Exponential and Logarithmic Functions

ARE YOU READY? PAGE 487

1. D

2. C

3. E

4. A

5. $x^2(x^3)(x) = x^5(x)$
$= x^6$

6. $3y^{-1}(5x^2y^2) = (3y^{-1}y^2)5x^2$
$= (3y)5x^2$
$= 15x^2y$

7. $\frac{a^8}{a^2} = a^{(8-2)}$
$= a^6$

8. $y^{15} \div y^{10} = y^{(15-10)}$
$= y^5$

9. $\frac{x^2y^5}{xy^6} = x^{(2-1)}y^{(5-6)}$
$= x^1y^{-1}$
$= \frac{x}{y}$

10. $\left(\frac{x}{3}\right)^{-3} = \left(\left(\frac{x}{3}\right)^{-1}\right)^3$
$= \left(\frac{3}{x}\right)^3$
$= \frac{27}{x^3}$

11. $(3x)^2(4x^3) = 9x^2(4x^3)$
$= 36x^5$

12. $\frac{a^{-2}b^3}{a^4b^{-1}} = a^{(-2-4)}b^{3-(-1)}$
$= a^{-6}b^4$
$= \frac{b^4}{a^6}$

13. $I = 3000(3\%)(2) = 180$
The simple interest is $180.

14. $2000(r)(3) = 90$
$6000r = 90$
$r = 0.015$ or 1.5%
The interest rate is 1.5%.

15. $P + P(6\%)(3) = 5310$
$P + P(0.18) = 5310$
$1.18P = 5310$
$P = 4500$
The loan is $4500.

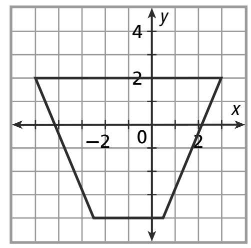
16. $3x - y = 4$
$3x = y + 4$
$x = \frac{y+4}{3}$

17. $y = -7x + 3$
$y + 7x = 3$
$7x = -y + 3$
$x = \frac{-y+3}{7}$

18. $\frac{x}{2} = 3y - 4$
$x = 2(3y - 4)$
$x = 6y - 8$

19. $y = \frac{3}{4}x - \frac{1}{2}$
$4y = 3x - 2$
$-3x = -4y - 2$
$x = \frac{4y+2}{3}$

20.



21. 7×10^9

22. 9.3×10^{-9}

23. 1.675×10^1

24. 0.0000094

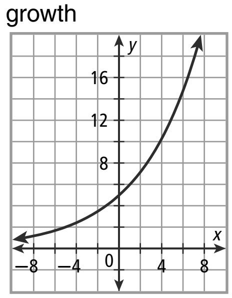
25. 470,000

26. 78,000

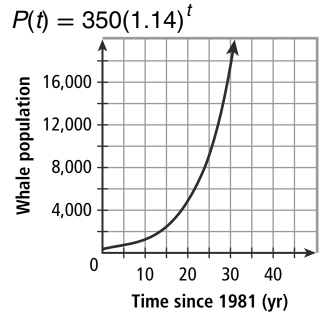
7-1 EXPONENTIAL FUNCTIONS, GROWTH, AND DECAY, PAGES 490–496

CHECK IT OUT!

1. growth

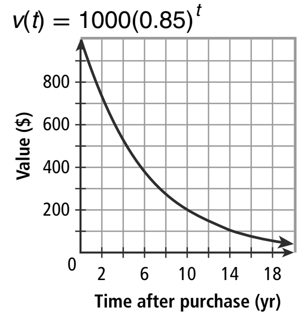


2. $P(t) = 350(1.14)^t$



The population will reach 20,000 in about 30.9 yr.

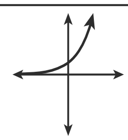
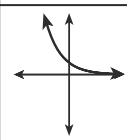
3. $v(t) = 1000(0.85)^t$



The value will fall below $100 in about 14.2 yr.

THINK AND DISCUSS

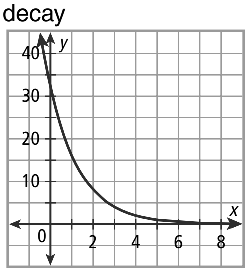
1. The base of the exponential function is between 0 and 1, so the function shows decay. An exponential decay function decreases over any interval in its domain.
2. Possible answer: $f(x) = 1.1^x$ shows growth, and $g(x) = 0.9^x$ shows decay. The graphs intersect at (0, 1).
3. Possible answer: exponential decay; exponential growth
4.

Exponential Functions $f(x) = ab^x$, where $a > 0$	Growth	Decay
Value of b	$b > 1$	$0 < b < 1$
General shape of the graph		
What happens to $f(x)$ as x increases?	$f(x)$ increases.	$f(x)$ decreases.
What happens to $f(x)$ as x decreases?	$f(x)$ decreases.	$f(x)$ increases.

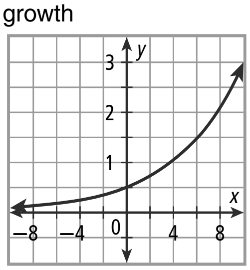
EXERCISES

GUIDED PRACTICE

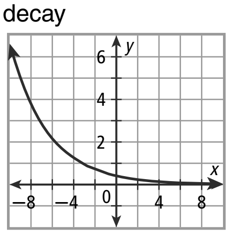
1. exponential decay
2. decay



3. growth

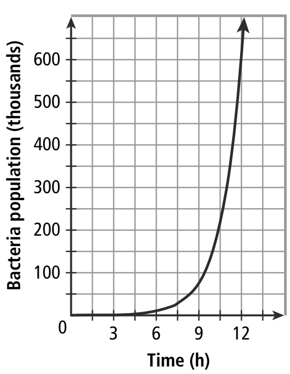


4. decay



5a. $f(x) = 150(2^x)$

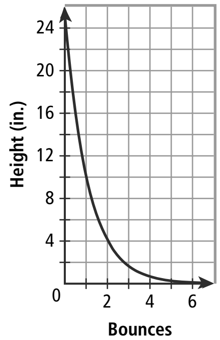
b.



c. The number of bacteria after 12 hours will be about 600,000.

6a. $f(x) = 25(0.4)^x$

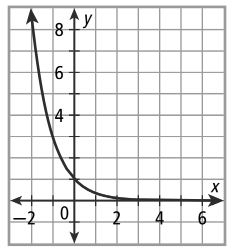
b.



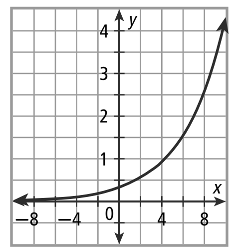
c. A new softball will rebound less than 1 inch after 4 bounces.

PRACTICE AND PROBLEM SOLVING

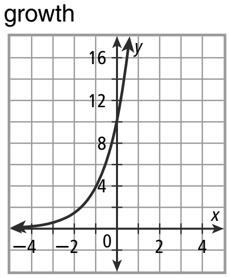
7. decay



8. growth

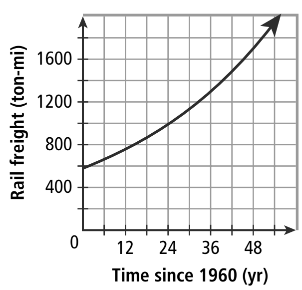


9. growth



10a. $f(t) = 580(1.0232)^t$

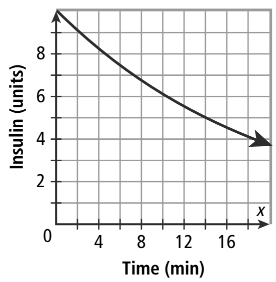
b.



c. The number of ton-miles would have exceeded or would exceed 1 trillion in year 24, or 1984.

11a. $f(x) = 10(0.95)^x$

b.



c. About 6 units will remain after 10 minutes.

d. It will take about 13.6 min for half of the dose to remain.

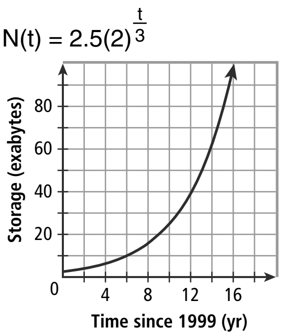
12. No; the variable does not contain an exponent.

13. No; 0^x is 0, a constant function.

14. Yes; the variable is in the exponent.

15. $2008 - 1626 = 382$
$24(1 + 3.5\%)^{382} = 24(1.035)^{382} = 12{,}229{,}955.1$
The balance in 2008 will be about $12,000,000.

16. $N(t) = 2.5(2)^{\frac{t}{3}}$

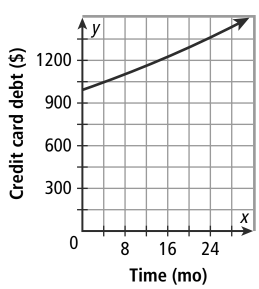


17. Let x be the number of years needed.
$2765(1 - 30\%)^x = 350$
$2765(0.7^x) = 350$
$0.7^x \approx 0.12658$
$x \approx 5.8$
It will take about 5.8 years for the computer's value to be less than $350.

18. 0.09; 0.21; 0.45; 1.00; 2.20; 4.84; 10.65; 23.43; 51.54

19. 15.63; 6.25; 2.50; 1.00; 0.40; 0.16; 0.06; 0.03; 0.01

20a.



b. You will owe $1195.62 after one year.

c. It will take about 18 months for the total amount to reach $1300.

21a. $12000(1 - 20\%)^6 = 12000(0.8)^6 \approx 3146$
The rep sold about 3146 animals in the 6th month after the peak.

b. Let x be the number of month that the rep first sold less than 1000 animals.
$12000(1 - 20\%)^x = 1000$
$12000(0.8^x) = 1000$
$0.8^x \approx 0.0833$
$x \approx 12$
The rep first sold less than 1000 animals in the 12th month.

22a. $A = P\left(1 + \frac{r}{n}\right)^{nt} = 5000\left(1 + \frac{5\%}{4}\right)^{4 \times 5}$
$= 5000(1.0125)^{20} = 6410.19$
The investment will be worth $6410.19 after 5 years.

b. Let x be the number of years needed.
$5000(1.0125^x) = 10000$
$1.0125^x = 2$
$x \approx 14$
The investment will be worth more than $10,000 after 14 years.

c. $A = P\left(1 + \frac{r}{n}\right)^{nt} = 5000\left(1 + \frac{5\%}{12}\right)^{12 \times 5}$
$= 5000(1.0041667)^{60} \approx 6416.79$
$6416.79 - 6410.19 = 6.60$
His investment will be worth $6.60 more after 5 years.

23. (0, 1)

24. (0, 58,025]

25. (34.868, 100]

26. $\left(\frac{3}{4}, 768\right]$

27a. $\frac{(500 - 415)}{500} = \frac{85}{500} = 0.17 = 17\%$
There is a 17% decrease in the amount each day.

b. $A(t) = 500(1 - 17\%)^t = 500(0.83)^t$

c. $A(14) = 500(0.83)^{14} \approx 36.8$
About 36.8 mg will remain after 14 days.

28. $N(t) = 6.1(1 + 1.4\%)^t = 6.1(1.014)^t$
$t = 2020 - 2000 = 20$
$N(20) = 6.1(1.014)^{20} \approx 8.1$
The population will be about 8.1 billion in 2020.

29. 3^x; when $x = 3$, they are equal, but 3^x becomes greater quickly as x increases.

30. Possible answer: A company doubles in size each year from an initial size of 12 people; $f(p) = 12(2)^x$; $f(3) = 12(2)^3$ means there are 96 people in 3 yr.

