{(1)} \\
-4x + y - 2z &= 3 \quad \text{(2)} \\
-2x - y &= -1 \quad \text{(3)} \\
2x + 2y &= 4 \quad \text{(4)} \\
-2x - y &= -1 \quad \text{(5)}
\end{align*}
\]

**Step 2** Eliminate another variable. Then solve for the remaining variable.
\[
\begin{align*}
2x + 2y &= 4 \quad \text{(4)} \\
-2x - y &= -1 \quad \text{(5)}
\end{align*}
\]
\[
y = 3
\]

**Step 3** Use one of the equations in your 2-by-2 system to solve for \(x\).
\[
2x + 2y = 4 \quad \text{(4)}
\]
\[
2x + 2(3) = 4
\]
\[
2x = -2
\]
\[
x = -1
\]

**Step 4** Substitute for \(x\) and \(y\) in one of the original equations to solve for \(z\).
\[
x - y + z = -2 \quad \text{(1)}
\]
\[
(-1) - (3) + z = -2
\]
\[
z = 2
\]
The solution is \((-1, 3, 2)\).

20. \[
\begin{align*}
x + y + 2z &= 8 \quad \text{(2)} \\
6x - 2y - 2z &= 5 \quad \text{(3)}
\end{align*}
\]
Multiply (2) by \(-2\) and eliminate \(z\) from (1) and (3).
\[
\begin{align*}
-6x + 2y + 2z &= 2 \quad \text{(1)} \\
6x - 2y - 2z &= 5 \quad \text{(3)}
\end{align*}
\]
\[
0 \neq -7
\]
inconsistent.