ARE YOU READY? PAGE 403

1. C

2. E

3. D

4. F

5. A

6. $6^4 = (6)(6)(6)(6) = 1296$

7. $-5^4 = -(5)(5)(5)(5) = -625$

8. $(-1)^5 = (-1)(-1)(-1)(-1)(-1) = -1$

9. $\left(-\frac{2}{3}\right)^2 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = \frac{4}{9}$

10. $x^4 - 5x^2 - 6x - 8$
$= (3)^4 - 5(3)^2 - 6(3) - 8$
$= 81 - 5(9) - 18 - 8$
$= 81 - 45 - 18 - 8$
$= 10$

11. $2x^3 - 3x^2 - 29x - 30$
$= 2(-2)^3 - 3(-2)^2 - 29(-2) - 30$
$= 2(-8) - 3(4) + 58 - 30$
$= -16 - 12 + 58 - 30$
$= 0$

12. $2x^3 - x^2 - 8x + 4$
$= 2\left(\frac{1}{2}\right)^3 - \left(\frac{1}{2}\right)^2 - 8\left(\frac{1}{2}\right) + 4$
$= 2\left(\frac{1}{8}\right) - \frac{1}{4} - \frac{8}{2} + 4$
$= \frac{2}{8} - \frac{1}{4} - 4 + 4$
$= 0$

13. $3x^4 + 5x^3 + 6x^2 + 4x - 1$
$= 3(-1)^4 + 5(-1)^3 + 6(-1)^2 + 4(-1) - 1$
$= 3(1) + 5(-1) + 6(1) - 4 - 1$
$= 3 - 5 + 6 - 4 - 1$
$= -1$

14. $2x^3y \cdot 4x^2$
$= 8x^5y$

15. $-5a^2b \cdot ab^4$
$= -5a^3b^5$

16. $\frac{-7t^4}{3t^2}$
$= -\frac{7}{3}t^2$

17. $\frac{3p^3q^2r}{12pr^4}$
$= \frac{p^2q^2}{4r^3}$

18. $A = 6s^2$
$= 6(4)^2$
$= 96 \text{ cm}^2$

19. $A = 2(hw + \ell w + h\ell)$
$= 2[(3)(1.5) + (8)(1.5) + (3)(8)]$
$= 2(4.5 + 12 + 24)$
$= 2(40.5)$
$= 81 \text{ ft}^2$

20. $V = \ell wh$
$= \left(\frac{2}{3}\right)(6)(1)$
$= 4 \text{ in}^3$

21. $V = ws^2$
$= (5)(2)^2$
$= 5(4)$
$= 20 \text{ cm}^3$

6-1 POLYNOMIALS, PAGES 406–412

CHECK IT OUT!

1a. x^3
The degree is 3.

b. 7
The degree is 0.

c. $5x^3y^2$
The degree is 5.

d. a^6bc^2
The degree is 9.

2a. Standard form: $-2x^2 + 4x + 2$
Leading coefficient: -2
Degree: 2
Terms: 3
Name: quadratic trinomial

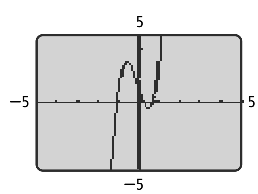
b. Standard form: $x^3 - 18x^2 + 2x - 5$
Leading coefficient: 1
Degree: 3
Terms: 4
Name: cubic polynomial with 4 terms

3a. $\left(-36x^2 + 6x - 11\right) + \left(6x^2 + 16x^3 - 5\right)$
$= \left(-36x^2 + 6x - 11\right) + \left(16x^3 + 6x^2 - 5\right)$
$= \left(16x^3\right) + \left(-36x^2 + 6x^2\right) + (6x) + (-11 - 5)$
$= 16x^3 - 30x^2 + 6x - 16$

b. $\left(5x^3 + 12 + 6x^2\right) - \left(15x^2 + 3x - 2\right)$
$= \left(5x^3 + 6x^2 + 12\right) + \left(-15x^2 - 3x + 2\right)$
$= \left(5x^3\right) + \left(6x^2 - 15x^2\right) + (-3x) + (12 + 2)$
$= 5x^3 - 9x^2 - 3x + 14$

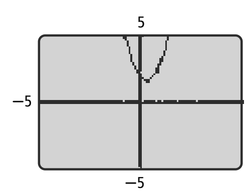
4. $f(4) = 0.000468(4)^4 - 0.016(4)^3 + 0.095(4)^2 + 0.806(4)$
$= 3.8398$
$f(17) = 0.000468(17)^4 - 0.016(17)^3 + 0.095(17)^2 + 0.806(17)$
$= 1.6368$
$f(4)$ represents the concentration of dye after 4 s.
$f(17)$ represents the concentration of dye after 17 s.

5a.



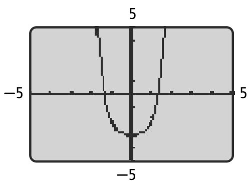
From left to right, the graph increases, decreases slightly, and then increases again. It crosses the x-axis 3 times, so there appear to be 3 real zeros.

b.



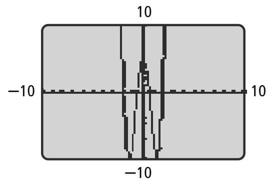
From right to left, the graph decreases and then increases. It does not cross the x-axis, so there are no real zeros.

c.



From left to right, the graph decreases and then increases. It crosses the *x*-axis twice, so there appear to be two real zeros.

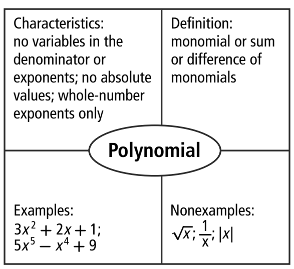
d.



From left to right, the graph alternately decreases and increases, changing direction three times. It crosses the *x*-axis 4 times, so there appear to be 4 real zeros.

THINK AND DISCUSS

1. Possible answer: yes; square roots of numbers are allowed.
2. Possible answer: 4, the sum of polynomials has the degree of the term with the greatest degree.
3. Possible answer: no; it depends on the degrees of the terms and their coefficients.
4.



EXERCISES

GUIDED PRACTICE

1. The leading coefficient of a polynomial is the number being multiplied by the variable with the greatest degree.

2. $-7x$
The degree is 1.

3. $4x^2y^3$
The degree is 5.

4. 13
The degree is 0.

5. m^3n^2p
The degree is 6.

6. Standard form: $x^3 + 2x^2 + 4x - 7$
Leading coefficient: 1
Degree: 3
Terms: 4
Name: cubic polynomial with 4 terms

7. Standard form: $3x^2 + 5x - 4$
Leading coefficient: 3
Degree: 2
Terms: 3
Name: quadratic trinomial

8. Standard form: $-4x^3 + 5x^2$
Leading coefficient: −4
Degree: 3
Terms: 2
Name: cubic binomial

9. Standard form: $4x^4 + 8x^2 - 3x + 1$
Leading coefficient: 4
Degree: 4
Terms: 4
Name: quartic polynomial with 4 terms

10. $\left(15x^2 - 3x + 11\right) + \left(2x^3 - x^2 + 6x + 1\right)$
$= \left(2x^3\right) + \left(15x^2 - x^2\right) + (-3x + 6x) + (11 + 1)$
$= 2x^3 + 14x^2 + 3x + 12$

11. $\left(12x - 1 + 2x^2\right) + \left(x^2 + 4\right)$
$= \left(2x^2 + 12x - 1\right) + \left(x^2 + 4\right)$
$= \left(2x^2 + x^2\right) + (12x) + (-1 + 4)$
$= 3x^2 + 12x + 3$

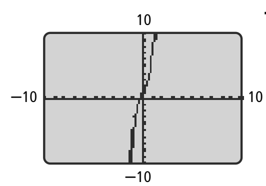
12. $\left(3x^2 - 5x\right) - \left(-4 + x^2 + x\right)$
$= \left(3x^2 - 5x\right) + \left(-x^2 - x + 4\right)$
$= \left(3x^2 - x^2\right) + (-5x - x) - (-4)$
$= 2x^2 - 6x + 4$

13. $\left(x^2 - 3x + 7\right) - \left(6x^2 + 4x + 12\right)$
$= \left(x^2 - 3x + 7\right) + \left(-6x^2 - 4x - 12\right)$
$= \left(x^2 - 6x^2\right) + (-3x - 4x) + (7 - 12)$
$= -5x^2 - 7x - 5$

14a. $F(5) = \frac{1}{3}(5)^3 + \frac{1}{2}(5)^2 + \frac{1}{6}(5)$
$= 55$
$F(10) = \frac{1}{3}(10)^3 + \frac{1}{2}(10)^2 + \frac{1}{6}(10)$
$= 385$

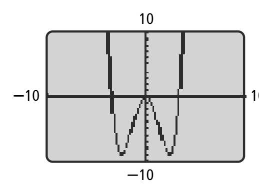
b. $F(5)$ represents the sum of squares of the first 5 natural numbers.
$F(10)$ represents the sum of squares of the first 10 natural numbers.

15.



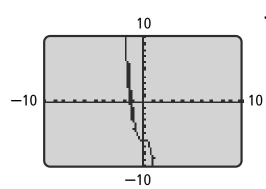
From left to right, the graph increases. It crosses the *x*-axis once, so there appears to be 1 real zero.

16.



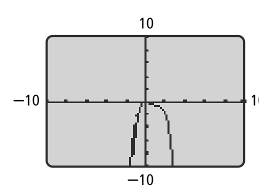
From left to right, the graph alternately decreases and increases, changing direction 3 times. It crosses the *x*-axis 3 times, so there appear to be 3 real zeros.

17.



From left to right, the graph decreases. It crosses the *x*-axis once, so there appears to be 1 real zero.

18.



From left to right, the graph increases and then decreases. It crosses the *x*-axis once, so there appears to be 1 real zero.

PRACTICE AND PROBLEM SOLVING

19. x^8
The degree is 8.

20. $6x^3y$
The degree is 4.

21. 8
The degree is 0.

22. $a^4b^6c^3$
The degree is 13.

23. Standard form: $2x^4 + 3x^3 + x^2 - 7x$
Leading coefficient: 2
Degree: 4
Terms: 4
Name: quartic polynomial with 4 terms

24. Standard form: $-4x^4 + 6x + 5^7$
Leading coefficient: −4
Degree: 4
Terms: 3
Name: quartic trinomial

25. Standard form: $2x^3 + 10x - 9$
Leading coefficient: 2
Degree: 3
Terms: 3
Name: cubic trinomial

26. Standard form: $2x^6 - 4x^4 + 3x^2 - 1$
Leading coefficient: 2
Degree: 6
Terms: 4
Name: sixth-degree polynomial with 4 terms

27. $\left(x^2 - 3x + 4\right) + \left(x^3 + 3x - 4\right)$
$= \left(x^3\right) + \left(x^2\right) + (-3x + 3x) + (4 - 4)$
$= x^3 + x^2$

28. $\left(x^2 - 3x + 4\right) - \left(3x + x^3 - 4\right)$
$= \left(x^2 - 3x + 4\right) + \left(-x^3 - 3x + 4\right)$
$= \left(-x^3\right) + \left(x^2\right) + (-3x - 3x) + (4 + 4)$
$= -x^3 + x^2 - 6x + 8$

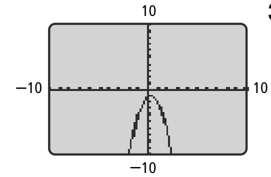
29. $\left(5y^3 - 2y^2 - 1\right) - \left(y^2 - 2y - 3\right)$
$= \left(5y^3 - 2y^2 - 1\right) + \left(-y^2 + 2y + 3\right)$
$= \left(5y^3\right) + \left(-2y^2 - y^2\right) + (2y) + (-1 + 3)$
$= 5y^3 - 3y^2 + 2y + 2$

30. $\left(2y^2 - 5y + 3\right) + \left(y^2 - 2y - 5\right)$
$= \left(2y^2 + y^2\right) + (-5y - 2y) + (3 - 5)$
$= 3y^2 - 7y - 2$

31a. $d(1) = -4(1)^3 + (1)^2$
$= -3$
$d(2) = -4(2)^3 + (2)^2$
$= -28$

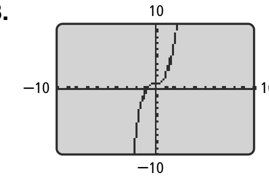
b. $d(1)$ represents a bend of 3 cm below the resting position for the stabilized point 1 m from the end of the board.
$d(2)$ represents a bend of 28 cm below the resting position for the stabilized point 2 m from the end of the board.

32.



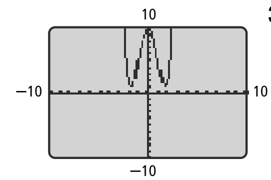
From left to right, the graph increases and then decreases. There are no real zeros.

33.



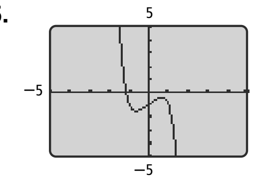
From left to right, the graph increases. There is 1 real zero.

34.



From left to right, the graph decreses, increases, decreases and then increases again. There are no zeros.

35.



From left to right, the graph decreases, increases and then decreases. There is 1 real zero.

	Polynomial	Standard Form	Leading Coefficient	Degree
36.	$8x + 3x^2 - 5$	$3x^2 + 8x - 5$	3	2
37.	$3x^2 + x^4 - 2$	$x^4 + 3x^2 - 2$	1	4
38.	$x^3 - x^4 + x - 1$	$-x^4 + x^3 + x - 1$	−1	4
39.	$64 + x^2$	$x^2 + 64$	1	2

40. Possible answer: $2x^4 + x^3 + 2$

41.
$S = 2\pi r\ell + 2\pi r^2$
$S(x) = 2\pi x(x + 4) + 2\pi x^2$
$= 2\pi x^2 + 8\pi x + 2\pi x^2$
$= 4\pi x^2 + 8\pi x$

42.
$S = 2s^2 + 4s\ell$
$S(x) = 2x^2 + 4(x)(x + 1)$
$= 2x^2 + 4x^2 + 4x$
$= 6x^2 + 4x$

43.
$S = \pi r^2 + \pi \ell r$
$S(x) = \pi(2x + 3)^2 + \pi\left(\frac{1}{2}x + 1\right)(2x + 3)$
$= \pi\left(4x^2 + 6x + 6x + 9\right) + \pi\left(x^2 + \frac{3}{2}x + 2x + 1\right)$
$= \pi\left(4x^2 + 12x + 9\right) + \pi\left(x^2 + \frac{7}{2}x + 3\right)$
$= 4\pi x^2 + 12\pi x + 9\pi + \pi x^2 + \frac{7}{2}\pi x + 3\pi$
$= 5\pi x^2 + \frac{31}{2}\pi x + 12\pi$

44. $S = s^2 + 4\left(\frac{bh}{2}\right)$

$$S(x) = (x-1)^2 + 4\left(\frac{(x-1)(3x)}{2}\right)$$
$$= \left(x^2 - x - x + 1\right) + 4\left(\frac{3x^2 - 3x}{2}\right)$$
$$= \left(x^2 - 2x + 1\right) + 2(3x^2 - 3x)$$
$$= x^2 - 2x + 1 + 6x^2 - 6x$$
$$= 7x^2 - 8x + 1$$

45a. $C(7) = 0.03(7)^3 - 0.75(7)^2 + 4.5(7) + 7$
$= \$12.04$

b. $C(19) = 0.03(19)^3 - 0.75(19)^2 + 4.5(19) + 7$
$= \$27.52$

46. 90

47. Sometimes true; possible answer: $x^2 + x + 1$ is a quadratic that is also a trinomial. $x^2 + 1$ is a quadratic that is not a trinomial.

48. always true

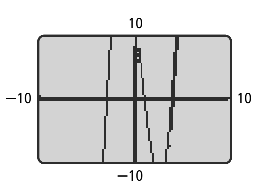
49. Sometimes true; possible answer: $5x^2 + 2x + 1$ is a polynomial in which the leading coefficient is also the greatest. $x^2 + 2x + 5$ is a polynomial in which the leading coefficient is not the greatest.

50a. $T(n+1) = \frac{1}{2}(n+1)^2 + \frac{1}{2}(n+1)$
$$= \frac{1}{2}\left(n^2 + 2n + 1\right) + \frac{1}{2}(n+1)$$
$$= \frac{1}{2}n^2 + n + \frac{1}{2} + \frac{1}{2}n + \frac{1}{2}$$
$$= \frac{1}{2}n^2 + \frac{3}{2}n + 1$$

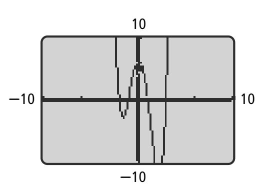
b. $T(n+1) - T(n) = \left(\frac{1}{2}n^2 + \frac{3}{2}n + 1\right) - \left(\frac{1}{2}n^2 + \frac{1}{2}n\right)$
$$= \left(\frac{1}{2}n^2 - \frac{1}{2}n^2\right) + \left(\frac{3}{2}n - \frac{1}{2}n\right) + (1)$$
$$= n + 1$$
The difference between the $(n+1)$th triangular number and the nth triangular number is $n + 1$.

51. Possible answer: The factors give the x-intercepts of the graphs.

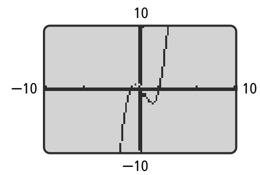
51a. The x-intercepts are -3, 1 and 4.



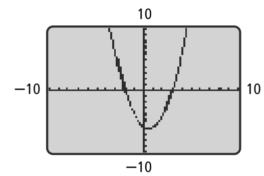
b. The x-intercepts are -1, -2, 3 and 1.



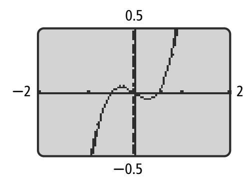
c. The x-intercepts are 0, -1 and 2.



d. The x-intercepts are -2, and 3.



e. The x-intercepts are $-\frac{1}{2}$, 0 and $\frac{1}{2}$.



52. Yes; possible answer: if you switch around the terms of a polynomial, it may no longer be in standard form, but it is still the same polynomial.

53. Yes; possible answer: if you are adding 3 polynomials, it does not matter which 2 you add first.

TEST PREP

54. A

55. J;
$$f(x) - g(x) = \left(2x^2 + 4x - 6\right) - \left(2x^2 + 2x + 8\right)$$
$$= \left(2x^2 + 4x - 6\right) + \left(-2x^2 - 2x - 8\right)$$
$$= \left(2x^2 - 2x^2\right) + (4x - 2x) + (-6 - 8)$$
$$= 2x - 14$$

56. C

57. J
Standard form: $-x^6 + 7x^3 + x$

58. $P(-2) = \frac{1}{2}(-2)^3 - (-2)^2 + 8$
$= 0$

CHALLENGE AND EXTEND

59. Possible answer: $P(x) = x^2 + x + 1$
$R(x) = \underline{x^2}$
$P(x) - R(x) = x + 1$

60. Possible answer: $P(x) = x^2 + x + 1$
$R(x) = \underline{-2x^2 - x - 1}$
$P(x) - R(x) = 3x^2 + 2x + 2$

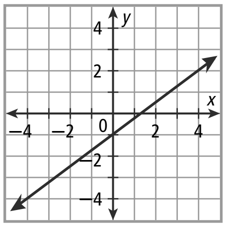
61. Possible answer: $P(x) = x^3 + x^2 + 2$
$R(x) = \underline{x + 1}$
$P(x) - R(x) = x^3 + x^2 - x + 1$

62. Possible answer: $P(x) = x^4 + x^2 + 1$
$R(x) = \underline{x^3 + 2}$
$P(x) - R(x) = x^4 - x^3 + x^2 - 1$

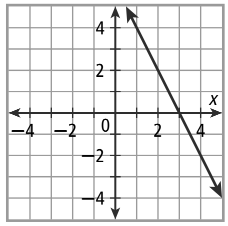
63. Possible answer: $P(x) = x^5 + x + 2$
$R(x) = \underline{-x^5 + 5}$
$P(x) - R(x) = 2x^5 + x - 3$

SPIRAL REVIEW

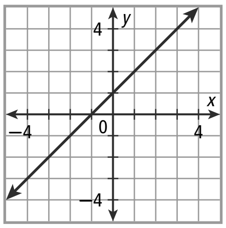
64.



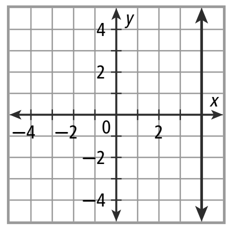
65.



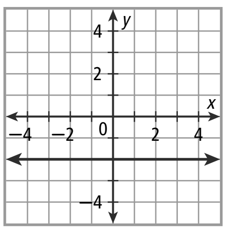
66.



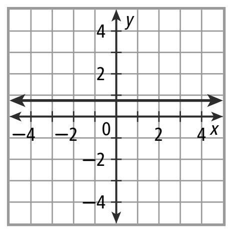
67. vertical



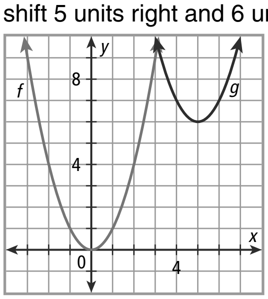
68. horizontal



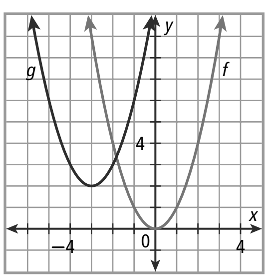
69. horizontal



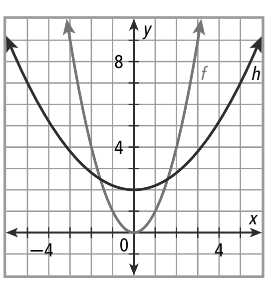
70. shift 5 units right and 6 units up



71. shift 3 units left and 2 units up



72. vertical compression by a factor of $\frac{1}{5}$ and shift up 2 units.



6-2 MULTIPLYING POLYNOMIALS, PAGES 414–420

CHECK IT OUT!

1a. $3cd^2(4c^2d - 6cd + 14cd^2)$

$= 3cd^2(4c^2d) + 3cd^2(-6cd) + 3cd^2(14cd^2)$

$= 12c^3d^3 - 18c^2d^3 + 42c^2d^4$

b. $x^2y(6y^3 + y^2 - 28y + 30)$

$= x^2y(6y^3) + x^2y(y^2) + x^2y(-28y) + x^2y(30)$

$= 6x^2y^4 + x^2y^3 - 28x^2y^2 + 30x^2y$

2a.

$$\begin{array}{r} 3b^2 - bc - 2c^2 \\ \underline{3b - 2c} \\ -6b^2c + 2bc^2 + 4c^3 \\ \underline{9b^3 - 3b^2c - 6bc^2 \qquad\qquad} \\ 9b^3 - 9b^2c - 4bc^2 + 4c^3 \end{array}$$

b. $(x^2 - 4x + 1)(x^2 + 5x - 2)$

	x^2	$5x$	-2
x^2	x^4	$+5x^3$	$-2x^2$
$-4x$	$-4x^3$	$-20x^2$	$+8x$
1	$+x^2$	$+5x$	-2

$= x^4 + x^3 - 21x^2 + 13x - 2$

3. $T(x) = N(x) \cdot C(x)$

$$\begin{array}{r} 0.02x^2 + 0.2x + 3 \\ \underline{\times -0.004x^2 - 0.1x + 3} \\ 0.06x^2 + 0.6x + 9 \\ -0.002x^3 - 0.02x^2 - 0.3x \qquad\quad \\ \underline{-0.00008x^4 - 0.0008x^3 - 0.012x^2 \qquad\qquad\qquad} \\ -0.00008x^4 - 0.0028x^3 + 0.028x^2 + 0.3x + 9 \end{array}$$

Mr. Silva's total manufacturing costs, in thousands of dollars, can be modeled by
$T(x) = -0.00008x^4 - 0.0028x^3 + 0.028x^2 + 0.3x + 9$

4. $(x + 4)^4$

$= (x + 4)(x + 4)(x + 4)(x + 4)$

$= (x^2 + 8x + 16)(x^2 + 8x + 16)$

	x^2	$8x$	16
x^2	x^4	$8x^3$	$16x^2$
$8x$	$8x^3$	$64x^2$	$128x$
16	$16x^2$	$128x$	256

$= x^4 + 16x^3 + 96x^2 + 256x + 256$

b. $(2x - 1)^3$

$= (2x - 1)(2x - 1)(2x - 1)$

$= (2x - 1)(4x^2 - 4x + 1)$

$= 2x(4x^2) + 2x(-4x) + 2x(1) - 1(4x^2) - 1(-4x) - 1(1)$

$= 8x^3 - 8x^2 + 2x - 4x^2 + 4x - 1$

$= 8x^3 - 12x^2 + 6x - 1$

5a. $(x + 2)^3$

$= [1(x)^3(2)^0] + [3(x)^2(2)^1] + [3(x)^1(2)^2] + [1(x)^0(2)^3]$

$= x^3 + 6x^2 + 12x + 8$

b. $(x-4)^5$

$= \left[1(x)^5(-4)^0\right] + \left[5(x)^4(-4)^1\right] + \left[10(x)^3(-4)^2\right] + \left[10(x)^2(-4)^3\right] + \left[5(x)^1(-4)^4\right] + \left[1(x)^0(-4)^5\right]$

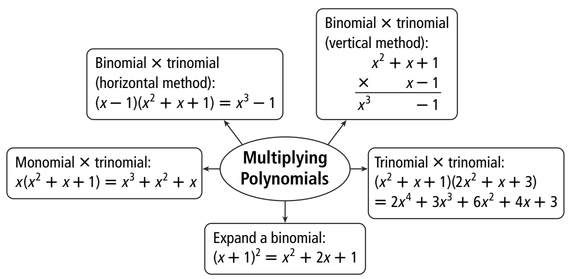
$= x^5 - 20x^4 + 160x^3 - 640x^2 + 1280x - 1024$

c. $(3x+1)^4$

$= \left[1(3x)^4(1)^0\right] + \left[4(3x)^3(1)^1\right] + \left[6(3x)^2(1)^2\right] + \left[4(3x)^1(1)^3\right] + \left[1(3x)^0(1)^4\right]$

$= 81x^4 + 108x^3 + 54x^2 + 12x + 1$

THINK AND DISCUSS

1. 5; possible answer: the degree of the product is the sum of the degrees of the 2 polynomials.
2. degree 7; 8 terms; the degree is given by the exponent of $(2x)^7$, and there are always $(n+1)$ terms. When expanding $(a+b)^n$
3. 

EXERCISES

GUIDED PRACTICE

1. $-4c^2d^3\left(5cd^2 + 3c^2d\right)$
   $= -4c^2d^3\left(5cd^2\right) - 4c^2d^3\left(3c^2d\right)$
   $= -20c^3d^5 - 12c^4d^4$
2. $3x^2(2y+5x)$
   $= 3x^2(2y) + 3x^2(5x)$
   $= 6x^2y + 15x^3$
3. $xy\left(5x^2 + 8x - 7\right)$
   $= xy\left(5x^2\right) + xy(8x) + xy(-7)$
   $= 5x^3y + 8x^2y - 7xy$
4. $2xy\left(3x^2 - xy + 7\right)$
   $= 2xy\left(3x^2\right) + 2xy(-xy) + 2xy(7)$
   $= 6x^3y - 2x^2y^2 + 14xy$
5. $(x-y)\left(x^2 + 2xy - y^2\right)$
   $= x\left(x^2\right) + x(2xy) + x\left(-y^2\right) - y\left(x^2\right) - y(2xy) - y\left(-y^2\right)$
   $= x^3 + 2x^2y - xy^2 - x^2y - 2xy^2 + y^3$
   $= x^3 + x^2y - 3xy^2 + y^3$
6. $(3x-2)\left(2x^2 + 3x - 1\right)$
   $= 3x\left(2x^2\right) + 3x(3x) + 3x(-1) - 2\left(2x^2\right) - 2(3x) - 2(-1)$
   $= 6x^3 + 9x^2 - 3x - 4x^2 - 6x + 2$
   $= 6x^3 + 5x^2 - 9x + 2$
7.
$$\begin{array}{rrrrrr}
 & & x^3 & & +3x^2 & +1 \\
 & & & 3x^2 & +6x & -2 \\ \hline
 & & -2x^3 & -6x^2 & & -2 \\
 & 6x^4 & +18x^3 & & +6x & \\
3x^5 & +9x^4 & & +3x^2 & & \\ \hline
3x^5 & +15x^4 & +16x^3 & -3x^2 & +6x & -2
\end{array}$$
8.
$$\begin{array}{rrrrr}
 & & x^2 & +9x & +7 \\
 & & 3x^2 & +9x & +5 \\ \hline
 & & 5x^2 & +45x & +35 \\
 & 9x^3 & +81x^2 & +63x & \\
3x^4 & +27x^3 & +21x^2 & & \\ \hline
3x^4 & +36x^3 & +107x^2 & +108x & +35
\end{array}$$
9. $R(x) = N(x) \cdot P(x)$
$$\begin{array}{rrrrr}
 & -0.1x^3 & +x^2 & -3x & +4 \\
 & & & 0.2x & +5 \\ \hline
 & -0.5x^3 & +5x^2 & -15x & +20 \\
-0.02x^4 & +0.2x^3 & -0.6x^2 & +0.8x & \\ \hline
-0.02x^4 & -0.3x^3 & +4.4x^2 & -14.2x & +20
\end{array}$$
   The revenue for this company can be modeled by $R(x) = -0.02x^4 - 0.3x^3 + 4.4x^2 - 14.2x + 20$
10. $(x+2)^3$
    $= (x+2)(x+2)(x+2)$
    $= (x+2)\left(x^2 + 4x + 4\right)$
    $= x\left(x^2\right) + x(4x) + x(4) + 2\left(x^2\right) + 2(4x) + 2(4)$
    $= x^3 + 4x^2 + 4x + 2x^2 + 8x + 8$
    $= x^3 + 6x^2 + 12x + 8$
11. $(x+y)^4$
    $= (x+y)(x+y)(x+y)(x+y)$
    $= \left(x^2 + 2xy + y^2\right)\left(x^2 + 2xy + y^2\right)$

	x^2	$2xy$	y^2
x^2	x^4	$2x^3y$	x^2y^2
$2xy$	$2x^3y$	$4x^2y^2$	$2xy^3$
y^2	x^2y^2	$2xy^3$	y^4

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

12. $(x+1)^4$
    $= (x+1)(x+1)(x+1)(x+1)$
    $= \left(x^2 + 2x + 1\right)\left(x^2 + 2x + 1\right)$

	x^2	$2x$	1
x^2	x^4	$2x^3$	x^2
$2x$	$2x^3$	$4x^2$	$2x$
1	x^2	$2x$	1

$= x^4 + 4x^3 + 6x^2 + 4x + 1$

13. $(x-3y)^3$
$= (x-3y)(x-3y)(x-3y)$
$= (x-3y)(x^2-6xy+9y^2)$
$= x(x^2) + x(-6xy) + x(9y^2) - 3y(x^2) - 3y(-6xy)$
$- 3y(9y^2)$
$= x^3 - 6x^2y + 9xy^2 - 3x^2y + 18xy^2 - 27y^3$
$= x^3 - 9x^2y + 27xy^2 - 27y^3$

14. $(x-2)^4$
$= [1(x)^4(-2)^0] + [4(x)^3(-2)^1] + [6(x)^2(-2)^2]$
$+ [4(x)^1(-2)^3] + [1(x)^0(-2)^4]$
$= x^4 - 8x^3 + 24x^2 - 32x + 16$

15. $(2x+y)^4$
$= [1(2x)^4(y)^0] + [4(2x)^3(y)^1] + [6(2x)^2(y)^2]$
$+ [4(2x)^1(y)^3] + [1(2x)^0(y)^4]$
$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$

16. $(x+2y)^3$
$= [1(x)^3(2y)^0] + [3(x)^2(2y)^1] + [3(x)^1(2y)^2]$
$+ [1(x)^0(2y)^3]$
$= x^3 + 6x^2y + 12xy^2 + 8y^3$

17. $(2x-y)^5$
$= [1(2x)^5(-y)^0] + [5(2x)^4(-y)^1] + [10(2x)^3(-y)^2]$
$+ [10(2x)^2(-y)^3] + [5(2x)^1(-y)^4] + [1(2x)^0(-y)^5]$
$= 32x^5 - 80x^4y + 80x^3y^2 - 40x^2y^3 + 10xy^4 - y^5$

PRACTICE AND PROBLEM SOLVING

18. $7x^3(2x+3)$
$= 7x^3(2x) + 7x^3(3)$
$= 14x^4 + 21x^3$

19. $3x^2(2x^2+9x-6)$
$= 3x^2(2x^2) + 3x^2(9x) + 3x^2(-6)$
$= 6x^4 + 27x^3 - 18x^2$

20. $xy^2(x^2+3xy+9)$
$= xy^2(x^2) + xy^2(3xy) + xy^2(9)$
$= x^3y^2 + 3x^2y^3 + 9xy^2$

21. $2r^2(6r^3+14r^2-30r+14)$
$= 2r^2(6r^3) + 2r^2(14r^2) + 2r^2(-30r) + 2r^2(14)$
$= 12r^5 + 28r^4 - 60r^3 + 28r^2$