TEST PREP

31. B

32. H

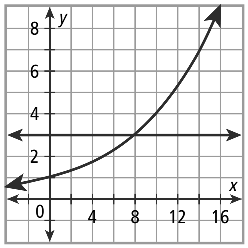
33. $a = 1$, $b = 2.5$

34. B

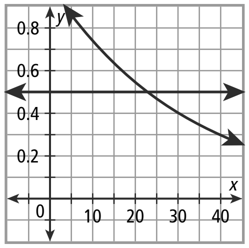
CHALLENGE AND EXTEND

35. The degree of a polynomial is the greatest exponent, but exponential functions have variable exponents that may be infinitely large.

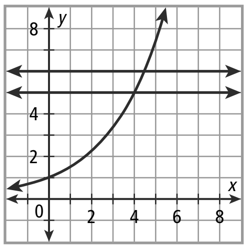
36. $x \geq 7.86$



37. $x > 22.76$



38. $3.97 < x < 4.42$



39. 2; (2, 4), (−0.767, 0.588)

40. $10^{\frac{1}{2}(0)+2} = 10^2 = 100$

$10^{\frac{1}{2}d+2} = 4(10)$

$10^{\frac{1}{2}d+2} = 40$

$\frac{1}{2}d + 2 \approx 2.6$

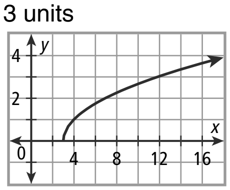
$d \approx 1.2$

There are 100 mosquitoes per acre at the time of the frost; it takes about 1.2 days for the population to quadruple.

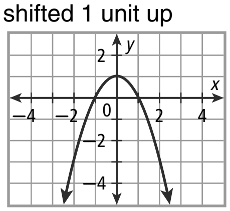
41. If $b = 0$, $f(x) = 0$; if $b = 1$, $f(x) = 1$. These are constant functions. If $b < 0$, noninteger exponents are not defined.

SPIRAL REVIEW

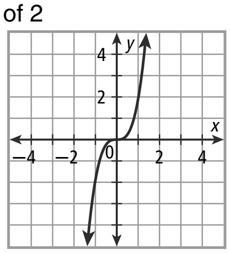
42. D: $\{x \mid x \geq 3\}$; use y; $f(x) = \sqrt{x}$ shifted right 3 units



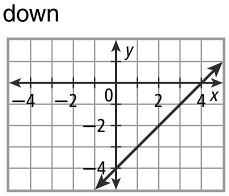
43. D: $\mathbb{R}$; R: $\{y \mid y \leq 1\}$; $f(x) = x^2$ reflected across x-axis and shifted 1 unit up



44. D: $\mathbb{R}$; R: $\mathbb{R}$; $f(x) = x^3$ stretched vertically by a factor of 2



45. D: $\mathbb{R}$; R: $\mathbb{R}$; $f(x) = x$ shifted 4 units down



46. Let x be the cost of a new video game, and y be the cost of an old one.

$3x + 2y = 235$ ①

$x + 4y = 195$ ②

Solve equation 1 and equation 2 for x and y using elimination.

① $2(3x + 2y) = 2(235) \rightarrow 6x + 4y = 470$

② $-(x + 4y) = -195 \rightarrow \underline{-x - 4y = -195}$

$5x = 275$

$x = 55$

Substitute 55 for x into equation 2 to find y.

$x + 4y = 195 \rightarrow (55) + 4y = 195$

$4y = 140$

$y = 35$

The cost of a new video game is \$55, and the cost of an old one is \$35.

47. odd; positive; 1

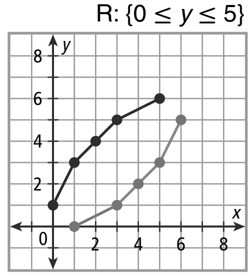
48. even; positive; 2

49. even; negative; 2

7-2 INVERSES OF RELATIONS AND FUNCTIONS, PAGES 498–504

CHECK IT OUT!

1. relation: D: $\{1 \leq x \leq 6\}$; R: $\{0 \leq y \leq 5\}$; inverse: D: $\{0 \leq y \leq 5\}$; R: $\{1 \leq x \leq 6\}$

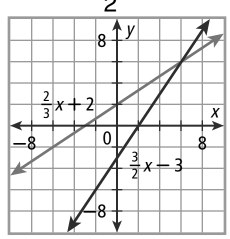


2a. $f^{-1}(x) = 3x$

b. $f^{-1}(x) = x - \frac{2}{3}$

3. $f^{-1}(x) = \frac{x+7}{5}$

4. $f^{-1}(x) = \frac{3}{2}x - 3$



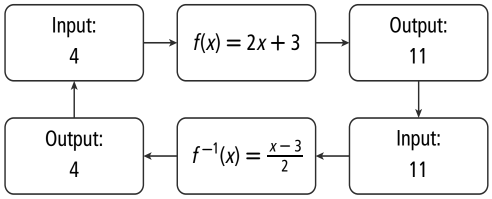
5. inverse: $z = 6t - 6 = 6(7) - 6 = 36$
36 oz of water are needed if 7 teaspoons of tea are used.

THINK AND DISCUSS

1. Possible answer: When x and y are interchanged, the inverse function is the same as the original function. The graph of an inverse is the reflection across $y = x$, but the original function is $y = x$.

2. Possible answer: $y = x$; $y = x^2$

3. Possible answer: You get the original function; yes, the original is a function.

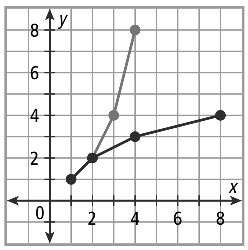
4. 

EXERCISES

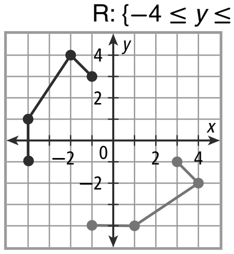
GUIDED PRACTICE

1. relation

2. relation: D: $\{1 \le x \le 4\}$; R: $\{1 \le y \le 8\}$; inverse: D: $\{1 \le x \le 8\}$; R: $\{1 \le y \le 4\}$;



3. relation: D: $\{-1 \le x \le 4\}$; R: $\{-4 \le y \le -1\}$; inverse: D: $\{-4 \le x \le -1\}$; R: $\{-1 \le y \le 4\}$;



4. $f^{-1}(x) = x - 3$

5. $f^{-1}(x) = \frac{1}{4}x$

6. $f^{-1}(x) = 2x$

7. $f^{-1}(x) = x + 2\frac{1}{2}$

8. $f^{-1}(x) = \frac{1}{5}(x + 1)$

9. $f^{-1}(x) = 2(x - 3)$

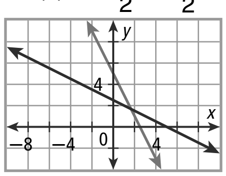
10. $f^{-1}(x) = -2x + 6$

11. $f^{-1}(x) = -\frac{2}{3}x + 1$

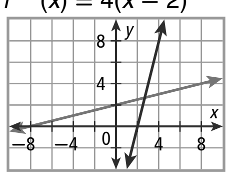
12. $f^{-1}(x) = \frac{1}{4}x - 1$

13. $f^{-1}(x) = \frac{2}{3}x + \frac{5}{3}$

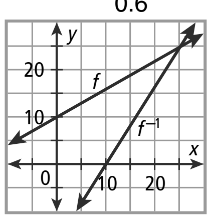
14. $f^{-1}(x) = -\frac{1}{2}x + \frac{5}{2}$



15. $f^{-1}(x) = 4(x - 2)$



16. $f^{-1}(x) = \dfrac{x - 10}{0.6}$

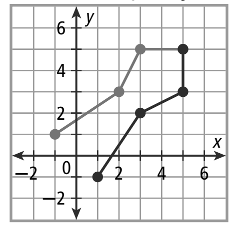


17. $F = \frac{9}{5}C + 32 = \frac{9}{5}(16) + 32 = 60.8$

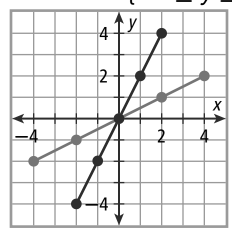
16°C is about 61°F.

PRACTICE AND PROBLEM SOLVING

18. relation: D: $\{-1 \le x \le 5\}$; R: $\{1 \le y \le 5\}$; inverse: D: $\{1 \le y \le 5\}$; R: $\{-1 \le x \le 5\}$



19. relation: D: $\{-4 \le x \le 4\}$; R: $\{-2 \le y \le 2\}$; inverse: D: $\{-2 \le y \le 2\}$; R: $\{-4 \le x \le 4\}$



20. $f^{-1}(x) = 1.\overline{21}x$

21. $f^{-1}(x) = x + 1\frac{3}{4}$

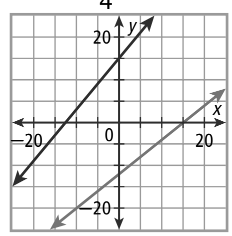
22. $f^{-1}(x) = 0.25x$

23. $f^{-1}(x) = -\frac{1}{32}x + \frac{21}{32}$

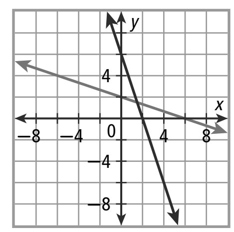
24. $f^{-1}(x) = 0.08x - 11.6$

25. $f^{-1}(x) = 5x - 60$

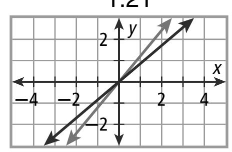
26. $f^{-1}(x) = \frac{5}{4}x + 15$



27. $f^{-1}(x) = -3x + 6$



28. $f^{-1}(x) = \dfrac{x}{1.21}$



29. $f(x) = 19500x + 1.28 \times 10^6$, where x is the number of years after 2001;

$f^{-1}(x) = \dfrac{x - 1.28 \times 10^6}{19500}$, where x is the number of bachelor's degrees awarded;

$f^{-1}(1.7 \times 10^6) = \dfrac{1.7 \times 10^6 - 1.28 \times 10^6}{19500} \approx 22$

About 22 years after 2001, 1.7 million bachelor's degrees will be awarded.

30a. $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{4 - 9}{3 - 2} = -5$

b. The slope of the inverse line is the negative reciprocal of the slope of the original line, which in this case is $-\left(\frac{1}{-5}\right) = -\frac{1}{5}$.

31a. $f^{-1}(x) = \dfrac{212 - x}{1.85}$

b. $f^{-1}(200) = \dfrac{212 - 200}{1.85} \approx 6.5$

$6.5 \times 1000 = 6500$

The boiling point of water will fall below 200°F above about 6500 ft.

c. $f^{-1}(160.3) = \dfrac{212 - 160.3}{1.85} = 27.946$

$27.946 \times 1000 = 27946$

The mountain peak's altitude is 27,946 ft.

32. (4, −3), (1, 4), and (−2, −4)

33. (4, 2), (2, 4), (−3, −1), and (−1, −3)

34. The inverse is $x = 3$, which is a line parallel to the y-axis. So it is not a function.

35. $f(x) = \dfrac{10}{12.59}x$; $f^{-1}(x) = \dfrac{12.59}{10}x = 1.259x$

$f^{-1}(25) = 1.259(25) = 31.48$

It will take Warhol 31.48 s to complete a 25 m race.

36a. $C = 22n + 3.5$

b. $n = \dfrac{C - 3.5}{22} = \dfrac{157.50 - 3.5}{22} = 7$

Seven tickets are purchased when the credit card bill is \$157.50.

c. $n = \dfrac{332.50 - 3.5}{22} = 14.95$

No; when $C = \$332.50$, n is not an integer.

37. B; the student may have found the inverse of each term.

38. The function is inversed, and the graph is reflected over the line $y = x$. The result may or may not be a function.

39. Yes; possible answer: for the ordered pairs (2, 1) and (2, 3), the inverse relation is (1, 2) and (3, 2), which is a function.

40a. $S(c) = \dfrac{1}{3}c - \dfrac{7}{24}$

b. $C(s) = \dfrac{s + \frac{7}{24}}{\frac{1}{3}} = 3\left(s + \dfrac{7}{24}\right) = 3s + \dfrac{7}{8}$;

yes; head circumference as a function of hat size.

c. $C\left(7\dfrac{3}{8}\right) = 3\left(7\dfrac{3}{8}\right) + \dfrac{7}{8} = 23$

The head circumference of the owner is 23 in.

41. always
42. sometimes
43. never
44. always
45. always
46. always

47a. $P = \dfrac{147}{340}d + 14.7$

b. D: $\{d \mid d \geq 0\}$;
R: $\{P \mid P \geq 14.7\}$

c. $d = \dfrac{P - 14.7}{\frac{147}{340}} = \dfrac{340}{147}(P - 14.7) = \dfrac{340}{147}P - 34$;

depth as a function of pressure

d. At 25.9 ft, the pressure is 25.9 psi.

TEST PREP

48. A;

$f^{-1}(x) = \dfrac{x + \frac{3}{4}}{4} = \dfrac{1}{4}\left(x + \dfrac{3}{4}\right) = \dfrac{1}{4}x + \dfrac{3}{16}$

49. F

50. C

51.

x	1	2	3	4	5
y	0	1	2	3	4

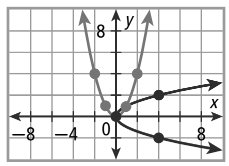
CHALLENGE AND EXTEND

52. $y = \dfrac{(x - b)}{m} = \dfrac{x}{m} - \dfrac{b}{m}$

53.
$ay + bx = c$
$ay = c - bx$
$y = \dfrac{c - bx}{a}$
$y = -\dfrac{b}{a}x + \dfrac{c}{a}$

54.
$x - y_1 = m(y - x_1)$
$x - y_1 = my - mx_1$
$x - y_1 + mx_1 = my$
$\dfrac{x - y_1 + mx_1}{m} = y$
$\dfrac{x - y_1}{m} + x_1 = y$

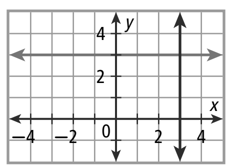
55.



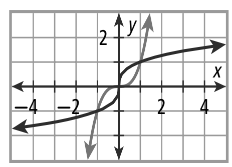
$y = x^2$; switch x and y: $x = y^2$

56. Either the function and its inverse are both $f(x) = f^{-1}(x) = x$, or the function and its inverse are both $f(x) = f^{-1}(x) = -x + k$, where k is any real number constant.

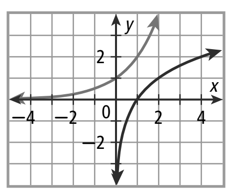
57.



58.



59.



SPIRAL REVIEW

60a. 44.95, 45.18, 46.04, 46.89, 47.53, 48.16

b. $\{v \mid 44.95 \leq v \leq 48.16\}$

61.
$2(x + 3)(x - 2)(x - 1) = 0$
$2\left(x^2 + x - 6\right)(x - 1) = 0$
$2\left(x^3 - 7x + 6\right) = 0$
$2x^3 - 14x + 12 = 0$

62.
$2\left(x - \sqrt{5}\right)\left(x + \sqrt{5}\right) = 0$
$2\left(x^2 - \left(\sqrt{5}\right)^2\right) = 0$
$2x^2 - 10 = 0$

63. $2[x - (1 - i)][x - (1 + i)](x - 2) = 0$
$2[(x - 1) + i][(x - 1) - i](x - 2) = 0$
$2\left((x - 1)^2 - i^2\right)(x - 2) = 0$
$2\left(x^2 - 2x + 1 - (-1)\right)(x - 2) = 0$
$2\left(x^2 - 2x + 2\right)(x - 2) = 0$
$2\left(x^3 - 4x^2 + 6x - 4\right) = 0$
$2x^3 - 8x^2 + 12x - 8 = 0$

64. $2(x + 3)(x - 8)(x - 9) = 0$
$2\left(x^2 - 5x - 24\right)(x - 9) = 0$
$2\left(x^3 - 14x^2 + 21x + 216\right) = 0$
$2x^3 - 28x^2 + 42x + 432 = 0$

65. decay **66.** decay

67. growth **68.** growth

7-3 LOGARITHMIC FUNCTIONS, PAGES 505–511

CHECK IT OUT!

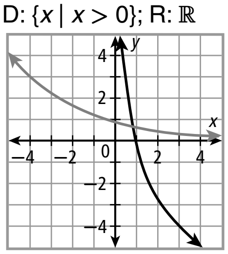
1a. $\log_9 81 = 2$ **b.** $\log_3 27 = 3$

c. $\log_x 1 = 0$ **2a.** $10^1 = 10$

b. $12^2 = 144$ **c.** $\left(\frac{1}{2}\right)^{-3} = 8$

3a. $\log 0.00001 = -5$ **b.** $\log_{25} 0.04 = -1$

4. D: $\{x \mid x > 0\}$; R: $\mathbb{R}$



5. pH $= -\log(0.000158) = 3.8$
The pH of the iced tea is 3.8.

THINK AND DISCUSS

1. The inverse of an exponential function is a logarithmic function and vice versa. Exponential functions have a vertical asymptote, and logarithmic functions have a horizontal asymptote. The domain of exponential functions is $\mathbb{R}$, and the range is restricted. The domain of logarithmic functions is restricted, and the domain is $\mathbb{R}$.

2. Possible answer: no; $\log_2 16 = 4$, but $\log_{16} 2 = 0.25$.

3.

Definition: the exponent to which a specified base is raised to obtain a given value	Characteristics: • the inverse of an exponential function • logarithm can be any real number • has a positive base not equal to 1 • written $f(x) = \log_b x$ • if the base is 10, $f(x) = \log x$
Logarithmic Function	
Examples: $3^2 = 9$, so $\log_3 9 = 2$. $4^{-3} = \frac{1}{64}$, so $\log_4 \frac{1}{64} = -3$. $10^0 = 1$, so $\log 1 = 0$.	Nonexamples: polynomial functions: $f(x) = x$, $f(x) = x^2$ exponential functions: $f(x) = 2^x$ root functions: $f(x) = \sqrt{x}$

EXERCISES

GUIDED PRACTICE

1. x

2. $\log_{2.4} 1 = 0$ **3.** $\log_4 8 = 1.5$

4. $\log 0.01 = -2$ **5.** $\log_3 243 = x$

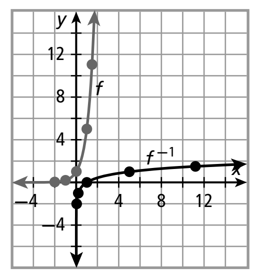
6. $4^{-2} = 0.0625$ **7.** $x^3 = -16$

8. $0.9^2 = 0.81$ **9.** $6^3 = x$

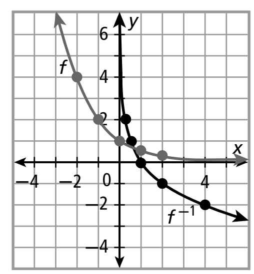
10. $\log_7 343 = 3$ **11.** $\log_3\left(\frac{1}{9}\right) = -2$

12. $\log_{0.5} 0.25 = 2$ **13.** $\log_{1.2} 1.44 = 2$

14. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$, R: $\mathbb{R}$



15. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$, R: $\mathbb{R}$



16. pOH $= -\log(0.000000004) = 8.4$
The pOH of the water is 8.4.

PRACTICE AND PROBLEM SOLVING

17. $\log_x 32 = 2.5$ **18.** $\log_6(216) = x$

19. $\log_{1.2} 1 = 0$ **20.** $\log_4 0.25 = -1$

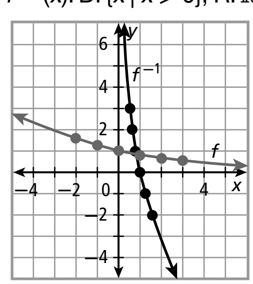
21. $5^4 = 625$ **22.** $2^6 = x$

23. $4.5^0 = 1$ **24.** $\pi^1 = \pi$

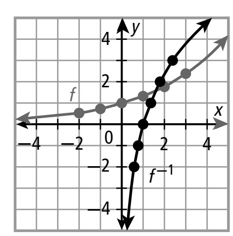
25. $\log_2 1 = 0$ **26.** $\log 0.001 = -3$

27. $\log_4 64 = 3$ **28.** $\log_{0.1} 100 = -2$

29. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$, R: $\mathbb{R}$



30. $f(x)$: D: $\mathbb{R}$, R: $\{y \mid y > 0\}$; $f^{-1}(x)$: D: $\{x \mid x > 0\}$, R: $\mathbb{R}$



31. pH $= -\log(0.0000006) = 6.2$
No; the pH is 6.2, so the flowers will be pink.

32a. $n = \dfrac{\log A - \log P}{\log(1.0175)} = \dfrac{\log 1210.26 - \log 1000}{\log(1.0175)} = 11$
The debt has been building for 11 months.

b. $n = \dfrac{\log A - \log P}{\log(1.0175)} = \dfrac{\log 1420 - \log 1210.26}{\log(1.0175)} = 9$
It will take 9 additional months until the debt exceeds \$1420.

c. As the debt builds, it takes a shorter time for the debt to increase by a relatively constant amount.

33. 1; $\log_a b = 0$ means $a^0 = b$ and a^0 is 1 for any $a \neq 0$.

34a. Jet: $L = 10\log\left(\dfrac{10^{15}I_0}{I_0}\right) = 10\log\left(10^{15}\right) = 10(15)$
$= 150$ dB;

Jackhammer: $L = 10\log\left(\dfrac{10^{12}I_0}{I_0}\right) = 10\log\left(10^{12}\right)$
$= 10(12) = 120$ dB;

Hair dryer: $L = 10\log\left(\dfrac{10^{7}I_0}{I_0}\right) = 10\log\left(10^{7}\right)$
$= 10(7) = 70$ dB;

Whisper: $L = 10\log\left(\dfrac{10^{3}I_0}{I_0}\right) = 10\log\left(10^{3}\right) = 10(3)$
$= 30$ dB;

Leaves rustling: $L = 10\log\left(\dfrac{10^{2}I_0}{I_0}\right) = 10\log\left(10^{2}\right)$
$= 10(2) = 20$ dB;

Softest audible sound: $L = 10\log\left(\dfrac{I_0}{I_0}\right) = 10\log(1)$
$= 10(0) = 0$ dB

b. Let the intensity of the rock concert be xI_0.

$$10\log\left(\frac{xI_0}{I_0}\right) = 110$$
$$10\log(x) = 110$$
$$\log(x) = 11$$
$$x = 10^{11}$$

The intensity of the rock concert is $10^{11}I_0$, and it should be placed between jackhammer and hair dryer.

c. No; 20 bels is 200 dB.

35. Yes; possible answer: $\log 10^3 = \log 1000 = 3$, and there are 3 zeros after the 1. $\log 10^{-2} = \log 0.01 = -2$, and there are 2 zeros.

36. $10^{2.3} \approx 199.5 \to \log 200 \approx 2.3$;
$10^{2.7} \approx 501.2 \to \log 500 \approx 2.7$

37a. orange **b.** lemon

c. grapefruit

38. $\log_0 3$ means $0^x = 3$, and $\log_1 3$ means $1^x = 3$. These have no solution.

TEST PREP

39. C

40. G

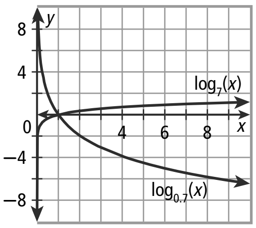
41. A

42. F

43. $2^6 = 64 \to \log_2 64 = 6$

CHALLENGE AND EXTEND

44.