22. $(x-y)(x^2-xy+y^2)$
$= x(x^2) + x(-xy) + x(y^2) - y(x^2) - y(-xy) - y(y^2)$
$= x^3 - x^2y + xy^2 - x^2y + xy^2 - y^3$
$= x^3 - 2x^2y + 2xy^2 - y^3$

23.
$$\begin{array}{r} 3x^2 - 4xy + 2y^2 \\ \underline{2x + 5y} \\ 15x^2y - 20xy^2 + 10y^3 \\ \underline{6x^3 - 8x^2y + 4xy^2 \qquad\qquad} \\ 6x^3 + 7x^2y - 16xy^2 + 10y^3 \end{array}$$

24.
$$\begin{array}{r} x^3 + x^2 \qquad + 1 \\ \underline{x^2 - x - 5} \\ -5x^3 - 5x^2 \qquad - 5 \\ -x^4 - x^3 \qquad\quad - x \qquad \\ \underline{x^5 + x^4 \qquad\quad + x^2 \qquad\qquad\quad} \\ x^5 \quad - 6x^3 - 4x^2 - x - 5 \end{array}$$

25. $(4x^2+3x+2)(3x^2+2x-1)$

	$4x^2$	$3x$	2
$3x^2$	$12x^4$	$9x^3$	$6x^2$
$2x$	$8x^3$	$6x^2$	$4x$
-1	$-4x^2$	$-3x$	-2

$= 12x^4 + 17x^3 + 8x^2 + x - 2$

26a. $V(x) = x(11-2x)(8.5-2x)$
$= (11x - 2x^2)(8.5-2x)$
$= 11x(8.5) + 11x(-2x) - 2x^2(8.5) - 2x^2(-2x)$
$= 93.5x - 22x^2 - 17x^2 + 4x^3$
$= 4x^3 - 39x^2 + 93.5x$

b. $V(1) = 4(1)^3 - 39(1)^2 + 93.5(1)$
$= 58.5 \text{ in}^3$

27. $(2x-2)^3$
$= (2x-2)(2x-2)(2x-2)$
$= (2x-2)(4x^2-8x+4)$
$= 2x(4x^2) + 2x(-8x) + 2x(4) - 2(4x^2) - 2(-8x)$
$- 2(4)$
$= 8x^3 - 16x^2 + 8x - 8x^2 + 16x - 8$
$= 8x^3 - 24x^2 + 24x - 8$

28. $\left(x+\frac{1}{3}\right)^4$
$= \left(x+\frac{1}{3}\right)\left(x+\frac{1}{3}\right)\left(x+\frac{1}{3}\right)\left(x+\frac{1}{3}\right)$
$= \left(x^2+\frac{2}{3}x+\frac{1}{9}\right)\left(x^2+\frac{2}{3}x+\frac{1}{9}\right)$

	x^2	$\frac{2}{3}x$	$\frac{1}{9}$
x^2	x^4	$\frac{2}{3}x^3$	$\frac{1}{9}x^2$
$\frac{2}{3}x$	$\frac{2}{3}x^3$	$\frac{4}{9}x^2$	$\frac{2}{27}x$
$\frac{1}{9}$	$\frac{1}{9}x^2$	$\frac{2}{27}x$	$\frac{1}{81}$

$= x^4 + \frac{4}{3}x^3 + \frac{2}{3}x^2 + \frac{4}{27}x + \frac{1}{81}$

29. $(x-y)^4$
$= (x-y)(x-y)(x-y)(x-y)$
$= (x^2-2xy+y^2)(x^2-2xy+y^2)$

	x^2	$-2xy$	y^2
x^2	x^4	$-2x^3y$	x^2y^2
$-2xy$	$-2x^3y$	$4x^2y^2$	$-2xy^3$
y^2	x^2y^2	$-2xy^3$	y^4

$= x^4 - 4x^3y + 6x^2y^2 - 4xy^3 + y^4$

30. $(4 + y)^3$
$= (4 + y)(4 + y)(4 + y)$
$= (4 + y)\left(16 + 8y + y^2\right)$
$= 4(16) + 4(8y) + 4\left(y^2\right) + y(16) + y(8y) + y\left(y^2\right)$
$= 64 + 32y + 4y^2 + 16y + 8y^2 + y^3$
$= 64 + 48y + 12y^2 + y^3$

31. $(x - 3y)^4$
$= \left[1(x)^4(-3y)^0\right] + \left[4(x)^3(-3y)^1\right] + \left[6(x)^2(-3y)^2\right]$
$+ \left[4(x)^1(-3y)^3\right] + \left[1(x)^0(-3y)^4\right]$
$= x^4 - 12x^3y + 54x^2y^2 - 108xy^3 + 81y^4$

32. $(x - 2)^5$
$= \left[1(x)^5(-2)^0\right] + \left[5(x)^4(-2)^1\right] + \left[10(x)^3(-2)^2\right]$
$+ \left[10(x)^2(-2)^3\right] + \left[5(x)^1(-2)^4\right] + \left[1(x)^0(-2)^5\right]$
$= x^5 - 10x^4 + 40x^3 - 80x^2 + 80x - 32$

33. $(x + y)^5$
$= \left[1(x)^5(y)^0\right] + \left[5(x)^4(y)^1\right] + \left[10(x)^3(-3y)^2\right]$
$+ \left[10(x)^2(y)^3\right] + \left[5(x)^1(y)^4\right] + \left[1(x)^0(y)^5\right]$
$= x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5$

34. $(2x - 3y)^4$
$= \left[1(2x)^4(-3y)^0\right] + \left[4(2x)^3(-3y)^1\right] + \left[6(2x)^2(-3y)^2\right]$
$+ \left[4(2x)^1(-3y)^3\right] + \left[1(2x)^0(-3y)^4\right]$
$= 16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4$

35. equivalent

36. equivalent

37. not equivalent

38. equivalent

39. $T(x) = N(x) \cdot C(x)$

$$\begin{array}{r} 0.3x^2 \ - 1.6x + 14 \\ \underline{-0.001x^2 - 0.06x + 8.3} \\ 2.49x^2 - 13.28x + 116.2 \\ -\ 0.018x^3 + 0.096x^2 \ - 0.84x \quad\ \ \\ \underline{-0.0003x^4 + 0.0016x^3 - 0.014x^2 \qquad\qquad\qquad\qquad} \\ -0.0003x^4 - 0.0164x^3 + 2.572x^2 - 14.12x + 116.2 \end{array}$$

Ms. Liao's dressmaking costs can be modeled by T(x) = $-0.0003x^4 - 0.0164x^3 + 2.572x^2 - 14.12x + 116.2$

40. $-6x^3\left(15y^4 - 7xy^3 + 2\right)$
$= -6x^3\left(15y^4\right) - 6x^3\left(-7xy^3\right) - 6x^3(2)$
$= -90x^3y^4 + 42x^4y^3 - 12x^3$

41. $(p - 2q)^3$
$= (p - 2q)(p - 2q)(p - 2q)$
$= (p - 2q)\left(p^2 - 4pq + 4q^2\right)$

$$\begin{array}{r} p^2 - 4pq + 4q^2 \\ \underline{p - 2q} \\ -2p^2q \ + 8pq^2 - 8q^3 \\ \underline{p^3 - 4p^2q \ + 4pq^2 \qquad\quad} \\ p^3 - 6p^2q + 12pq^2 - 8q^3 \end{array}$$

42. $\left(x^2 - 2yz - y^2\right)\left(y^2 + x\right)$
$= x^2\left(y^2\right) + x^2(x) - 2yz\left(y^2\right) - 2yz(x) - y^2\left(y^2\right) - y^2(x)$
$= x^2y^2 + x^3 - 2y^3z - 2xyz - y^4 - xy^2$

43. $\left(x^4 + xy^3\right)\left(x^2 + y^3\right)$
$= x^4\left(x^2\right) + x^4\left(y^3\right) + xy^3\left(x^2\right) + xy^3\left(y^3\right)$
$= x^6 + x^4y^3 + x^3y^3 + xy^6$

44. $(3 - 3y)^4$
$= \left(1(3)^4(-3y)^0\right) + \left(4(3)^3(-3y)^1\right) + \left(6(3)^2(-3y)^2\right)$
$+ \left(4(3)^1(-3y)^3\right) + \left(1(3)^0(-3y)^4\right)$
$= 81 - 324y + 486y^2 - 324y^3 + 81y^4$

45. $\left(5x^3 + x^2 - 9x\right)(y + 2)$
$= 5x^3(y) + x^2(y) - 9x(y) + 5x^3(2) + x^2(2) - 9x(2)$
$= 5x^3y + x^2y - 9xy + 10x^3 + 2x^2 - 18x$

46. $\left(3 + x - 2x^2\right)(x - 1)$
$= 3(x) + 3(-1) + x(x) + x(-1) - 2x^2(x) - 2x^2(-1)$
$= 3x - 3 + x^2 - x - 2x^3 + 2x^2$
$= -2x^3 + 3x^2 + 2x - 3$

47. $3(x - 2)^4$
$= 3(x - 2)(x - 2)(x - 2)(x - 2)$
$= 3\left(x^2 - 4x + 4\right)\left(x^2 - 4x + 4\right)$

	x^2	$-4x$	4
x^2	x^4	$-4x^3$	$4x^2$
$-4x$	$-4x^3$	$16x^2$	$-16x$
4	$4x^2$	$-16x$	16

$= 3\left(x^4 - 8x^3 + 24x^2 - 32x + 16\right)$
$= 3x^4 - 24x^3 + 72x^2 - 96x + 48$

48.

$$\begin{array}{r} x^4 \ - 2x^3 \ + x^2 \qquad + 1 \\ \underline{x - 6} \\ -6x^4 + 12x^3 - 6x^2 \qquad - 6 \\ \underline{x^5 - 2x^4 \quad + x^3 \qquad + x \qquad} \\ x^5 - 8x^4 + 13x^3 - 6x^2 + x - 6 \end{array}$$

49. $\left(30 + x^3 + x^2\right)\left(x - 15 - x^2\right)$

	30	x^3	x^2
x	$30x$	x^4	x^3
-15	-450	$-15x^3$	$-15x^2$
$-x^2$	$-30x^2$	$-x^5$	$-x^4$

$= -x^5 - 14x^3 - 45x^2 + 30x - 450$

50. $\left(\frac{1}{2} + z\right)^4$
$= \left(\frac{1}{2} + z\right)\left(\frac{1}{2} + z\right)\left(\frac{1}{2} + z\right)\left(\frac{1}{2} + z\right)$
$= \left(\frac{1}{4} + z + z^2\right)\left(\frac{1}{4} + z + z^2\right)$

	$\frac{1}{4}$	z	z^2
$\frac{1}{4}$	$\frac{1}{16}$	$\frac{1}{4}z$	$\frac{1}{4}z^2$
z	$\frac{1}{4}z$	z^2	z^3
z^2	$\frac{1}{4}z^2$	z^3	z^4

$= \frac{1}{16} + \frac{1}{2}z + \frac{3}{2}z^2 + 2z^3 + z^4$

51.

$$\begin{array}{r} x^5 - 4x^3 + 7 \\ 2x - 3 \\ \hline -3x^5 \quad + 12x^3 \quad - 21 \\ 2x^6 \quad - 8x^4 \quad + 14x \\ \hline 2x^6 - 3x^5 - 8x^4 + 12x^3 + 14x - 21 \end{array}$$

52. The coefficients would be the numbers in the seventh row of a Pascal Triangle:
1, 7, 21, 35, 35, 21, 7, 1

53. Kinetic energy as a function of time:

$$\begin{aligned} K &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(2)(-9.8t + 24)^2 \\ &= (-9.8t + 24)^2 \\ &= 96.04t^2 - 470.4t + 576 \end{aligned}$$

Potential energy as a function of time:

$$\begin{aligned} U &= 9.8mh \\ &= 9.8(2)\left(-4.9t^2 + 24t + 60\right) \\ &= 19.6\left(-4.9t^2 + 24t + 60\right) \\ &= -96.04t^2 + 470.4t + 1176 \end{aligned}$$

$K + U$
$= 96.04t^2 - 470.4t + 576 - 96.04t^2 + 470.4t + 1176$
$= 1752$

Possible answer: No, you get a constant 1752 joules, which is independent of time *t*.

54. Student B; the error is that the powers of *b* should begin at 0 and increase, not decrease.

55a.

$$\begin{aligned} f(n) &= \frac{n(n+1)^2(n+2)}{4} \\ &= \frac{n\left(n^2 + 2n + 1\right)(n+2)}{4} \\ &= \frac{\left(n^3 + 2n^2 + n\right)(n+2)}{4} \\ &= \frac{n^4 + 2n^3 + 2n^3 + 4n^2 + n^2 + 2n}{4} \\ &= \frac{n^4 + 4n^3 + 5n^2 + 2n}{4} \\ &= \frac{n^4}{4} + \frac{4n^3}{4} + \frac{5n^2}{4} + \frac{2n}{4} \\ &= \frac{1}{4}n^4 + n^3 + \frac{5}{4}n^2 + \frac{1}{2}n \end{aligned}$$

b. $f(12) = \frac{1}{4}(12)^4 + (12)^3 + \frac{5}{4}(12)^2 + \frac{1}{2}(12)$
$= 7098$

c. $f(20) = \frac{1}{4}(20)^4 + (20)^3 + \frac{5}{4}(20)^2 + \frac{1}{2}(20)$
$= 48{,}510$

$f(20)$ represents the product of the 20th and 21st triangular numbers.

56. Possible answer: The powers of $-y$ alternate between positive and negative.

57. Possible answer: Find the row of Pascal's triangle that corresponds to the power of the binomial. Write the decreasing powers of the first term of the binomial times the increasing powers of the second term, and multiply by the appropriate value from the row of Pascal's triangle.

TEST PREP

58. D

$$\begin{aligned} &(y-3)\left(y^2 - 6y - 9\right) \\ &= y\left(y^2\right) + y(-6y) + y(-9) - 3\left(y^2\right) - 3(-6y) - 3(-9) \\ &= y^3 - 9y^2 + 9y + 27 \end{aligned}$$

59. J

$\ell = 2x(2x)$ and $w = 2x(y)$
$= 4x^2$ $= 2xy$

$$\begin{aligned} P &= 2\ell + 2w \\ &= 2\left(4x^2\right) + 2(2xy) \\ &= 8x^2 + 4xy \end{aligned}$$

60. A

$15(x)^4(-4)^2$
$= 240x^4$

61. H

$$\begin{aligned} &a^2b\left(2a^3b - 5ab^4\right) \\ &= a^2b\left(2a^3b\right) + a^2b\left(-5ab^4\right) \\ &= 2a^5b^2 - 5a^3b^5 \end{aligned}$$

62. $(4 - x)^4$

$$\begin{aligned} &= \left[1(4)^4(-x)^0\right] + \left[4(4)^3(-x)^1\right] + \left[6(4)^2(-x)^2\right] \\ &\quad + \left[4(4)^1(-x)^3\right] + \left[1(4)^0(-x)^4\right] \\ &= 256 - 256x + 96x^2 - 16x^3 + x^4 \end{aligned}$$

CHALLENGE AND EXTEND

63. $(x - 1)^{10}$

$$\begin{aligned} &= \left[1(x)^{10}(-1)^0\right] + \left[10(x)^9(-1)^1\right] + \left[45(x)^8(-1)^2\right] \\ &\quad + \left[120(x)^7(-1)^3\right] + \left[210(x)^6(-1)^4\right] \\ &\quad + \left[252(x)^5(-1)^5\right] + \left[210(x)^4(-1)^6\right] \\ &\quad + \left[120(x)^3(-1)^7\right] + \left[45(x)^2(-1)^8\right] \\ &\quad + \left[10(x)^1(-1)^9\right] + \left[1(x)^0(-1)^{10}\right] \\ &= x^{10} - 10x^9 + 45x^8 - 120x^7 + 210x^6 - 252x^5 \\ &\quad + 210x^4 - 120x^3 + 45x^2 - 10x + 1 \end{aligned}$$

64. $(14 + y)^5$

$$\begin{aligned} &= \left[1(14)^5(y)^0\right] + \left[5(14)^4(y)^1\right] + \left[10(14)^3(y)^2\right] \\ &\quad + \left[10(14)^2(y)^3\right] + \left[5(14)^1(y)^4\right] + \left[1(14)^0(y)^5\right] \\ &= 537{,}824 + 192{,}080y + 27{,}440y^2 + 1960y^3 \\ &\quad + 70y^4 + y^5 \end{aligned}$$

65. $(m - n)^3(m + n)^3$

$$\begin{aligned} &= \Big(\left[1(m)^3(-n)^0\right] + \left[3(m)^2(-n)^1\right] + \left[3(m)^1(-n)^2\right] \\ &\quad + \left[1(m)^0(-n)^3\right]\Big)\Big(\left[1(m)^3(n)^0\right] + \left[3(m)^2(n)^1\right] \\ &\quad + \left[3(m)^1(n)^2\right] + \left[1(m)^0(n)^3\right]\Big) \\ &= \left(m^3 - 3m^2n + 3mn^2 - n^3\right) \\ &\quad \left(m^3 + 3m^2n + 3mn^2 + n^3\right) \end{aligned}$$

	m^3	$-3m^2n$	$3mn^2$	$-n^3$
m^3	m^6	$-3m^5n$	$3m^4n^2$	$-m^3n^3$
$3m^2n$	$3m^5n$	$-9m^4n^2$	$9m^3n^3$	$-3m^2n^4$
$3mn^2$	$3m^4n^2$	$-9m^3n^3$	$9m^2n^4$	$-3mn^4$
n^3	m^3n^3	$-3m^2n^4$	$3mn^5$	$-n^6$

$= m^6 - 3m^4n^2 + 3m^2n^4 - n^6$

66. $(ab+2c)^4$

$= \left[1(ab)^4(2c)^0\right] + \left[4(ab)^3(2c)^1\right] + \left[6(ab)^2(2c)^2\right]$
$+ \left[4(ab)^1(2c)^3\right] + \left[1(ab)^0(2c)^4\right]$
$= a^4b^4 + 8a^3b^3c + 24a^2b^2c^2 + 32abc^3 + 16c^4$

67.
$$\begin{array}{rr} P(x) = & x+3 \\ \text{Possible answer: } B(x) = & \underline{x-3} \\ & -3x-9 \\ & \underline{x^2+3x} \\ P(x)\cdot B(x) = & x^2-9 \end{array}$$

68.
$$\begin{array}{rr} P(x) = & x+3 \\ \text{Possible answer: } B(x) = & \underline{x+1} \\ & x+3 \\ & \underline{x^2+3x} \\ P(x)\cdot B(x) = & x^2+4x+3 \end{array}$$

69.
$$\begin{array}{rr} P(x) = & x+3 \\ \text{Possible answer: } B(x) = & \underline{x^3+1} \\ & x+3 \\ & \underline{x^4+3x^3} \\ P(x)\cdot B(x) = & x^4+3x^3+x+3 \end{array}$$

SPIRAL REVIEW

70. Let x represent the number of lost points.
$f(x) = 5x$
$f(11) = 5(11) = 55$
Players must run 55 sprints for a game with a score of 84–73.

71. $A^2 = \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 4 & 13 \end{bmatrix}$

72. $CA = \begin{bmatrix} 6 & 3 \\ -1 & 5 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 4 & 3 \end{bmatrix} = \begin{bmatrix} 0 & 15 \\ 22 & 14 \\ 28 & 21 \end{bmatrix}$

73. $B^2 = \begin{bmatrix} 0 & 4 & 2 \\ 2 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 0 & 4 & 2 \\ 2 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 10 \\ -4 & 10 & 6 \\ -4 & -6 & 6 \end{bmatrix}$

74. $BC = \begin{bmatrix} 0 & 4 & 2 \\ 2 & -1 & 1 \\ -2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 6 & 3 \\ -1 & 5 \\ 0 & 7 \end{bmatrix} = \begin{bmatrix} -4 & 34 \\ 13 & 8 \\ -13 & 20 \end{bmatrix}$

75. Standard form: $4x^4 - 6x^3 + 5x^2 + 3x$
Leading coefficient: 4
Degree: 4
Terms: 4
Name: quartic polynomial with 4 terms

76. Standard form: $5x^3 + 10x^2 - x$
Leading coefficient: 5
Degree: 3
Terms: 3
Name: cubic trinomial

77. Standard form: $3x^5 - 4x^2 - 2x + 9$
Leading coefficient: 3
Degree: 5
Terms: 4
Name: quintic polynomial with 4 terms

6-3 DIVIDING POLYNOMIALS, PAGES 422–428

CHECK IT OUT!

1a.
$$\begin{array}{r} 5x+1 \\ 3x+1\overline{)15x^2+8x-12} \\ \underline{-\left(15x^2+5x\right)} \\ 3x-12 \\ \underline{-(3x+1)} \\ -13 \end{array}$$

$$\frac{15x^2+8x-12}{3x+1} = 5x+1-\frac{13}{3x+1}$$

b.
$$\begin{array}{r} x+8 \\ x-3\overline{)x^2+5x-28} \\ \underline{-\left(x^2-3x\right)} \\ 8x-28 \\ \underline{-(8x-24)} \\ -4 \end{array}$$

$$\frac{x^2+5x-28}{x-3} = x+8-\frac{4}{x-3}$$

2a.
$$\begin{array}{r|rrr} \underline{-3} & 6 & -5 & -6 \\ & & -18 & 69 \\ \hline & 6 & -23 & \boxed{63} \end{array}$$

$$\frac{6x^2-5x-6}{x+3} = 6x-23+\frac{63}{x+3}$$

b.
$$\begin{array}{r|rrr} \underline{6} & 1 & -3 & -18 \\ & & 6 & 18 \\ \hline & 1 & 3 & \boxed{0} \end{array}$$

$$\frac{x^2-3x-18}{x-6} = x+3$$

3a.
$$\begin{array}{r|rrrr} \underline{-3} & 1 & 3 & 0 & 4 \\ & & -3 & 0 & 0 \\ \hline & 1 & 0 & 0 & \boxed{4} \end{array}$$

$P(-3) = 4$

b.
$$\begin{array}{r|rrr} \underline{\tfrac{1}{5}} & 5 & 9 & 3 \\ & & 1 & 2 \\ \hline & 5 & 10 & \boxed{5} \end{array}$$

$P\left(\frac{1}{5}\right) = 5$

4. The area A is related to length ℓ and width w by the equation $A = \ell \cdot w$

$$\ell(y) = \frac{A}{w} = \frac{y^2-14y+45}{y-9}$$

$$\begin{array}{r|rrr} \underline{9} & 1 & -14 & 45 \\ & & 9 & -45 \\ \hline & 1 & -5 & \boxed{0} \end{array}$$

The length can be represented by $\ell(y) = y - 5$.

THINK AND DISCUSS

1. Possible answer: No; the divisor must be a linear binomial in the form $x - a$; $x^2 + 3$ is a quadratic binomial.

2. Possible answer: Use synthetic substitution; $P(6) = 8$.

3.

Long Division and Synthetic Division

- Similarities: Both are used to divide polynomials.
- Differences: Long division works for all divisors. Synthetic substitution only works for linear binomial divisors.

EXERCISES

GUIDED PRACTICE

1. Possible answer: Synthetic division is a method of division that uses only the coefficients for linear binomial divisors.

2.

$$\begin{array}{r} 5x \quad - 2 \\ 4x-1 \overline{)\,20x^2 - 13x + 2} \\ -\left(20x^2 - 5x\right) \\ -8x + 2 \\ -(-8x+2) \\ \hline 0 \end{array}$$

$$\frac{20x^2 - 13x + 2}{4x - 1} = 5x - 2$$

3.

$$\begin{array}{r} x \quad + 2 \\ x-1 \overline{)\,x^2 + x - 1} \\ -\left(x^2 - x\right) \\ 2x - 1 \\ -(2x-2) \\ \hline 1 \end{array}$$

$$\frac{x^2 + x - 1}{x - 1} = x + 2 + \frac{1}{x - 1}$$

4.

$$\begin{array}{r} x \quad - 7 \\ x+5 \overline{)\,x^2 - 2x + 3} \\ -\left(x^2 + 5x\right) \\ -7x + 3 \\ -(-7x-35) \\ \hline 38 \end{array}$$

$$\frac{x^2 - 2x + 3}{x + 5} = x - 7 + \frac{38}{x + 5}$$

5.

$$\begin{array}{r|rrr} 3 & 7 & -23 & 6 \\ & & 21 & -6 \\ \hline & 7 & -2 & \boxed{0} \end{array}$$

$$\frac{7x^2 - 23x + 6}{x - 3} = 7x - 2$$

6.

$$\begin{array}{r|rrrrr} -3 & 1 & 0 & 0 & -5 & 10 \\ & & -3 & 9 & -27 & 96 \\ \hline & 1 & -3 & 9 & -32 & \boxed{106} \end{array}$$

$$\frac{x^4 - 5x + 10}{x + 3} = x^3 - 3x^2 + 9x - 32 + \frac{106}{x + 3}$$

7.

$$\begin{array}{r|rrr} -7 & 1 & 1 & -42 \\ & & -7 & 42 \\ \hline & 1 & -6 & \boxed{0} \end{array}$$

$$\frac{x^2 + x - 42}{x + 7} = x - 6$$

8.

$$\begin{array}{r|rrrr} 2 & 2 & -9 & 0 & 27 \\ & & 4 & -10 & -20 \\ \hline & 2 & -5 & -10 & \boxed{7} \end{array}$$

$P(2) = 7$

9.

$$\begin{array}{r|rrr} -8 & 1 & -1 & -30 \\ & & -8 & 72 \\ \hline & 1 & -9 & \boxed{42} \end{array}$$

$P(-8) = 42$

10.

$$\begin{array}{r|rrrr} \frac{1}{3} & 3 & 5 & 4 & 2 \\ & & 1 & 2 & 2 \\ \hline & 3 & 6 & 6 & \boxed{4} \end{array}$$

$P\left(\frac{1}{3}\right) = 4$

11.

$$\begin{array}{r|rrrrrr} -1 & 3 & 0 & 0 & 4 & 1 & 6 \\ & & -3 & 3 & -3 & -1 & 0 \\ \hline & 3 & -3 & 3 & 1 & 0 & \boxed{6} \end{array}$$

$P(-1) = 6$

12. The area A is related to length ℓ and width w by the equation $A = \ell \cdot w$

$$w(y) = \frac{A}{\ell} = \frac{2x^3 - 8x^2 + 2x + 12}{x - 2}$$

$$\begin{array}{r|rrrr} 2 & 2 & -8 & 2 & 12 \\ & & 4 & -8 & -12 \\ \hline & 2 & -4 & -6 & \boxed{0} \end{array}$$

The length can be represented by $w(y) = 2x^2 - 4x - 6$.

PRACTICE AND PROBLEM SOLVING

13.

$$\begin{array}{r} x \; + \; 4 \\ 2x+2 \overline{)\,2x^2 + 10x + 8} \\ -\left(2x^2 + 2x\right) \\ 8x + 8 \\ -(8x+8) \\ \hline 0 \end{array}$$

$$\frac{2x^2 + 10x + 8}{2x + 2} = x + 4$$

14.

$$\begin{array}{r} 3x \; - \; 6 \\ 3x+0 \overline{)\,9x^2 - 18x} \\ -\left(9x^2 + 0x\right) \\ -18x \\ -(-18x) \\ \hline 0 \end{array}$$

$$\frac{9x^2 - 18x}{3x} = 3x - 6$$

15.

$$\begin{array}{r} x^2 - 1 \\ x+2 \overline{)\,x^3 + 2x^2 - x - 2} \\ -\left(x^3 + 2x^2\right) \\ -x - 2 \\ -(-x-2) \\ \hline 0 \end{array}$$

$$\frac{x^3 + 2x^2 - x - 2}{x + 2} = x^2 - 1$$

16.

$$\begin{array}{r} x^3 \; + x^2 + 4x \; + 9 \\ x-4 \overline{)\,x^4 - 3x^3 + 0x^2 \; - 7x - 14} \\ -\left(x^4 - 4x^3\right) \\ x^3 + 0x^2 \\ -\left(x^3 - 4x^2\right) \\ 4x^2 \; - 7x \\ -\left(4x^2 - 16x\right) \\ 9x - 14 \\ -(9x - 36) \\ \hline 22 \end{array}$$

$$\frac{x^4 - 3x^3 - 7x - 14}{x - 4} = x^3 + x^2 + 4x + 9 + \frac{22}{x - 4}$$

17.
$$\begin{array}{r} \frac{1}{2}x^3 - 2x^2 + 0x - \frac{7}{2} \\ 2x^3 + 0x^2 + 0x + 0 \overline{\big)\, x^6 - 4x^5 + 0x^4 - 7x^3} \\ -\left(x^6 + 0x^5 + 0x^4 + 0x^3\right) \\ \hline -4x^5 + 0x^4 - 7x^3 \\ -\left(-4x^5 + 0x^4 + 0x^3\right) \\ \hline -7x^3 \end{array}$$

$$\frac{x^6 - 4x^5 - 7x^3}{2x^3} = \frac{1}{2}x^3 - 2x^2 - \frac{7}{2}$$

18.
$$\begin{array}{r} 2x + 1 \\ 3x - 5 \overline{\big)\, 6x^2 - 7x - 5} \\ -\left(6x^2 - 10x\right) \\ \hline 3x - 5 \\ -(3x - 5) \\ \hline 0 \end{array}$$

$$\frac{6x^2 - 7x - 5}{3x - 5} = 2x + 1$$

19.
$$\begin{array}{r|rrr} -1 & 1 & 5 & 6 \\ & & -1 & -4 \\ \hline & 1 & 4 & \boxed{2} \end{array}$$
$$\frac{x^2 + 5x + 6}{x + 1} = x + 4 + \frac{2}{x + 1}$$

20.
$$\begin{array}{r|rrrrr} -5 & 1 & 6 & 6 & 0 & 0 \\ & & -5 & -5 & -5 & 25 \\ \hline & 1 & 1 & 1 & -5 & \boxed{25} \end{array}$$
$$\frac{x^4 + 6x^3 + 6x^2}{x + 5} = x^3 + x^2 + x - 5 + \frac{25}{x + 5}$$

21.
$$\begin{array}{r|rrr} -8 & 1 & 9 & 6 \\ & & -8 & -8 \\ \hline & 1 & 1 & \boxed{-2} \end{array}$$
$$\frac{x^2 + 9x + 6}{x + 8} = x + 1 - \frac{2}{x + 8}$$

22.
$$\begin{array}{r|rrr} 2 & 2 & 3 & -20 \\ & & 4 & 14 \\ \hline & 2 & 7 & \boxed{-6} \end{array}$$
$$\frac{2x^2 + 3x - 20}{x - 2} = 2x + 7 - \frac{6}{x - 2}$$

23.
$$\begin{array}{r|rrr} \frac{1}{2} & 2 & 13 & -8 \\ & & 1 & 7 \\ \hline & 2 & 14 & \boxed{-1} \end{array}$$
$$\frac{2x^2 + 13x - 8}{x - \frac{1}{2}} = 2x + 14 - \frac{1}{x - \frac{1}{2}}$$

24.
$$\begin{array}{r|rrr} -1 & 4 & 5 & 1 \\ & & -4 & -1 \\ \hline & 4 & 1 & \boxed{0} \end{array}$$
$$\frac{4x^2 + 5x + 1}{x + 1} = 4x + 1$$

25.
$$\begin{array}{r|rrr} 4 & 2 & -5 & -3 \\ & & 8 & 12 \\ \hline & 2 & 3 & \boxed{9} \end{array}$$
$P(4) = 9$

26.
$$\begin{array}{r|rrrr} -1 & 4 & -5 & 0 & 3 \\ & & -4 & 9 & -9 \\ \hline & 4 & -9 & 9 & \boxed{-6} \end{array}$$
$P(-1) = -6$

27.
$$\begin{array}{r|rrrr} -\frac{1}{3} & 3 & -5 & -1 & 2 \\ & & -1 & 2 & -\frac{1}{3} \\ \hline & 3 & -6 & 1 & \boxed{\frac{5}{3}} \end{array}$$
$P\left(-\frac{1}{3}\right) = \frac{5}{3}$

28.
$$\begin{array}{r|rrr} \frac{4}{5} & 25 & 0 & -16 \\ & & 20 & 16 \\ \hline & 25 & 20 & \boxed{0} \end{array}$$
$P\left(\frac{4}{5}\right) = 0$

29. $I(t) = \dfrac{0.5t^3 + 4.5t^2 + 4t}{t + 1}$
$$\begin{array}{r|rrrr} -1 & 0.5 & 4.5 & 4 & 0 \\ & & -0.5 & -4 & 0 \\ \hline & 0.5 & 4 & 0 & \boxed{0} \end{array}$$
The current in the system can be represented by $I(t) = 0.5t^2 + 4t$

30. Possible answer: The divisor is a factor of the polynomial.

31. $a = 2$; $b = 8$; $c = 29$ **32.** $a = -2$; $b = 3$; $c = -6$

33. $a = 3$; $b = 9$; $c = -4$

34. Divide $P(x)$ by $(x - 2)$:
$$\begin{array}{r|rrr} 2 & 1 & 3 & -7 \\ & & 2 & 10 \\ \hline & 1 & 5 & \boxed{3} \end{array}$$
$$\frac{P(x)}{x - 2} = x + 5 + \frac{3}{x - 2}$$

35. Multiply both sides by $x - 2$:
$$(x - 2)\frac{P(x)}{x - 2} = (x - 2)\left(x + 5 + \frac{3}{x - 2}\right)$$
$$P(x) = (x + 5)(x - 2) + 3$$

36. Evaluate P(2):
$P(2) = (2 + 5)(2 - 2) + 3 = 3$

37. The density D is related to mass M and volume V by the equation $D = \frac{M}{V}$.
$$D(h) = \left(\frac{1}{4}h^3 - h^2 + 5h\right) \div \left(\frac{1}{4}\pi h^3\right)$$
$$\begin{array}{r} \frac{1}{\pi} - \frac{4}{\pi h} + \frac{20}{\pi h^2} \\ \frac{1}{4}\pi h^3 + 0h^2 + 0h + 0 \overline{\big)\, \frac{1}{4}h^3 - h^2 + 5h + 0} \\ -\left(\frac{1}{4}h^3 + 0h^2 + 0h + 0\right) \\ \hline -h^2 + 5h + 0 \\ -\left(-h^2 + 0h + 0\right) \\ \hline 5h + 0 \\ -(5h + 0) \\ \hline 0 \end{array}$$
The density can be represented by $D(h) = \frac{1}{\pi} - \frac{4}{\pi h} + \frac{20}{\pi h^2}$.