The range of $\log_7 x$ is negative for $0 < x < 1$ and positive for $x > 1$. The range of $\log_{0.7} x$ is positive for $0 < x < 1$ and negative for $x > 1$.

45. $\log_3 9 = 2$; $\log_3 27 = 3$; $\log_3 243 = 5$
$\log_3 9 + \log_3 27 = \log_3 243$
$\log_b\left(b^x\right) + \log_b\left(b^y\right) = \log_b\left(b^{x+y}\right)$

46. Let $\log_7 7^{2x+1} = a$.
Write the above exponental equation as a logarithmic equation, we obtain
$7^a = 7^{2x+1} \to a = 2x + 1$
Hence we have proved that $\log_7 7^{2x+1} = 2x + 1$.

47a. $2^{11} = 2048$ Hz, $\log_2 2048 = 11$

b. 3 octaves lower; $\log_2 256 = 8$, $\log_2 32 = 5$,
$8 - 5 = 3$

SPIRAL REVIEW

48. $\left((2a^4)(5b^2)\right)^2$
$\left(10a^4b^2\right)^2$
$10^2\left(a^4\right)^2\left(b^2\right)^2$
$100a^8b^4$

49. $\dfrac{8s^2t^6}{4st^8}$
$\dfrac{8s^2}{4s} \cdot \dfrac{t^6}{t^8}$
$2s \cdot \dfrac{1}{t^2}$
$\dfrac{2s}{t^2}$

50. $-2t^2\left(5st^{-1}\right)$
$(-2 \cdot 5)s\left(t^2 \cdot t^{-1}\right)$
$-10st$

51. $7a^{-2}b^3\left(3ab + 4a^{-1}b^2\right)$
$7a^{-2}b^3 \cdot 3ab + 7a^{-2}b^3 \cdot 4a^{-1}b^2$
$21a^{-1}b^4 + 28a^{-3}b^5$

52. $0 = 25 - 16t^2$
$16t^2 = 25$
$t^2 = \dfrac{25}{16}$
$t = \sqrt{\dfrac{25}{16}}$
$t = \dfrac{5}{4}$
It took $\dfrac{5}{4}$ s or 1.25 s for the brick to hit the ground.

53. 0.35, 0.59, 1, 1.7, 2.89

54. 2.78, 1.67, 1, 0.6, 0.36

55. 11.11, 3.33, 1, 0.3, 0.09

7-4 PROPERTIES OF LOGARITHMS, PAGES 512–519

CHECK IT OUT!

1a. $\log_5 625 + \log_5 25$
$\log_5 (625 \cdot 25)$
$\log_5 15625$
6

b. $\log_{\frac{1}{3}} 27 + \log_{\frac{1}{3}} \frac{1}{9}$
$\log_{\frac{1}{3}} \left(27 \cdot \frac{1}{9}\right)$
$\log_{\frac{1}{3}} 3$
-1

2. $\log_7 49 - \log_1 7$
$\log_7 \left(\frac{49}{7}\right)$
$\log_7 7$
1

3a. $\log 10^4$
$4 \log 10$
$4(1)$
4

b. $\log_5 25^2$
$2 \log_5 25$
$2(2)$
4

c. $\log_2 \left(\frac{1}{2}\right)^5$
$5 \log_2 \left(\frac{1}{2}\right)$
$5(-1)$
-5

4a. 0.9

b. $8x$

5a. $\log_9 27$
$\frac{\log_3 27}{\log_3 9}$
1.5

b. $\log_8 16$
$\frac{\log_2 16}{\log_2 8}$
$1.\overline{3}$

6.
$8 = \frac{2}{3} \log \left(\frac{E}{10^{11.8}}\right)$
$\left(\frac{3}{2}\right)8 = \log \left(\frac{E}{10^{11.8}}\right)$
$12 = \log \left(\frac{E}{10^{11.8}}\right)$
$12 = \log E - \log 10^{11.8}$
$12 = \log E - 11.8$
$23.8 = \log E$
$10^{23.8} = E$
$10^{25.6} \div 10^{23.8} = 10^{1.8} \approx 63$
About 63 times as much energy is released by an earthquake with a magnitude of 9.2 than by one with a magnitude of 8.

THINK AND DISCUSS

1. Change the base and enter Y=log(X)/log(5).
2. $10^{25.6}$ is $10^{0.6} \times 10^{25}$, and $10^{0.6}$ is about 3.98, so $10^{25.6}$ is about 3.98×10^{25}.
3. You get $\log_b a = \frac{1}{\log_a b}$.

4.

Property of Exponents	Property of Logarithms
$b^m b^n = b^{m+n}$	$\log_b mn = \log_b m + \log_b n$
$\frac{b^m}{b^n} = b^{m-n}$	$\log_b \frac{m}{n} = \log_b m - \log_b n$
$(b^a)^p = b^{ap}$	$\log_b a^p = p \log_b a$
$b^{\log_b x} = x$	$\log_b b^x = x$
	$\log_b x = \frac{\log_a x}{\log_a b}$

EXERCISES

GUIDED PRACTICE

1. $\log_5 50 + \log_5 62.5$
$\log_5 (50 \cdot 62.5)$
$\log_5 3125$
5

2. $\log 100 + \log 1000$
$\log (100 \cdot 1000)$
$\log 100000$
5

3. $\log_3 3 + \log_3 27$
$\log_3 (3 \cdot 27)$
$\log_3 81$
4

4. $\log_4 320 - \log_4 5$
$\log_4 \left(\frac{320}{5}\right)$
$\log_4 64$
3

5. $\log 5.4 - \log 0.054$
$\log \left(\frac{5.4}{0.054}\right)$
$\log 100$
2

6. $\log_6 496.8 - \log_6 2.3$
$\log_6 \left(\frac{496.8}{2.3}\right)$
$\log_6 216$
3

7. 2

8. 5

9. $\log_7 49^3$
$\log_7 (7^2)^3$
$\log_7 7^6$
6

10. $\log_{\frac{1}{2}} (0.25)^4$
$\log_{\frac{1}{2}} (0.5^2)^4$
$\log_{\frac{1}{2}} 0.5^8$
8

11. $\frac{x}{2} + 5$

12. 19

13. $\log_4 1024$
$\log_4 4^5$
5

14. $\log_2 (0.5)^4$
$\log_2 (2^{-1})^4$
$\log_2 2^{-4}$
-4

15. $\log_9 \frac{1}{27}$
$\frac{\log_3 \frac{1}{27}}{\log_3 9}$
-1.5

16. $\log_8 32$
$\frac{\log_2 32}{\log_2 8}$
$1.\overline{6}$

17. $\log_5 10$
$\frac{\log 10}{\log 5}$
≈ 1.43

18. $\log_2 27$
$\frac{\log 27}{\log 2}$
≈ 4.75

19. $8.1 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$ $7.9 = \frac{2}{3}\log\left(\frac{E}{10^{11.8}}\right)$

$\left(\frac{3}{2}\right)8.1 = \log\left(\frac{E}{10^{11.8}}\right)$ $\left(\frac{3}{2}\right)7.9 = \log\left(\frac{E}{10^{11.8}}\right)$

$12.15 = \log\left(\frac{E}{10^{11.8}}\right)$ $11.85 = \log\left(\frac{E}{10^{11.8}}\right)$

$12.15 = \log E - \log 10^{11.8}$ $11.85 = \log E - \log 10^{11.8}$

$12.15 = \log E - 11.8$ $11.85 = \log E - 11.8$

$23.95 = \log E$ $23.65 = \log E$

$10^{23.95} = E$ $10^{23.65} = E$

$10^{23.95} \div 10^{23.65} = 10^{0.3} \approx 2$

About 2 times as much energy was released by the 1811 earthquake than by the 1957 one.

PRACTICE AND PROBLEM SOLVING

20. $\log_8 4 + \log_8 16$
$\log_8 (4 \cdot 16)$
$\log_8 64$
2

21. $\log 2 + \log 5$
$\log (2 \cdot 5)$
$\log 10$
1

22. $\log_{2.5} 3.125 + \log_{2.5} 5$
$\log_{2.5} (3.125 \cdot 5)$
$\log_{2.5} 15.625$
3

23. $\log 1000 - \log 100$
$\log\left(\frac{1000}{100}\right)$
$\log 10$
1

24. $\log_2 16 - \log_2 2$
$\log_2\left(\frac{16}{2}\right)$
$\log_2 8$
3

25. $\log_{1.5} 6.75 - \log_{1.5} 2$
$\log_{1.5}\left(\frac{6.75}{2}\right)$
$\log_{1.5} 3.375$
3

26. $\log_2 16^3$
$\log_2 \left(2^4\right)^3$
$\log_2 2^{12}$
12

27. $\log (100)^{0.1}$
$\log \left(10^2\right)^{0.1}$
$\log 10^{0.2}$
0.2

28. $\log_5 125^{\frac{1}{3}}$
$\log_5 \left(5^3\right)^{\frac{1}{3}}$
$\log_5 5^1$
1

29. $7 + x$

30. 4.52

31. $\log_9 6561$
$\log_9 9^4$
4

32. $\log_{\frac{1}{2}} 16$
$\frac{\log_2 16}{\log_2 \frac{1}{2}}$
$\frac{4}{-1}$
-4

33. $\log_{25} 125$
$\frac{\log_5 125}{\log_5 25}$
$\frac{3}{2}$

34. $\log_4 9$
$\frac{\log 9}{\log 4}$
≈ 1.58

35. $100 = 10\log\left(\frac{I}{I_0}\right)$ $105 = 10\log\left(\frac{I}{I_0}\right)$

$10 = \log\left(\frac{I}{I_0}\right)$ $10.5 = \log\left(\frac{I}{I_0}\right)$

$10^{10} = \frac{I}{I_0}$ $10^{10.5} = \frac{I}{I_0}$

$10^{10} I_0 = I$ $10^{10.5} I_0 = I$

$10^{10.5} I_0 \div 10^{10} I_0 = 10^{0.5} I_0 \approx 3.16$

The concert sound is bout 3.16 times more intense than the allowable level.

36a. $1 - (-5.3) = 5 \log \frac{d}{10}$
$6.3 = 5 \log \frac{d}{10}$
$1.26 = \log \frac{d}{10}$
$10^{1.26} = \frac{d}{10}$
$18.2 \approx \frac{d}{10}$
$182 = d$

The distance of Antares from Earth is about 182 parsecs.

b. $2.9 - M = 5 \log \frac{225}{10}$
$2.9 - M = 5 \log 22.5$
$2.9 - M \approx 5(1.35)$
$2.9 - M = 6.75$
$M = -3.85$

The absolute magnitude is about −3.9.

c. $5 - (-0.4) = 5 \log \frac{d}{10}$
$5.4 = 5 \log \frac{d}{10}$
$1.08 = \log \frac{d}{10}$
$10^{1.08} = \frac{d}{10}$
$12.0 \approx \frac{d}{10}$
$120 = d$
$182 \div 120 \approx 1.5$

The distance to Antares is about 1.5 times as great as the distance to Rho Oph.

37. $\log_b m + \log_b n = \log_b mn$

38. $\log_b m - \log_b n = \log_b \frac{m}{n}$

39. $\log_b \left(b^m\right)^n = \log_b mn$

$n \log_b b^m = mn$

40. $\log_2 32 - \log_2 128$
$\log_2\left(\frac{32}{128}\right)$
$\log_2 \frac{1}{4}$
-2

41. $\log 0.1 + \log 1 + \log 10$
$\log (0.1 \cdot 1 \cdot 10)$
$\log 1$
0

42. $2 - \log_{11} 121$

$2 - \log_{11} 11^2$

$2 - 2$

0

43. $\log_{\frac{1}{2}} 2 + \log_{\frac{1}{2}} 2^{\frac{1}{2}}$

$\log_{\frac{1}{2}}\left(2 \cdot 2^{\frac{1}{2}}\right)$

$\log_{\frac{1}{2}} 2^{\frac{3}{2}}$

$-\frac{3}{2}$

44. $7^{\log_7 7} - \log_7 7^7$

$7(1) - 7$

$7 - 7$

0

45. $\frac{10^{\log 10}}{\log 10^{10}}$

$\frac{10(1)}{10}$

$\frac{10}{10}$

1

46a. $\log 20 = \log(2 \cdot 10) = \log 2 + \log 10 \approx 0.301 + 1 \approx 1.301$

b. $\log 200 = \log\left(2 \cdot 10^2\right) = \log 2 + \log 10^2 \approx 0.301 + 2 \approx 2.301$

c. $\log 2000 = \log\left(2 \cdot 10^3\right) = \log 2 + \log 10^3 \approx 0.301 + 3 \approx 3.301$

47. $10^{-7} - 10^{-7.6}$

48a. $P = 143(1 - 4\%)^t = 143(0.96)^t$

b. $t = \log_{0.96}\left(\frac{P}{143}\right)$

c. $\frac{\log\left(\frac{x}{143}\right)}{\log 0.96}$

d. $t = \log_{0.96}\left(\frac{30}{143}\right) = \frac{\log\left(\frac{30}{143}\right)}{\log 0.96} \approx 39$

The population will drop below 30 after about 39 years.

49. $P = 40(1 + 8\%)^t = 40(1.08)^t$

$t = \log_{1.08}\left(\frac{P}{40}\right) = \log_{1.08}\left(\frac{50}{40}\right) = \frac{\log\left(\frac{50}{40}\right)}{\log 1.08} \approx 2.9$

It will take about 2.9 years for the value of the stock to reach $50.

50a.

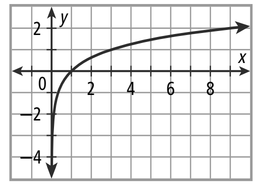
$$2(500) = 500(1.016)^n$$
$$2 = (1.016)^n$$
$$\log 2 = \log(1.016)^n$$
$$\log 2 = n \log(1.016)$$
$$\frac{\log 2}{\log(1.016)} = n$$
$$43.7 \approx n$$

It will take about 43.7 months for the debt to double.

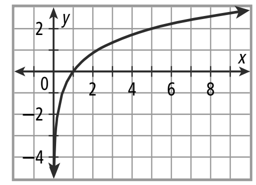
b. It will take another 43.7 months for the debt to double again.

c. no

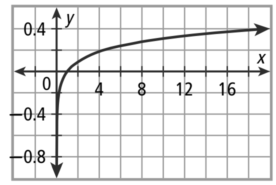
51.



52.



53.



54. Possible answer: Change the base from b to 10 by writing $\log_b x$ as $\frac{\log x}{\log b}$. Enter log(X)/log(b), using the calculator's **LOG** key.

55a. $\log_{12} 1.65 = \log_{12}\left(\frac{33}{20}\right) = \log_{12} 33 - \log_{12} 20 \approx 1.4 - 1.2 \approx 0.2$

b. $\log_{12} 660 = \log_{12}(20 \cdot 33) = \log_{12} 20 + \log_{12} 33 \approx 1.2 + 1.4 \approx 2.6$

c. $\log_{12} 400 = \log_{12} 20^2 = 2\log_{12} 20 \approx 2(1.2) \approx 2.4$

56a. $\log 2.5 \approx 0.398$

b. $\log\left(2.5 \times 10^6\right) = \log 2.5 + \log 10^6 \approx 0.398 + 6 \approx 6.398$

c. $\log\left(2.5 \times 10^2\right) = \log 2.5 + \log 10^2 \approx 0.398 + 2 \approx 2.398$

$\log\left(a \times 10^x\right) = x + \log a$

d. $\log\left(2.5 \times 10^{-3}\right) = \log 2.5 + \log 10^{-3} \approx 0.398 - 3 \approx -2.602$

Yes, the conjecture holds for scientific notation with negative exponents.

57. sometimes

58. always

59. always

60. never

61. always

62. never

63. sometimes

64. never

65. B; $\log 80 + \log 20 \neq \log(80 + 20)$

TEST PREP

66. B

67. H;

$\log_9 x^2 + \log_9 x$

$2\log_9 x + \log_9 x$

$3\log_9 x$

68. A;

$\log 6$

$\log(2 \cdot 3)$

$\log 2 + \log 3$

CHALLENGE AND EXTEND

69a. Possible answer: the 3 on the top scale is lined up with the 1 on the lower scale. At 2 on the lower scale, the product, 6, is read on the top scale. So the sum of log3 units and log2 units is log 6 units.

b. The lengths show $\log 3 + \log 2 = \log(3 \cdot 2) = \log 6$

70. $\{x \mid x < -2 \cup x > 2\}$

71. $\{x \mid x > 1\}$

72. $(x^2 > 1 \text{ and } x > 0)$ or $(x^2 < 1 \text{ and } x < 0)$
$\{x \mid -1 < x < 0 \cup x > 1\}$

73. $\{x \mid x > 0\}$

74. $\{x \mid x > -1\}$

75. $\{x \mid -1 \le x < 0\}$

76. Let $\log_b a^p = m \rightarrow b^m = a^p$ ①
Let $\log_b a = n \rightarrow b^n = a$ ②
Substitute b^n for a into equation 1.
$b^m = (b^n)^p \rightarrow b^m = b^{np} \rightarrow m = np$ ③
Substitute $\log_b a^p$ for m and $\log_b a$ for n into equation 3.
$\log_b a^p = p\log_b a$

77. $\log_9 3^{2x}$
$\log_9 (3^2)^x$
$x \log_9 9$
$x(1)$
x

78. $x^2 = 25$
$x = \sqrt{25}$
$x = 5$

79. $x^3 = -8$
$x = \sqrt[3]{-8}$
$x = -2$
Since $x > 0$, there is no solution for the equation.

80. $x^0 = 1$
The solution for the equation is $\{x \mid x > 0 \text{ and } x \neq 1\}$.

SPIRAL REVIEW

81. $9 = 3(x - 14)$
$9 = 3x - 42$
$-3x = -51$
$x = 17$

82. $4(x + 1) = 3(2x - 6)$
$4x + 4 = 6x - 18$
$-2x = -22$
$x = 11$

83. $-20 + 8n = n + 29$
$7n = 49$
$n = 7$

84. $8\left(n + \frac{3}{4}\right) = 10n - 4$
$8n + 6 = 10n - 4$
$-2n = -10$
$n = 5$

85. $3\sqrt{-16}$
$3(\sqrt{16})(\sqrt{-1})$
$3(4)(i)$
$12i$

86. $-\frac{1}{2}\sqrt{-40}$
$-\frac{1}{2}(\sqrt{40})(\sqrt{-1})$
$-\frac{1}{2}(2\sqrt{10})(i)$
$-i\sqrt{10}$

87. $4\sqrt{-8}$
$4(\sqrt{8})(\sqrt{-1})$
$4(2\sqrt{2})(i)$
$8i\sqrt{2}$

88. $\sqrt{-125}$
$(\sqrt{125})(\sqrt{-1})$
$(5\sqrt{5})(i)$
$5i\sqrt{5}$

89. $\log_5 125 = 3$

90. $\log 0.1 = -1$

91. $\log_{36} 6 = 0.5$

92. $\log_4 256 = x$

93. $12^0 = 1$
$\log_1 12 = 0$

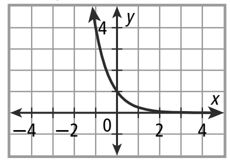
94. $5^2 = 25$
$\log_5 25 = 2$

95. $16^{0.5} = 4$
$\log_{16} 4 = 0.5$

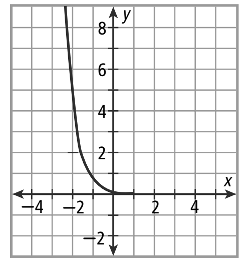
96. $625^{-0.5} = 0.04$
$\log_{625} 0.04 = -0.5$

READY TO GO ON? PAGE 521

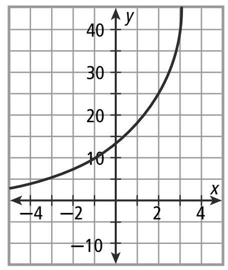
1. decay



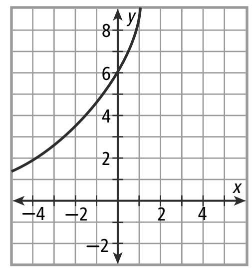
2. decay



3. growth

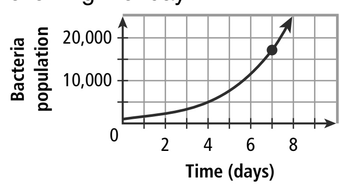


4. growth

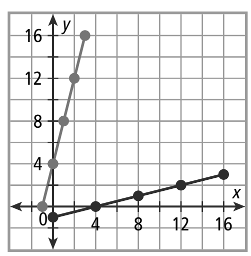


5a. $p = 1000(1.5)^d$

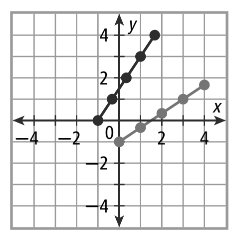
b. There will be about 17,086 bacteria in the culture the following Monday.



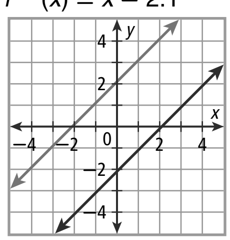
6.



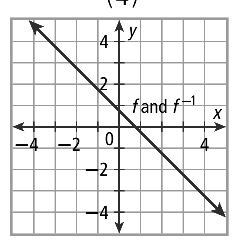
7.



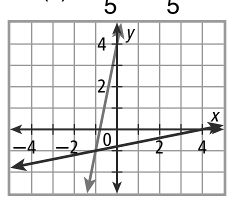
8. $f^{-1}(x) = x - 2.1$



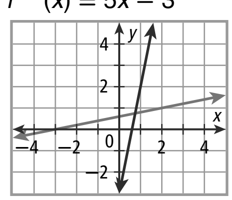
9. $f^{-1}(x) = \left(\frac{3}{4}\right) - x$



10. $f^{-1}(x) = \frac{1}{5}x - \frac{4}{5}$



11. $f^{-1}(x) = 5x - 3$



12. $f^{-1}(x) = \dfrac{(x - 210)}{55}$

$f^{-1}(402.50) = \dfrac{(402.50 - 210)}{55} = \dfrac{192.50}{55} = 3.5$

The number of hours of labor is 3.5 h.

13. $\log_3 9 = 2$

14. $\log_{17.6} 1 = 0$

15. $\log_2 0.25 = -2$

16. $\log_{0.5} 0.0625 = x$

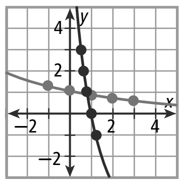
17. $4^3 = 64$

18. $\left(\dfrac{1}{5}\right)^{-2} = 25$

19. $0.99^0 = 1$

20. $e^5 = x$

21.