38. The volume V is related to height h and base B by the equation $V = \frac{1}{3}Bh.$

$$B(x) = \frac{3V}{h} = \frac{3\left(\frac{1}{3}x^3 + \frac{4}{3}x^2 + \frac{2}{3}x - \frac{1}{3}\right)}{x+1}$$
$$= \frac{x^3 + 4x^2 + 2x - 1}{x+1}$$

$$\begin{array}{r|rrrr} -1 & 1 & 4 & 2 & -1 \\ & & -1 & -3 & 1 \\ \hline & 1 & 3 & -1 & \boxed{0} \end{array}$$

The area of the base can be represented by $B(x) = x^2 + 3x - 1$.

39.
$$\begin{array}{r} y^2 + 5 \\ y^2 + 0y + 4\overline{\big)\, y^4 + 0x^3 + 9y^2 + 0x + 20} \\ \underline{-\left(y^4 + 0y^3 + 4y^2\right)} \\ 5y^2 + 0x + 20 \\ \underline{-\left(5y^2 + 0y + 20\right)} \\ 0 \end{array}$$
$$\frac{y^4 + 9y^2 + 20}{y^2 + 4} = y^2 + 5$$

40.
$$\begin{array}{r|rrr} \frac{1}{2} & 2 & -5 & 2 \\ & & 1 & -2 \\ \hline & 2 & -4 & \boxed{0} \end{array}$$
$$\frac{2x^2 - 5x + 2}{x - \frac{1}{2}} = 2x - 4$$

41.
$$\begin{array}{r} x^2 - 5x - 12 \\ 3x + 4\overline{\big)\, 3x^3 - 11x^2 - 56x - 48} \\ \underline{-\left(3x^3 + 4x^2\right)} \\ -15x^2 - 56x \\ \underline{-\left(-15x^2 - 20x\right)} \\ -36x - 48 \\ \underline{-(-36x - 48)} \\ 0 \end{array}$$
$$\frac{3x^3 - 11x^2 - 56x - 48}{3x + 4} = x^2 - 5x - 12$$

42.
$$\begin{array}{r} -y^2 + 6 \\ -y^2 + 0y + 10\overline{\big)\, y^4 + 0y^3 - 16y^2 - 0y + 60} \\ \underline{-\left(y^4 + 0y^3 - 10y^2\right)} \\ -6y^2 + 0y + 60 \\ \underline{-\left(-6y^2 + 0y + 60\right)} \\ 0 \end{array}$$
$$\frac{60 - 16y^2 + y^4}{10 - y^2} = -y^2 + 6$$

43.
$$\begin{array}{r} t - 4 \\ t^2 - 3t + 0\overline{\big)\, t^3 - 7t^2 + 12t} \\ \underline{-\left(t^3 - 3t^2 + 0t\right)} \\ -4t^2 + 12t \\ \underline{-\left(-4t^2 + 12t\right)} \\ 0 \end{array}$$
$$\frac{t^3 - 7t^2 + 12t}{t^2 - 3t} = t - 4$$

44.
$$\begin{array}{r|rrr} 1 & 1 & -18 & 14 \\ & & 1 & -17 \\ \hline & 1 & -17 & \boxed{-3} \end{array}$$
$$\frac{y^2 - 18y + 14}{y - 1} = y - 17 - \frac{3}{y - 1}$$

45.
$$\begin{array}{r|rrrrr} 6 & 1 & -3 & -28 & 59 & 6 \\ & & 6 & 18 & -60 & -6 \\ \hline & 1 & 3 & -10 & -1 & \boxed{0} \end{array}$$
$$\frac{x^4 - 3x^3 - 28x^2 + 59x + 6}{x - 6} = x^3 + 3x^2 - 10x - 1$$

46.
$$\begin{array}{r} d + 4 \\ 2d + 2\overline{\big)\, 2d^2 + 10d + 8} \\ \underline{-\left(2d^2 + 2d\right)} \\ 8d + 8 \\ \underline{-(8d + 8)} \\ 0 \end{array}$$
$$\frac{2d^2 + 10d + 8}{2d + 2} = d + 4$$

47.
$$\begin{array}{r|rrrrr} 6 & 1 & -7 & 9 & -22 & 25 \\ & & 6 & -6 & 18 & -24 \\ \hline & 1 & -1 & 3 & -4 & \boxed{1} \end{array}$$
$$\frac{x^4 - 7x^3 + 9x^2 - 22x + 25}{x - 6} = x^3 - x^2 + 3x - 4 + \frac{1}{x - 6}$$

48.
$$\begin{array}{r|rrrr} 1 & 6 & -14 & 10 & -4 \\ & & 6 & -8 & 2 \\ \hline & 6 & -8 & 2 & \boxed{-2} \end{array}$$
$$\frac{6x^3 - 14x^2 + 10x - 4}{x - 1} = 6x^2 - 8x + 2 - \frac{2}{x - 1}$$

49. Solution B is correct. In solution A, the second row is subtracted from the first row instead of being added.

50. No;
$$\begin{array}{r|rrrr} -3 & 3 & 5 & 2 & -12 \\ & & -9 & 12 & -42 \\ \hline & 3 & -4 & 14 & \boxed{54} \end{array}$$
possible answer: by synthetic substitution, the remainder is not 0, so $x + 3$ is not a factor of the polynomial.

51. Possible answer: The divisor must be a linear binomial with a leading coefficient of 1. The dividend must be written in standard form with 0 representing any missing terms.

52a. 35;
$$g(5) = \frac{1}{6}(5)^3 + \frac{1}{2}(5)^2 + \frac{1}{3}(5) = 35$$

b.
$$\begin{array}{r|rrrr} 24 & \frac{1}{6} & \frac{1}{2} & \frac{1}{3} & 0 \\ & & 4 & 108 & 2600 \\ \hline & \frac{1}{6} & \frac{9}{2} & \frac{325}{3} & \boxed{2600} \end{array}$$
$g(24) = 2600$

TEST PREP

53. B
$$\begin{array}{r|rrr} -3 & 2 & 6 & 3 \\ & & -6 & 0 \\ \hline & 2 & 0 & \boxed{3} \end{array}$$

54. H
$$\frac{6a^2b + 9b^2}{3a^2} = \frac{6a^2b}{3a^2} + \frac{9b^2}{3a^2}$$
$$= 2b + \frac{3b^2}{a^2}$$

55. D

$$\begin{array}{r|rrr} 4 & 1 & 3 & -28 \\ & & 4 & 28 \\ \hline & 1 & 7 & \boxed{0} \end{array}$$

$$\frac{x^2 + 3x - 28}{x - 4} = x + 7$$

56.

$$\begin{array}{r|rrrrr} -2 & 3 & 0 & -6 & 0 & 12 \\ & & -6 & 12 & -12 & 24 \\ \hline & 3 & -6 & 6 & -12 & \boxed{36} \end{array}$$

$f(-2) = 36$

CHALLENGE AND EXTEND

57. $P(-4) = 4(-4)^9 + 7(-4)^7 - 6(-4)^5 - 5(-4)^4 - (-4)^2 + 3(-4) - 2$
$= -1{,}189{,}150$

58. $P(-1) = 4(-1)^9 + 7(-1)^7 - 6(-1)^5 - 5(-1)^4 - (-1)^2 + 3(-1) - 2$
$= -28$

59. $P(1) = 4(1)^9 + 7(1)^7 - 6(1)^5 - 5(1)^4 - (1)^2 + 3(1) - 2$
$= 0$

60. $P(3) = 4(3)^9 + 7(3)^7 - 6(3)^5 - 5(3)^4 - (3)^2 + 3(3) - 2$
$= 89{,}260$

61.

$$\begin{aligned} P(-3) &= 0 \\ 2(-3)^3 + 3(-3)^2 - k(-3) - 27 &= 0 \\ -54 + 27 + 3k - 27 &= 0 \\ -54 + 3k &= 0 \\ +54 \qquad & \;\; +54 \\ 3k &= 54 \\ \frac{3k}{3} &= \frac{54}{3} \\ k &= 18 \end{aligned}$$

62.

$$\begin{array}{r} 5a \quad + 2b \\ ab - b^2 \overline{)\, 5a^2b - 3ab^2 - 2b^3} \\ \underline{-\left(5a^2b - 5ab^2\right)} \qquad\quad \\ 2ab^2 - 2b^3 \\ \underline{-\left(2ab^2 - 2b^3\right)} \\ 0 \end{array}$$

$$\frac{5a^2b - 3ab^2 - 2b^3}{ab - b^2} = 5a + 2b$$

63a. The density can be modeled by

$$\begin{aligned} D(d) &= \frac{M(d)}{V(d)} \\ &= \frac{\left(3.96 \times 10^{12}\right)d^3 - \left(6.50 \times 10^{17}\right)d^2 + \left(2.56 \times 10^{22}\right)d - 5.56 \times 10^{25}}{\frac{1}{6}\pi d^3} \\ &= \frac{\left(3.96 \times 10^{12}\right)d^3}{\frac{1}{6}\pi d^3} + \frac{\left(-6.50 \times 10^{17}\right)d^2 + \left(2.56 \times 10^{22}\right)d - 5.56 \times 10^{25}}{\frac{1}{6}\pi d^3} \\ &= \frac{\left(2.376 \times 10^{13}\right)}{\pi} + \frac{(-3.90 \times 10^{18})d^2 + \left(1.536 \times 10^{23}\right)d - 3.336 \times 10^{26}}{\pi d^3} \end{aligned}$$

b. $\dfrac{2.376 \times 10^{13}}{\pi} + \dfrac{(-3.90 \times 10^{18})(142{,}984)^2 + \left(1.536 \times 10^{23}\right)(142{,}984) - 3.336 \times 10^{26}}{\pi(142{,}984)^3}$

$\approx 1.236 \times 10^{12}$ kg/km^3

c. $\dfrac{2.376 \times 10^{13}}{\pi} + \dfrac{(-3.90 \times 10^{18})(49{,}528)^2 + \left(1.536 \times 10^{23}\right)(49{,}528) - 3.336 \times 10^{26}}{\pi(49{,}528)^3}$

$\approx 1.556 \times 10^{12}$ kg/km^3

SPIRAL REVIEW

64. Let x represent the number of students who took the survey.

70% of $x = 448$
$0.7x = 448$
$x = 640$

Therefore, 640 students took the survey.

65. $x = -\dfrac{b}{2a} = -\dfrac{2}{2(-4)} = \dfrac{-2}{-8} = 0.25$

$$\begin{aligned} f(0.25) &= -4(0.25)^2 + 2(0.25) + 1 \\ &= -0.25 + 0.5 + 1 \\ &= 1.25 \end{aligned}$$

The maximum is 1.25.
D: $\{x \mid x \in \mathbb{R}\}$;
R: $\{y \mid y \le 1.25\}$.

66. $x = -\dfrac{b}{2a} = -\dfrac{(-5)}{2\left(\frac{1}{2}\right)} = 5$

$$\begin{aligned} g(5) &= \frac{1}{2}(5)^2 - 5(5) + 6 \\ &= 12.5 - 25 + 6 \\ &= -6.5 \end{aligned}$$

The minimum is -6.5.
D: $\{x \mid x \in \mathbb{R}\}$;
R: $\{y \mid y \ge -6.5\}$.

67. $x = -\dfrac{b}{2a} = -\dfrac{(-4)}{2\left(\frac{1}{3}\right)} = 6$

$$\begin{aligned} f(6) &= \frac{1}{3}(6)^2 - 4(6) + 10 \\ &= 12 - 24 + 10 \\ &= -2 \end{aligned}$$

The minimum is -2.
D: $\{x \mid x \in \mathbb{R}\}$;
R: $\{y \mid y \ge -2\}$.

68. $x = -\dfrac{b}{2a} = -\dfrac{(-2)}{2\left(-\frac{1}{4}\right)} = -4$

$$\begin{aligned} g(-4) &= -\frac{1}{4}(-4)^2 - 2(-4) + 6 \\ &= -4 + 8 + 6 \\ &= 10 \end{aligned}$$

The maximum is 10.
D: $\{x \mid x \in \mathbb{R}\}$;
R: $\{y \mid y \le 10\}$.

69. $4x^2y\left(3xy^2 + 6x + 5y^3\right)$
$= 4x^2y\left(3xy^2\right) + 4x^2y(6x) + 4x^2y\left(5y^3\right)$
$= 12x^3y^3 + 24x^3y + 20x^2y^4$

70. $5y^2\left(3xy + 4x^2y^2 - 8x^3y\right)$
$= 5y^2(3xy) + 5y^2\left(4x^2y^2\right) + 5y^2\left(-8x^3y\right)$
$= 15xy^3 + 20x^2y^4 - 40x^3y^3$

71.

$$\begin{array}{r} 2x^2 - 2xy + 2y^2 \\ \underline{2x - 2y} \\ -4x^2y + 4xy^2 - 4y^3 \\ \underline{4x^3 - 4x^2y + 4xy^2 \qquad\qquad} \\ 4x^3 - 8x^2y + 8xy^2 - 4y^3 \end{array}$$

72. $2(y-2)^4$
$= 2(y-2)(y-2)(y-2)(y-2)$
$= 2\left(y^2-4y+4\right)\left(y^2-4y+4\right)$

	y^2	$-4y$	4
y^2	y^4	$-4y^3$	$4y^2$
$-4y$	$-4y^3$	$16y^2$	$-16y$
4	$4y^2$	$-16y$	16

$= 2\left(y^4-8y^3+24y^2-32y+16\right)$
$= 2y^4-16y^3+48y^2-64y+32$

6-4 FACTORING POLYNOMIALS, PAGES 430–435

CHECK IT OUT!

1a.
$$\begin{array}{r|rrr} -2 & 4 & -2 & 5 \\ & & -8 & 20 \\ \hline & 4 & -10 & \boxed{25} \end{array}$$
$P(-2) \neq 0$, so $(x+2)$ is not a factor of $P(x) = 4x^2-2x+5$

b. $(3x-6) = 3(x-2)$
$$\begin{array}{r|rrrrr} 2 & 3 & -6 & 6 & 3 & -30 \\ & & 6 & 0 & 12 & 30 \\ \hline & 3 & 0 & 6 & 15 & \boxed{0} \end{array}$$
$P(2) = 0$, so $(3x-6)$ is a factor of $P(x) = 3x^4-6x^3+6x^2+3x-30$

2a. $x^3-2x^2-9x+18$
$= \left(x^3-2x^2\right)+(-9x+18)$
$= x^2(x-2)-9(x-2)$
$= (x-2)\left(x^2-9\right)$
$= (x-2)(x+3)(x-3)$

b. $2x^3+x^2+8x+4$
$= \left(2x^3+x^2\right)+(8x+4)$
$= x^2(2x+1)+4(2x+1)$
$= (2x+1)\left(x^2+4\right)$

3a. $8+z^6$
$= 2^3+\left(z^2\right)^3$
$= \left(2+z^2\right)\left[2^2-2\cdot z^2+\left(z^2\right)^2\right]$
$= \left(2+z^2\right)\left(4-2z^2+z^4\right)$

b. $2x^5-16x^2$
$= 2x^2\left(x^3-8\right)$
$= 2x^2\left(x^3-2^3\right)$
$= 2x^2(x-2)\left(x^2+x\cdot 2+2^2\right)$
$= 2x^2(x-2)\left(x^2+2x+4\right)$

4. $V(x)$ has 3 real zeros at $x = 1, 3, 4$.
The corresponding factors are $(x-1)$, $(x-3)$, and $(x-4)$.
$$\begin{array}{r|rrrr} 1 & 1 & -8 & 19 & -12 \\ & & 1 & -7 & 12 \\ \hline & 1 & -7 & 12 & \boxed{0} \end{array}$$
$V(x) = (x-1)\left(x^2-7x+12\right)$
$$\begin{array}{r|rrr} 3 & 1 & -7 & 12 \\ & & 3 & -12 \\ \hline & 1 & -4 & \boxed{0} \end{array}$$
$V(x) = (x-1)(x-3)(x-4)$

THINK AND DISCUSS

1. Possible answer: For a linear binomial $x-a$, use synthetic substitution to find $P(a)$. If $P(a) = 0$, then the linear binomial is a factor.
2. Possible answer: First, factor out any common monomial. Then if each term of the binomial is a perfect cube, you can use the sum or difference of cubes to factor.
3.

Method	Polynomial	Factored Form
Difference of two squares	(s^2-t^2)	$(s+t)(s-t)$
Difference of two cubes	(s^3-t^3)	$(s-t)(s^2+st+t^2)$
Sum of two cubes	(s^3+t^3)	$(s+t)(s^2-st+t^2)$

EXERCISES

GUIDED PRACTICE

1.
$$\begin{array}{r|rrrrr} -2 & 2 & 2 & -1 & -5 & -4 \\ & & -2 & 0 & 1 & 4 \\ \hline & 2 & 0 & -1 & -4 & \boxed{0} \end{array}$$
$P(-1) = 0$, so $(x+1)$ is a factor of $P(x)$

2.
$$\begin{array}{r|rrrr} -2 & 5 & 1 & 0 & -7 \\ & & 10 & 22 & 44 \\ \hline & 5 & 11 & 22 & \boxed{37} \end{array}$$
$P(2) \neq 0$, so $(x-2)$ is not a factor of $P(x) = 5x^3+x^2-7$

3. $(2x-4) = 2(x-2)$
$$\begin{array}{r|rrrrrr} 2 & 2 & -4 & 0 & 2 & -2 & -4 \\ & & 4 & 0 & 0 & 4 & 4 \\ \hline & 2 & 0 & 0 & 2 & 2 & \boxed{0} \end{array}$$
$P(2) = 0$, so $(2x-4)$ is a factor of $P(x) = 2x^5-4x^4+2x^2-2x-4$

4. x^3+x^2-x-1
$= \left(x^3+x^2\right)+(-x-1)$
$= x^2(x+1)-1(x+1)$
$= (x+1)\left(x^2-1\right)$
$= (x+1)(x+1)(x-1)$

5. $x^3+5x^2-4x-20$
$= \left(x^3+5x^2\right)+(-4x-20)$
$= x^2(x+5)-4(x+5)$
$= (x+5)\left(x^2-4\right)$
$= (x+5)(x+2)(x-2)$

6. $8x^3 + 4x^2 - 2x - 1$
$= (8x^3 + 4x^2) + (-2x - 1)$
$= 4x^2(2x + 1) - 1(2x + 1)$
$= (2x + 1)(4x^2 - 1)$
$= (2x + 1)(2x + 1)(2x - 1)$

7. $2x^3 - 2x^2 - 8x + 8$
$= (2x^3 - 2x^2) + (-8x + 8)$
$= 2x^2(x - 1) - 8(x - 1)$
$= (x - 1)(2x^2 - 8)$
$= 2(x - 1)(x^2 - 4)$
$= 2(x - 1)(x + 2)(x - 2)$

8. $2x^3 - 3x^2 - 2x + 3$
$= (2x^3 - 3x^2) + (-2x + 3)$
$= x^2(2x - 3) - 1(2x + 3)$
$= (2x - 3)(x^2 - 1)$
$= (2x - 3)(x + 1)(x - 1)$

9. $12x^2 + 3x - 24x - 6$
$= (12x^2 + 3x) + (-24x - 6)$
$= 3x(4x + 1) - 6(4x + 1)$
$= (4x + 1)(3x - 6)$
$= 3(4x + 1)(x - 2)$

10. $8 - m^6$
$= 2^3 - (m^2)^3$
$= (2 - m^2)[2^2 + 2 \cdot m^2 + (m^2)^2]$
$= (2 - m^2)(4 + 2m^2 + m^4)$

11. $2t^7 + 54t^4$
$= 2t^4(t^3 + 27)$
$= 2t^4(t^3 + 3^3)$
$= 2t^4(t + 3)(t^2 - t \cdot 3 + 3^2)$
$= 2t^4(t + 3)(t^2 - 3t + 9)$

12. $x^3 + 64$
$= x^3 + 4^3$
$= (x + 4)(x^2 - x \cdot 4 + 4^2)$
$= (x + 4)(x^2 - 4x + 16)$

13. $27 + x^3$
$= 3^3 + x^3$
$= (3 + x)(3^2 - 3 \cdot x + x^2)$
$= (3 + x)(9 - 3x + x^2)$

14. $4t^5 - 32t^2$
$= 4t^2(t^3 - 8)$
$= 4t^2(t^3 - 2^3)$
$= 4t^2(t - 2)(t^2 + t \cdot 2 + 2^2)$
$= 4t^2(t - 2)(t^2 + 2t + 4)$

15. $y^3 - 125$
$= y^3 - 5^3$
$= (y - 5)(y^2 + y \cdot 5 + 5^2)$
$= (y - 5)(y^2 + 5y + 25)$

16. $V(x)$ has 3 real zeros at $x = -2, -5, 7$.
The corresponding factors are
$(x + 2)$, $(x + 5)$, and $(x - 7)$.

−2	1	0	−39	−70
		−2	4	70
	1	−2	−35	0

$V(x) = (x + 2)(x^2 - 2x - 35)$

−5	1	−2	−35
		−5	35
	1	−7	0

$V(x) = (x + 2)(x + 5)(x - 7)$

PRACTICE AND PROBLEM SOLVING

17.

3	4	−12	0	2	−6	−5	10
		12	0	0	6	0	−15
	4	0	0	2	0	−5	−5

$P(3) \neq 0$, so $(x - 5)$ is not a
factor of $P(x) = 4x^6 - 12x^5 + 2x^3 - 6x^2 - 5x + 10$

18.

8	1	−8	0	0	8	−64
		8	0	0	0	64
	1	0	0	0	8	0

$P(8) = 0$, so $(x - 8)$ is a
factor of $P(x) = x^5 - 8x^4 + 8x - 64$

19. $(3x + 12) = 3(x + 4)$

−4	3	12	0	6	24
		−12	0	0	−24
	3	0	0	6	0

$P(-4) = 0$, so $(3x + 12)$ is a
factor of $P(x) = 3x^4 + 12x^3 + 6x + 24$

20. $8y^3 - 4y^2 - 50y + 25$
$= (8y^3 - 4y^2) + (-50y + 25)$
$= 4y^2(2y - 1) - 25(2y - 1)$
$= (2y - 1)(4y^2 - 25)$
$= (2y - 1)(2y + 5)(2y - 5)$

21. $4b^3 + 3b^2 - 16b - 12$
$= (4b^3 + 3b^2) + (-16b - 12)$
$= b^2(4b + 3) - 4(4b + 3)$
$= (4b + 3)(b^2 - 4)$
$= (4b + 3)(b + 2)(b - 2)$

22. $3p^3 - 21p^2 - p + 7$
$= (3p^3 - 21p^2) + (-p + 7)$
$= 3p^2(p - 7) - (p - 7)$
$= (p - 7)(3p^2 - 1)$

23. $3x^3 + x^2 - 27x - 9$
$= (3x^3 + x^2) + (-27x - 9)$
$= x^2(3x + 1) - 9(3x + 1)$
$= (3x + 1)(x^2 - 9)$
$= (3x + 1)(x + 3)(x - 3)$

24. $8z^2 - 4z + 10z - 5$
$= (8z^2 - 4z) + (10z - 5)$
$= 4z(2z - 1) + 5(2z - 1)$
$= (2z - 1)(4z + 5)$

25. $5x^3 - x^2 - 20x + 4$
$= (5x^3 - x^2) + (-20x + 4)$
$= x^2(5x - 1) - 4(5x - 1)$
$= (5x - 1)(x^2 - 4)$
$= (5x - 1)(x + 2)(x - 2)$

26. $125 + z^3$
$= 5^3 + z^3$
$= (5 + z)(5^2 - 5 \cdot z + z^2)$
$= (5 + z)(25 - 5z + z^2)$

27. $s^6 - 1$
$= (s^2)^3 - 1^3$
$= (s^2 - 1)\left[(s^2)^2 + s^2 \cdot 1 + 1^2\right]$
$= (s + 1)(s - 1)s^4 + s^2 + 1$
$= (s + 1)(s - 1)(s^4 + s^2 + 1)$
$= (s + 1)(s - 1)(s^2 + s + 1)(s^2 - s + 1)$

28. $24n^2 + 3n^5$
$= 3n^2(8 + n^3)$
$= 3n^2(2^3 + n^3)$
$= 3n^2(2 + n)(2^2 - 2 \cdot n + n^2)$
$= 3n^2(2 + n)(4 - 2n + n^2)$

29. $6x^4 - 162x$
$= 6x(x^3 - 27)$
$= 6x(x^3 - 3^3)$
$= 6x(x - 3)(x^2 + x \cdot 3 + 3^2)$
$= 6x(x - 3)(x^2 + 3x + 9)$

30. $40 - 5t^3$
$= 5(8 - t^3)$
$= 5(2^3 - t^3)$
$= 5(2 - t)(2^2 + 2 \cdot t + t^2)$
$= 5(2 - t)(4 + 2t + t^2)$

31. $y^5 + 27y^2$
$= y^2(y^3 + 27)$
$= y^2(y^3 + 3^3)$
$= y^2(y + 3)(y^2 - y \cdot 3 + 3^2)$
$= y^2(y + 3)(y^2 - 3y + 9)$

32. $V(x)$ has 1 real zero at $x = -3, 2$.
The corresponding factors are $(x + 3)$, and $(x - 2)$.

-3	-28	-28	168
		84	-168
	-28	56	0

$V(x) = (x + 3)(-28x + 56)$
$V(x) = -28(x + 3)(x - 2)$

33. $x^6 - 14x^4 + 49x^2$
$= x^2(x^4 - 14x^2 + 49)$
$= x^2(x^2 - 7)(x^2 - 7)$

34. $2x^3 + x^2 - 72x - 36$
factors of -36 include $\pm1, \pm2, \pm3$ and $\pm4, \pm6$ etc.
it appears that $f(6) = 0$ and $f(-6) = 0$;
so $(x + 6)$ and $(x - 6)$ are factors.
Use long division to get the remaining factor.
We see that
$2x^3 + x^2 - 72x - 36 = (x + 6)(x - 6)(2x + 1)$

35. $4x^3 + x^2 - 16x - 4$
factors of 4 are ±1, ±2 and ±4.
it appears that $f(2) = 0$, so $(x - 2)$ is a factor.
use long division to find the other factors.
we see that $4x^2 + 9x + 2 = (x + 2)(4x + 1)$ are the other factors.
so $4x^3 + x^2 - 16x - 4 = (x - 2)(x + 2)(4x + 1)$

36. $9x^9 - 16x^7 + 9x^6 - 16x^4$
$= x^4(9x^5 - 16x^3 + 9x^2 - 16)$
$= x^4\left[(9x^5 - 16x^3) + (9x^2 - 16)\right]$
$= x^4\left[x^3(9x^2 - 16) + (9x^2 - 16)\right]$
$= x^4(9x^2 - 16)(x^3 + 1)$
$= x^4(3x + 4)(3x - 4)(x + 1)(x^2 - x \cdot 1 + 1^2)$
$= x^4(3x + 4)(3x - 4)(x + 1)(x^2 - x + 1)$

37. $8x^7 - 4x^5 - 18x^3 + 9x$
$= x(8x^6 - 4x^4 - 18x^2 + 9)$
$= x\left[(8x^6 - 4x^4) + (-18x^2 + 9)\right]$
$= x\left[4x^4(2x^2 - 1) - 9(2x^2 - 1)\right]$
$= x(2x^2 - 1)(4x^4 - 9)$
$= x(2x^2 - 1)(2x^2 + 3)(2x^2 - 3)$

38. $x^{13} - 15x^9 - 16x^5$
$= x^5(x^8 - 15x^4 - 16)$
$= x^5(x^4 + 1)(x^4 - 16)$
$= x^5(x^4 + 1)(x^2 + 4)(x^2 - 4)$
$= x^5(x^4 + 1)(x^2 + 4)(x + 2)(x - 2)$

39. $a = 3$; $d = 72$;
possible answer: the value of a is the leading coefficient, 3; the value of d is the product of the constant terms of each factor and the leading coefficient 3.

40a.

7	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{1}{3}$	0
		$\frac{7}{6}$	$\frac{35}{3}$	84
	$\frac{1}{6}$	$\frac{5}{3}$	12	84

$g(7) = 84$

b. Possible answer: For $n = 7$, $g(7) = 84$.
So if $\frac{1}{6}n^3 + \frac{1}{2}n^2 + \frac{1}{3}n = 84$
then $n^3 + 3n^2 + 2n = 504$
and $n^3 + 3n^2 + 2n - 504 = 0$
The equation is true for $n = 7$, so $n - 7$ must be a factor of $n^3 + 3n^2 + 2n - 504$.

c.

7	1	3	2	−504
		7	70	504
	1	10	72	0

$(n-7)(n^2+10n+72)$

41.

2	1	−2	5	−9	−2
		2	0	10	2
	1	0	5	1	0

$(x-2)$ is a factor of $P(x)$.
$P(x)=(x-2)(x^3+5x+1)$

42.

1	4	−4	−2	3	−1	−7	7
		4	0	−2	1	0	−7
	4	0	−2	1	0	−7	0

$(x-1)$ is a factor of $P(x)$.
$P(x)=(x-1)(4x^5-2x^3+x^2-7)$

43.

−2	2	4	0	−6	−9	6
		−4	0	0	12	−6
	2	0	0	−6	3	0

$(x+2)$ is a factor of $P(x)$.
$P(x)=(x+2)(2x^4-6x+3)$

44.

4	2	−9	7	−14	8
		8	−4	12	−8
	2	−1	3	−2	0

$(x-4)$ is a factor of $P(x)$.
$P(x)=(x-4)(2x^3-x^2+3x-2)$

45a. $f(t)=-t^4+44t^3-612t^2+2592t$
$=-t(t^3-44t^2+612t-2592)$
$=-t(t^3-8t^2-36t^2+612t-2592)$
$=-t[(t^3-8t^2)+(-36t^2+612t-2592)]$
$=-t[t^2(t-8)+(-36t+324)(t-8)]$
$=-t(t-8)(t^2-36t+324)$
$=-t(t-8)(t-18)(t-18)$

b. 1985 − 1980 = 5 yrs

5	−1	44	−612	2592	0
		−5	195	−2085	2535
	−1	39	−417	507	2535

In 1985, the company's profit was $2,535,000.

c.

15	−1	44	−612	2592	0
		−15	435	−2655	−945
	−1	29	−177	−63	−945

$f(15)=-945$;
The company lost $945,000 in 1980 + 15 = 1995.

d. Possible answer: the company will continue to lose money after breaking even in 1998.

46. $B(x)=\dfrac{V(x)}{h(x)}=\dfrac{2x^3-17x^2+27x+18}{x-6}$

6	2	−17	27	18
		12	−30	−18
	2	−5	−3	0

$B(x)=2x^2-5x-3$

47. $B(x)=\dfrac{V(x)}{h(x)}=\dfrac{x^4-16}{x+2}$

−2	1	0	0	0	−16
		−2	4	−8	16
	1	−2	4	−8	0

$B(x)=x^3-2x^2+4x-8$

48. $B(x)=\dfrac{V(x)}{h(x)}=\dfrac{3x^6-3x^3}{x+1}$

−1	3	0	0	3	0	0	0
		−3	3	−3	0	0	0
	3	−3	3	0	0	0	0

$B(x)=3x^5-3x^4+3x^3$

49. Possible answer: If you know a possible root of the related equation, you can use synthetic division to test the value. If the remainder is 0, the value corresponds to a factor of the polynomial.

TEST PREP

50. C

−5	1	2	−9	30
		−5	15	−30
	1	−3	6	0

51. J
Find if $(x-4)$, $(x+2)$, and $(x+1)$ are factors of $x^3-x^2-10x-8$

4	1	−1	−10	−8
		4	12	8
	1	3	2	0

So, $(x-4)$ is a factor.

−2	1	3	2
		−2	−2
	1	1	0

So $(x+2)$ and $(x+1)$ are factors.

52. $4p^5-16p^3-20p$
$=4p(p^4-4p^2-5)$
$=4p(p^4-5p^2+p^2-5)$
$=4p[(p^4-5p^2)+(p^2-5)]$
$=4p(p^4-4p^2-5)$
$=4p(p^2-5)(p^2+1)$

CHALLENGE AND EXTEND

53. Factor as a sum of two cubes:
$(x-3)^3+8$
$=(x-3)^3+2^3$
$=[(x-3)+2][(x-3)^2-(x-3)\cdot 2+2^2]$
$=[(x-3)+2][(x-3)^2-2(x-3)+4]$
Simplify:
$=[(x-3)+2][(x-3)^2-2(x-3)+4]$
$=(x-3+2)(x^2-6x+9-2x+6+4)$
$=(x-1)(x^2-8x+19)$

54. Factor as a difference of two cubes:
$(2a+b)^3-b^3$
$=[(2a+b)-b][(2a+b)^2+(2a+b)\cdot b+b^2]$
$=[(2a+b)-b][(2a+b)^2+(2a+b)b+b^2]$
Simplify:
$=[(2a+b)-b][(2a+b)^2+(2a+b)b+b^2]$
$=(2a+b-b)(4a^2+4ab+b^2+2ab+b^2+b^2)$
$=(2a)(4a^2+6ab+3b^2)$

55. $$\begin{array}{r|rrr} 1 & 1 & 0 & -1 \\ & & 1 & 1 \\ \hline & 1 & 1 & \boxed{0} \end{array}$$
$\left(x^2-1\right) \div (x-1) = (x+1)$
$$\begin{array}{r|rrrr} 1 & 1 & 0 & 0 & -1 \\ & & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & \boxed{0} \end{array}$$
$\left(x^3-1\right) \div (x-1) = \left(x^2+x+1\right)$
$$\begin{array}{r|rrrrr} 1 & 1 & 0 & 0 & 0 & -1 \\ & & 1 & 1 & 1 & 1 \\ \hline & 1 & 1 & 1 & 1 & \boxed{0} \end{array}$$
$\left(x^4-1\right) \div (x-1) = \left(x^3+x^2+x+1\right)$
$\left(x^n-1\right) \div (x-1) = \left(x^{n-1}+x^{n-2}+\ldots+x+1\right)$,
with all coefficients of the quotient equal to 1.

56. $u = \sqrt{x}$;
$u^2 + 3u + 2$
$= (u+2)(u+1)$
$= (\sqrt{x}+2)(\sqrt{x}+1)$

57. $u = 3x - 8$;
$u^2 + 6u + 9$
$= (u+3)(u+3)$
$= [(3x-8)+3][(3x-8)+3]$
$= (3x-5)(3x-5)$

58. $u = x^{\frac{1}{4}}$
$2\left(u^2 - u - 6\right)$
$= 2(u-3)(u+2)$
$= 2\left(x^{\frac{1}{4}} - 3\right)\left(x^{\frac{1}{4}} + 2\right)$

59. $u = x - \frac{1}{3}$
$\frac{1}{2}u^2 + \frac{5}{2}u - 42$
$= \frac{1}{2}\left(u^2 + 5u - 84\right)$
$= \frac{1}{2}(u+12)(u-7)$
$= \frac{1}{2}\left[\left(x-\frac{1}{3}\right)+12\right]\left[\left(x-\frac{1}{3}\right)-7\right]$
$= \frac{1}{2}\left(x+\frac{35}{3}\right)\left(x-\frac{22}{3}\right)$

SPIRAL REVIEW

60. $20b + 15t + 25z = 100$

61. $(2+4i)(2-4i)$
$= 4 - 8i + 8i - 16i^2$
$= 4 - 16(-1)$
$= 4 + 16$
$= 20$

62. $4i(6+9i)$
$= 24i + 36i^2$
$= 24i + 36(-1)$
$= -36 + 24i$

63. $\frac{3+4i}{5+i}$
$= \frac{3+4i}{5+i}\left(\frac{5-i}{5-i}\right)$
$= \frac{15 - 3i + 20i - 4i^2}{25 - 5i + 5i - i^2}$
$= \frac{15 + 17i + 4}{25 + 1}$
$= \frac{19 + 17i}{26}$
$= \frac{19}{26} + \frac{17}{26}i$

64. $\frac{8-2i}{i}$
$= \frac{8-2i}{i}\left(\frac{-i}{-i}\right)$
$= \frac{-8i + 2i^2}{-i^2}$
$= \frac{-8i - 2}{1}$
$= -2 - 8i$

65. $$\begin{array}{r|rrr} 5 & 3 & -2 & -1 \\ & & 15 & 65 \\ \hline & 3 & 13 & \boxed{64} \end{array}$$
$P(5) = 64$

66. $$\begin{array}{r|rrrr} -2 & 1 & -4 & 1 & -2 \\ & & -2 & 12 & -26 \\ \hline & 1 & -6 & 13 & \boxed{-28} \end{array}$$
$P(-2) = -28$

67. $$\begin{array}{r|rrr} -1 & 8 & -5 & 7 \\ & & -8 & 13 \\ \hline & 8 & -13 & \boxed{20} \end{array}$$
$P(-1) = 20$

68. $$\begin{array}{r|rrr} 3 & 6 & -3 & -8 \\ & & 18 & 45 \\ \hline & 6 & 15 & \boxed{37} \end{array}$$
$P(3) = 37$

READY TO GO ON? PAGE 437

1. Standard form: $3x^5 + 4x^2 - 5$
Leading coefficient: 3
Degree: 5
Terms: 3
Name: quintic trinomial

2. Standard form: $13x + 7$
Leading coefficient: 13
Degree: 1
Terms: 2
Name: linear binomial

3. Standard form: $5x^3 + x^2 - 3x + 1$
Leading coefficient: 5
Degree: 3
Terms: 4
Name: cubic polynomial with 4 terms

4. Standard form: $2x^4 - 5x^3 + 8x$
Leading coefficient: 2
Degree: 4
Terms: 3
Name: quartic trinomial

5. $\left(3x^2+1\right) + \left(4x^2+3\right)$
$= \left(3x^2 + 4x^2\right) + (1+3)$
$= 7x^2 + 4$

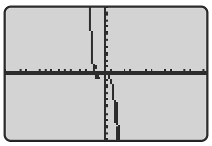
6. $\left(9x^3 - 6x^2\right) - \left(2x^3 + x^2 + 2\right)$
$= \left(9x^3 - 2x^3\right) + \left(-6x^2 - x^2\right) + (-2)$
$= 7x^3 - 7x^2 - 2$

7. $\left(11x^2 + x^3 + 7\right) + \left(5x^3 + 4x^2 - 2x\right)$
$= \left(x^3 + 11x^2 + 7\right) + \left(5x^3 + 4x^2 - 2x\right)$
$= \left(x^3 + 5x^3\right) + \left(11x^2 + 4x^2\right) + (-2x) + (7)$
$= 6x^3 + 15x^2 - 2x + 7$

8. $\left(x^5 - 4x^4 + 1\right) - \left(-7x^4 + 11\right)$
$= \left(x^5\right) + \left(-4x^4 + 7x^4\right) + (1 - 11)$
$= x^5 + 3x^4 - 10$

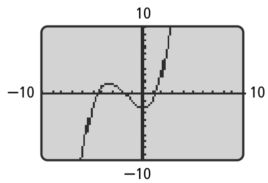
9. $C(100) = (100)^3 - 15(100) + 15 = \$998{,}515$
$C(100)$ represents the cost of manufacturing 100 units, which is $998,515.

10.



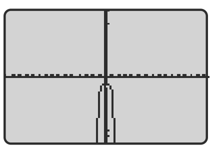
From left to right, it alternately increases and decreases, changing directions twice and crossing the *x*-axis twice. There appear to be 2 real zeros.

11.



From left to right, it alternately increases and decreases, changing direction twice and crossing the *x*-axis 3 times. There appear to be 3 real zeros.

12.



From left to right, its increases then decreases, and never crosses the *x*-axis. There appear to be no real zeros.

13. $2y\left(4x^2 + 7xy\right)$
$= 2y\left(4x^2\right) + 2y(7xy)$
$= 8x^2y + 14xy^2$

14. $(a + b)\left(3ab + b^2\right)$
$= a(3ab) + a\left(b^2\right) + b(3ab) + b\left(b^2\right)$
$= 3a^2b + ab^2 + 3ab^2 + b^3$
$= 3a^2b + 4ab^2 + b^3$

15. $\left(2x + \frac{1}{3}\right)^2$
$= 4x^2 + \frac{4}{3}x + \frac{1}{9}$

16.

$$\begin{array}{r} x^3 - x^2 + 3x + 5 \\ \underline{2x - 3} \\ -3x^3 + 3x^2 - 9x - 15 \\ \underline{2x^4 - 2x^3 + 6x^2 + 10x \qquad\;} \\ 2x^4 - 5x^3 + 9x^2 + x - 15 \end{array}$$

17. $(x - 3)^4$
$= \left[1(x)^4(-3)^0\right] + \left[4(x)^3(-3)^1\right] + \left[6(x)^2(-3)^2\right]$
$+ \left[4(x)^1(-3)^3\right] + \left[1(x)^0(-3)^4\right]$
$= x^4 - 12x^3 + 54x^2 - 108x + 81$

18. $(x + 2y)^3$
$= \left[1(x)^3(2y)^0\right] + \left[3(x)^2(2y)^1\right] + \left[3(x)^1(2y)^2\right]$
$+ \left[1(x)^0(2y)^3\right]$
$= x^3 + 6x^2y + 12xy^2 + 8y^3$

19. $(4x - 1)^4$
$= \left[1(4x)^4(-1)^0\right] + \left[4(4x)^3(-1)^1\right] + \left[6(4x)^2(-1)^2\right]$
$+ \left[4(4x)^1(-1)^3\right] + \left[1(4x)^0(-1)^4\right]$
$= 256x^4 - 256x^3 + 96x^2 - 16x + 1$

20. $V(x) = 3x(x - 4)(2x + 1)$
$= \left(3x^2 - 12x\right)(2x + 1)$
$= 3x^2(2x) + 3x^2(1) - 12x(2x) - 12x(1)$
$= 6x^3 + 3x^2 - 24x^2 - 12x$
$= 6x^3 - 21x^2 - 12x$

21.

$$\begin{array}{r} 3y + 8 \\ 2y - 1\overline{\big)6y^2 + 13y - 8} \\ \underline{-\left(6y^2 - 3y\right)} \\ 16y - 8 \\ \underline{-(16y - 8)} \\ 0 \end{array}$$

$\frac{6y^2 + 13y - 8}{2y - 1} = 3y + 8$

22.

−3	3	11	11	15
		−9	−6	−15
	3	2	5	0

$\frac{3x^3 + 22x^2 + 11x + 15}{x + 3} = 3x^2 + 2x + 5$

23.

−1	1	2	−5	6
		−1	−1	6
	1	1	−6	12

$P(-1) = 12$

24.

2	1	0	1	1	−6
		2	4	10	22
	1	2	5	11	16

$P(2) = 16$

25. $3t^3 - 21t^2 - 12t$
$= 3t\left(t^2 - 7t - 4\right)$

26. $16y^2 - 49$
$= (4y - 7)(4y + 7)$

27. $y^3 + 7y^2 + 2y + 14$
$= \left(y^3 + 7y^2\right) + (2y + 14)$
$= y^2(y + 7) + 2(y + 7)$
$= (y + 7)\left(y^2 + 2\right)$

28. $a^6 + 125$
$= \left(a^2\right)^3 + 5^3$
$= \left(a^2 + 5\right)\left[\left(a^2\right)^2 - a^2 \cdot 5 + 5^2\right]$
$= \left(a^2 + 5\right)\left(a^4 - 5a^2 + 25\right)$

29. $V(x)$ has 3 real zeros at $x = -4, -1, 3$.
The corresponding factors are
$(x + 4)$, $(x + 1)$, and $(x - 3)$.

-4	1	2	-11	-12
		-4	8	12
	1	-2	-3	0

$V(x) = (x + 4)\left(x^2 - 2x - 3\right)$

-1	1	-2	-3
		-1	3
	1	-3	0

$V(x) = (x + 4)(x + 1)(x - 3)$

6-5 FINDING REAL ROOTS OF POLYNOMIAL EQUATIONS, PAGES 438–444

CHECK IT OUT!

1a. $2x^6 - 10x^5 - 12x^4 = 0$
$2x^4\left(x^2 - 5x - 6\right) = 0$
$2x^4(x + 1)(x - 6) = 0$
$2x^4 = 0$, $x + 1 = 0$, or $x - 6 = 0$
$x = 0$ $\quad x = -1$ $\quad x = 6$
The roots are 0, -1, and 6.

b. $x^3 - 2x^2 - 25x = -50$
$x^3 - 2x^2 - 25x + 50 = 0$
$x^2(x - 2) - 25(x - 2) = 0$
$(x - 2)\left(x^2 - 25\right) = 0$
$(x - 2)(x + 5)(x - 5) = 0$
$x - 2 = 0$, $x + 5 = 0$, or $x - 5 = 0$
$x = 2$ $\quad x = -5$ $\quad x = 5$
The roots are 2, 5, and -5.

2a. $x^4 - 8x^3 + 24x^2 - 32x + 16 = 0$
$(x - 2)(x - 2)(x - 2)(x - 2) = 0$
root 2 with multiplicity 4

b. $2x^6 - 22x^5 + 48x^4 + 72x^3 = 0$
$2x^3\left(x^3 - 11x^2 + 24x + 36\right) = 0$
$2x^3\left(x^3 + x^2 - 12x^2 + 24x + 36\right) = 0$
$2x^3\left[x^2(x + 1) + (-12x + 36)(x + 1)\right] = 0$
$2x^3(x + 1)\left(x^2 - 12x + 36\right) = 0$
$2x^3(x + 1)(x - 6)(x - 6) = 0$
root 0 with multiplicity 3; root -1 with multiplicity 1; root 6 with multiplicity 2

3. Let x represent the length in feet.
Then the width is $x - 1$, and the height is $x + 4$.
$x(x - 1)(x + 4) = 12$
$\left(x^2 - x\right)(x + 4) = 12$
$x^3 + 3x^2 - 4x = 12$
$x^3 + 3x^2 - 4x - 12 = 0$
Factors of -12: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm12$

$\frac{p}{q}$	**1**	**3**	**-4**	**-12**
1	1	4	0	-12
2	1	5	6	0
3	1	6	14	30

$(x - 2)\left(x^2 + 5x + 6\right) = 0$
$(x - 2)(x + 2)(x + 3) = 0$
$x - 2 = 0$, $x + 2 = 0$, or $x + 3 = 0$
$x = 2$ $\quad x = -2$ $\quad x = -3$
The length must be positive, so the length should be 2 ft.

4. Possible rational roots: $\pm\frac{1}{2}, \pm 1, \pm2, \pm 4$

$-\frac{1}{2}$	2	-3	-10	-4
		-1	2	4
	2	-4	-8	0

$\left(x + \frac{1}{2}\right)\left(2x^2 - 4x - 8\right) = 0$
Solve $2x^2 - 4x - 8 = 0$
$2\left(x^2 - 2x - 4\right) = 0$
$x = \frac{2 \pm \sqrt{4 + 16}}{2} = 1 \pm \sqrt{5}$
The fully factored equation is:
$2\left(x + \frac{1}{2}\right)\left[x - \left(1 + \sqrt{5}\right)\right]\left[x - \left(1 - \sqrt{5}\right)\right] = 0$
The real roots are $-\frac{1}{2}$ and $1 \pm \sqrt{5}$.

THINK AND DISCUSS

1. Possible answer: The multiplicity of a root is equal to the number of times the factor corresponding to the root appears in the factored form of the equation.

2.

Theorem	Roots	Polynomial
Rational Root Theorem	1, 2, 3	$(x-1)(x-2)(x-3)$
Irrational Root Theorem	$1, \pm\sqrt{2}$	$(x-1)(x^2-2)$

EXERCISES

GUIDED PRACTICE

1. Possible answer: A root with a multiplicity greater than 1 appears as a factor multiple times.

2. $2x^4 + 16x^3 + 32x^2 = 0$
$2x^2\left(x^2 + 8x - 16\right) = 0$
$2x^2(x + 4)^2 = 0$
$2x^2 = 0$, or $(x + 4)^2 = 0$
$x = 0$ $\quad x = -4$
The roots are 0, and -4.

3. $x^4 - 37x^2 + 36 = 0$
$(x^2 - 36)(x^2 - 1) = 0$
$(x - 6)(x + 6)(x - 1)(x + 1) = 0$
$x - 6 = 0, x + 6 = 0, x - 1 = 0, \text{ or } x + 1 = 0$
$x = 6 \quad x = -6 \quad x = 1 \quad x = -1$
The roots are 6, −6, 1, and −1.

4. $4x^7 - 28x^6 = -48x^5$
$4x^7 - 28x^6 + 48x^5 = 0$
$4x^5(x^2 - 7x + 12) = 0$
$4x^5(x - 3)(x - 4) = 0$
$4x^5 = 0, x - 3 = 0, \text{ or } x - 4 = 0$
$x = 0 \quad x = 3 \quad x = 4$
The roots are 0, 3, and 4.

5. $3x^4 + 11x^3 = 4x^2$
$3x^4 + 11x^3 - 4x^2 = 0$
$x^2(3x^2 + 11x - 4) = 0$
$x^2(3x - 1)(x + 4) = 0$
$x^2 = 0, 3x - 1 = 0, \text{ or } x + 4 = 0$
$x = 0 \quad x = \frac{1}{3} \quad x = -4$
The roots are 0, $\frac{1}{3}$, and −4.

6. $2x^3 - 12x^2 = 32x - 192$
$2x^3 - 12x^2 - 32x + 192 = 0$
$2(x^3 - 6x^2 - 16x + 96) = 0$
$2[x^2(x - 6) - 16(x - 6)] = 0$
$2(x - 6)(x^2 - 16) = 0$
$2(x - 6)(x + 4)(x - 4) = 0$
$x - 6 = 0, x + 4 = 0, \text{ or } x - 4 = 0$
$x = 6 \quad x = -4 \quad x = 4$
The roots are 6, −4, and 4.

7. $x^4 + 100 = 29x^2$
$x^4 - 29x^2 + 100 = 0$
$(x^2 - 4)(x^2 - 25) = 0$
$(x + 2)(x - 2)(x + 5)(x - 5) = 0$
$x + 2 = 0, x - 2 = 0, x + 5 = 0, \text{ or } x - 5 = 0$
$x = -2 \quad x = 2 \quad x = -5 \quad x = 5$
The roots are −2, 2, −5, and 5.

8. $2x^5 + 12x^4 + 16x^3 - 12x^2 - 18x = 0$
$x(2x^4 + 12x^3 + 16x^2 - 12x - 18) = 0$
$x(2x^4 + 12x^3 + 18x^2 - 2x^2 - 12x - 18) = 0$
$x[2x^2(x^2 + 6x + 9) - 2(x^2 + 6x + 9)] = 0$
$x(2x^2 - 2)(x^2 + 6x + 9) = 0$
$2x(x^2 - 1)(x + 3)(x + 3) = 0$
$2x(x + 1)(x - 1)(x + 3)(x + 3) = 0$
roots 0, −1, and 1 with multiplicity 1;
root −3 with multiplicity 2.

9. $x^6 - 12x^4 + 48x^2 - 64 = 0$
$(x^2 - 4)(x^2 - 4)(x^2 - 4) = 0$
$(x + 2)(x - 2)(x + 2)(x - 2)(x + 2)(x - 2) = 0$
roots −2, and 2 with multiplicity 3.

10. Let x represent the width in feet.
Then the length is $x + 3$, and the height is $x + 1$.
$x(x + 3)(x + 1) = 30$
$(x^2 + 3x)(x + 1) = 30$
$x^3 + 4x^2 + 3x = 30$
$x^3 + 4x^2 + 3x - 30 = 0$
Factors of −30: ±1, ±2, ±3, ±5, ±6, ±10, ±15, ±30

$\frac{p}{q}$	**1**	**4**	**3**	**−30**
1	1	5	8	−22
2	1	6	15	0
3	1	7	24	42

$(x - 2)(x^2 + 6x + 15) = 0$
$x - 2 = 0 \text{ or } x^2 + 6x + 15 = 0$
$x = 2 \qquad x = -3 \pm \sqrt{6}i$
The width must be positive and real,
so the width should be 2 ft.

11. Possible rational roots:
±1, ±2, ±3, ±5, ±6, ±10, ±15, ±30

−6	1	6	−5	−30
		−6	0	30
	1	0	−5	0

$(x + 6)(x^2 - 5) = 0$
Solve $x^2 - 5 = 0$
$x = \frac{-0 \pm \sqrt{0 + 20}}{2} = \pm\sqrt{5}$
The fully factored equation is:
$(x + 6)(x - \sqrt{5})(x + \sqrt{5}) = 0$
The real roots are −6, and $\pm\sqrt{5}$.

12. Possible rational roots:
±1, ±2, ±3, ±4, ±6, ±11, ±12, ±22, ±33, ±44,
$\pm132, \pm\frac{1}{3}, \pm\frac{4}{3}, \pm\frac{11}{3}, \pm\frac{22}{3}, \pm\frac{44}{3}$

4	3	−18	−9	132
		12	−24	−132
	3	−6	−33	0

$(x - 4)(3x^2 - 6x - 33) = 0$
Solve $3x^2 - 6x - 33 = 0$
$3(x^2 - 2x - 11) = 0$
$x = \frac{2 \pm \sqrt{4 + 44}}{2} = 1 \pm 2\sqrt{3}$
The fully factored equation is:
$3(x - 4)[x - (1 + 2\sqrt{3})][x - (1 - 2\sqrt{3})] = 0$
The real roots are 4, and $1 \pm 2\sqrt{3}$.

13. Possible rational roots:
$\pm1, \pm2, \pm4, \pm5, \pm8, \pm10, \pm20, \pm40, \pm\frac{1}{2}, \pm\frac{5}{2}$

1	2	0	−42	40
		2	2	−40
	2	2	−40	0

$(x - 1)(2x^2 + 2x - 40) = 0$
$2(x - 1)(x^2 + x - 20) = 0$
$2(x - 1)(x - 4)(x + 5) = 0$
The real roots are 1, 4, and −5.

14. Possible rational roots: $\pm1, \pm2, \pm4, \pm5, \pm10$

2	1	0	−9	0	20
		2	4	−10	−20
	1	2	−5	−10	0

$(x-2)(x^3+2x^2-5x-10)=0$

$(x-2)(x+2)(x^2-5)=0$

Solve $x^2-5=0$

$x=\dfrac{-0\pm\sqrt{0+20}}{2}=\pm\sqrt{5}$

The fully factored equation is:

$(x-2)(x+2)(x-\sqrt{5})(x+\sqrt{5})=0$

The real roots are ±2, and $\pm\sqrt{5}$.

PRACTICE AND PROBLEM SOLVING

15. $x^3+3x^2-9x=27$

$x^3+3x^2-9x-27=0$

$(x^2-9)(x+3)=0$

$(x+3)(x-3)(x+3)=0$

$x+3=0$, $x-3=0$, or $x+3=0$

$x=-3$ $\quad x=3$ $\quad x=-3$

The roots are −3, and 3.

16. $4x^5-8x^3+4x=0$

$4x(x^4-2x^2+1)=0$

$4x(x^2-1)(x^2-1)=0$

$4x(x+1)(x-1)(x+1)(x-1)=0$

$4x=0$, $x+1=0$, $x-1=0$, $x+1=0$, or $x-1=0$

$x=0$ $\quad x=-1$ $\quad x=1$ $\quad x=-1$ $\quad x=1$

The roots are 0, −1, and 1.

17. $10x^3-640x=0$

$10x(x^2-64)=0$

$10x(x+8)(x-8)=0$

$10x=0$, $x+8=0$, or $x-8=0$

$x=0$ $\quad x=-8$ $\quad x=8$

The roots are 0, −8, and 8.

18. $x^4-12x^2=-36$

$x^4-12x^2+36=0$

$(x^2-6)(x^2-6)=0$

$x^2-6=0$, or $x^2-6=0$

$x=\pm\sqrt{6}$ $\quad x=\pm\sqrt{6}$

The roots are $\pm\sqrt{6}$.

19. $2x^3-5x^2-4x+10=0$

$(x^2-2)(2x-5)=0$

$x^2-2=0$, or $2x-5=0$

$x=\pm\sqrt{2}$ $\quad x=\dfrac{5}{2}$

The roots are $\pm\sqrt{2}$, and $\dfrac{5}{2}$.

20. $4x^3+7x^2-5x=6$

$4x^3+7x^2-5x-6=0$

$(4x^3-4x^2)+(11x^2-5x-6)=0$

$4x^2(x-1)+(11x+6)(x-1)=0$

$(x-1)(4x^2+11x+6)=0$

$(x-1)(x+2)(4x+3)=0$

$x-1=0$, $x+2=0$, or $4x+3=0$

$x=1$ $\quad x=-2$ $\quad x=-\dfrac{3}{4}$

The roots are 1, −2, and $-\dfrac{3}{4}$.

21. $8x^5-192x^4+1536x^3-4096x^2=0$

$8x^2(x^3-24x^2+192x-512)=0$

$8x^2(x-8)(x-8)(x-8)=0$

root 0 with multiplicity 2; root 8 with multiplicity 3

22. $x^4+2x^3-11x^2-12x+36=0$

Possible factors of 36:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

2	1	2	−11	−12	36
		2	8	−6	−36
	1	4	−3	−18	0

$(x-2)(x^3+4x^2-3x-18)=0$

Possible factors of 18: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

2	1	4	−3	−18
		2	12	18
	1	6	9	0

$(x-2)(x-2)(x^2+6x+9)=0$

$(x-2)(x-2)(x+3)(x+3)=0$

roots 2, and −3 with multiplicity 2

23. Let x represent the side length of the cut out square. Then the side length of the box is $10-2x$,and the height is x.

$x(10-2x)(10-2x)=48$

$(10x-2x^2)(10-2x)=48$

$4x^3-40x^2+100x=48$

$4x^3-40x^2+100x-48=0$

Possible rational factors: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48, \pm\frac{1}{4}, \pm\frac{1}{2}, \pm\frac{4}{3}, \pm\frac{3}{2}$

$\frac{p}{q}$	**4**	**−40**	**100**	**−48**
1	4	−36	64	16
2	4	−32	36	24
3	4	−28	16	0

$(x-3)(4x^2-28x+16)=0$

$4(x-3)(x^2-7x+4)=0$

Solve $x^2-7x+4=0$

$x=\dfrac{7\pm\sqrt{49-16}}{2}=\dfrac{7\pm\sqrt{33}}{2}$

$4(x-3)\left(x-\dfrac{7+\sqrt{33}}{2}\right)\left(x-\dfrac{7-\sqrt{33}}{2}\right)=0$

The side length should be rational; you must cut out a 3 in. by 3 in. square.

24. $x^4 - 3x^2 - 4 = 0$

$(x^2 - 4)(x^2 + 1) = 0$

$(x + 2)(x - 2)(x^2 + 1) = 0$

Solve $x^2 + 1 = 0$

$x = \frac{0 \pm \sqrt{0 - 4}}{2} = -i$

The fully factored equation is:

$(x + 2)(x - 2)(x - i)(x + i) = 0$

The real roots are −2, and 2.

25. $3x^3 + 4x^2 - 6x - 8 = 0$

$x^2(3x + 4) - 2(3x + 4) = 0$

$(3x + 4)(x^2 - 2) = 0$

Solve $x^2 - 2 = 0$

$x = \frac{0 \pm \sqrt{0 + 8}}{2} = \pm\sqrt{2}$

The fully factored equation is:

$(3x + 4)(x - \sqrt{2})(x + \sqrt{2}) = 0$

The real roots are $-\frac{4}{3}$, and $\pm\sqrt{2}$.

26. $x^4 - 2x^3 - 2x^2 = 0$

$x^2(x^2 - 2x - 2) = 0$

Solve $x^2 - 2x - 2 = 0$

$x = \frac{2 \pm \sqrt{4 + 8}}{2} = 1 \pm \sqrt{3}$

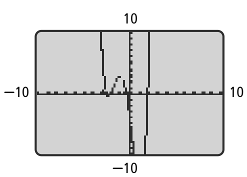
The fully factored equation is:

$x^2[x - (1 + \sqrt{3})][x - (1 - \sqrt{3})] = 0$

The real roots are 0, and $1 \pm \sqrt{3}$.

27a. Possible rational roots: ±1, ±2, ±4

b.



$x = -2$, and $x = 2$

c. 2

d. $x = -0.38$, and $x = -2.62$

28a. $V(x) = x^3$

b. $125 = x^3$

$0 = x^3 - 125$

p, factors of 125: ±1, ±5, ±25, ±125

q, factors of 1: ±1

The possible rational roots are:

±1, ±5, ±25, ±125.

c. The only root is 5, with a multiplicity of 3.

d. The side length of the box is the value of *x*, or 5 in.

e. The number of square inches needed to make the box is the surface area. There are 6 faces, each with an area of $5 \cdot 5 = 25$ in^2.
So, the surface area is $6 \cdot 25 = 150$ in^2.

29. Possible rational roots: ±1, ±2, ±3, ±6

3	1	−7	14	−6
		3	−12	6
	1	−4	2	0

$(x - 3)(x^2 - 4x + 2) = 0$

Solve $x^2 - 4x + 2 = 0$

$x = \frac{4 \pm \sqrt{16 - 8}}{2} = 2 \pm \sqrt{2}$

The fully factored equation is:

$(x - 3)[x - (2 + \sqrt{2})][x - (2 + \sqrt{2})] = 0$

The real roots are 3, and $2 \pm \sqrt{2}$.

30. $\frac{5}{3}x^3 + \frac{8}{3}x^2 - \frac{4}{3}x = 0$

$5x^3 + 8x^2 - 4x = 0$

$x(5x^2 + 8x - 4) = 0$

$x(x + 2)(5x - 2) = 0$

The real roots are 0, −2, and $\frac{2}{5}$.

31. Possible rational roots: ±1, ±2, ±3, ±5, ±6, ±10, ±15, ±25, ±30, ±50, ±75, ±150

3	1	−1	−31	25	150
		3	6	−75	−150
	1	2	−25	−50	0

$(x - 3)(x^3 + 2x^2 - 25x - 50) = 0$

$(x - 3)(x + 2)(x^2 - 25) = 0$

$(x - 3)(x + 2)(x + 5)(x - 5) = 0$

The real roots are −5, −2, 3, and 5.

32. $3x^4 + 19x^2 + 27x + 6 = 23x^3$

$3x^4 - 23x^3 + 19x^2 + 27x + 6 = 0$

Possible rational roots: ±1, ±2, ±3, ±6, $\pm\frac{1}{3}$, $\pm\frac{2}{3}$

2	3	−23	19	27	6
		6	−34	−30	−6
	3	−17	−15	−3	0

$(x - 2)(3x^3 - 17x^2 - 15x - 3) = 0$

Possible rational roots: $\pm\frac{1}{3}$, ±1, ±3

$-\frac{1}{3}$	3	−17	−15	−3
		−1	6	3
	3	−18	−9	0

$(x - 2)\left(x + \frac{1}{3}\right)(3x^2 - 18x - 9) = 0$

Solve $3x^2 - 18x - 9 = 0$

$3(x^2 - 6x - 3) = 0$

$x = \frac{6 \pm \sqrt{36 + 12}}{2} = 3 \pm 2\sqrt{3}$

The fully factored equation is:

$3(x - 2)\left(x + \frac{1}{3}\right)[x - (3 + 2\sqrt{3})][x - (3 - 2\sqrt{3})] = 0$

The real roots are 2, $-\frac{1}{3}$, and $3 \pm 2\sqrt{3}$.

33. $x^5 - 4x^4 - 2x^3 + 4x^2 + x = 0$

$x\left(x^4 - 4x^3 - 2x^2 + 4x + 1\right) = 0$

Possible rational factors: ± 1

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & -2 & 4 & 1 \\ & & 1 & -3 & -5 & -1 \\ \hline & 1 & -3 & -5 & -1 & \boxed{0} \end{array}$$

$x(x - 1)\left(x^3 - 3x^2 - 5x - 1\right) = 0$

Possible rational factors: ± 1

$$\begin{array}{r|rrrr} -1 & 1 & -3 & -5 & -1 \\ & & -1 & 4 & 1 \\ \hline & 1 & -4 & -1 & \boxed{0} \end{array}$$

$x(x - 1)(x + 1)\left(x^2 - 4x - 1\right) = 0$

Solve $x^2 - 4x - 1 = 0$

$x = \dfrac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}$

The fully factored equation is:

$x(x - 1)(x + 1)\left[x - \left(2 + \sqrt{5}\right)\right]\left[x - \left(2 - \sqrt{5}\right)\right] = 0$

The real roots are -1, 0, 1, and $2 \pm \sqrt{5}$.