22. $\log_3 81 + \log_3 9$
$\log_3 (81 \cdot 9)$
$\log_3 729$
6

23. $\log_{\frac{1}{5}} 25 + \log_{\frac{1}{5}} 5$
$\log_{\frac{1}{5}} (25 \cdot 5)$
$\log_{\frac{1}{5}} 125$
-3

24. $\log_{1.2} 2.16 - \log_{1.2} 1.5$
$\log_{1.2}\left(\dfrac{2.16}{1.5}\right)$
$\log_{1.2} 1.44$
2

25. $\log_4 256^2$
$2 \log_4 256$
$2(4)$
8

26. $\log_7 343$
$\log_7 7^3$
3

27. 0.73

28. $\log_{27} 243$
$\dfrac{\log_3 243}{\log_3 27}$
$\dfrac{5}{3}$

29. $\log_{10} 0.01$
$\log_{10} 10^{-2}$
-2

30. $\log_5 625$
$\log_5 5^4$
4

7-5 EXPONENTIAL AND LOGARITHMIC EQUATIONS AND INEQUALITIES, PAGES 522–528

CHECK IT OUT!

1a. $3^{2x} = 27$
$3^{2x} = 3^3$
$2x = 3$
$x = 1.5$

b. $7^{-x} = 21$
$\log 7^{-x} = \log 21$
$-x \log 7 = \log 21$
$x = \dfrac{-\log 21}{\log 7} \approx -1.565$

c. $2^{3x} = 15$
$\log 2^{3x} = \log 15$
$3x \log 2 = \log 15$
$x = \dfrac{\log 15}{3 \log 2} \approx 1.302$

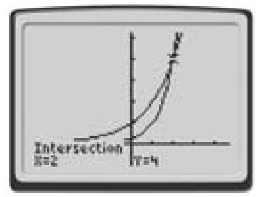
2. $3^{n-1} \geq 10^8$
$\log 3^{n-1} \geq \log 10^8$
$(n - 1)\log 3 \geq 8$
$n - 1 \geq \dfrac{8}{\log 3}$
$n \geq \approx \dfrac{8}{\log 3} + 1$
$n \geq \approx 18$

You would receive at least a million dollars on day 18.

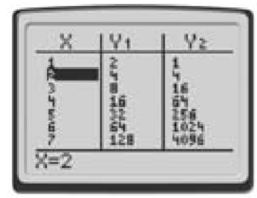
3a. $3 = \log 8 + 3 \log x$
$3 = \log 8 + \log x^3$
$3 = \log 8x^3$
$10^3 = 10 \log 8x^3$
$10^3 = 8x^3$
$125 = x^3$
$5 = x$

b. $2\log x - \log 4 = 0$
$2 \log x = \log 4$
$\log x^2 = \log 4$
$x^2 = 4$
$x = 2$

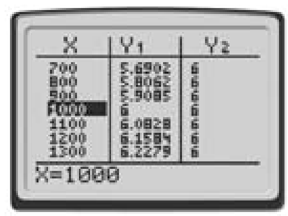
4a. $x = 2$



b. $x < 2$



c. $x = 1000$



THINK AND DISCUSS

1. If $\log a = \log b$, then $10^a = 10^b$. The exponents must be equal for the exponential expressions to be equal.

2a. Write $\log x^5 = 10$ as $5 \log x$.

b. Write $\log 2x + \log 2$ as $\log 4x$

c. Take the log of both sides.

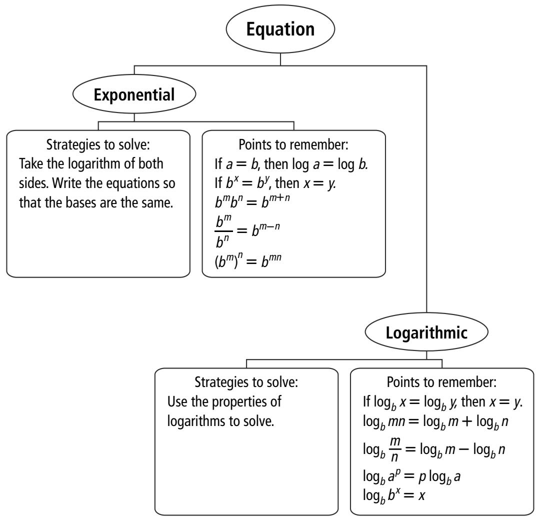
d. Use 10 as a base for each side.

e. Rewrite $\log (x + 4) + \log (x)$ as $\log \left(x^2 + 4x\right)$.

f. Use 6 as a base for each side.

3. Yes; possible answer: $\log (-x) = 1$ has -10 as a solution.

4.



EXERCISES

GUIDED PRACTICE

1. exponential equation

2.
$$4^{2x} = 32^{\frac{1}{2}}$$
$$(2^2)^{2x} = (2^5)^{\frac{1}{2}}$$
$$2^{4x} = 2^{\frac{5}{2}}$$
$$4x = \frac{5}{2}$$
$$x = \frac{5}{8}$$

3.
$$9^x = 3^{x-2}$$
$$(3^2)^x = 3^{x-2}$$
$$3^{2x} = 3^{x-2}$$
$$2x = x - 2$$
$$x = -2$$

4.
$$2^x = 4^{x+1}$$
$$2^x = (2^2)^{x+1}$$
$$2^x = 2^{2x+2}$$
$$x = 2x + 2$$
$$x = -2$$

5.
$$4^x = 10$$
$$\log 4^x = \log 10$$
$$x \log 4 = 1$$
$$x = \frac{1}{\log 4} \approx 1.661$$

6.
$$\left(\frac{1}{4}\right)^{2x} = \left(\frac{1}{2}\right)^x$$
$$\left(\left(\frac{1}{2}\right)^2\right)^2 x = \left(\frac{1}{2}\right)^x$$
$$\left(\frac{1}{2}\right)^{4x} = \left(\frac{1}{2}\right)^x$$
$$4x = x$$
$$x = 0$$

7.
$$2.4^{3x+1} = 9$$
$$\log 2.4^{3x+1} = \log 9$$
$$(3x+1)\log 2.4 = \log 9$$
$$3x + 1 = \frac{\log 9}{\log 2.4}$$
$$x = \frac{1}{3}\left(\frac{\log 9}{\log 2.4} - 1\right)$$
$$x \approx 0.503$$

8.
$$10000 = 3400(1 + 0.03)^t$$
$$2.94 \approx 1.03^t$$
$$\log 2.94 \approx \log 1.03^t$$
$$\log 2.94 \approx t \log 1.03$$
$$\frac{\log 2.94}{\log 1.03} \approx t$$
$$36.48 \approx t$$

It will take 37 years for the population to exceed 10,000 people.

9.
$$\log_2 (7x + 1) = \log_2 (2 - x)$$
$$7x + 1 = 2 - x$$
$$8x = 1$$
$$x = \frac{1}{8}$$

10.
$$\log_6 (2x + 3) = 3$$
$$6^{\log_6 (2x+3)} = 6^3$$
$$2x + 3 = 216$$
$$2x = 213$$
$$x = 106.5$$

11.
$$\log 72 - \log\left(\frac{2x}{3}\right) = 0$$
$$\log 72 = \log\left(\frac{2x}{3}\right)$$
$$72 = \frac{2x}{3}$$
$$216 = 2x$$
$$108 = x$$

12.
$$\log_3 x^9 = 12$$
$$9 \log_3 x = 12$$
$$\log_3 x = \frac{4}{3}$$
$$x = 3^{\frac{4}{3}}$$
$$x \approx 4.33$$

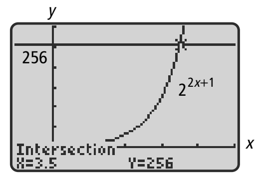
13.
$$\log_7 (3 - 4x) = \log_7\left(\frac{x}{3}\right)$$
$$3 - 4x = \frac{x}{3}$$
$$3 = \frac{13x}{3}$$
$$x = \frac{9}{13}$$

14.
$$\log 50 + \log\left(\frac{x}{2}\right) = 2$$
$$\log 25x = 2$$
$$25x = 10^2$$
$$x = 4$$

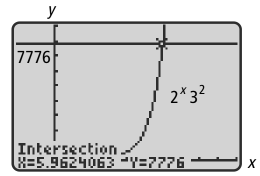
15.
$$\log x + \log (x + 48) = 0$$
$$\log x(x + 48) = 0$$
$$x(x + 48) = 10^2$$
$$x^2 + 48x - 100 = 0$$
$$(x - 2)(x + 50) = 0$$
$$x = 2$$

16.
$$\log\left(x + \frac{3}{10}\right) + \log x + 1 = 0$$
$$\log x\left(x + \frac{3}{10}\right) = -1$$
$$x\left(x + \frac{3}{10}\right) = 10^{-1}$$
$$x^2 + \frac{3}{10}x = \frac{1}{10}$$
$$10x^2 + 3x - 1 = 0$$
$$(5x - 1)(2x + 1) = 0$$
$$x = \frac{1}{5}$$

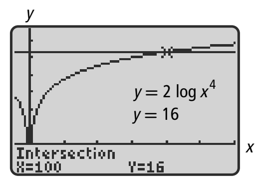
17. $x = 3.5$



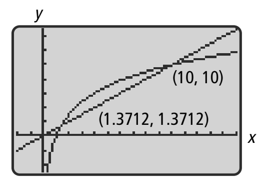
18. $x \le 5$



19. $x = 100$



20. $x > 10$



$y = x$ Answer
$y = 10 \log x$ $x < 1.3712$
$x > 10$

PRACTICE AND PROBLEM SOLVING

21.
$$2^{x-1} = \frac{1}{64}$$
$$2^{x-1} = 2^{-6}$$
$$x - 1 = -6$$
$$x = -5$$

22.
$$\left(\frac{1}{4}\right)^x = 8^{x-1}$$
$$\left(2^{-2}\right)^x = \left(2^3\right)^{x-1}$$
$$2^{-2x} = 2^{3x-3}$$
$$-2x = 3x - 3$$
$$-5x = -3$$
$$x = 0.6$$

23.
$$\left(\frac{1}{5}\right)^{x-2} = 125^{\frac{x}{2}}$$
$$\left(5^{-1}\right)^{x-2} = \left(5^3\right)^{\frac{x}{2}}$$
$$5^{2-x} = 5^{\frac{3}{2}x}$$
$$2 - x = \frac{3}{2}x$$
$$2 = \frac{5}{2}x$$
$$x = 0.8$$

24.
$$\left(\frac{1}{2}\right)^{-x} = 1.6$$
$$\log\left(\frac{1}{2}\right)^{-x} = \log 1.6$$
$$-x \log\frac{1}{2} = \log 1.6$$
$$x = -\frac{\log 1.6}{\log\frac{1}{2}}$$
$$x \approx 0.678$$

25.
$$(1.5)^{x-1} = 14.5$$
$$\log(1.5)^{x-1} = \log 14.5$$
$$(x-1)\log 1.5 = \log 14.5$$
$$x - 1 = \frac{\log 14.5}{\log 1.5}$$
$$x = \frac{\log 14.5}{\log 1.5} + 1$$
$$x \approx 7.595$$

26.
$$3^{\frac{x}{2}+1} = 12.2$$
$$\log 3^{\frac{x}{2}+1} = \log 12.2$$
$$\left(\frac{x}{2} + 1\right)\log 3 = \log 12.2$$
$$\frac{x}{2} + 1 = \frac{\log 12.2}{\log 3}$$
$$x = 2\left(\frac{\log 12.2}{\log 3} - 1\right)$$
$$x \approx 2.554$$

27.
$$50 > 325\left(\frac{1}{2}\right)^{\frac{t}{15}}$$
$$0.154 > \approx \frac{1}{2}^{\frac{t}{15}}$$
$$\log 0.154 > \approx \log\left(\frac{1}{2}\right)^{\frac{t}{15}}$$
$$\log 0.154 > \approx \frac{t}{15}\log\frac{1}{2}$$
$$\frac{\log 0.154}{\log\frac{1}{2}} > \approx \frac{t}{15}$$
$$\frac{15\log 0.154}{\log\frac{1}{2}} > \approx t$$
$$40.48 > \approx t$$

It takes 41 min for the amount to drop below 50 mg.