34. $x^3 + 9 - 6x^2 = -4\left(11x - 2x^2\right)$

$x^3 + 9 - 6x^2 + 4\left(11x - 2x^2\right) = 0$

$x^3 - 14x^2 + 44x + 9 = 0$

Possible rational roots: $\pm 1, \pm 3, \pm 9$

$$\begin{array}{r|rrrr} 9 & 1 & -14 & 44 & 9 \\ & & 9 & -45 & -9 \\ \hline & 1 & -5 & -1 & \boxed{0} \end{array}$$

$(x - 9)\left(x^2 - 5x - 1\right) = 0$

Solve $x^2 - 5x - 1 = 0$

$x = \dfrac{5 \pm \sqrt{25 + 4}}{2} = \dfrac{5}{2} \pm \dfrac{\sqrt{29}}{2}$

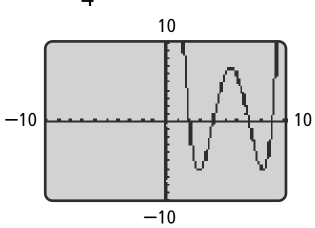
The fully factored equation is:

$(x - 9)\left[x - \left(\dfrac{5}{2} - \dfrac{\sqrt{29}}{2}\right)\right]\left[x - \left(\dfrac{5}{2} + \dfrac{\sqrt{29}}{2}\right)\right] = 0$

The real roots are 9, and $\dfrac{5}{2} \pm \dfrac{\sqrt{29}}{2}$.

35a. $h(0) = \dfrac{1}{4}(0 - 2)(0 - 4)(0 - 7)(0 - 9) = 126$ ft

b.



Possible answer: The coaster passes through 2 tunnels within the first 100 s.

c. The coaster enters a tunnel at 30 s, and leaves it at 50 s. Therefore the roots are 3, and 5.

$h(t) = a(t - 3)(t - 5)$

Since the starting height is 45

$45 = a(0 - 3)(0 - 5)$

$a = 3$

The roller coaster ride can be modelled by

$h(t) = 3(t - 3)(t - 5)$

36a. $V(x) = \dfrac{1}{3}\ell wh$

$= \dfrac{1}{3}(x)(x)(x + 2)$

$= \dfrac{1}{3}\left(x^3 + 2x^2\right)$

$= \dfrac{1}{3}x^3 + \dfrac{2}{3}x^2$

b. $\dfrac{1}{3}x^3 + \dfrac{2}{3}x^2 = 147$

$\dfrac{1}{3}x^3 + \dfrac{2}{3}x^2 - 147 = 0$

$x^3 + 2x^2 - 441 = 0$

c. $x^3 + 2x^2 - 441 = 0$

Possible factors of -441: $\pm 1, \pm 3, \pm 7, \pm 9, \pm 49, \pm 63, \pm 147, \pm 441$

$$\begin{array}{r|rrrr} 7 & 1 & 2 & 0 & -441 \\ & & 7 & 63 & 441 \\ \hline & 1 & 9 & 63 & \boxed{0} \end{array}$$

$(x - 7)\left(x^2 + 9x + 63\right) = 0$

Solve $x^2 + 9x + 63 = 0$

$x = \dfrac{-9 \pm \sqrt{81 - 256}}{2} = -\dfrac{9}{2} \pm \dfrac{3\sqrt{19}}{2}$

The fully factored equation is:

$(x - 7)\left[x - \left(-\dfrac{9}{2} - \dfrac{3\sqrt{19}}{2}\right)\right]\left[x - \left(-\dfrac{9}{2} + \dfrac{3\sqrt{19}}{2}\right)\right] = 0$

The dimensions are 7 cm × 7 cm × 9 cm.

37. Possible answer: The zero on the graph does not represent a rational root of the related equation. $P(4)$ must equal 0 in order for the root to correspond to a zero at $x = 4$.

38. Possible answer: $x = 2$ is a root with a multiplicity of 2 for both f and g. The graphs of f and g each intersect the x-axis at (2, 0) but do not cross it. $x = -2$ is a root with a multiplicity of 3 for f and h. The graphs of f and h each bend at the intersection with the x-axis at $(-2, 0)$.

39. Possible answer: The graph bends near (0, 0). Since $5x^4 - 20x^3 = 5x^3(x - 4)$, 0 is a root with multiplicity 3, and roots with odd multiplicities have this behavior when graphed.

TEST PREP

40. B

Possible rational roots:

$\pm 1, \pm 2, \pm 3, \pm 5, \pm 6, \pm 10, \pm 15, \pm 30, \pm\dfrac{1}{2}, \pm\dfrac{1}{4}, \pm\dfrac{1}{8}, \pm\dfrac{3}{2}, \pm\dfrac{3}{4}, \pm\dfrac{3}{8}, \pm\dfrac{5}{2}, \pm\dfrac{5}{4}, \pm\dfrac{5}{8}, \pm\dfrac{15}{2}, \pm\dfrac{15}{4}, \pm\dfrac{15}{8}$

$$\begin{array}{r|rrrr} 2 & 8 & -2 & -43 & 30 \\ & & 16 & 24 & -30 \\ \hline & 8 & 14 & -15 & \boxed{0} \end{array}$$

$(x - 2)\left(8x^2 + 14x - 15\right) = 0$

$(x - 2)(2x - 5)(4x - 3) = 0$

41. F

Use a graphing calculator to find the roots.

42. C

$\dfrac{p}{q}$	6	13	0	−4
−2	3	1	−2	0
$\dfrac{1}{2}$	6	16	8	0
3	6	31	93	275
$-\dfrac{2}{3}$	6	9	−6	0

43. H

CHALLENGE AND EXTEND

44. The factors are
$x, x-1, x-\left(3-\sqrt{5}\right)$, and $x-\left(3+\sqrt{5}\right)$.
$f(x)=x(x-1)\left[x-\left(3-\sqrt{5}\right)\right]\left[x-\left(3+\sqrt{5}\right)\right]$
$=\left(x^2-x\right)\left(x^2-6x+4\right)$
$=x^4-7x^3+10x^2-4x$

45. $(2)^3+3(2)^2-(2)+k=0$
$18+k=0$
$k=-18$

46. $k(-3)^3-2(-3)^2+(-3)-6=0$
$-27k-27=0$
$-27k=27$
$k=-1$

47. $6(4)^3-23(4)^2-k(4)+8=0$
$24-4k=0$
$-4k=-24$
$k=6$

SPIRAL REVIEW

48. $h(2)=85-16(2)^2$
$=85-64$
$=21$
The height of the water ballon after 2 s is 21 ft.

49. $x^2+x+5=11$
$x^2+x-6=0$
$(x+3)(x-2)=0$
$x+3=0$ or $x-2=0$
$x=-3$ $\quad x=2$
x-value test: $(-4)^2+(-4)+5>11$ ✓
$(-3)^2+(-3)+5>11$ ✗
$(3)^2+(3)+5>11$ ✓
Therefore $x<-3$ or $x>2$.

50. $x^2-10x+2=-23$
$x^2-10x+25=0$
$(x-5)(x-5)=0$
$x-5=0$ or $x-5=0$
$x=5$ $\quad x=5$
x-value test: $(4)^2-10(4)+2\le-23$ ✗
$(6)^2-10(6)+2\le-23$ ✗
Therefore $x=5$.

51. $x^2=1$
$x^2-1=0$
$(x+1)(x-1)=0$
$x+1=0$ or $x-1=0$
$x=-1$ $\quad x=1$
x-value test: $(-2)^2<1$ ✗
$(0)^2<1$ ✓
$(2)^2<1$ ✗
Therefore $-1<x<1$.

52. $x^3+3x^2-4x-12$
$=\left(x^3+3x^2\right)+(-4x-12)$
$=x^2(x+3)-4(x+3)$
$=(x+3)\left(x^2-4\right)$
$=(x+3)(x+2)(x-2)$

53. $8x^3+4x^2-8x-4$
$=4\left(2x^3+x^2-2x-1\right)$
$=4\left(\left(2x^3+x^2\right)-(2x+1)\right)$
$=4\left(x^2(2x+1)-(2x+1)\right)$
$=4(2x+1)\left(x^2-1\right)$
$=4(2x+1)(x+1)(x-1)$

54. x^3+27
$=x^3+3^3$
$=(x+3)\left(x^2-x\cdot3+3^2\right)$
$=(x+3)\left(x^2-3x+9\right)$

6-6 FUNDAMENTAL THEOREM OF ALGEBRA, PAGES 445–451

CHECK IT OUT!

1a. $P(x)=(x+2)(x-2)(x-4)$
$=\left(x^2-4\right)(x-4)$
$=x^3-4x^2-4x+16$

b. $P(x)=x\left(x-\frac{2}{3}\right)(x-3)$
$=\left(x^2-\frac{2}{3}x\right)(x-3)$
$=x^3-\frac{11}{3}x^2+2x$

2. $x^4+4x^3-x^2+16x-20=0$
Possible rational roots: ±1, ±2, ±4, ±5, ±10, ±20

1	1	4	−1	16	−20
		1	5	4	20
	1	5	4	20	0

$(x-1)\left(x^3+5x^2+4x+20\right)=0$
$(x-1)\left[x^2(x+5)+4(x+5)\right]=0$
$(x-1)(x+5)\left(x^2+4\right)=0$
Solve $x^2+4=0$
$x^2=-4$
$x=\pm2i$
The fully factored equation is:
$(x-1)(x+5)(x+2i)(x-2i)=0$
The solutions are 1, −5, and $\pm2i$.

3. The roots are $2i$, $-2i$, $1+\sqrt{2}$, $1-\sqrt{2}$, and 3.
$P(x)=(x-2i)(x+2i)\left(x-1-\sqrt{2}\right)\left(x-1+\sqrt{2}\right)(x-3)$
$=\left(x^2+4\right)\left(x^2-2x-1\right)(x-3)$
$=\left(x^4-2x^3+3x^2-8x-4\right)(x-3)$
$=x^5-5x^4+9x^3-17x^2+20x+12$

4. Let x represent the radius of the silo.

$V = V_{hemisphere} + V_{cylinder}$

$V(x) = \frac{2}{3}\pi x^3 + 20\pi x^2$

Set the volume equal to 2106π.

$$\frac{2}{3}\pi x^3 + 20\pi x^2 = 2106\pi$$

$$\frac{2}{3}\pi x^3 + 20\pi x^2 - 2106\pi = 0$$

$$\frac{2}{3}x^3 + 20x^2 - 2106 = 0$$

$$\frac{2}{3}\left(x^3 + 30x^2 - 3159\right) = 0$$

9	1	30	0	−3159
		9	351	3159
	1	39	351	0

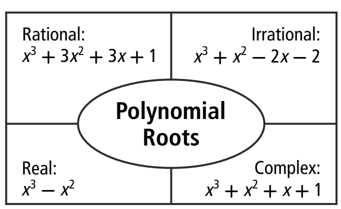
$$\frac{2}{3}(x - 9)\left(x^2 + 39x + 351\right) = 0$$

Solve $x^2 + 39x + 351 = 0$

$$x = \frac{-39 \pm \sqrt{1521 - 1404}}{2} = -\frac{39}{2} \pm \frac{3\sqrt{13}}{2}$$

The roots are ≈ -24.9, ≈ -14.1, and 9.
The radius must be a positive number, so the radius of the silo is 9 ft.

THINK AND DISCUSS

1. Possible answer: Complex roots come in conjugate pairs for polynomial equations with real coefficients. The second root is $1 + i$, and polynomials that have at least 2 roots must have a degree of at least 2.
2. The linear polynomial has 1 root, and the cubic polynomial has 3 roots. The product will have 4 roots, of which at least 2 roots will be real.
3. 

EXERCISES

GUIDED PRACTICE

1. $P(x) = \left(x - \frac{1}{3}\right)(x - 1)(x - 2)$
$= \left(x^2 - \frac{4}{3}x + \frac{1}{3}\right)(x - 2)$
$= x^3 - \frac{10}{3}x^2 + 3x - \frac{2}{3}$

2. $P(x) = (x + 2)(x - 2)(x + 3)$
$= \left(x^2 - 4\right)(x + 3)$
$= x^3 - 3x^2 - 4x + 12$

3. $P(x) = (x + 2)\left(x - \frac{1}{2}\right)(x - 2)$
$= \left(x^2 + \frac{3}{2}x - 1\right)(x - 2)$
$= x^3 - \frac{1}{2}x^2 - 4x + 2$

4. $x^4 - 81 = 0$
$\left(x^2 - 9\right)\left(x^2 + 9\right) = 0$
$(x - 3)(x + 3)\left(x^2 + 9\right) = 0$
Solve $x^2 + 9 = 0$
$x^2 = -9$
$x = \pm 3i$
The fully factored equation is:
$(x - 3)(x + 3)(x - 3i)(x + 3i) = 0$
The solutions are 3, −3, and $\pm 3i$.

5. $3x^3 - 10x^2 + 10x - 4 = 0$
Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm\frac{1}{3}, \pm\frac{2}{3}, \pm\frac{4}{3}$

2	3	−10	10	−4
		6	−8	4
	3	−4	2	0

$(x - 2)\left(3x^2 - 4x + 2\right) = 0$
Solve $3x^2 - 4x + 2 = 0$
$$x = \frac{4 \pm \sqrt{16 - 24}}{6} = \frac{2 \pm i\sqrt{2}}{3}$$
The fully factored equation is:
$$(x - 2)\left(x - \frac{2 + i\sqrt{2}}{3}\right)\left(x - \frac{2 - i\sqrt{2}}{3}\right) = 0$$
The solutions are 2, and $\frac{2 \pm i\sqrt{2}}{3}$.

6. $x^3 - 3x^2 + 4x - 12 = 0$
$x^2(x - 3) + 4(x - 3) = 0$
$(x - 3)\left(x^2 + 4\right) = 0$
Solve $x^2 + 4 = 0$
$x^2 = -4$
$x = \pm 2i$
The fully factored equation is:
$(x - 3)(x + 2i)(x - 2i) = 0$
The solutions are 3, and $\pm 2i$.

7. The roots are $1 - i$, $1 + i$, and 2.
$P(x) = [x - (1 - i)][x - (1 + i)](x - 2)$
$= \left(x^2 - 2x + 2\right)(x - 2)$
$= x^3 - 4x^2 + 6x - 4$

8. The roots are $1 + \sqrt{5}$, $1 - \sqrt{5}$, and 3.
$P(x) = \left[x - \left(1 + \sqrt{5}\right)\right]\left[x - \left(1 - \sqrt{5}\right)\right](x - 3)$
$= \left(x^2 - 2x - 4\right)(x - 3)$
$= x^3 - 5x^2 + 2x + 12$

9. The roots are $2i$, $-2i$, $\sqrt{2}$, $-\sqrt{2}$, and 2.
$P(x) = (x - 2i)(x + 2i)\left(x - \sqrt{2}\right)\left(x + \sqrt{2}\right)(x - 2)$
$= \left(x^2 + 4\right)\left(x^2 - 2\right)(x - 2)$
$= \left(x^4 + 2x^2 - 8\right)(x - 2)$
$= x^5 - 2x^4 + 2x^3 - 4x^2 - 8x + 16$

10. Let x represent the radius of the silo.

$V = V_{cone} + V_{cylinder}$

$V(x) = \frac{1}{3}\pi x^3 + 30\pi x^2$

Set the volume equal to 1152π.

$$\frac{1}{3}\pi x^3 + 30\pi x^2 = 1152\pi$$
$$\frac{1}{3}\pi x^3 + 30\pi x^2 - 1152\pi = 0$$
$$\frac{1}{3}x^3 + 30x^2 - 1152 = 0$$
$$\frac{1}{3}\left(x^3 + 90x^2 - 3456\right) = 0$$

6	1	90	0	−3456
		6	576	3456
	1	96	576	0

$$\frac{1}{3}(x-6)\left(x^2 + 96x + 576\right) = 0$$

Solve $x^2 + 96x + 576 = 0$

$$x = \frac{-96 \pm \sqrt{9216 - 2304}}{2} = -48 \pm 24\sqrt{3}$$

The roots are ≈ -89.57, ≈ -6.43, and 6.
The radius must be a positive number, so the radius of the silo is 6 ft.

PRACTICE AND PROBLEM SOLVING

11. $P(x) = (x+1)(x+1)(x-2)$
$= \left(x^2 + 2x + 1\right)(x-2)$
$= x^3 + 2x^2 + x$
$\quad -2x^2 - 4x - 2$
$= x^3 - 3x - 2$

12. $P(x) = (x-2)(x-1)\left(x - \frac{2}{3}\right)$
$= \left(x^2 - 3x + 2\right)\left(x - \frac{2}{3}\right)$
$= x^3 - \frac{11}{3}x^2 + 4x - \frac{4}{3}$

13. $P(x) = (x+4)(x+1)(x-2)$
$= \left(x^2 + 5x + 4\right)(x-2)$
$= x^3 + 3x^2 - 6x - 8$

14. $x^4 - 16 = 0$
$\left(x^2 - 4\right)\left(x^2 + 4\right) = 0$
$(x-2)(x+2)\left(x^2 + 4\right) = 0$
Solve $x^2 + 4 = 0$
$x^2 = -4$
$x = \pm 2i$
The fully factored equation is:
$(x-2)(x+2)(x-2i)(x+2i) = 0$
The solutions are 2, −2, and $\pm 2i$.

15. $x^3 - 7x^2 + 15x - 9 = 0$
Possible roots are: $\pm 1, \pm 3, \pm 9$

1	1	−7	15	−9
		1	−6	9
	1	−6	9	0

$(x-1)\left(x^2 - 6x + 9\right) = 0$
$(x-1)(x-3)(x-3) = 0$
The solutions are 1, and 3.

16. $x^4 + 5x^2 - 36 = 0$
$\left(x^2 - 4\right)\left(x^2 + 9\right) = 0$
$(x-2)(x+2)\left(x^2 + 9\right) = 0$
Solve $x^2 + 9 = 0$
$x^2 = -9$
$x = \pm 3i$
The fully factored equation is:
$(x-2)(x+2)(x-3i)(x+3i) = 0$
The solutions are 2, −2, and $\pm 3i$.

17. $2x^3 - 3x^2 + 8x - 12 = 0$
$x^2(2x-3) + 4(2x-3) = 0$
$(2x-3)\left(x^2 + 4\right) = 0$
Solve $x^2 + 4 = 0$
$x^2 = -4$
$x = \pm 2i$
The fully factored equation is:
$(2x-3)(x-2i)(x+2i) = 0$
The solutions are $\frac{3}{2}$ and $\pm 2i$.

18. $x^4 - 5x^3 + 3x^2 + x = 0$
$x\left(x^3 - 5x^2 + 3x + 1\right) = 0$
Possible roots are: ± 1

1	1	−5	3	1
		1	−4	−1
	1	−4	−1	0

$x(x-1)\left(x^2 - 4x - 1\right) = 0$
Solve $x^2 - 4x - 1 = 0$
$x = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5}$
The fully factored equation is:
$x(x-1)\left(x - 2 - \sqrt{5}\right)\left(x - 2 + \sqrt{5}\right) = 0$
The solutions are 0, 1, and $2 \pm \sqrt{5}$.

19. $x^4 - 4x^2 + 3 = 0$
$\left(x^2 - 1\right)\left(x^2 - 3\right) = 0$
$(x-1)(x+1)\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right) = 0$
The solutions are 1, −1, and $\pm\sqrt{3}$.

20. The roots are $2 - i$, $2 + i$, $\sqrt{3}$, $-\sqrt{3}$, and 2.
$P(x) = [x - (2-i)][x - (2+i)]\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x-2)$
$= \left(x^2 - 4x + 5\right)\left(x^2 - 3\right)(x-2)$
$= \left(x^4 - 4x^3 + 2x^2 + 12x - 15\right)(x-2)$
$= x^5 - 6x^4 + 10x^3 + 8x^2 - 39x + 30$

21. The roots are $2\sqrt{2}$, $-2\sqrt{2}$, $\sqrt{5}$, $-\sqrt{5}$, and −3.
$P(x) = \left(x - 2\sqrt{2}\right)\left(x + 2\sqrt{2}\right)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right)(x+3)$
$= \left(x^2 - 8\right)\left(x^2 - 5\right)(x+3)$
$= \left(x^4 - 13x^2 + 40\right)(x+3)$
$= x^5 + 3x^4 - 13x^3 - 39x^2 + 40x + 120$

22. The roots are $-2i$, $2i$, $1 + i$, and $1 - i$.
$P(x) = (x-2i)(x+2i)[x - (1+i)][x - (1-i)]$
$= \left(x^2 + 4\right)\left(x^2 - 2x + 2\right)$
$= x^4 - 2x^3 + 6x^2 - 8x + 8$

23. Let x represent the radius of the bin.

$V = V_{hemisphere} + V_{cylinder}$

$V(x) = \frac{2}{3}\pi x^3 + 45\pi x^2$

Set the volume equal to 4131π.

$\frac{2}{3}\pi x^3 + 45\pi x^2 = 4131\pi$

$\frac{2}{3}\pi x^3 + 45\pi x^2 - 4131\pi = 0$

$\frac{2}{3}x^3 + 45x^2 - 4131 = 0$

9	$\frac{2}{3}$	45	0	−4131
		6	459	4131
	$\frac{2}{3}$	51	459	0

$(x - 9)\left(\frac{2}{3}x^2 + 51x + 459\right) = 0$

Solve $\frac{2}{3}x^2 + 51x + 459 = 0$

$x = \frac{-51 \pm \sqrt{2601 - 1224}}{2\left(\frac{2}{3}\right)} = \frac{9}{4}\left(-17 \pm 3\sqrt{17}\right)$

The roots are ≈ -66.08, ≈ -10.42, and 9.
The radius must be a positive number, so the radius of the bin is 9 ft.

24. $x^4 - 3x^3 + 5x^2 - 27x - 36 = 0$

Possible rational roots:
$\pm1, \pm2, \pm3, \pm4, \pm6, \pm9, \pm12, \pm18, \pm36$

4	1	−3	5	−27	−36
		4	4	36	36
	1	1	9	9	0

$(x - 4)\left(x^3 + x^2 + 9x + 9\right) = 0$

$(x - 4)\left[x^2(x + 1) + 9(x + 1)\right] = 0$

$(x - 4)(x + 1)\left(x^2 + 9\right) = 0$

Solve $x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

The fully factored equation is:

$(x - 4)(x + 1)(x - 3i)(x + 3i) = 0$

The solutions are 4, −1, and $\pm 3i$.

25. $x^4 + 4x^3 - 3x^2 - 14x - 8 = 0$

Possible rational roots: $\pm1, \pm2, \pm4, \pm8$

2	1	4	−3	−14	−8
		2	12	18	8
	1	6	9	4	0

$(x - 2)\left(x^3 + 6x^2 + 9x + 4\right) = 0$

Possible rational roots: $\pm1, \pm2, \pm4$

−1	1	6	9	4
		−1	−5	−4
	1	5	4	0

$(x - 2)(x + 1)\left(x^2 + 5x + 4\right) = 0$

$(x - 2)(x + 1)(x + 1)(x + 4) = 0$

The solutions are 2, −1, and −4.

26. $x^3 + 3x^2 + 3x + 1 = 0$

$(x + 1)(x + 1)(x + 1) = 0$

The solution is −1.

27. $x^4 + 4x^3 + 6x^2 + 4x + 1 = 0$

$(x + 1)(x + 1)(x + 1)(x + 1) = 0$

The solution is −1.

28. $6x^3 + 11x^2 - 3x - 2 = 0$

Possible rational roots: $\pm1, \pm2, \pm\frac{1}{2}, \pm\frac{1}{3}, \pm\frac{1}{6}, \pm\frac{2}{3}$

−2	6	11	−3	−2
		−12	2	2
	6	−1	−1	0

$(x + 2)\left(6x^2 - x - 1\right) = 0$

$(x + 2)(2x - 1)(3x + 1) = 0$

The solutions are −2, $\frac{1}{2}$, and $-\frac{1}{3}$.

29. $x^3 - 2x^2 - 2x - 3 = 0$

Possible rational roots: $\pm1, \pm3$

3	1	−2	−2	−3
		3	3	3
	1	1	1	0

$(x - 3)\left(x^2 + x + 1\right) = 0$

Solve $x^2 + x + 1 = 0$

$x = \frac{-1 \pm \sqrt{1 - 4}}{2} = \frac{-1 \pm i\sqrt{3}}{2}$

The fully factored equation is:

$(x - 3)\left[x - \left(\frac{-1 + i\sqrt{3}}{2}\right)\right]\left[x - \left(\frac{-1 - i\sqrt{3}}{2}\right)\right] = 0$

The solutions are 3, and $\frac{-1 \pm i\sqrt{3}}{2}$.

30. $x^3 - 6x^2 + 11x - 6 = 0$

Possible rational roots: $\pm1, \pm2, \pm3, \pm6$

1	1	−6	11	−6
		1	−5	6
	1	−5	6	0

$(x - 1)\left(x^2 - 5x + 6\right) = 0$

$(x - 1)(x - 2)(x - 3) = 0$

The solutions are 1, 2, and 3.

31. $x^4 - 13x^3 + 55x^2 - 91x = 0$

$x\left(x^3 - 13x^2 + 55x - 91\right) = 0$

Possible rational roots: $\pm1, \pm7, \pm13, \pm91$

7	1	−13	55	−91
		7	−42	91
	1	−6	13	0

$x(x - 7)\left(x^2 - 6x + 13\right) = 0$

Solve $x^2 - 6x + 13 = 0$

$x = \frac{6 \pm \sqrt{36 - 52}}{2} = 3 \pm 2i$

The fully factored equation is:

$x(x - 7)[x - (3 + 2i)][x - (3 - 2i)] = 0$

The solutions are 0, 7, and $3 \pm 2i$.

32. $x^4 + x^2 - 12 = 0$

$\left(x^2 - 3\right)\left(x^2 + 4\right) = 0$

Solve $x^2 - 3 = 0$ $x^2 + 4 = 0$

$x^2 = 3$ $x^2 = -4$

$x = \pm\sqrt{3}$ $x = \pm 2i$

The fully factored equation i s:

$\left(x - \sqrt{3}\right)\left(x + \sqrt{3}\right)(x - 2i)(x + 2i) = 0$

The solutions are $\pm\sqrt{3}$, and $\pm 2i$.

33. $x^4 + 14x^2 + 45 = 0$
$(x^2 + 9)(x^2 + 5) = 0$
Solve $x^2 + 9 = 0$ $\quad x^2 + 5 = 0$
$x^2 = -9 \quad x^2 = -5$
$x = \pm 3i \quad x = \pm i\sqrt{5}$
The fully factored equation is:
$(x - 3i)(x + 3i)(x - i\sqrt{5})(x + i\sqrt{5}) = 0$
The solutions are $\pm 3i$, and $\pm i\sqrt{5}$.

34. $x^3 + 13x - 85 = 31$
$x^3 + 13x - 116 = 0$
Possible rational roots are:
$\pm 1, \pm 2, \pm 4, \pm 29, \pm 58, \pm 116$

4	1	0	13	−116
		4	16	116
	1	4	29	0

$(x - 4)(x^2 + 4x + 29) = 0$
Solve $x^2 + 4x + 29 = 0$
$x = \dfrac{-4 \pm \sqrt{16 - 116}}{2} = -2 \pm 5i$
The fully factored equation is:
$(x - 4)[x - (-2 + 5i)][x - (-2 - 5i)] = 0$
The solutions are 4, and $-2 \pm 5i$.

35. $x^3 - 4x^2 + x + 14 = 8$
$x^3 - 4x^2 + x + 6 = 0$
Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 6$

−1	1	−4	1	6
		−1	5	−6
	1	−5	6	0

$(x + 1)(x^2 - 5x + 6) = 0$
$(x + 1)(x - 2)(x - 3) = 0$
The solutions are −1, 2, and 3.

36. $V = \ell \bullet w \bullet h$
$V(x) = (x + 3)(x + 1)(x - 1)$
$= (x^2 + 4x + 3)(x - 1)$
$= x^3 + 3x^2 - x - 3$
Set volume equal to 105.
$x^3 + 3x^2 - x - 3 = 105$
$x^3 + 3x^2 - x - 108 = 0$
Possible rational roots: $\pm 1, \pm 2, \pm 3, \pm 4, \pm 6, \pm 9, \pm 12, \pm 18, \pm 27, \pm 36, \pm 54, \pm 108$

4	1	3	−1	−108
		4	28	108
	1	7	27	0

$(x - 4)(x^2 + 7x + 27) = 0$
Solve $x^2 + 7x + 27 = 0$
$x = \dfrac{-7 \pm \sqrt{49 - 108}}{2} = \dfrac{-7 \pm i\sqrt{59}}{2}$
The roots are 4, and $\dfrac{-7 \pm i\sqrt{59}}{2}$.
The dimension must be a real number, take $x = 4$.
So the dimensions of the prism are
$(x + 3)(x + 1)(x - 1)$
$= 7 \times 5 \times 3$

37a. Set volume equal to 81.
$\frac{1}{3}x^3 - 2x^2 = 81$
$\frac{1}{3}x^3 - 2x^2 - 81 = 0$
$x^3 - 6x^2 - 243 = 0$

b. Possible rational roots:
$\pm 1, \pm 3, \pm 9, \pm 27, \pm 81, \pm 243$

9	1	−6	0	−243
		9	27	243
	1	3	27	0

$(x - 9)(x^2 + 3x + 27) = 0$
Solve $x^2 + 3x + 27 = 0$
$x = \dfrac{-3 \pm \sqrt{9 - 108}}{2} = \dfrac{-3 \pm 3i\sqrt{11}}{2}$
The roots are 9, and $\dfrac{-3 \pm 3i\sqrt{11}}{2}$.
The length must be a real positive number, therefore take $x = 9$. So the length is 9 m.

c. The other roots are complex. After $x - 9$ is factored out, the remaining quadratic equation has a negative discriminant.

38. The roots are 0, $\sqrt{5}$, $-\sqrt{5}$, and 2.
$P(x) = x(x - 2)(x - \sqrt{5})(x + \sqrt{5})$
$= (x^2 - 2x)(x^2 - 5)$
$= x^4 - 2x^3 - 5x^2 + 10x$

39. The roots are $4i$, $-4i$, 2, and −2.
$P(x) = (x - 4i)(x + 4i)(x - 2)(x + 2)$
$= (x^2 + 16)(x^2 - 4)$
$= x^4 + 12x^2 - 64$

40. The roots are 1, −1, $3i$, and $-3i$.
$P(x) = (x - 1)(x + 1)(x + 1)(x + 1)(x + 3i)(x - 3i)$
$= (x^2 - 1)(x^2 + 2x + 1)(x^2 + 9)$
$= (x^4 + 2x^3 - 2x - 1)(x^2 + 9)$
$= x^6 + 2x^5 + 9x^4 + 16x^3 - x^2 - 18x - 9$

41. The roots are 1, and 2.
$P(x) = (x - 1)(x - 1)(x - 2)$
$= (x^2 - 2x + 1)(x - 2)$
$= x^3 - 4x^2 + 5x - 2$

42. The roots are $1 - \sqrt{2}$, $1 + \sqrt{2}$, $2i$, and $-2i$.
$P(x) = [x - (1 - \sqrt{2})][x - (1 + \sqrt{2})](x - 2i)(x + 2i)$
$= (x^2 - 2x - 1)(x^2 + 4)$
$= x^4 - 2x^3 + 3x^2 - 8x - 4$

43. The roots are 3, $3i$, and $-3i$.
$P(x) = (x - 3)(x - 3)(x - 3i)(x + 3i)$
$= (x^2 - 6x + 9)(x^2 + 9)$
$= x^4 - 6x^3 + 18x^2 - 54x + 81$

44. never true

45. never true

46. always true

47. Sometimes true; $(x - 1)^2 = 0$ has root 1 with multiplicity 2 and degree 2. $x(x - 1)^2 = 0$ has root 1 with multiplicity 2 and degree 3.

48. $x \approx -1.748, -0.420, 0.625, 1.542$

49. $x = 0$ or $x \approx \pm 0.537$

50. $x \approx -0.3614, -0.4998, 1.3050$

51. $x \approx -0.782, 0.975, 3.965$

52a. Set the volume equal to 39,186.

$0.485h^3 - 362h^2 + 89{,}889h - 7{,}379{,}874 = 39{,}186$

$0.485h^3 - 362h^2 + 89{,}889h - 7{,}419{,}060 = 0$

$$\begin{array}{r|rrrr} 220 & 0.485 & -362 & 89{,}889 & -7{,}419{,}060 \\ & & 106.7 & -56{,}166 & 7{,}419{,}060 \\ \hline & 0.485 & -255.3 & 33{,}723 & 0 \end{array}$$

$h - 2200.485h^2 - 255.3h + 33{,}723 = 0$

Solving $0.485h^2 - 255.3h + 33{,}723 = 0$, the discriminant will be negative, and h will be a complex number.

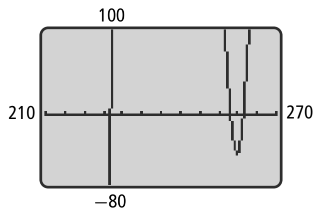
Therefore, the height of the tree is 220 ft.

b. Set the volume equal to 44,471.

$0.485h^3 - 362h^2 + 89{,}889h - 7{,}379{,}874 = 44{,}471$

$0.485h^3 - 362h^2 + 89{,}889h - 7{,}424{,}345 = 0$

From the graph, the possible heights of this tree are 226.9 ft, 258 ft, and 261.4 ft.



c. $V(255.8) = 0.485(255.8)^3 - 362(255.8)^2 + 89{,}889(255.8) - 7{,}379{,}874 \approx 44{,}648.2$

True volume of tree 44,471

Volume from model 44,648.2

The difference is 177.2 ft^3.

53. Set the volume equal to 1476π.

$$33\pi r^2 + \frac{4}{3}\pi r^3 = 1476\pi$$

$$\frac{4}{3}\pi r^3 + 33\pi r^2 - 1476\pi = 0$$

$$\frac{4}{3}r^3 + 33r^2 - 1476 = 0$$

$$\begin{array}{r|rrrr} 6 & \frac{4}{3} & 33 & 0 & -1476 \\ & & 8 & 246 & 1476 \\ \hline & \frac{4}{3} & 41 & 246 & 0 \end{array}$$

$$(x-6)\left(\frac{4}{3}x^2 + 41x + 246\right) = 0$$

$$\frac{1}{3}(x-6)\left(4x^2 + 123x + 738\right) = 0$$

Solve $4x^2 + 123x + 738 = 0$

$$x = \frac{-123 \pm \sqrt{15{,}129 - 11{,}808}}{2} = -\frac{123}{2} \pm \frac{\sqrt{3321}}{2}$$

The roots are ≈ -90.3, ≈ -32.7, and 6.

The radius must be a positive number, so the radius of the tank is 6 in.

54. 6; since $3i$ is a root 3 times, $-3i$ must also be a root 3 times.

55. Possible answer: $\pm 2.25, \frac{2}{3}$;

$$3x^3 - 2x^2 - 15x + 10 = 0$$

$$\left(3x^3 - 2x^2\right) + (-15x + 10) = 0$$

$$x^2(3x-2) - 5(3x-2) = 0$$

$$(3x-2)\left(x^2 - 5\right) = 0$$

Solve $3x - 2 = 0$

$3x = 2$

$x = \frac{2}{3}$

Solve $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

The fully factored equation is:

$(3x-2)\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) = 0$

The exact roots are $\frac{2}{3}$, and $+\sqrt{5}$.

56. Possible answer: Graph the polynomial to find the number of real zeros. Factor out the real zeros of the polynomial by using a graph, the Rational Root Theorem, or synthetic division. Use the quadratic equation to factor the remaining part of the polynomial. There may be 0, 2, or 4 imaginary zeros.

TEST PREP

57. B

$$\begin{array}{r|rrrrr} -1 & 1 & -2 & -3 & 4 & 4 \\ & & -1 & 3 & 0 & -4 \\ \hline & 1 & -3 & 0 & 4 & 0 \end{array}$$

$y = (x-1)\left(x^3 - 3x^2 + 4\right)$

$$\begin{array}{r|rrrr} -1 & 1 & -3 & 0 & 4 \\ & & -1 & 4 & -4 \\ \hline & 1 & -4 & 4 & 0 \end{array}$$

$y = (x-1)(x-1)\left(x^2 - 4x + 4\right)$

$= (x-1)(x-1)(x-2)(x-2)$

58. G

59. D

$P(x) = x(x-i)(x+i)$

$= x\left(x^2 + 1\right)$

$= x^3 + x$

60. H

Complex and irrational roots come in pairs, so the roots are $3 - \sqrt{2}$, $3 + \sqrt{2}$, $6i$, $-6i$ and 4.

61. D

$f(x) = \left[x - \left(1 + \sqrt{3}\right)\right]\left[x - \left(1 - \sqrt{3}\right)\right]$

$= x^2 - 2x - 2$

62.

$$2x^3 - 6x^2 + 8x - 24 = 0$$

$$2\left(x^3 - 3x^2 + 4x - 12\right) = 0$$

$$2\left[x^2(x-3) + 4(x-3)\right] = 0$$

$$2(x-3)\left(x^2 + 4\right) = 0$$

Solve $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

The fully factored equation is:

$2(x-3)(x-2i)(x+2i) = 0$

The solutions are 3, and $\pm 2i$.

CHALLENGE AND EXTEND

63.
$$\begin{array}{r|rrrr} 3i & 1 & -3 & 9 & -27 \\ & & 3i & -9-9i & 27 \\ \hline & 1 & -3+3i & -9i & \boxed{0} \end{array}$$
$f(3i) = 0$
$$\begin{array}{r|rrrr} -\sqrt{3} & 1 & -3 & 9 & -27 \\ & & -\sqrt{3} & 3+3\sqrt{3} & -9-12\sqrt{3} \\ \hline & 1 & -3-\sqrt{3} & 12+3\sqrt{3} & \boxed{-36-12\sqrt{3}} \end{array}$$
$f(-\sqrt{3}) = -36 - 12\sqrt{3}$

yes; $x = 3i$ is a zero.

64.
$$\begin{array}{r|rrrr} 2i & 1 & -2i & -4 & 8i \\ & & 2i & 0 & -8i \\ \hline & 1 & 0 & -4 & \boxed{0} \end{array}$$
$(x - 2i)(x^2 - 4) = 0$
$(x - 2i)(x - 2)(x + 2) = 0$
$x - 2i = 0$ or $x - 2 = 0$ or $x + 2 = 0$
$x = 2i \quad x = 2 \quad x = -2$

65.
$$\begin{array}{r|rrrr} \sqrt{2} & 1 & -\sqrt{2} & 9 & -9\sqrt{2} \\ & & \sqrt{2} & 0 & 9\sqrt{2} \\ \hline & 1 & 0 & 9 & \boxed{0} \end{array}$$
$(x - \sqrt{2})(x^2 + 9) = 0$
$(x - \sqrt{2})(x - 3i)(x + 3i) = 0$
$x - \sqrt{2} = 0$ or $x - 3i = 0$ or $x + 3i = 0$
$x = \sqrt{2} \quad x = 3i \quad x = -3i$

66. Possible answer: Polynomials with nonreal or irrational coefficients do not have complex or irrational roots in conjugate pairs.

67. Solve $a^2 + b^2 = 0$
$a^2 = -b^2$
$a = \pm bi$
The factored equation is:
$= (a + bi)(a - bi)$

68. $a^4 + b^4$
$= (a^2)^2 + (b^2)^2$
$= (a^2 + b^2 i)(a^2 - b^2 i)$
Solve $a^2 + b^2 i = 0 \qquad a^2 - b^2 i = 0$
$a^2 = -b^2 i \qquad a^2 = b^2 i$
$a = \pm bi\sqrt{i} \qquad a = \pm b\sqrt{i}$
The factored equation is:
$= (a + bi\sqrt{i})(a - bi\sqrt{i})(a + b\sqrt{i})(a - b\sqrt{i})$

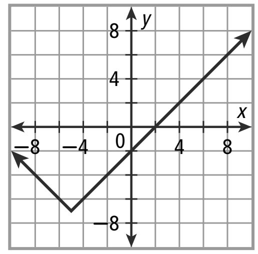
69. $a^6 + b^6$
$= (a^2)^3 + (b^2)^3$
$= (a^2 + b^2)\left[(a^2)^2 - a^2 \cdot b^2 + (b^2)^2\right]$
$= (a + bi)(a - bi)(a^4 - a^2b^2 + b^4)$

70. Possible answer: yes; $a^2 + b^2$ is a factor of both $a^4 + b^4$ and $a^6 + b^6$ after the first step.

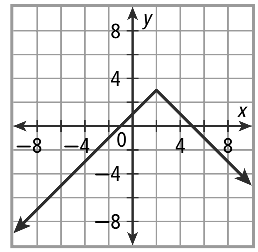
71. $x^4 + 2x^2 + 1 = 0$
$(x^2 + 1)(x^2 + 1) = 0$
Solve $x^2 + 1 = 0$
$x^2 = -1$
$x = \pm i$
The roots are i and $-i$.

SPIRAL REVIEW

72. shift 5 units left and 7 units down



73. shift 2 units right, reflection across x-axis, and shift 3 units up



74. $g(x) = |x + 3| - 4$
vertex $= (-3, -4)$

75. $f(x) = |x - 9| - 3$
vertex $= (9, -3)$

76. $4x^4 + 32x^3 + 64x^2 = 0$
$4x^2(x^2 + 8x + 16) = 0$
$4x^2(x + 4)(x + 4) = 0$
$4x^2 = 0$ or $x + 4 = 0$ or $x + 4 = 0$
$x = 0 \quad x = -4 \quad x = -4$

77. $x^3 - 43x^2 + 42x = 0$
$x(x^2 - 43x + 42) = 0$
$x(x - 1)(x - 42) = 0$
$x = 0$ or $x - 1 = 0$ or $x - 42 = 0$
$x = 0 \quad x = 1 \quad x = 42$

78. $3x^5 + 18x^4 - 81x^3 = 0$
$3x^3(x^2 + 6x - 27) = 0$
$3x^3(x - 3)(x + 9) = 0$
$3x^3 = 0$ or $x - 3 = 0$ or $x + 9 = 0$
$x = 0 \quad x = 3 \quad x = -9$

79. $2x^3 + 12x^2 = 32x$
$2x^3 + 12x^2 - 32x = 0$
$2x(x^2 + 6x - 16x) = 0$
$2x(x - 2)(x + 8) = 0$
$2x = 0$ or $x - 2 = 0$ or $x + 8 = 0$
$x = 0 \quad x = 2 \quad x = -8$

6-7 INVESTIGATING GRAPHS OF POLYNOMIAL FUNCTIONS, PAGES 454–459

CHECK IT OUT!

1a. The leading coefficient is 2, which is positive.
The degree is 5, which is odd.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.

b. The leading coefficient is −3, which is negative.
The degree is 2, which is even.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.

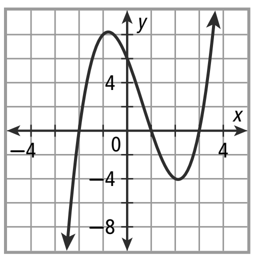
2a. As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of odd degree with
a negative leading coefficient.

b. As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
$P(x)$ is of even degree with
a positive leading coefficient.

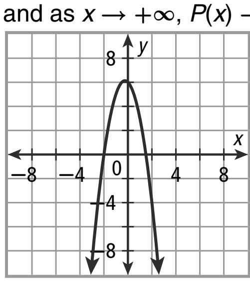
3a. $f(x) = x^3 - 2x^2 - 5x + 6$
Possible rational roots: $\pm1, \pm2, \pm3, \pm6$

$$\begin{array}{r|rrrr} 1 & 1 & -2 & -5 & 6 \\ & & 1 & -1 & -6 \\ \hline & 1 & -1 & -6 & \boxed{0} \end{array}$$

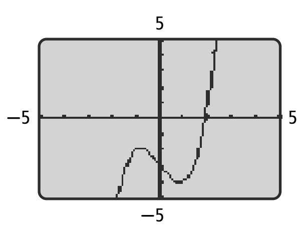
$f(x) = (x - 1)(x^2 - x - 6)$
$= (x - 1)(x - 3)(x + 2)$
The zeros are 1, 3, and −2.
Plot other points: $f(0) = 6$, so the y-intercept is 6; $f(-1) = 8$, and $f(2) = -4$.
The degree is odd and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.



b. $f(x) = -2x^2 - x + 6$
$= (x + 2)(2x - 3)$
The zeros are −2, and $\frac{3}{2}$.
Plot other points: $f(0) = 6$, so the y-intercept is 6.
The degree is even and the leadind coefficient is negative, so as $x \to -\infty$, $P(x) \to -\infty$,
and as $x \to +\infty$, $P(x) \to -\infty$.

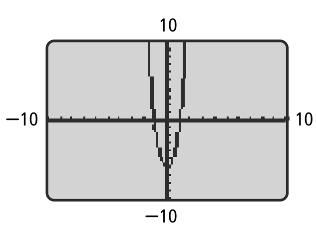


4a.



local minimum: ≈ -4.0887
local maximum: ≈ -1.9113

b.



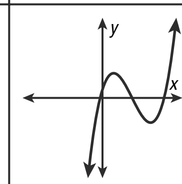
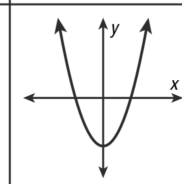
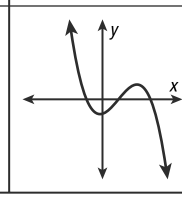
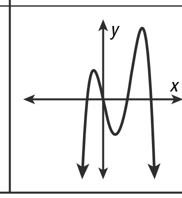
local minimum: −6

5. Let x represent the side length of the cut-out square.
So, $V(x) = x(16 - 2x)(20 - 2x)$
Values of x greater than 8, or less than 0 do not make sense for this problem.
The graph has a local maximum of about 420.1 when $x \approx 2.94$. So, the largest open box will have a volume of 420.1 ft^3.

THINK AND DISCUSS

1. Possible answer: The graph must turn between each pair of roots in order to cross the x-axis again.

2.

Leading Coefficient	Odd Degree	Even Degree
Positive		
Negative		

EXERCISES

GUIDED PRACTICE

1. A graph "turns around" at a turning point.

2. The leading coefficient is −4, which is negative.
The degree is 4, which is even.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.

3. The leading coefficient is −2, which is negative.
The degree is 7, which is odd.
As $x \to -\infty$, $Q(x) \to +\infty$, and
as $x \to +\infty$, $Q(x) \to -\infty$.

4. The leading coefficient is 1, which is positive.
The degree is 5, which is odd.
As $x \to -\infty$, $R(x) \to -\infty$, and
as $x \to +\infty$, $R(x) \to +\infty$.

5. The leading coefficient is 3, which is positive.
The degree is 2, which is even.
As $x \to -\infty$, $S(x) \to +\infty$, and
as $x \to +\infty$, $S(x) \to +\infty$.

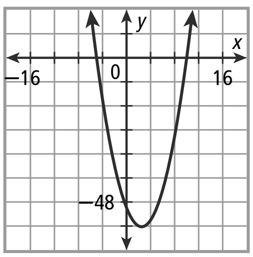
6. As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
$P(x)$ is of odd degree with
a positive leading coefficient.

7. As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
$P(x)$ is of even degree with
a positive leading coefficient.

8. As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of odd degree with
a negative leading coefficient.

9. As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of even degree with
a negative leading coefficient.

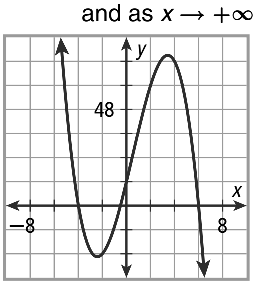
10. $f(x) = x^2 - 5x - 50$
$= (x + 5)(x - 10)$
The zeros are −5, and 10.
Plot other points: $f(0) = -50$, so the y-intercept is −50
The degree is even and the leading coefficient is
positive, so as $x \to -\infty$, $P(x) \to +\infty$,
and as $x \to +\infty$, $P(x) \to +\infty$.

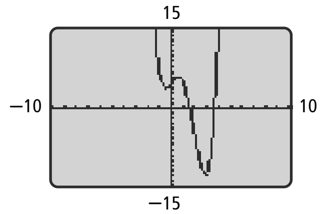


11. $f(x) = -x^3 + \frac{3}{2}x^2 + 25x + 12$
Possible rational roots: ±1, ±2, ±3, ±4, ±6, ±12

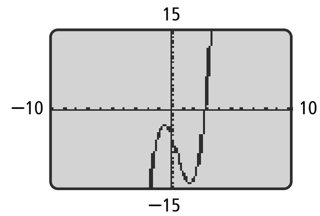
$$\begin{array}{r|rrrr} 6 & -1 & \frac{3}{2} & 25 & 12 \\ & & -6 & -27 & -12 \\ \hline & -1 & -\frac{9}{2} & -2 & 0 \end{array}$$

$f(x) = (x - 6)\left(-x^2 - \frac{9}{2}x - 2\right)$
$= -\frac{1}{2}(x - 6)\left(2x^2 + 9x + 4\right)$
$= \frac{1}{2}(x - 6)(x + 4)(2x + 1)$
The zeros are 6, −4, and $-\frac{1}{2}$.
Plot other points: $f(0) = 12$, so the y-intercept is 12;
$f(-1) = -\frac{21}{2}$, and $f(4) = 72$.
The degree is odd and the leading coefficient is
negative, so as $x \to -\infty$, $P(x) \to +\infty$,
and as $x \to +\infty$, $P(x) \to -\infty$.



12.


local minimum: ≈ 4.0539, and −13.13
local maximum: ≈ 6.0761

13.


local minimum: ≈ −14.0902
local maximum: ≈ −2.9098

14. $A(x) = 60x - 3x^2$
Values of x greater than 15, or less than 0 do not
make sense for this problem.
The graph has a local maximum of about 300 when
$x \approx 10$. So, the maximum area of the patio is 300 ft^2.

PRACTICE AND PROBLEM SOLVING

15. The leading coefficient is 2, which is positive.
The degree is 3, which is odd.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.

16. The leading coefficient is −3, which is negative.
The degree is 4, which is even.
As $x \to -\infty$, $Q(x) \to -\infty$, and
as $x \to +\infty$, $Q(x) \to -\infty$.

17. The leading coefficient is −1, which is negative.
The degree is 5, which is odd.
As $x \to -\infty$, $R(x) \to +\infty$, and
as $x \to +\infty$, $R(x) \to -\infty$.

18. The leading coefficient is 5.5, which is positive.
The degree is 8, which is even.
As $x \to -\infty$, $S(x) \to +\infty$, and
as $x \to +\infty$, $S(x) \to +\infty$.

19. As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of even degree with
a negative leading coefficient.

20. As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
$P(x)$ is of odd degree with
a positive leading coefficient.

21. As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of odd degree with
a negative leading coefficient.

22. As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of even degree with
a negative leading coefficient.

23. $f(x) = x^3 - \frac{7}{3}x^2 - \frac{43}{3}x + 5$

$= \frac{1}{3}\left(3x^3 - 7x^2 - 43x + 15\right)$

Possible rational roots: $\pm1, \pm3, \pm5, \pm15, \pm\frac{1}{3}, \pm\frac{5}{3}$

5	3	−7	−43	15
		15	40	−15
	3	8	−3	0

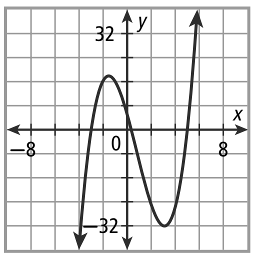
$f(x) = \frac{1}{3}(x - 5)\left(3x^2 + 8x - 3\right)$

$= \frac{1}{3}(x - 5)(3x - 1)(x + 3)$

The zeros are 5, $\frac{1}{3}$, and −3.

Plot other points: $f(0) = 5$, so the y-intercept is 5; $f(-1) = 16$, and $f(3) = -32$.

The degree is odd and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.



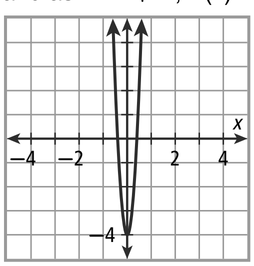
24. $f(x) = 25x^2 - 4$

$= (5x - 4)(5x + 4)$

The zeros are $\frac{4}{5}$, and $-\frac{4}{5}$.

Plot other points: $f(0) = -4$, so the y-intercept is −4;

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to +\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.



25. $f(x) = x^4 + x^3 - 28x^2 + 20x + 48$

Possible rational roots:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm16, \pm24, \pm48$

2	1	1	−28	20	48
		2	6	−44	−48
	1	3	−22	−24	0

$f(x) = (x - 2)\left(x^3 + 3x^2 - 22x - 24\right)$

Possible rational roots:

$\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

4	1	3	−22	−24
		4	28	24
	1	7	6	0

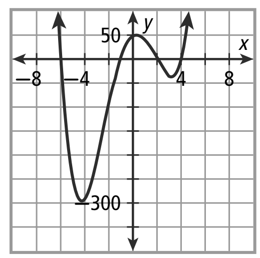
$f(x) = (x - 2)(x - 4)\left(x^2 + 7x + 6\right)$

$= (x - 2)(x - 4)(x + 1)(x + 6)$

The zeros are 2, 4, −1, and −6.

Plot other points: $f(0) = 48$, so the y-intercept is 48; $f(-4) = -288$, and $f(3) = -36$.

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to +\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.



26. $f(x) = x^3 + \frac{13}{2}x^2 + 11x + 4$

$= \frac{1}{2}\left(2x^3 + 13x^2 + 22x + 8\right)$

Possible rational roots: $\pm1, \pm2, \pm4, \pm8, \pm\frac{1}{2}$

−2	2	13	22	8
		−4	−18	−8
	2	9	4	0

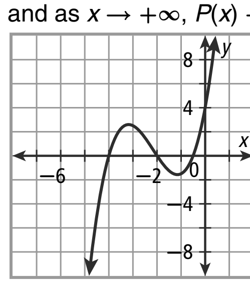
$f(x) = \frac{1}{2}(x + 2)\left(2x^2 + 9x + 4\right)$

$= \frac{1}{2}(x + 2)(x + 4)(2x + 1)$

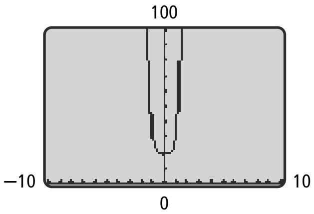
The zeros are −2, −4, and $-\frac{1}{2}$.

Plot other points: $f(0) = 4$, so the y-intercept is 4; $f(-3) = \frac{5}{2}$, and $f(-1) = -\frac{3}{2}$.

The degree is odd and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.

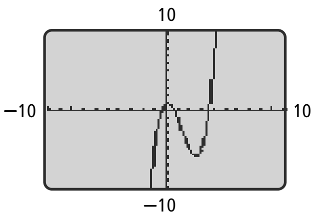


27.



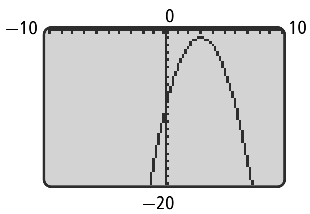
local minimum: 20

28.



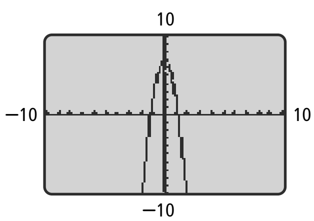
local minimum: ≈ -5.88
local maximum: ≈ 1

29.



local maximum: ≈ -1

30.



local maximum: 7

31. $V(t) = -1.7t^2 + 1.7t + 3$
Values of t greater than 1, or less than 0 are not considered for this problem.
The graph has a local maximum of 3.425 when $x \approx 0.5$. So, the maximum volume of the air is 3.425 L, and it occurs at 0.5 s

32. The leading coefficient is 5, which is positive.
The degree is 3, which is odd.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
Therefore, graph C.

33. The leading coefficient is 2, which is positive.
The degree is 6, which is even.
As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to +\infty$.
Therefore, graph B.

34. The leading coefficient is −1, which is negative.
The degree is 5, which is odd.
As $x \to -\infty$, $P(x) \to +\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
Therefore, graph D.

35. The leading coefficient is −4, which is negative.
The degree is 2, which is even.
As $x \to -\infty$, $P(x) \to -\infty$, and
as $x \to +\infty$, $P(x) \to -\infty$.
Therefore, graph A.

36. $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to +\infty$ as $x \to +\infty$.

37. $f(x) \to +\infty$ as $x \to -\infty$, and $f(x) \to +\infty$ as $x \to +\infty$.

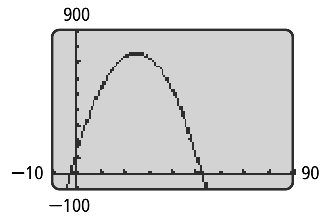
38. $f(x) \to +\infty$ as $x \to -\infty$, and $f(x) \to -\infty$ as $x \to +\infty$.

39. $f(x) \to +\infty$ as $x \to -\infty$, and $f(x) \to +\infty$ as $x \to +\infty$.

40. $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to +\infty$ as $x \to +\infty$.

41. $f(x) \to -\infty$ as $x \to -\infty$, and $f(x) \to -\infty$ as $x \to +\infty$.

42a.



b. \$756.25

c. The maximum revenue occurs at $x \approx 25$.
Therefore, it would take 25 reductions of \$0.25.
The price of a shirt will be $13 - 0.25(25) = \$6.75$.

43. Possible answer: For very large values of x, the leading term overpowers the other terms, so its degree and coefficient are all that matter.

44. g; possible answer: The degrees of f and h are odd so their range is $\mathbb{R}$. The degree of g is even, so its range is not $\mathbb{R}$.

45a. Dimensions of the base:

$$2x + 2w = 20$$
$$x + w = 10$$
$$w = 10 - x$$

Volume of the pyramid:

$$V = \frac{1}{3}\ell wh$$
$$V(x) = \frac{1}{3}(x^2)(10 - x)$$
$$= \frac{1}{3}(10x^2 - x^3)$$
$$= -\frac{1}{3}x^3 + \frac{10}{3}x^2$$

b. Values of x greater than 10, or less than 0 do not make sense for this problem.
The graph has a local maximum of about 49.4 when $x \approx 6.7$. So, the maximum area of the box is 49.4 in^3.

c. The maximum volume occurs when $x \approx 6.7$.
So dimensions are approximately: $\ell \times w \times h$
$= 6.7 \text{ in.} \times 3.3 \text{ in.} \times 6.7 \text{ in.}$

46. No; possible answer: after the first turning pt., there may be another turning pt. before the graph reaches the x-axis.

47. Possible answer: Identify possible rational roots and use them to factor the polynomial. Plot the zeros of the function and a few other points as guidelines. Determine the end behavior, and graph.

TEST PREP

48. C

If there are n distinct real roots, then there are exactly $n - 1$ turning points.

49. H

The function will have an odd degree and a positive leading coefficient.

50a. $f(x) = 2x^4 + 12x^3 + 24x^2 + 16x$

$= 2x(x^3 + 6x^2 + 12x + 8)$

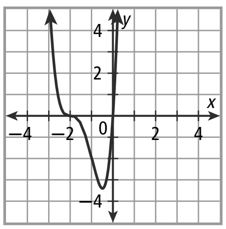
$= 2x(x + 2)(x + 2)(x + 2)$

The solutions are 0, and −2.

b. As $x \to -\infty$, $f(x) \to +\infty$, and as $x \to +\infty$, $f(x) \to +\infty$.

There is a positive leading coefficient, and an even degree.

c.



CHALLENGE AND EXTEND

51. $3x^6 - 57x^4 + 6x^3 + 144x^2 - 96x = 0$

$3x(x^5 - 19x^3 + 2x^2 + 48x - 32) = 0$

Possible rational roots: ±1, ±2, ±4, ±8, ±16, ±32

1	1	0	−19	2	48	−32
		1	1	−18	−16	32
	1	1	−18	−16	32	0

$3x(x - 1)(x^4 + x^3 - 18x^2 - 16x + 32) = 0$

Possible rational roots: ±1, ±2, ±4, ±8, ±16, ±32

1	1	1	−18	−16	32
		1	2	−16	−32
	1	2	−16	−32	0

$3x(x - 1)(x - 1)(x^3 + 2x^2 - 16x - 32) = 0$

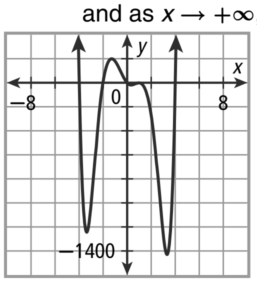
$3x(x - 1)(x - 1)(x + 2)(x^2 - 16) = 0$

$3x(x - 1)(x - 1)(x + 2)(x + 4)(x - 4) = 0$

The zeros are 0, 1, −2, −4, and 4.

Plot other points: $f(0) = 0$, so the y-intercept is 0; $f(-3) = -1008$, $f(3) = -1260$, $f(-1) = 80$

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $f(x) \to +\infty$, and as $x \to +\infty$, $f(x) \to +\infty$.



52. $-2x^5 - 14x^4 - 30x^3 - 18x^2 = 0$

$(-2x^2)(x^3 + 7x^2 + 15x + 9) = 0$

Possible rational roots: ±1, ±3, ±9,

−1	1	7	15	9
		−1	−6	−9
	1	6	9	0

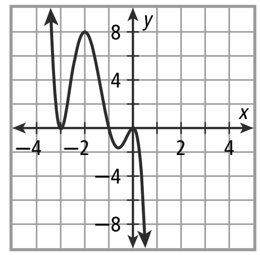
$(-2x^2)(x + 1)(x^2 + 6x + 9) = 0$

$(-2x^2)(x + 1)(x + 3)(x + 3) = 0$

The zeros are 0, −1, and −3

Plot other points: $f(0) = 0$, so the y-intercept is 0; $f(-2) = 8$, $f(-0.5) = -1.5625$

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $f(x) \to -\infty$, and as $x \to +\infty$, $f(x) \to +\infty$.



53.

x	$f(x)$	$g(x)$	$\frac{f(x)}{g(x)}$
5	125	−25	−5
10	1000	420	2.3810
50	125,000	110,180	1.1345
100	1,000,000	940,380	1.0634
500	125,000,000	123,501,980	1.0121
1000	1,000,000,000	994,003,980	1.0060
5000	1.25×10^{11}	1.2485×10^{11}	1.0012

54. As $x \to +\infty$, $\frac{f(x)}{g(x)} \to 1$

55. Possible answer: the same end behavior

SPIRAL REVIEW

56.

$2x + y = 2$	
2(2) + (−2)	2
2	2 ✓

$6x - 2y = 16$	
6(2) − 2(−2)	16
16	16 ✓

(2, −2) is a solution.

57.

$x + 3y = 0$	
3 + 3(−1)	0
0	0 ✓

$8x + 4y = 21$	
8(3) + 4(−1)	21
20	21 ✗

(3, −1) is not a solution.

58.

$x + y = 5$	
5 + (−5)	5
0	5 ✗

(5, −5) is not a solution.

59. $h(t) = -16t^2 + 64t + 6$

$= -16(t^2 - 4t) + 6$

$= -16\left(t^2 - 4t + \left(\frac{4}{2}\right)^2\right) + 6 + 16\left(\frac{4}{2}\right)^2$

$= -16(t^2 - 4t + 4) + 6 + 64$

$= -16(t - 2)^2 + 70$

The vertex is (2, 70).

60.

$$\begin{array}{r} x + 1 \\ x + 3 \overline{)\, x^2 + 4x + 10} \\ -(x^2 + 3x) \\ \hline x + 10 \\ -(x + 3) \\ \hline 7 \end{array}$$

$\frac{x^2 + 4x + 10}{x + 3} = x + 1 + \frac{7}{x + 3}$

61.

$$\begin{array}{r} 10x - 2 \\ x + 1 \overline{)\, 10x^2 + 8x + 6} \\ -(10x^2 + 10x) \\ \hline -2x + 6 \\ -(-2x - 2) \\ \hline 8 \end{array}$$

$\frac{10x^2 + 8x + 6}{x + 1} = 10x - 2 + \frac{8}{x + 1}$

62.

$$\begin{array}{r} x - 7 \\ x + 8 \overline{)\, x^2 + x - 64} \\ -(x^2 + 8x) \\ \hline -7x - 64 \\ -(-7x - 56) \\ \hline -8 \end{array}$$

$\frac{x^2 + x - 64}{x + 8} = x - 7 - \frac{8}{x + 8}$

6-8 TRANSFORMING POLYNOMIAL FUNCTIONS, PAGES 460–465

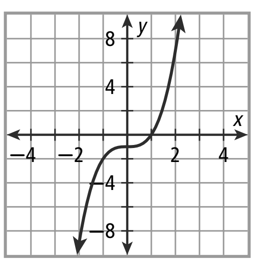
CHECK IT OUT!

1a. $g(x) = f(x) - 5$

$= (x^3 + 4) - 5$

$= x^3 - 1$

To graph $g(x)$, translate the graph of $f(x)$ 5 units down. This is a vertical translation.

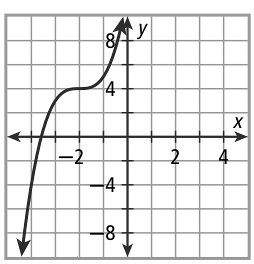


b. $g(x) = f(x + 2)$

$= (x + 2)^3 + 4$ or

$x^3 + 6x^2 + 12x + 12$

To graph $g(x)$, translate the graph of $f(x)$ 2 units left. This is a horizontal translation.



2a. $g(x) = -f(x)$

$= -(x^3 - 2x^2 - x + 2)$

$= -x^3 + 2x^2 + x - 2$

b. $g(x) = f(-x)$

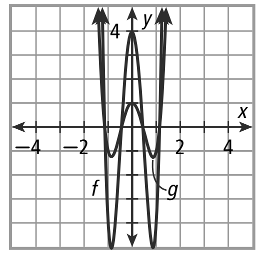
$= (-x)^3 - 2(-x)^2 - (-x) + 2$

$= -x^3 - 2x^2 + x + 2$

3a. $g(x) = \frac{1}{4}f(x)$

$= \frac{1}{4}(16x^4 - 24x^2 + 4)$

$= 4x^4 - 6x^2 + 1$

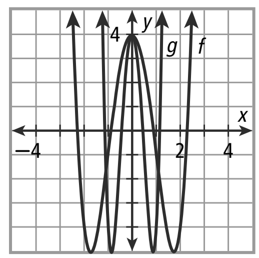


$g(x)$ is a vertical compression of $f(x)$.

b. $g(x) = f\left(\frac{1}{2}x\right)$

$= 16\left(\frac{1}{2}x\right)^4 - 24\left(\frac{1}{2}x\right)^2 + 4$

$= x^4 - 6x^2 + 4$



$g(x)$ is a horizontal stretch of $f(x)$.