28.
$$\log_3(7x) = \log_3(2x + 0.5)$$
$$7x = 2x + 0.5$$
$$5x = 0.5$$
$$x = 0.1$$

29.
$$\log_2\left(1 + \frac{x}{2}\right) = 4$$
$$1 + \frac{x}{2} = 2^4$$
$$\frac{x}{2} = 15$$
$$x = 30$$

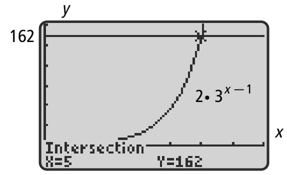
30.
$$\log 5x - \log(15.5) = 2$$
$$\log\frac{5x}{15.5} = 2$$
$$\frac{x}{3.1} = 10^2$$
$$x = 310$$

31.
$$\log_5 x^4 = 2.5$$
$$4\log_5 x = 2.5$$
$$\log_5 x = 0.625$$
$$x = 560.625 \approx 2.73$$

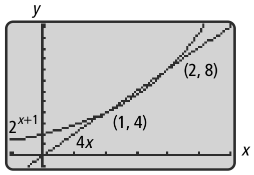
32.
$$\log x - \log\left(\frac{x}{100}\right) = x$$
$$\log\left(\frac{x}{\frac{x}{100}}\right) = x$$
$$\log 100 = x$$
$$2 = x$$

33.
$$2 - \log 3x = \log\left(\frac{x}{12}\right)$$
$$2 = \log 3x + \log\left(\frac{x}{12}\right)$$
$$2 = \log\left(\frac{x^2}{4}\right)$$
$$10^2 = \frac{x^2}{4}$$
$$400 = x^2$$
$$20 = x$$

34. $x = 5$

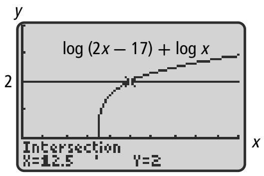


35. $x < 1$



Answer
$x < 1$ or $x > 2$

36. $x \geq 12.5$



37. $\log x = \log\left(x^2 - 12\right)$
$x = x^2 - 12$
$0 = x^2 - x - 12$
$0 = (x - 4)(x + 3)$
$x = 4$ or -3
$\log(-3)$ is undefined, so the only solution is $x = 4$.

38. $5^{2x} = 100$
$\log 5^{2x} = \log 100$
$2x \log 5 = 2$
$x \log 5 = 1$
$x = \dfrac{1}{\log 5} \approx 1.43$

39. $2^{x+2} = 64$
$2^{x+2} = 2^6$
$x + 2 = 6$
$x = 4$

40a. $3 = \log_2 \dfrac{1}{\ell}$
$2^3 = \dfrac{1}{\ell}$
$\dfrac{1}{8} = \ell$

b. $0 = \log_2 \dfrac{1}{\ell}$
$2^0 = \dfrac{1}{\ell}$
$1 = \ell$
$n = \log_2 \dfrac{1}{2}(1) = \log_2 \dfrac{1}{2} = -1$
The *f*-stop setting is *f*/4.

41. $110 = 440 \cdot 2^{\frac{n}{12}}$
$\dfrac{1}{4} = 2^{\frac{n}{12}}$
$2^{-2} = 2^{\frac{n}{12}}$
$-2 = \dfrac{n}{12}$
$-24 = n$
The position is 24 keys below concert A.

42. $500 = 250(1 + 4.5\%)^n$
$2 = (1.045)^n$
$\log 2 = \log (1.045)^n$
$\log 2 = n \log 1.045$
$\dfrac{\log 2}{\log 1.045} = n$
$15.75 \approx n$
It will take at least 16 quarters or 4 yr.

43. 0; $\log x^2 = 2 \log x$, no value of x satisfies the inequality; the graphs coincide, so there is no region where $\log x^2 < 2 \log x$.

44. The student solved $\log (x + 4) = 8$.

45. Method 1: Try to write them so that the bases are all the same.
$2^x = 8^3$
$2^x = \left(2^3\right)^3$
$2^x = 2^9$
$x = 9$

Method 2: Take the logarithm of both sides.
$5^x = 10$
$\log 5^x = \log 10$
$x \log 5 = 1$
$x = \dfrac{1}{\log 5}$

46a. Decreasing; 0.987 is less than 1.

b. $t = 1980 - 1980 = 0$
$N(0) = 119(0.987)^0 = 119(1) = 119$
$t = 2000 - 1980 = 20$
$N(20) = 119(0.987)^{20} \approx 92$
There are 119,000 farms in 1980 and 92,000 in 2000.

c. $80000 = 119(0.987)^t$
$672.27 \approx (0.987)^t$
$30 \approx t$
$1980 + 30 = 2010$
The number of farms will be about 80,000 in 2010.

47a. $2.55 = 128(10)^{-0.0682h}$
$0.02 \approx 10^{-0.0682h}$
$\log 0.02 \approx -0.0682h$
$25 \approx h$
$22.9 = 128(10)^{-0.0682h}$
$0.18 \approx 10^{-0.0682h}$
$\log 0.18 \approx -0.0682h$
$11 \approx h$
The lowest altitude is 11 km, and the highest is 25 km; the model is useful in lower stratosphere and upper troposphere.

b. $P(0) = 128(10)^{-0.0682(0)} = 128(10)^0 = 128(1) = 128$
128 *kPa* = 128 × 0.145 psi = 18.56 psi
The model would predict a sea-level pressure greater than the actual one.

TEST PREP

48. B;
$b^x = c$
$\log b^x = \log c$
$x \log b = \log c$
$x = \dfrac{\log c}{\log b}$

49. J;
$\log (x - 21) = 2 - \log x$
$\log (x - 21) + \log x = 2$
$\log x(x - 21) = 2$
$x(x - 21) = 10^2$
$x - 21x - 100 = 0$
$(x - 25)(x + 4) = 0$
$x = 25$

50. B

CHALLENGE AND EXTEND

51. Possible answer: no; $x = x^x$, and $x^1 = x^x$, so $x = 1$, but $\log_1$ is not defined.

52. $x = 0.125^{\log_2 5}$

$x = \left(2^{-3}\right)^{\log_2 5}$

$x = 2^{-3\log_2 5}$

$x = 2^{\log_2 5^{-3}}$

$x = 5^{-3}$

$x = \frac{1}{125}$ or 0.008

53. $\log_3 36 - \log_3 x > 1$

$\log_3\left(\frac{36}{x}\right) > 1$

$\frac{36}{x} > 3^1$

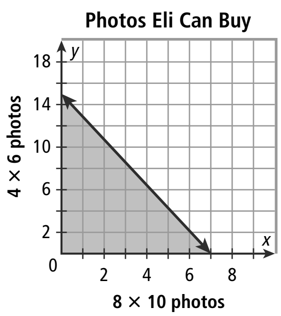
$x < 12$

$\{x \mid x < 12\}$

SPIRAL REVIEW

54a. $0.75x + 0.35y \le 5.25$

b.



He can buy six 4-by-6-inch photographs.

55. $\det A = 4(7) - 2(1) = 26$

56. $\det A = -1(10) - 9(-5) = 35$

57. $\det A = \frac{1}{2}(6) - 0(-1) = 3$

58. $\det A = \frac{2}{3}(9) - \frac{1}{3}(6) = 4$

59. $f^{-1}(x) = \frac{x-3}{4} = \frac{1}{4}x - \frac{3}{4}$

60. $f^{-1}(x) = \frac{1}{6}x + 2$

61. $f^{-1}(x) = 3(x - 9) = 3x - 27$

62. $f^{-1}(x) = \frac{5x+1}{7} = \frac{5}{7}x + \frac{1}{7}$

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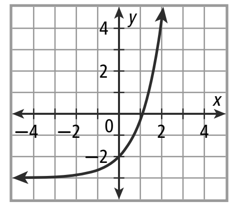
CHECK IT OUT!

1.



2a. $\ln e^{3.2} = 3.2$

b. $e^{2\ln x}$

$e^{\ln x^2}$

x^2

c. $\ln e^{x+4y} = x + 4y$

3. $A = Pe^{rt} = 100e^{0.035(8)} \approx 132.31$

The total amount is $132.31.

4. $\frac{1}{2} = 1e^{-k(28)}$

$\ln\frac{1}{2} = \ln e^{-28k}$

$\ln 2^{-1} = -28k$

$-\ln 2 = -28k$

$k = \frac{\ln 2}{28} \approx 0.0248$

$200 = 650e^{-0.0248t}$

$\frac{200}{650} = e^{-0.0248t}$

$\ln\frac{200}{650} = \ln e^{-0.0248t}$

$\ln\frac{200}{650} = -0.0248t$

$t = \frac{\ln\frac{200}{650}}{-0.0248} \approx 47.5$

I will take about 47.6 days to decay.

THINK AND DISCUSS

1. Possible answer: *e* and π are irrational constants. π is a ratio of parts of a circle and is greater than *e*.

2. e^x and $\ln x$ are inverse functions. ln represents the logarithm, or exponent, when *e* is used as a base.

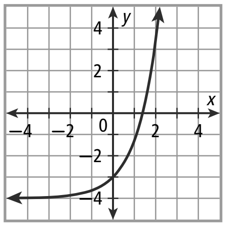
3.

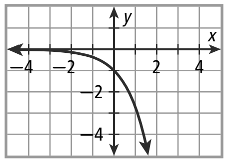
	Natural Logarithms	Common Logarithms
Base	$e = 2.718\ldots$	10
Logarithmic Form	$\ln x = y$ $\ln 100 \approx 4.6$	$\log x = y$ $\log 100 = 2$
Exponential Form	$x = e^y$ $100 \approx e^{4.6}$	$x = 10^y$ $100 = 10^2$
$\log_b 1$	$\ln 1 = 0$	$\log 1 = 0$
$\log_b b$	$\ln e = 1$	$\log 10 = 1$
$\log_b b^x$	$\ln e^x = x$	$\log 10^x = x$
$b^{\log_b x}$	$e^{\ln x} = x$	$10^{\log x} = x$

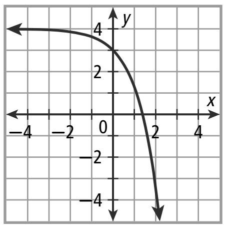
EXERCISES

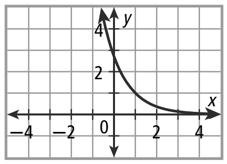
GUIDED PRACTICE

1. $f(x) = \ln x$; natural logarithm

2.


3.


4.


5.


6. $\ln e^1 = 1$

7. $\ln e^{x-y} = x - y$

8. $\ln e^{\left(\frac{-x}{3}\right)} = -\frac{x}{3}$

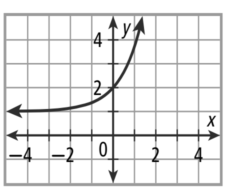
9. $e^{\ln 2x} = 2x$

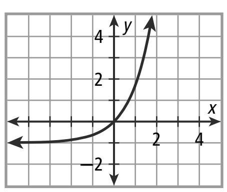
10. $e^{3\ln x} = e^{\ln x^3} = x^3$

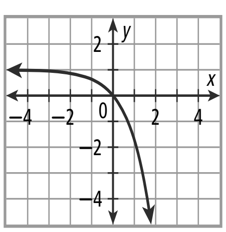
11. $A = Pe^{rt} = 7750e^{0.04(5)} \approx 9465.87$
The total amount is \$9465.87.

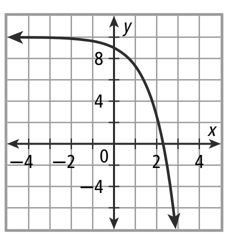
12.
$$\frac{1}{2} = 1e^{-k(6)}$$
$$\ln\frac{1}{2} = \ln e^{-6k}$$
$$\ln 2^{-1} = -6k$$
$$-\ln 2 = -6k$$
$$k = \frac{\ln 2}{6} \approx 0.1155$$
$$N(24) = 250e^{-0.1155(24)} \approx 16$$
The amount remaining after 24 hours is about 16 mg.