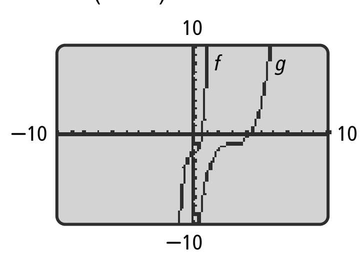
4a. A vertical compression is represented by $af(x)$, and a horizontal shift is represented by $f(x - h)$. Combining the two transformations gives $g(x) = af(x - h)$.

Substitute $\frac{1}{2}$ for a and -3 for h.

$g(x) = \frac{1}{2}f(x - 3)$

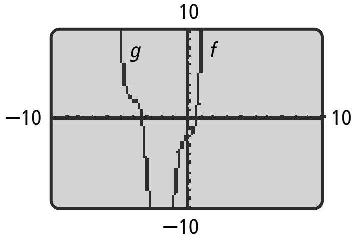
$= \frac{1}{2}(8(x - 3)^3 - 2)$

$= 4(x - 3)^3 - 1$ or $4x^3 - 36x^2 + 108x - 109$



b. A reflection across the x-axis is represented by $-f(x)$, and a horizontal shift is represented by $f(x - h)$. Combining the two transformations gives $g(x) = -f(x - h)$.
Substitute 4 for h.

$$g(x) = -f(x - (-4))$$
$$= -\left(8(x + 4)^3 - 2\right)$$
$$= -8(x + 4)^3 + 2 \text{ or } -8x^3 - 96x^2 - 384x - 510$$



5. $g(x) = f(x - 5)$
$$= 0.01(x - 5)^3 + 0.7(x - 5)^2 + 0.4(x - 5) + 120$$
$$= 0.01\left(x^3 - 15x^2 + 75x - 125\right) + 0.7\left(x^2 - 10x + 25\right) + 0.4(x - 5) + 120$$
$$= 0.01x^3 + 0.55x^2 - 5.85x + 134.25$$
Possible answer: The model represents the number of sales since March.

THINK AND DISCUSS

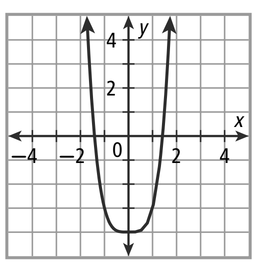
1. Possible answer: It eliminates all real zeros. It does not change the number of real zeros.
2. No; possible answer: the zeros change location, not quantity.
3.

Transformation	Vertical shift	Horizontal shift	Vertical stretch	Horizontal compression
Example	$f(x) + 2$	$f(x - 2)$	$2f(x)$	$f(2x)$

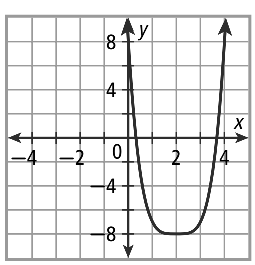
EXERCISES

GUIDED PRACTICE

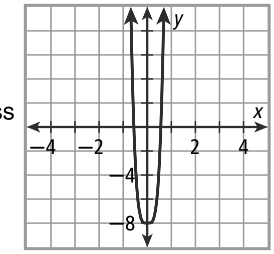
1. $g(x) = f(x) + 4$
$$= \left(x^4 - 8\right) + 4$$
$$= x^4 - 4$$
To graph $g(x)$, translate the graph of $f(x)$ 4 units up. This is a vertical translation.



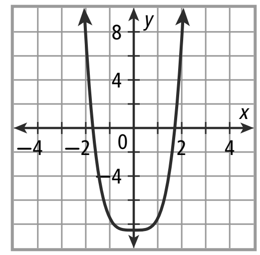
2. $h(x) = f(x - 2)$
$$= (x - 2)^4 - 8 \text{ or } x^4 - 8x^3 + 24x^2 - 32x + 8$$
To graph $h(x)$, translate the graph of $f(x)$ 2 units right. This is a horizontal translation.



3. $j(x) = f(3x)$
$$= (3x)^4 - 8$$
$$= 81x^4 - 8$$
To graph $j(x)$, compress the graph of $f(x)$ by $\frac{1}{3}$. This is a horizontal compression.



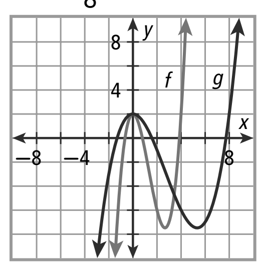
4. $k(x) = f(x) - \frac{1}{2}$
$$= \left(x^4 - 8\right) - \frac{1}{2}$$
$$= x^4 - 8.5$$
To graph $k(x)$, translate the graph of $f(x)$ $\frac{1}{2}$ a unit up. This is a vertical translation.



5. $g(x) = f(-x)$
$$= -(-x)^3 + 3(-x)^2 - 2(-x) + 1$$
$$= x^3 + 3x^2 + 2x + 1$$

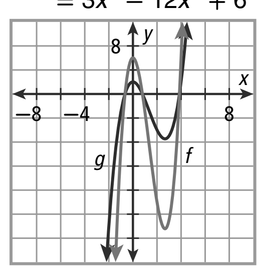
6. $g(x) = -f(x)$
$$= -\left(-x^3 + 3x^2 - 2x + 1\right)$$
$$= x^3 - 3x^2 + 2x - 1$$

7. $g(x) = f\left(\frac{1}{2}x\right)$
$$= \left(\frac{1}{2}x\right)^3 - 4\left(\frac{1}{2}x\right)^2 + 2$$
$$= \frac{1}{8}x^3 - x^2 + 2$$



$g(x)$ is a horizontal stretch of $f(x)$.

8. $g(x) = 3f(x)$
$$= 3\left(x^3 - 4x^2 + 2\right)$$
$$= 3x^3 - 12x^2 + 6$$

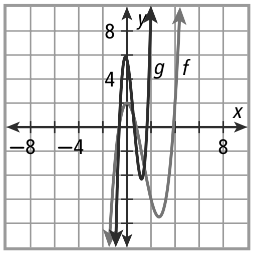


$g(x)$ is a vertical stretch of $f(x)$.

9. $g(x) = f(2x) + 4$

$= \left((2x)^3 - 4(2x)^2 + 2\right) + 4$

$= 8x^3 - 16x^2 + 6$



$g(x)$ is a horizontal compression and a vertical shift of $f(x)$.

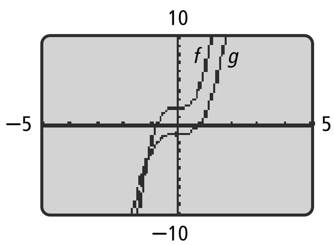
10. A vertical compression is represented by $af(x)$, and a vertical shift is represented by $f(x) + k$. Combining the two transformations gives $g(x) = af(x) + k$.

Substitute $\frac{1}{2}$ for a and -2 for k.

$g(x) = \frac{1}{2}f(x) - 2$

$= \frac{1}{2}\left(4x^3 + 2\right) - 2$

$= 2x^3 - 1$



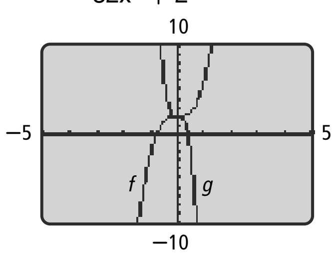
11. A reflection across the y-axis is represented by $f(-x)$, and a horizontal compression is represented by $f\left(\frac{1}{b}x\right)$. Combining the two transformations gives $g(x) = f\left(-\frac{1}{b}x\right)$.

Substitute $\frac{1}{2}$ for b.

$g(x) = f(-2x)$

$= 4(-2x)^3 + 2$

$= -32x^3 + 2$



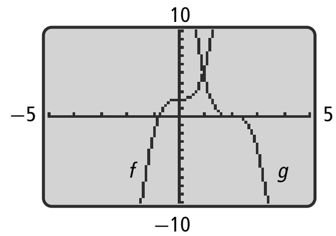
12. A horizontal shift is represented by $f(x - h)$, a vertical shift is represented by $f(x) + k$, and reflection across the x-axis is represented by $-f(x)$. Combining the three transformations gives $g(x) = -f(x - h) + k$.

Substitute 2 for h and -3 for k.

$g(x) = -\left(f(x - 2) - 3\right)$

$= -\left(4(x - 2)^3 + 2 - 3\right)$

$= -4(x - 2)^3 + 1$ or $-4x^3 + 24x^2 - 48x + 33$



13. $c(x) = 2C(x)$

$= 2\left(2x^3 - 3x + 30\right)$

$= 4x^3 - 6x + 60$

The cost has doubled.

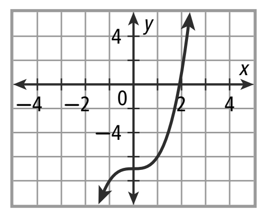
PRACTICE AND PROBLEM SOLVING

14. $g(x) = f(x) - 3$

$= \left(x^3 - 4\right) - 3$

$= x^3 - 7$

To graph $g(x)$, translate the graph of $f(x)$ 3 units down. This is a vertical translation.

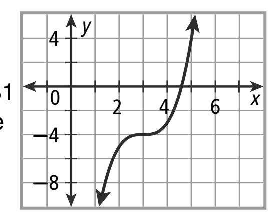


15. $h(x) = f(x - 3)$

$= (x - 3)^3 - 4$ or

$x^3 - 9x^2 + 27x - 31$

To graph $h(x)$, translate the graph of $f(x)$ 3 units right. This is a horizontal translation.

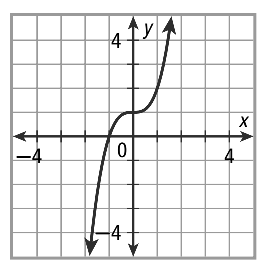


16. $j(x) = f(x) + 5$

$= \left(x^3 - 4\right) + 5$

$= x^3 + 1$

To graph $j(x)$, translate the graph of $f(x)$ 5 units up. This is a vertical translation.



17. $g(x) = -f(x)$

$= -\left(x^3 - 2x^2 + 5x - 3\right)$

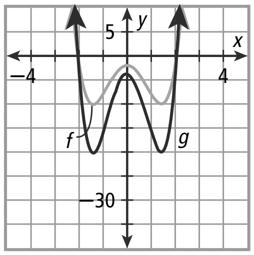
$= -x^3 + 2x^2 - 5x + 3$

18. $g(x) = f(-x)$

$= (-x)^3 - 2(-x)^2 + 5(-x) - 3$

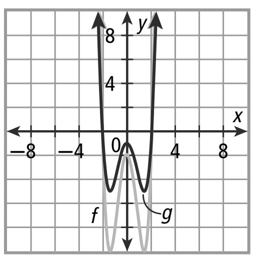
$= -x^3 - 2x^2 - 5x - 3$

19. $g(x) = 2f(x)$
$= 2\left(2x^4 - 8x^2 - 2\right)$
$= 4x^4 - 16x^2 - 4$



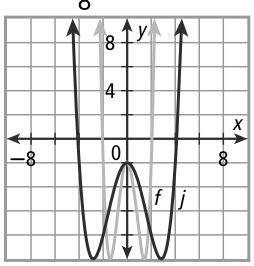
$g(x)$ is a vertical stretch of $f(x)$.

20. $g(x) = \frac{1}{2}f(x)$
$= \frac{1}{2}\left(2x^4 - 8x^2 - 2\right)$
$= x^4 - 4x^2 - 1$



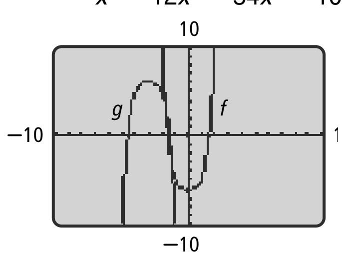
$g(x)$ is a vertical compression of $f(x)$.

21. $g(x) = f\left(\frac{1}{2}x\right)$
$= 2\left(\frac{1}{2}x\right)^4 - 8\left(\frac{1}{2}x\right)^2 - 2$
$= \frac{1}{8}x^4 - 2x^2 - 2$

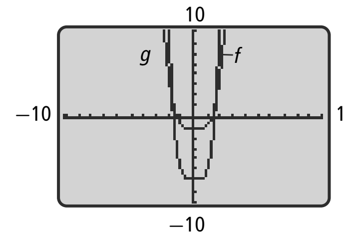


$g(x)$ is a horizontal stretch of $f(x)$.

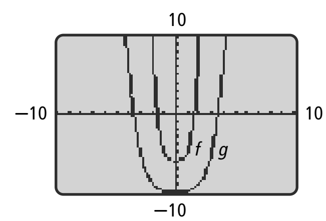
22. A reflection across the x-axis is represented by $-f(x)$, and a horizontal shift is represented by $f(x - h)$. Combining the two transformations gives $g(x) = -f(x - h)$.
Substitute −3 for h.
$g(x) = -\left((x + 3)^4 - 6\right)$
$= -(x + 3)^4 + 6$ or
$-x^4 - 12x^3 - 54x^2 - 108x - 75$



23. A vertical compression is represented by $af(x)$, and vertical shift is represented by $f(x) + k$. Combining the two transformations gives $g(x) = af(x) + k$.
Substitute $\frac{1}{3}$ for a and 1 for k.
$g(x) = \frac{1}{3}\left(x^4 - 6\right) + 1$
$= \frac{1}{3}x^4 - 1$



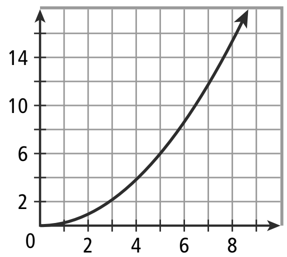
24. A horizontal stretch is represented by $f\left(\frac{1}{b}x\right)$, and horizontal shift is represented by $f(x) + k$, and a reflection across the y-axis is represented by $f(-x)$. Combining the three transformations gives
$g(x) = f\left(-\frac{1}{b}x\right) + k$
Substitute 2 for b and −4 for k.
$g(x) = \left(\frac{1}{2}(-x)\right)^4 - 6 - 4$
$g(x) = \frac{1}{16}x^4 - 10$



25. $V\left(\frac{2}{3}x\right) = \left(\frac{2}{3}x\right)^3 + 3\left(\frac{2}{3}x\right)^2 + \left(\frac{2}{3}x\right) + 8$
$= \frac{8}{27}x^3 + \frac{4}{3}x^2 + \frac{2}{3}x + 8$
possible answer: the length of x is stretched by a factor of $\frac{3}{2}$.

26. B; possible answer: a shift to the right is represented by $f(x - 3)$, not $f(x + 3)$.

27a.



Set $F > W$
$0.24v^2 > 1$
Solve $0.24v^2 = 1$
$0.24v^2 - 1 = 0$
$(v + 2.04)(v - 2.04) = 0$
$v + 2.04 = 0$ or $v - 2.04 = 0$
$v = -2.04$ $v = 2.04$
x-value test: $0.24(-2.05)^2 > 1$ ✓
$0.24(2.03)^2 > 1$ ✗
$0.24(2.05)^2 > 1$ ✓
Therefore $v < -2.04$, or $v > 2.04$.
Since v can not be negative, $v > 2.04$.

b. $G(v) = F(v - (-5))$
$= 0.24(v + 5)^2$
$= 0.24(v^2 + 10v + 25)$
$= 0.24v^2 + 2.4v + 6$
The transformation represents a shift 5 units left.

c. Since v cannot be negative, and when $v \geq 0$, $0.24(v + 5)^2 \geq 0.24(5)^2 = 6 > W$, the values of v for which $G > M$ are $v \geq 0$.

d. $H(v) = 20v^2$
$= 83\frac{1}{3}(0.24v^2)$
$= 83\frac{1}{3}F(v)$
$H(v)$ is a vertical stretch of $F(v)$.

28. Possible answer: 2 real solutions for $k < 0$, no real solutions for $k > 0$; negative k shifts the graph down, so it crosses the x-axis; positive k shifts the graph up and it will never cross the x-axis.

29. Possible answer: The portion of the graph above the x-axis is flipped below the axis, and the portion below is flipped above.

30a. $W(x) = \frac{1}{3}(12x)^3 + (12x)^2$
$= 576x^3 + 144x^2$

b. $W(x) = V(12x)$

c. horizontal compression by $\frac{1}{12}$

d. Possible answer: Expressing the length of the base in centimeters corresponds to a horizontal stretch rather than a compression.

TEST PREP

31. B

32. J

33. B

34a. The leading coefficient is positive, and the degree is odd; as $x \to +\infty$, $f(x) \to +\infty$, and as $x \to -\infty$, $f(x) \to -\infty$.

b. $g(x) = f(-x)$
$= 3(-x)^3 - 9(-x)^2 - 3(-x) + 9$
$= -3x^3 - 9x^2 + 3x + 9$

c. As $x \to -\infty$, $f(x) \to +\infty$, and as $x \to +\infty$, $f(x) \to -\infty$.
Possible answer: A reflection across the y-axis of an odd function changes the end behavior by changing the sign of the leading coefficient.

CHALLENGE AND EXTEND

35. $g(x) = x^3 - 6$
$= ((x + 2) - 2)^3 - 6$
$= f(x - 2)$
shift right 2 units

36. $g(x) = (x + 2)^3$
$= ((x + 2)^3 - 6) + 6$
$= f(x) + 6$
shift up 6 units

37. $g(x) = (x - 1)^3 + 2$
$= (((x + 2) - 3)^3 - 6) + 8$
$= f(x - 3) + 8$
shift right 3 units and up 8 units

38. Possible answer: shift down 6 units or vertical compression by a factor of 3 or shift left 2 units

SPIRAL REVIEW

39. Let t represent the number of minutes, and let $w(t)$ represent the number of words typed.
$w(t) = 60t$
Yes, this is a function.

40. $(6y + 4y^2 - 3) + (9y^2 - 5 + 8y)$
$= (4y^2 + 6y - 3) + (9y^2 + 8y - 5)$
$= (4y^2 + 9y^2) + (6y + 8y) + (-3 - 5)$
$= 13y^2 + 14y - 8$

41. $(2x^5 - 4x + 8x^2) - (3x^4 - x^5 + 3x^2)$
$= (2x^5 + 8x^2 - 4x) - (-x^5 + 3x^4 + 3x^2)$
$= (2x^5 + x^5) + (-3x^4) + (8x^2 - 3x^2) + (-4x)$
$= 3x^5 - 3x^4 + 5x^2 - 4x$

42. $x^4 - 81 = 0$
$(x^2 - 9)(x^2 + 9) = 0$
$(x - 3)(x + 3)(x^2 + 9) = 0$
Solve $x^2 + 9 = 0$
$x^2 = -9$
$x = \pm 3i$
The fully factored equation is:
$(x - 3)(x + 3)(x - 3i)(x + 3i) = 0$
The roots are ± 3, and $\pm 3i$.

43. $x^4 + x^3 + 3x^2 + 5x - 10 = 0$

Possible rational roots: $\pm 1, \pm 2, \pm 5, \pm 10$

-2	1	1	3	5	-10
		-2	2	-10	10
	1	-1	5	-5	0

$$(x + 2)(x^3 - 1x^2 + 5x - 5) = 0$$
$$(x + 2)[x^2(x - 1) + 5(x - 1)] = 0$$
$$(x + 2)(x - 1)(x^2 + 5) = 0$$

Solve $x^2 + 5 = 0$

$$x^2 = -5$$
$$x = \pm i\sqrt{5}$$

The fully factored equation is:

$$(x + 2)(x - 1)(x - i\sqrt{5})(x + i\sqrt{5}) = 0$$

The roots are 1, -2, and $\pm i\sqrt{5}$.

44. $x^3 - 5x^2 - 17x + 21 = 0$

Possible rational roots: $\pm 1, \pm 3, \pm 7, \pm 21$

1	1	-5	-17	21
		1	-4	-21
	1	-4	-21	0

$$(x - 1)(x^2 - 4x - 21) = 0$$
$$(x - 1)(x + 3)(x - 7) = 0$$

The roots are 1, -3, and 7.

45. $x^5 + 3x^4 + 2x^3 + 16x^2 - 48x - 64 = 0$

Possible rational roots:
$\pm 1, \pm 2, \pm 4, \pm 8, \pm 16, \pm 32, \pm 64$

-4	1	3	2	16	-48	-64
		-4	4	-24	32	64
	1	-1	6	-8	-16	0

$$(x + 4)(x^4 - 1x^3 + 6x^2 - 8x - 16) = 0$$

Possible rational roots: $\pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

-1	1	-1	6	-8	-16
		-1	2	-8	16
	1	-2	8	-16	0

$$(x + 4)(x + 1)(x^3 - 2x^2 + 8x - 16) = 0$$
$$(x + 4)(x + 1)[x^2(x - 2) + 8(x - 2)] = 0$$
$$(x + 4)(x + 1)(x - 2)(x^2 + 8) = 0$$

Solve $x^2 + 8 = 0$

$$x^2 = -8$$
$$x = \pm 2i\sqrt{2}$$

The fully factored equation is:

$$(x + 4)(x + 1)(x - 2)(x - 2i\sqrt{2})(x + 2i\sqrt{2}) = 0$$

The roots are -4, -1, 2, and $\pm 2i\sqrt{2}$.

6-9 CURVE FITTING WITH POLYNOMIAL MODELS, PAGES 466–471

CHECK IT OUT!

1. The x-values increase by a constant, 3. Find the differences of the y-values.
   First differences: 20 6 0 2 12
   Second differences: -14 -6 2 10
   Third differences: 8 8 8
   The third differences are constant. A cubic polynomial best describes the data.
2. Let x represent the speed of the car. The speed increases by a constant amount of 5. The gas consumptions are the y-values.
   First differences: 1.2 0.2 -0.2 0.4 1.6 3.6 6.4
   Second differences: -1 -0.4 0.6 1.2 2 2.8
   Third differences: 0.6 1 0.6 0.8 0.8
   The third differences are relatively close, a cubic function should be a good model.
   $f(x) = 0.001x^3 - 0.113x^2 + 4.134x - 24.867$
3. Let x represent the number of years since 1994. From a scatter plot, the function appears to be either cubic or quartic.
   cubic: $R^2 \approx 0.8625$ quartic: $R^2 \approx 0.9959$
   The quartic function is a more appropriate choice.
   The data can be modeled by
   $f(x) \approx 19.09x^4 - 377.90x^3 + 2153.24x^2 - 2183.29x + 3871.46$
   1999 is 5 years after 1994.
   $f(5) \approx 19.09(5)^4 - 377.90(5)^3 + 2153.24(5)^2 - 2183.29(5) + 3871.46$
   $\approx 11{,}482.84$
   Based on the model, the opening value was about \$11,482.84 in 1999.

THINK AND DISCUSS

1. At each step, there is one fewer value with which to compute. To have values left after 4 finite differences, there must be at least 5 data points.
2. **Polynomial Models**

Linear: 1; first;

x	1	2
y	1	2

Quadratic: 2; second;

x	1	2	3
y	1	4	9

Cubic: 3; third;

x	1	2	3	4
y	1	8	27	64

Quartic: 4; fourth;

x	-1	0	1	2	3
y	1	0	1	16	81

EXERCISES

GUIDED PRACTICE

1. The x-values increase by a constant, 1. Find the differences of the y-values.
   First differences: -6 -6 -6 -6 -6
   The first differences are constant. A linear polynomial best describes the data.
2. The x-values increase by a constant, 1. Find the differences of the y-values.
   First differences: -5 2 8 13 17
   Second differences: 7 6 5 4
   Third differences: -1 -1 -1
   The third differences are constant. A cubic polynomial best describes the data.

3. The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	−25	3	1	−7	3
Second differences:	28	−2	−8	10	
Third differences:	−30	−6	18		
Fourth differences:	24	24			

The fourth differences are constant. A quartic polynomial best describes the data.

4. Let x represent the number of years since 1957. The years increase by a constant amount of 10. The amounts of retail space available are the y-values.

First differences:	3.9	8	12.6	17.6	23
Second differences:	4.1	4.6	5	5.4	
Third differences:	0.5	0.4	0.4		

A cubic function should be a good model.

$f(x) \approx -0.000071x^3 + 0.0186x^2 + 0.196x + 2.804$, where x is the number of years since 1957.

5. Let x represent the number of hours since 12 hours. From a scatter plot, the function appears to be either cubic or quartic.

cubic: $R^2 \approx 0.9848$ quartic: $R^2 \approx 0.999$

The quartic function is a more appropriate choice. The data can be modeled by

$f(x) = 0.00000424x^4 - 0.001052x^3 - 0.07216x^2 + 22.145x + 29.701$

120 is 108 hours after 12 hours.

$f(108) = 0.00000424(108)^4 - 0.001052(108)^3 - 0.07216(108)^2 + 22.145(108) + 29.701 \approx 831$

Based on the model, the number of infected patients after 120 hours was 831.

PRACTICE AND PROBLEM SOLVING

6. The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	1	10	14.5	14.5	10
Second differences:	9	4.5	0	−4.5	
Third differences:	−4.5	−4.5	−4.5		

The third differences are constant. A cubic polynomial best describes the data.

7. The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	−4	6	10	10	8
Second differences:	10	4	0	−2	
Third differences:	−6	−4	−2		
Fourth differences:	2	2			

The fourth differences are constant. A quartic polynomial best describes the data.

8. The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	4	3.3	2.6	1.9	1.2
Second differences:	−0.7	−0.7	−0.7	−0.7	

The second differences are constant. A quadratic polynomial best describes the data.

9. Let x represent the number of years since 2002. The years increase by a constant amount of 1. The number of Chess Club members are the y-values.

First differences:	0	0	2	4	6
Second differences:	0	2	2	2	

The second differences are relatively close, a quadratic function should be a good model.

$f(x) = 0.821x^2 - 1.821x + 23.357$

10. Let x represent the number of years since 1996. From a scatter plot, the function appears to be either cubic or quartic.

cubic: $R^2 \approx 0.7825$ quartic: $R^2 \approx 0.9685$

The quartic function is a more appropriate choice. The data can be modeled by

$f(x) = 0.03225x^4 - 0.4998x^3 + 2.4507x^2 - 4.0892x + 15.2948$

2005 is 9 years after 1996.

$f(9) = 0.03225(9)^4 - 0.4998(9)^3 + 2.4507(9)^2 - 4.0892(9) + 15.2948 \approx 24.2$

Possible answer: The number of visitors in 2005 will be about 24.2 million.

11. Let x represent the number of years since 1989. From a scatter plot, the function appears to be either cubic or quartic.

cubic: $R^2 \approx 0.6832$ quartic: $R^2 \approx 1$

The quartic function is a more appropriate choice. The data can be modeled by

$f(x) = -10.0274x^4 + 238.816x^3 - 1750.66x^2 + 3593.26x + 59{,}336$

1999 is 10 years after 1996.

$f(10) = -10.0274(10)^4 + 238.816(10)^3 - 1750.66(10)^2 + 3593.26(10) + 59{,}336 \approx 58{,}745$

Possible answer: The number of graduates in 1999 was about 58,745.

12a. Latitude and temperature; possible answer: for latitude and temperature, quartic $R^2 = 0.9981$, but for longitude and temperature, quartic $R^2 = 0.513$.

b. No; possible answer: the latitude value 30 corresponds to 2 different temperature values, 40 and 42.

c. No; possible answer: the longitude value 83 corresponds to 2 different temperature values, 20 and 52.

13a. Let x represent number of years since December 1997.

$f(x) = 0.019x^3 + 0.185x^2 + 0.95x + 12.056$; $R^2 = .9944$

b. $f(x) = 0.0075x^4 - 0.071x^3 - 0.143x^2 + 0.604x + 12.083$; $R^2 = .9967$

c. no

d. Possible answer: The cubic and quartic polynomials are almost equally appropriate models.

14a. The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	3	6	10	15
Second differences:	3	4	5	
Third differences:	1	1		

The third differences are constant. A cubic polynomial best describes the data.

b. $f(x) = \frac{1}{6}x^3 + \frac{1}{2}x^2 + \frac{1}{3}x$

c. $f(12) = \frac{1}{6}(12)^3 + \frac{1}{2}(12)^2 + \frac{1}{3}(12) = 364$

15. Yes; possible answer: the step after a constant difference is 0 difference. Since the fourth differences are almost 0, the third differences must be nearly constant, so a cubic polynomial is an appropriate model.

16. Possible answer: Compute finite differences until the values are approximately constant, or enter the data into a calculator and check the R^2-values for several different degree polynomial models.

TEST PREP

17. C

The x-values increase by a constant, 1. Find the differences of the y-values.

First differences:	20	6	0	2	12
Second differences:	−14	−6	2	10	
Third differences:	8	8	8		

18. F

The graph has 3 roots: −2, 1, and 3. Therefore the factors are $(x + 2)(x - 1)(x - 3)$. As $x \to -\infty$, $f(x) \to +\infty$, and as $x \to +\infty$, $f(x) \to -\infty$. Therefore the function is reflected across the x-axis, which is represented by $-f(x)$.

19. $f(x) = x^3 - 5x + 4$

CHALLENGE AND EXTEND

20. $f(-2) = -1$ and $f(2) = -3$;

So $(a, f(a)) = (-2, -1)$ and $(b, f(b)) = (2, -3)$

$$\text{Average slope} = \frac{f(b) - f(a)}{b - a} = \frac{-3 - (-1)}{2 - (-2)} = -\frac{1}{2}$$

21. Possible answer: Divide the first differences by the change in the x-values.

22. It approaches 0.

23. Possible answer:

x	1	2	3	4	5	6
y	60	54	40	28	30	60

SPIRAL REVIEW

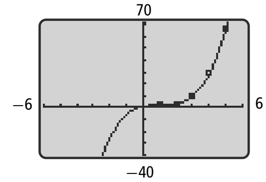
24. $-b^2(2b^2 + 5b - 3)$
$= -2b^4 - 5b^3 + 3b^2$

25. $a + 3a - 5a^2(6a - a)$
$= a + 3a - 30a^3 + 5a^3$
$= 4a - 25a^3$

26. $\dfrac{u^2 - v + 3v}{v(v^2 - u)}$

$= \dfrac{u^2 + 2v}{v^3 - uv}$

27. The data points resemble a cubic function. The data set is a horizontal shift 1 unit right, and vertical shift 2 units up.



28. The leading coefficient is 1, which is positive.
The degree is 4, which is even.
As $x \to -\infty$, $f(x) \to +\infty$, and
as $x \to +\infty$, $f(x) \to +\infty$.

29. The leading coefficient is 4, which is positive.
The degree is 5, which is odd.
As $x \to -\infty$, $f(x) \to -\infty$, and
as $x \to +\infty$, $f(x) \to +\infty$.

30. The leading coefficient is −3, which is negative.
The degree is 6, which is even.
As $x \to -\infty$, $f(x) \to -\infty$, and
as $x \to +\infty$, $f(x) \to -\infty$.

READY TO GO ON? PAGE 473

1. Set profit equal to 0.

$t^4 - 10t^2 + 9 = 0$
$(t^2 - 1)(t^2 - 9) = 0$
$(t + 1)(t - 1)(t + 3)(t - 3) = 0$
$t + 1 = 0$ or $t - 1 = 0$ or $t + 3 = 0$ or $t - 3 = 0$
$t = -1$ $\quad t = 1$ $\quad t = -3$ $\quad t = 3$

Since time can not be negative, in the years 2001 and 2003, the profit was 0.

2. $x^3 + 6x^2 + 12x + 8 = 0$
$(x + 2)(x + 2)(x + 2) = 0$
root −2 with multiplicity 3

3. $2x^3 + 8x^2 - 32x - 128 = 0$
$2(x^3 + 4x^2 - 16x - 64) = 0$
$2(x^2 - 16)(x + 4) = 0$
$2(x - 4)(x + 4)(x + 4) = 0$
root 4 with multiplicity 1; root −4 with multiplicity 2

4. $x^4 - 6x^3 + 9x^2 = 0$
$x^2(x^2 - 6x + 9) = 0$
$x^2(x - 3)(x - 3) = 0$
root 0 with multiplicity 2; root 3 with multiplicity 2

5. $P(x) = (x - 1)(x - 1)(x - 2)$
$= (x^2 - 2x + 1)(x - 2)$
$= x^3 - 4x^2 + 5x - 2$

6. $P(x) = (x - i)(x + i)(x + 1)(x)$
$= (x^2 + 1)(x^2 + x)$
$= x^4 + x^3 + x^2 + x$

7. $x^4 - 2x^3 + 6x^2 - 18x - 27 = 0$

Possible rational roots: $\pm1, \pm3, \pm9, \pm27$

3	1	−2	6	−18	−27
		3	3	27	27
	1	1	9	9	0

$(x - 3)(x^3 + x^2 + 9x + 9) = 0$

$(x - 3)(x + 1)(x^2 + 9) = 0$

Solve $x^2 + 9 = 0$

$x^2 = -9$

$x = \pm 3i$

The fully factored equation is:

$(x - 3)(x + 1)(x - 3i)(x + 3i) = 0$

The roots are 3, −1, and $\pm 3i$.

8. $f(x) = x^4 - 13x^2 + 36$

$= (x^2 - 4)(x^2 - 9)$

$= (x + 2)(x - 2)(x + 3)(x - 3)$

The zeros are −2, 2, −3, and 3.

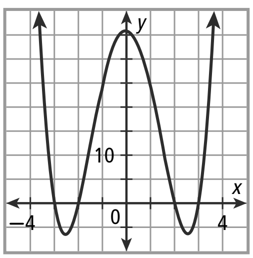
Plot other points: $f(0) = 36$, so the y-intercept is 36;

$f(-2.5) = -6.1875$,

$f(2.5) = -6.1875$

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $f(x) \to +\infty$,

and as $x \to +\infty$, $f(x) \to +\infty$.



9. $f(x) = x^3 - 4x^2 - 15x + 18$

Possible rational roots: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

1	1	−4	−15	18
		1	−3	−18
	1	−3	−18	0

$f(x) = (x - 1)(x^2 - 3x - 18)$

$= (x - 1)(x - 6)(x + 3)$

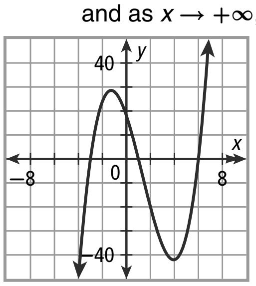
The zeros are 1, 6, and −3.

Plot other points: $f(0) = 18$, so the y-intercept is 18;

$f(-1) = 28$, and $f(4) = -42$

The degree is odd and the leading coefficient is positive, so as $x \to -\infty$, $f(x) \to -\infty$,

and as $x \to +\infty$, $f(x) \to +\infty$.



10. As $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to +\infty$.
$P(x)$ is of odd degree with a positive leading coefficient.

11. As $x \to -\infty$, $P(x) \to +\infty$, and as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of odd degree with a negative leading coefficient.

12. As $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to -\infty$.
$P(x)$ is of even degree with a negative leading coefficient.

13. $g(x) = -f(x)$

$= -(x^4 - 3x^2 + 6)$

$= -x^4 + 3x^2 - 6$

14. $g(x) = f(-x)$

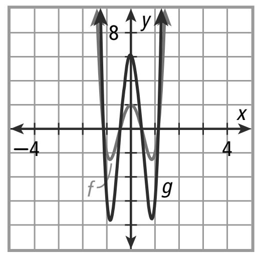
$= (-x)^4 - 3(-x)^2 + 6$

$= x^4 - 3x^2 + 6$

15. $g(x) = 3f(x)$

$= 3(8x^4 - 12x^2 + 2)$

$= 24x^4 - 36x^2 + 6$

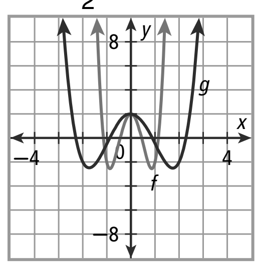


$g(x)$ is a vertical stretch of $f(x)$.

16. $g(x) = f\left(\frac{1}{2}x\right)$

$= 8\left(\frac{1}{2}x\right)^4 - 12\left(\frac{1}{2}x\right)^2 + 2$

$= \frac{1}{2}x^4 - 3x^2 + 2$

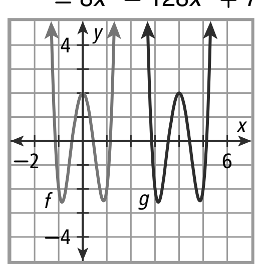


$g(x)$ is a horizontal stretch of $f(x)$.

17. $g(x) = f(x - 4)$

$= 8(x - 4)^4 - 12(x - 4)^2 + 2$

$= 8x^4 - 128x^3 + 756x^2 - 1952x + 1858$



$g(x)$ is a horizontal shift of $f(x)$.

18. Let x represent time. The time increases by a constant amount of 1. The bacteria populations are the y-values.

First differences:	68	140	263	434
Second differences:	72	123	171	
Third differences:	51	48		

The third differences are relatively close, a cubic function should be a good model.
$f(x) = 8.25x^3 - 13.18x^2 + 49.58x - 0.6$

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1. monomial
2. synthetic division
3. multiplicity
4. end behavior

LESSON 6-1

5. Standard form: $-3x^3 + 4x^2 + 6x + 7$
Leading coefficient: -3
Degree: 3
Terms: 4
Name: cubic polynomial with 4 terms

6. Standard form: $-x^5 + 2x^4 + 5x^3 + 8x$
Leading coefficient: -1
Degree: 5
Terms: 4
Name: quintic polynomial with 4 terms

7. Standard form: $9x^2 - 11x + 1$
Leading coefficient: 9
Degree: 2
Terms: 3
Name: quadratic trinomial

8. Standard form: $x^4 - 6x^2$
Leading coefficient: 1
Degree: 4
Terms: 2
Name: quartic binomial

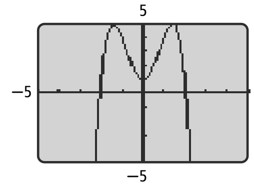
9. $(8x^3 - 4x^2 - 3x + 1) - (1 - 5x^2 + x)$
$= (8x^3 - 4x^2 - 3x + 1) + (5x^2 - x - 1)$
$= (8x^3) + (-4x^2 + 5x^2) + (-3x - x) + (1 - 1)$
$= 8x^3 + x^2 - 4x$

10. $(6x^2 + 7x - 2) + (1 - 5x^3 + 3x)$
$= (6x^2 + 7x - 2) + (-5x^3 + 3x + 1)$
$= (-5x^3) + (6x^2) + (7x + 3x) + (-2 + 1)$
$= -5x^3 + 6x^2 + 10x - 1$

11. $(5x - 2x^2) - (4x^2 + 6x - 9)$
$= (-2x^2 + 5x) + (-4x^2 - 6x + 9)$
$= (-2x^2 - 4x^2) + (5x - 6x) + 9$
$= -6x^2 - x + 9$

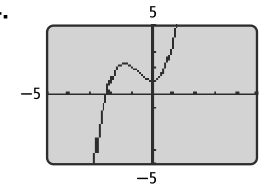
12. $(x^4 - x^2 + 4) + (x^2 - x^3 - 5x^4 - 7)$
$= (x^4 - x^2 + 4) + (-5x^4 - x^3 + x^2 - 7)$
$= (x^4 - 5x^4) + (-x^3) + (-x^2 + x^2) + (4 - 7)$
$= -4x^4 - x^3 - 3$

13.



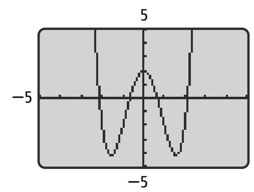
From left to right, it alternately increases and decreases, changing direction 3 times and crossing the x-axis 2 times. There appear to be 2 real zeros.

14.



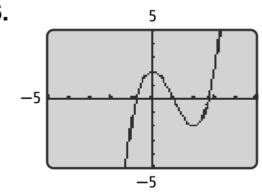
From left to right, it increases, decreases slightly, and then increases again. It crosses the x-axis 1 time. There appears to be 1 real zero.

15.



From left to right, it alternately decreases and increases, changing direction 3 times. It crosses the x-axis 4 times. There appear to be 4 real zeros.

16.



From left to right, it increases, decreases, and then increases again. It crosses the x-axis 3 times. There appear to be 3 real zeros.

LESSON 6-2

17. $5x^2(3x - 2)$
$= 5x^2(3x) + 5x^2(-2)$
$= 15x^3 - 10x^2$

18. $-3t(2t^2 - 6t + 1)$
$= -3t(2t^2) - 3t(-6t) - 3t(1)$
$= -6t^3 + 18t^2 - 3t$

19. $ab^2(a^2 - a + ab)$
$= ab^2(a^2) + ab^2(-a) + ab^2(ab)$
$= a^3b^2 - a^2b^2 + a^2b^3$

20. $(x - 2)(x^2 - 2x - 3)$
$= x(x^2) + x(-2x) + x(-3) - 2(x^2) - 2(-2x) - 2(-3)$
$= x^3 - 2x^2 - 3x - 2x^2 + 4x + 6$
$= x^3 - 4x^2 + x + 6$

21. $(2x + 5)(x^3 - x^2 + 1)$
$= 2x(x^3) + 2x(-x^2) + 2x(1) + 5(x^3) + 5(-x^2) + 5(1)$
$= 2x^4 - 2x^3 + 2x + 5x^3 - 5x^2 + 5$
$= 2x^4 + 3x^3 - 5x^2 + 2x + 5$

22. $(x - 3)^3$
$= [1(x)^3(-3)^0] + [3(x)^2(-3)^1] + [3(x)^1(-3)^2]$
$+ [1(x)^0(-3)^3]$
$= x^3 - 9x^2 + 27x - 27$

23. $(x+4)(x^4-3x^2+x)$
$= x(x^4) + x(-3x^2) + x(x) + 4(x^4) + 4(-3x^2) + 4(x)$
$= x^5 - 3x^3 + x^2 + 4x^4 - 12x^2 + 4x$
$= x^5 + 4x^4 - 3x^3 - 11x^2 + 4x$

24. $(2x+1)^4$
$= [1(2x)^4(1)^0] + [4(2x)^3(1)^1] + [6(2x)^2(1)^2]$
$\quad + [4(2x)^1(1)^3] + [1(2x)^0(1)^4]$
$= 16x^4 + 32x^3 + 24x^2 + 8x + 1$

25. $V = \pi r^2 \cdot h$
$= (\pi(2x)^2)(x^2 - x - 3)$
$= 4\pi x^2(x^2 - x - 3)$
$= 4\pi x^2(x^2) + 4\pi x^2(-x) + 4\pi x^2(-3)$
$= 4\pi x^4 - 4\pi x^3 - 12\pi x^2$

LESSON 6-3

26.

$$\begin{array}{r}
x^2 - 7x + 16 \\
x+2\,\overline{)\,x^3 - 5x^2 + 2x - 7} \\
-(x^3 + 2x^2) \\
\hline
-7x^2 + 2x \\
-(-7x^2 - 14x) \\
\hline
16x - 7 \\
-(16x + 32) \\
\hline
-39
\end{array}$$

$$\frac{x^3 - 5x^2 + 2x - 7}{x+2} = x^2 - 7x + 16 - \frac{39}{x+2}$$

27.

$$\begin{array}{r}
4x^3 + 2x^2 + 4x + 1 \\
2x-1\,\overline{)\,8x^4 + 0x^3 + 6x^2 - 2x + 4} \\
-(8x^4 - 4x^3) \\
\hline
4x^3 + 6x^2 \\
-(4x^3 - 2x^2) \\
\hline
8x^2 - 2x \\
-(8x^2 - 4x) \\
\hline
2x + 4 \\
-(2x - 1) \\
\hline
5
\end{array}$$

$$\frac{8x^4 + 6x^2 - 2x + 4}{2x - 1} = 4x^3 + 2x^2 + 4x + 1 + \frac{5}{2x-1}$$

28.

$$\begin{array}{r|rrrr}
3 & 1 & -4 & 3 & 2 \\
 & & 3 & -3 & 0 \\
\hline
 & 1 & -1 & 0 & \boxed{2}
\end{array}$$

$$\frac{x^3 - 4x^2 + 3x + 2}{x-3} = x^2 - x + \frac{2}{x-3}$$

29.

$$\begin{array}{r|rrrr}
2 & 1 & 0 & 2 & -1 \\
 & & 2 & 4 & 12 \\
\hline
 & 1 & 2 & 6 & \boxed{11}
\end{array}$$

$$\frac{x^3 + 2x - 1}{x-2} = x^2 + 2x + 6 + \frac{11}{x-2}$$

30. Number of ribbons $= \dfrac{\text{length of spool}}{\text{length of ribbon}} = \dfrac{x^3 + x^2}{x-1}$

$$\begin{array}{r|rrrr}
1 & 1 & 1 & 0 & 0 \\
 & & 1 & 2 & 2 \\
\hline
 & 1 & 2 & 2 & \boxed{2}
\end{array}$$

Yes. The number of strips of ribbons can be represented by $x^2 + 2x + 2$. Possible answer: with a remainder of 2 in.

LESSON 6-4

31.

$$\begin{array}{r|rrrr}
-3 & 1 & 2 & 0 & -5 \\
 & & -3 & 3 & -9 \\
\hline
 & 1 & -1 & 3 & \boxed{-14}
\end{array}$$

$x + 3$ is not a factor of $P(x)$.

32.

$$\begin{array}{r|rrrrr}
1 & 4 & 0 & -5 & 3 & -2 \\
 & & 4 & 4 & -1 & 2 \\
\hline
 & 4 & 4 & -1 & 2 & \boxed{0}
\end{array}$$

$x - 1$ is a factor of $P(x)$.

33.

$$\begin{array}{r|rrrr}
2 & 2 & -3 & 1 & -6 \\
 & & 4 & 2 & 6 \\
\hline
 & 2 & 1 & 3 & \boxed{0}
\end{array}$$

$x - 2$ is a factor of $P(x)$.

34. $x^3 - x^2 - 16x + 16$
$= (x^3 - x^2) + (-16x + 16)$
$= x^2(x-1) - 16(x-1)$
$= (x-1)(x^2 - 16)$
$= (x-1)(x-4)(x+4)$

35. $4x^3 - 8x^2 - x + 2$
$= (4x^3 - 8x^2) + (-x + 2)$
$= 4x^2(x-2) - (x-2)$
$= (x-2)(4x^2 - 1)$
$= (x-2)(2x-1)(2x+1)$

36. $3x^3 + 81$
$= 3(x^3 + 27)$
$= 3(x^3 + 3^3)$
$= 3(x+3)(x^2 - x \cdot 3 + 3^2)$
$= 3(x+3)(x^2 - 3x + 9)$

37. $16x^3 - 2$
$= 2(8x^3 - 1)$
$= 2[(2x)^3 - 1^3]$
$= 2(2x-1)[(2x)^2 + 2x \cdot 1 + 1^2]$
$= 2(2x-1)(4x^2 + 2x + 1)$

LESSON 6-5

38. $x^3 - 5x^2 + 8x - 4 = 0$
Possible rational roots: $\pm 1, \pm 5$

$$\begin{array}{r|rrrr}
1 & 1 & -5 & 8 & -4 \\
 & & 1 & -4 & 4 \\
\hline
 & 1 & -4 & 4 & \boxed{0}
\end{array}$$

$(x-1)(x^2 - 4x + 4) = 0$
$(x-1)(x-2)(x-2) = 0$
The roots are 1 and 2.

39. $x^3 + 6x^2 + 9x + 2 = 0$

Possible rational roots: ±1, ±2

−2	1	6	9	2
		−2	−8	−2
	1	4	1	0

$(x + 2)(x^2 + 4x + 1) = 0$

Solve $x^2 + 4x + 1 = 0$

$x = \dfrac{-4 \pm \sqrt{16 - 4}}{2} = -2 \pm \sqrt{3}$

The fully factored equation is:

$(x + 2)[x - (-2 + \sqrt{3})][x - (-2 - \sqrt{3})] = 0$

The roots are −2 and $-2 \pm \sqrt{3}$.

40. $x^3 + 3x^2 + 3x + 1 = 0$

$(x + 1)^3 = 0$

The roots are −1 with multiplicity 3.

41. $x^4 - 12x^2 + 27 = 0$

$(x^2 - 9)(x^2 - 3) = 0$

$(x + 3)(x - 3)(x^2 - 3) = 0$

Solve $x^2 - 3 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

The fully factored equation is:

$(x + 3)(x - 3)(x + \sqrt{3})(x - \sqrt{3}) = 0$

The roots are −3, 3, and $\pm\sqrt{3}$.

42. $x^3 + x^2 - 2x - 2 = 0$

$x^2(x + 1) - 2(x + 1) = 0$

$(x + 1)(x^2 - 2) = 0$

Solve $x^2 - 2 = 0$

$x^2 = 2$

$x = \pm\sqrt{2}$

The fully factored equation is:

$(x + 1)(x + \sqrt{2})(x - \sqrt{2}) = 0$

The roots are −1, and $\pm\sqrt{2}$.

43. $x^3 - 5x^2 + 4 = 0$

Possible rational roots: ±1, ±2, ±4

1	1	−5	0	4
		1	−4	−4
	1	−4	−4	0

$(x - 1)(x^2 - 4x - 4) = 0$

Solve $x^2 - 4x - 4 = 0$

$x = \dfrac{4 \pm \sqrt{16 + 16}}{2} = 2 \pm 2\sqrt{2}$

The fully factored equation is:

$(x - 1)[x - (2 + 2\sqrt{2})][x - (2 - 2\sqrt{2})] = 0$

The roots are 1, and $2 \pm 2\sqrt{2}$.

44. Let x represent the width in meters.

Then the length is $2x$, and the height is $x + 4$.

$2x(x)(x + 4) = 48$

$(2x^2)(x + 4) = 48$

$2x^3 + 8x^2 = 48$

$2x^3 + 8x^2 - 48 = 0$

$2(x^3 + 4x^2 - 24) = 0$

Factors of −24: ±1, ±2, ±3, ±4, ±6, ±8, ±12, ±24

$\frac{p}{q}$	1	4	0	−24
1	1	5	5	−19
2	1	6	12	0
3	1	7	21	39

$2(x - 2)(x^2 + 6x + 12) = 0$

Solve $x^2 + 6x + 12 = 0$

$x = \dfrac{-6 \pm \sqrt{36 - 48}}{2} = -3 \pm i\sqrt{3}$

The width must be positive and real, so the width should be 2 m.

LESSON 6-6

45. $P(x) = (x + 3)(x - 2)(x - 4)$
$= (x^2 + x - 6)(x - 4)$
$= x^3 - 3x^2 - 10x + 24$

46. $P(x) = \left(x + \frac{1}{2}\right)(x + 2)(x - 3)$
$= \left(x^2 + \frac{5}{2}x + 1\right)(x - 3)$
$= x^3 - \frac{1}{2}x^2 - \frac{13}{2}x - 3$

47. $P(x) = (x - \sqrt{2})(x + \sqrt{2})(x + 1)$
$= (x^2 - 2)(x + 1)$
$= x^3 + x^2 - 2x - 2$

48. $P(x) = (x + 3)(x - i)(x + i)$
$= (x + 3)(x^2 + 1)$
$= x^3 + 3x^2 + x + 3$

49. $P(x) = (x - \sqrt{2})(x + \sqrt{2})(x - \sqrt{3})(x + \sqrt{3})$
$= (x^2 - 2)(x^2 - 3)$
$= x^4 - 5x^2 + 6$

50. $P(x) = [x - (1 + \sqrt{3})][x - (1 - \sqrt{3})](x - 2i)(x + 2i)$
$= (x^2 - 2x - 2)(x^2 + 4)$
$= x^4 - 2x^3 + 2x^2 - 8x - 8$

51. $x^3 - x^2 + 4x - 4 = 0$

$x^2(x - 1) + 4(x - 1) = 0$

$(x - 1)(x^2 + 4) = 0$

Solve $x^2 + 4 = 0$

$x^2 = -4$

$x = \pm 2i$

The fully factored equation is:

$(x - 1)(x - 2i)(x + 2i) = 0$

The solutions are 1, $2i$, and $-2i$.

52. $x^4 - x^2 - 2 = 0$

$(x^2 + 1)(x^2 - 2) = 0$

Solve $x^2 + 1 = 0$ | $x^2 - 2 = 0$

$x^2 = -1$ | $x^2 = 2$

$x = \pm i$ | $x = \pm\sqrt{2}$

The fully factored equation is:

$(x - i)(x + i)(x - \sqrt{2})(x + \sqrt{2}) = 0$

The solutions are i, $-i$, $\sqrt{2}$, and $-\sqrt{2}$.

53. $x^4 - \frac{63}{4}x^2 - 4 = 0$

Possible rational roots: $\pm1, \pm2, \pm4$

4	1	0	$-\frac{63}{4}$	0	-4
		4	16	1	4
	1	4	$\frac{1}{4}$	1	0

$(x - 4)\left(x^3 + 4x^2 + \frac{1}{4}x + 1\right) = 0$

$(x - 4)(x + 4)\left(x^2 + \frac{1}{4}\right) = 0$

Solve $x^2 + \frac{1}{4} = 0$

$x^2 = -\frac{1}{4}$

$x = \pm\frac{1}{2}i$

The fully factored equation is:

$(x - 4)(x + 4)\left(x - \frac{1}{2}i\right)\left(x + \frac{1}{2}i\right) = 0$

The solutions are 4, -4, $\frac{1}{2}i$, and $-\frac{1}{2}i$.

54. $x^3 + 3x^2 - 5x - 15 = 0$

$(x + 3)(x^2 - 5) = 0$

Solve $x^2 - 5 = 0$

$x^2 = 5$

$x = \pm\sqrt{5}$

The fully factored equation is:

$(x + 3)(x - \sqrt{5})(x + \sqrt{5}) = 0$

The solutions are -3, $\sqrt{5}$, and $-\sqrt{5}$.

LESSON 6-7

55. Leading coefficient: -2; Degree: 3;
End behavior: $x \to -\infty$, $f(x) \to +\infty$
$x \to +\infty$, $f(x) \to -\infty$.

56. Leading coefficient: 1; Degree: 4;
End behavior: $x \to -\infty$, $f(x) \to +\infty$
$x \to +\infty$, $f(x) \to +\infty$.

57. Leading coefficient: -3; Degree: 6;
End behavior: $x \to -\infty$, $f(x) \to -\infty$
$x \to +\infty$, $f(x) \to -\infty$.

58. Leading coefficient: 7; Degree: 5;
End behavior: $x \to -\infty$, $f(x) \to -\infty$
$x \to +\infty$, $f(x) \to +\infty$.

59. $f(x) = x^3 - x^2 - 5x + 6$

Possible rational roots: $\pm1, \pm2, \pm3, \pm6$

2	1	-1	-5	6
		2	2	-6
	1	1	-3	0

$f(x) = (x - 2)(x^2 + x - 3)$

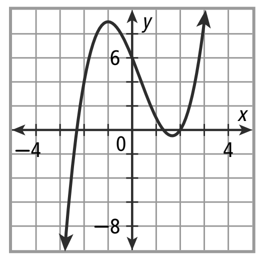
Solve $x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1 + 12}}{2} = \frac{-1 \pm \sqrt{13}}{2}$

The zeros are 2, ≈ 1.303, and ≈ -2.303

Plot other points: $f(0) = 6$, so the y-intercept is 6;
$f(-1) = 9$, and $f(1.5) = -0.375$.

The degree is odd and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to -\infty$,
and as $x \to +\infty$, $P(x) \to +\infty$.



60. $f(x) = x^4 - 10x^2 + 9$

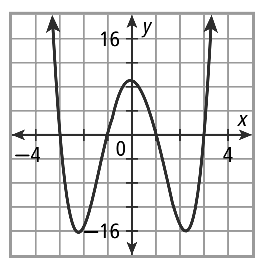
$= (x^2 - 1)(x^2 - 9)$

$= (x - 1)(x + 1)(x - 3)(x + 3)$

The zeros are 1, -1, 3, and -3.

Plot other points: $f(0) = 9$, so the y-intercept is 9;
$f(-2) = -15$, and $f(2) = -15$.

The degree is even and the leading coefficient is positive, so as $x \to -\infty$, $P(x) \to +\infty$,
and as $x \to +\infty$, $P(x) \to +\infty$.



61. $f(x) = -x^3 + 5x^2 + x - 5$

$= -(x^3 - 5x^2 - x + 5)$

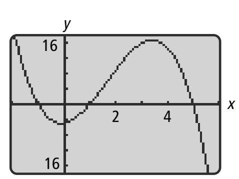
$= -(x - 5)(x^2 - 1)$

$= -(x - 5)(x - 1)(x + 1)$

The zeros are 5, 1, and -1.

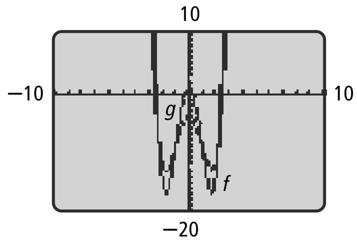
Plot other points: $f(0) = -5$, so the y-intercept is -5;
$f(3) = 16$

The degree is odd and the leading coefficient is negative, so as $x \to -\infty$, $P(x) \to +\infty$,
and as $x \to +\infty$, $P(x) \to -\infty$.

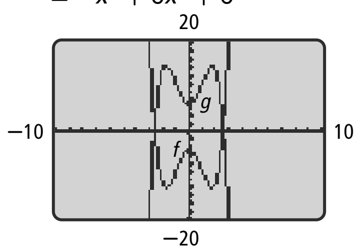


LESSON 6-8

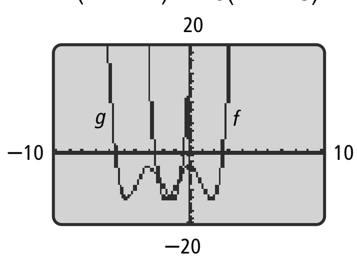
62. $g(x) = 2f(x) + 9$
$= 2(x^4 - 6x^2 - 4) + 9$
$= 2x^4 - 12x^2 - 8 + 9$
$= 2x^4 - 12x^2 + 1$



63. $g(x) = -(f(x) - 2)$
$= -(x^4 - 6x^2 - 4 - 2)$
$= -(x^4 - 6x^2 - 6)$
$= -x^4 + 6x^2 + 6$



64. $g(x) = f(-x - 3)$
$= (-x - 3)^4 - 6(-x - 3)^2 - 4$



LESSON 6-9

65. Let x represent the number of days. The days increase by a constant amount of 1. The attendances for the movie are the y-values.

First differences:	50	20	70	40
Second differences:	−30	50	−30	
Third differences:	80	−80		
Fourth differences:	−160			

The fourth differences are constant, a quartic will be the best model.

$f(x) \approx -6\frac{2}{3}x^4 + 80x^3 - 328\frac{1}{3}x^2 + 572x - 72$

66. Let x represent the number of years. The years increase by a constant amount of 1. The poulations are the y-values.

First differences:	783	702	1104	1989
Second differences:	−81	402	885	
Third differences:	483	483		

The third differences are constant, a cubic function should be a good model.

$f(x) \approx 80.5x^3 - 523.5x^2 + 1790x + 544$

CHAPTER TEST, PAGE 478

1. $(3x^2 - x + 1) + (x)$
$= (3x^2) + (-x + x) + (1)$
$= 3x^2 + 1$

2. $(6x^3 - 3x + 2) - (7x^3 + 3x + 7)$
$= (6x^3 - 3x + 2) + (-7x^3 - 3x - 7)$
$= (6x^3 - 7x^3) + (-3x - 3x) + (2 - 7)$
$= -x^3 - 6x - 5$

3. $(y^2 + 3y^2 + 2) + (y^4 + y^3 - y^2 + 5)$
$= (4y^2 + 2) + (y^4 + y^3 - y^2 + 5)$
$= (y^4) + (y^3) + (4y^2 - y^2) + (2 + 5)$
$= y^4 + y^3 + 3y^2 + 7$

4. $(4x^4 + x^2) - (x^3 - x^2 - 1)$
$= (4x^4 + x^2) + (-x^3 + x^2 + 1)$
$= (4x^4) + (-x^3) + (x^2 + x^2) + (1)$
$= 4x^4 - x^3 + 2x^2 + 1$

5. $C(15) = \frac{1}{10}(15)^3 - (15)^2 + 25 = 137.50$
The cost of manufacturing 15 units is $137.50.

6. $xy(2x^4y + x^2y^2 - 3xy^3)$
$= xy(2x^4y) + xy(x^2y^2) + xy(-3xy^3)$
$= 2x^5y^2 + x^3y^3 - 3x^2y^4$

7. $(t + 3)(2t^2 - t + 3)$
$= t(2t^2) + t(-t) + t(3) + 3(2t^2) + 3(-t) + 3(3)$
$= 2t^3 - t^2 + 3t + 6t^2 - 3t + 9$
$= 2t^3 + 5t^2 + 9$

8. $(x + 5)^3$
$= [1(x)^3(5)^0] + [3(x)^2(5)^1] + [3(x)^1(5)^2] + [1(x)^0(5)^3]$
$= x^3 + 15x^2 + 75x + 125$

9. $(2y + 3)^4$
$= [1(2y)^4(3)^0] + [4(2y)^3(3)^1] + [6(2y)^2(3)^2]$
$+ [4(2y)^1(3)^3] + [1(2y)^0(3)^4]$
$= 16y^4 + 96y^3 + 216y^2 + 216y + 81$

10.

2	5	−6	−8
		10	8
	5	4	0

$\frac{5x^2 - 6x - 8}{x - 2} = 5x + 4$

11.

$$\begin{array}{r} x^2 - 3x + 3 \\ 2x - 1\,\overline{)\,2x^3 - 7x^2 + 9x - 4} \\ -(2x^3 - x^2) \\ -6x^2 + 9x \\ -(-6x^2 + 3x) \\ 6x - 4 \\ -(6x - 3) \\ -1 \end{array}$$

$\frac{2x^3 - 7x^2 + 9x - 4}{2x - 1} = x^2 - 3x + 3 - \frac{1}{2x - 1}$

12.

3	1	3	−1	2	−6
		3	18	51	159
	1	6	17	53	153

13. $-2x^2 - 6x + 56$
$= -2(x^2 + 3x - 28)$
$= -2(x + 7)(x - 4)$

14. $m^5 + m^4 - 625m - 625$
$= m^4(m + 1) - 625(m + 1)$
$= (m + 1)(m^4 - 625)$
$= (m + 1)(m^2 - 25)(m^2 + 25)$
$= (m + 1)(m + 5)(m - 5)(m^2 + 25)$

15. $4x^3 - 32$
$= 4(x^3 - 8)$
$= 4(x^3 - 2^3)$
$= 4(x - 2)(x^2 + x \cdot 2 + 2^2)$
$= 4(x - 2)(x^2 + 2x + 4)$

16. $2x^4 - 9x^3 + 7x^2 + 2x - 2 = 0$

Possible rational roots: $\pm 1, \pm 2, \pm\frac{1}{2}$

1	2	−9	7	2	−2
		2	−7	0	2
	2	−7	0	2	0

$(x - 1)(2x^3 - 7x^2 + 2) = 0$

Possible rational roots: $\pm 1, \pm 2, \pm\frac{1}{2}$

$-\frac{1}{2}$	2	−7	0	2
		−1	4	−2
	2	−8	4	0

$(x - 1)\left(x + \frac{1}{2}\right)(2x^2 - 8x + 4) = 0$

$2(x - 1)\left(x + \frac{1}{2}\right)(x^2 - 4x + 2) = 0$

Solve $x^2 - 4x + 2 = 0$

$x = \frac{4 \pm \sqrt{16 - 8}}{2} = 2 \pm \sqrt{2}$

The fully factored equation is:

$2(x - 1)\left(x + \frac{1}{2}\right)\left[x - (2 + \sqrt{2})\right]\left[x - (2 - \sqrt{2})\right] = 0$

The roots are 1, $-\frac{1}{2}$, $2 + \sqrt{2}$, and $2 - \sqrt{2}$, all with multiplicity 1.

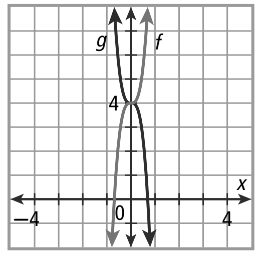
17. $P(x) = (x - 1)(x - 4)(x + 5)$
$= (x^2 - 5x + 4)(x + 5)$
$= x^3 - 21x + 20$

18. As $x \to -\infty$, $P(x) \to +\infty$, and as $x \to +\infty$, $P(x) \to -\infty$. $P(x)$ is of odd degree with a negative leading coefficient.

19. As $x \to -\infty$, $P(x) \to -\infty$, and as $x \to +\infty$, $P(x) \to +\infty$. $P(x)$ is of odd degree with a positive leading coefficient.

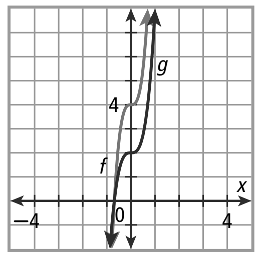
20. As $x \to -\infty$, $P(x) \to +\infty$, and as $x \to +\infty$, $P(x) \to +\infty$. $P(x)$ is of even degree with a positive leading coefficient.

21. $g(x) = f(-x)$
$= 12(-x)^3 + 4$
$= -12x^3 + 4$



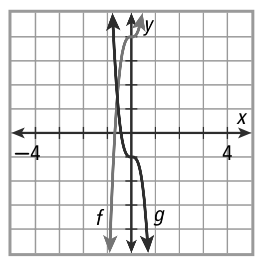
$g(x)$ is a reflection across the y-axis of $f(x)$.

22. $g(x) = \frac{1}{2}f(x)$
$= \frac{1}{2}(12x^3 + 4)$
$= 6x^3 + 2$



$g(x)$ is a vertical compression of $f(x)$ by a factor of $\frac{1}{2}$.

23. $g(x) = -f(x) + 3$
$= -(12x^3 + 4) + 3$
$= -12x^3 - 4 + 3$
$= -12x^3 - 1$



$g(x)$ is a reflection across the x-axis, and vertical shift up of $f(x)$ by 3 units.

24. Let x represent the number of hours. The hours increase by a constant amount of 1. The number of bracelets are the y-values.

First differences: 2 6 10 14 18

Second differences: 4 4 4 4

The second differences are constant, a quadratic function should be a good model.

$f(x) = 2x^2 - 4x + 5$

25. Let x represent the number of days.

From a scatter plot, the function appears to be either cubic or quartic.

cubic: $R^2 \approx 0.8740$ quartic: $R^2 = 1$

The quartic function is a more appropriate choice.

$f(x) = 2.87x^4 - 37.83x^3 + 250.17x^2 + 179.8$