PRACTICE AND PROBLEM SOLVING

13.


14.


15.


16.


17. $\ln e^0 = 0$

18. $\ln e^{2a} = 2a$

19. $e^{\ln(c+2)} = c + 2$

20. $e^{4\ln x} = e^{\ln x^4} = x^4$

21. $A = 5000e^{0.035(3)} \approx 5553.55$
The total amount of his investment after 3 years is about \$5553.55.

22.
$$\frac{1}{2} = 1e^{-k(24110)}$$
$$\ln\frac{1}{2} = \ln e^{-24110k}$$
$$\ln 2^{-1} = -24110k$$
$$-\ln 2 = -24110k$$
$$k = \frac{\ln 2}{24110} \approx 0.000029$$
$$N(5000) = 20e^{-0.000029(5000)} \approx 17$$
$$1 = 20e^{-0.000029t}$$
$$\frac{1}{20} = e^{-0.000029t}$$
$$\ln\frac{1}{20} = \ln e^{-0.000029t}$$
$$\ln\frac{1}{20} = -0.000029t$$
$$t = \frac{\ln\frac{1}{20}}{-0.000029} \approx 100000$$
The decay constant is 0.000029; the amount remaining is 17 g after 5000 years; it takes 100,000 years for the 20 grams to decay to 1 gram.

23a. $\ln 10 \approx 2.30$; $\log e \approx 0.43$; they are reciprocals.

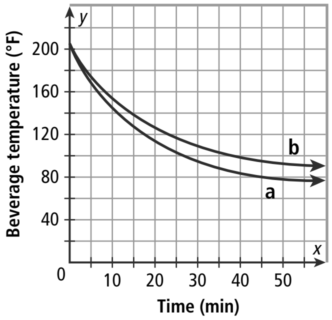
b. $\ln 10 = \frac{\log 10}{\log e} = \frac{1}{\log e}$

24. $\log x = \frac{\ln x}{\ln 10}$ by change of base, so
$$\ln 10(\log x) = \ln 10\left(\frac{\ln x}{\ln 10}\right) = \ln x$$

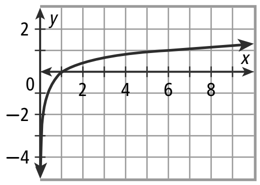
25a.
$$140 = (206 - 70)e^{-0.283t}$$
$$140 = 136e^{-0.283t}$$
$$\frac{140}{136} = e^{-0.283t}$$
$$\ln\frac{140}{136} = -0.283t$$
$$t = \frac{\ln\frac{140}{136}}{00.283} \approx 2.4$$
It takes about 2.4 min for the coffee to reach its best temperature.

b.
$$140 = (206 - 86)e^{-0.283t}$$
$$140 = 120e^{-0.283t}$$
$$\frac{140}{120} = e^{-0.283t}$$
$$\ln\frac{140}{120} = -0.283t$$
$$t = \frac{\ln\frac{140}{120}}{00.283} \approx 2.8$$
It takes about 2.8 min for the coffee to reach its best temperature.

c. room: ≈ 17.4 min;
patio: ≈ 16.9 min



26.



Possible answer: They are the same. $y = \log_6 x$ is changed to base e by $y = \frac{\ln x}{\ln 6}$ and to base 10 by $y = \frac{\log x}{\log 6}$.

27. B

28. A

29. C

30a. $t = 1984 - 1954 = 30$

$$472000 = 4700e^{30k}$$
$$\frac{472000}{4700} = e^{30k}$$
$$\ln\frac{472}{47} = 30k$$
$$k = \frac{\ln\frac{472}{47}}{30} \approx 0.154$$

The growth factor k is about 0.154.

b. $t = 2010 - 1954 = 56$

$P(56) = 4700e^{0.154(56)} \approx 25600000$

The population would have been about 25.6 million in 2010.

31. $\ln 5 + \ln x = 1$
$$\ln 5x = 1$$
$$5x = e^1$$
$$x = \frac{e}{5} \approx 0.54$$

32. $\ln 5 - \ln x = 3$
$$\ln\frac{5}{x} = 3$$
$$\frac{5}{x} = e^3$$
$$x = \frac{5}{e^3} \approx 0.25$$

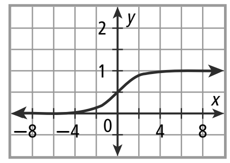
33. $\ln 10 + \ln x^2 = 10$
$$\ln 10x^2 = 10$$
$$10x^2 = e^{10}$$
$$x^2 = \frac{e^{10}}{10}$$
$$x = \pm\sqrt{\frac{e^{10}}{10}}$$
$$x = \pm\frac{e^5}{\sqrt{10}} \approx \pm 47$$

34. $2\ln x - 2 = 0$
$$2\ln x = 2$$
$$\ln x = 1$$
$$x = e$$

35. $4\ln x - \ln x^4 = 0$
$$4\ln x - 4\ln x = 0$$
$$0 = 0$$
$\{x \mid x > 0\}$

36. $e^{\ln x^3} = 8$
$$\ln e^{\ln x^3} = \ln 8$$
$$\ln x^3 = \ln 8$$
$$x^3 = 8$$
$$x = 2$$

37a.



b. 2: $y = 0$ and $y = 1$

c. Possible answer: The epidemic spreads slowly at first, then steadily, and then it tapers off slowly at the end.

38a. 1: $f(x) = 10^x$;
2: $f(x) = e^x$;
3: $f(x) = 2^x$;

b. (0, 1)

c. $2^0 = 10^0 = e^0 = 1$

39. Possible answer: a little more;
\$1000 at 8% interest compounded daily for 1 year:
$A = 1000\left(1 + \frac{0.08}{365}\right)^{365} \approx \$1083.28$;
compounded continuously:
$A = 1000e^{0.08} \approx \$1083.29$

40a. $t = 2000 - 1990 = 10$
$$30800 = 33500e^{10k}$$
$$\frac{30800}{33500} = e^{10k}$$
$$\ln\frac{308}{335} = 10k$$
$$k = \frac{\ln\frac{308}{335}}{10} \approx -0.0084$$

b. $t = 2010 - 1990 = 20$

$N(20) = 33500e^{-0.0084(20)} \approx 28000$

There will be about 28,000 farms in 2010.

c.
$$1279 = 1209e^{10k}$$
$$\frac{1279}{1209} = e^{10k}$$
$$\ln\frac{1279}{1209} = 10k$$
$$k = \frac{\ln\frac{1279}{1209}}{10} \approx 0.0056$$

$A(t) = A_0e^{kt} = 1209e^{0.0056(20)} \approx 1350$

The average size will be about 1350 acres in 2010.

TEST PREP

41. C

42. J

43. A

44. Possible answer: $\ln\left(\frac{1}{x}\right)$

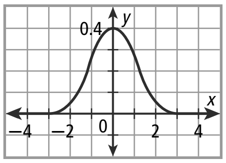
CHALLENGE AND EXTEND

45. Let n be the number of periods needed in one year.

$(1 + \frac{0.08}{n})^n \geq 0.999e^{0.08}$

Solve the equation by trial and error to get $n = 4$.
Possible answer: 4; yes, at 18% interest it takes 17 periods.

46. D: $\mathbb{R}$; R: $\left\{0 < y \leq \frac{1}{\sqrt{2\pi}}\right\}$

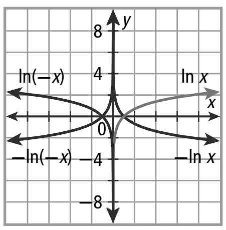


47a. $f(x) = \ln(-x)$

b. $f(x) = -\ln x$

c. $f(x) = -\ln(-x)$

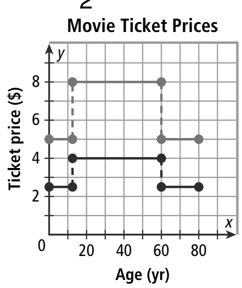
d.



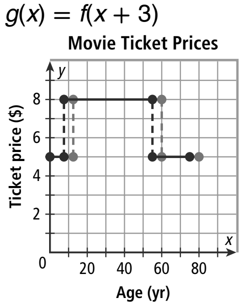
one asymptote: $x = 0$

SPIRAL REVIEW

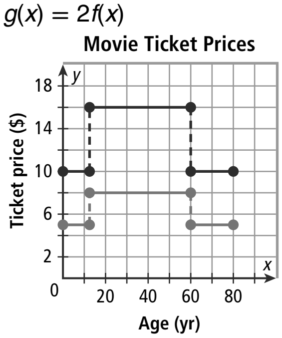
48a. $g(x) = \frac{1}{2}f(x)$



b. $g(x) = f(x + 3)$



c. $g(x) = 2f(x)$



49. $g(x) = f(x) + 5 = -2x^2 + 3x + 1$

50. $g(x) = f(x + 2) = -2x^2 - 5x - 6$

51. $g(x) = -f(x) = 2x^2 - 3x + 4$

52. $g(x) = f\left(\frac{x}{2}\right) = -\frac{1}{2}x^2 + \frac{3}{2}x - 4$

53. $\log_2 8 + \log_2 \frac{1}{2}$

$\log_2 8 \cdot \frac{1}{2}$

$\log_2 4$

2

54. $\log_4 64 - \log_4 1$

$\log_4 \frac{64}{1}$

$\log_4 64$

3

55. $\log_3 243 - \log_3 2187$

$\log_3 \frac{243}{2187}$

$\log_3 \left(\frac{1}{9}\right)$

-2

56. $\log_5 25 + \log_5 125$

$\log_5 25 \cdot 125$

$\log_5 5^5$

5

57. $\log_8 8 + \log_8 \frac{1}{8}$

$\log_8 8 \cdot \frac{1}{8}$

$\log_8 1$

0

58. $\log x^2 - \log x$

$\log \frac{x^2}{x}$

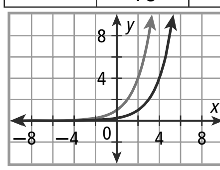
$\log x$

7-7 TRANSFORMING EXPONENTIAL AND LOGARITHMIC FUNCTIONS, PAGES 537–544

CHECK IT OUT!

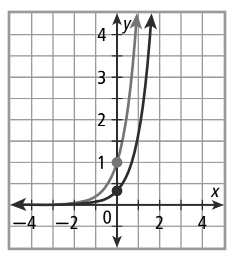
1.

x	−2	−1	0	1	2
$j(x)$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	1



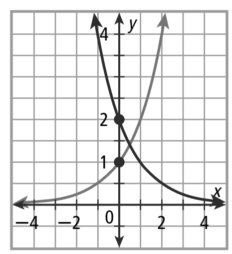
$y = 0$; translation 2 units right

2a.



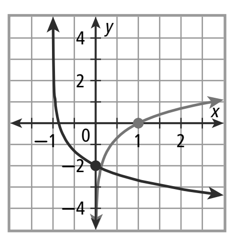
$\frac{1}{3}$; $y = 0$; $f(x) = 5^x$; vertical compression by a factor of $\frac{1}{3}$.

b.



2; $y = 0$; $j(x) = 2^x$; reflection across y-axis and vertical stretch by a factor of 2

3.



$x = -1$; the graph of $f(x) = \ln x$ is translated 1 unit left, reflected across the x-axis, and translated 2 units down.

4. $g(x) = 2\log(x + 3)$

5. $0 > -15 \log(t + 1) + 85$
$-85 > -15 \log(t + 1)$
$\frac{17}{3} < \log(t + 1)$
$10^{\frac{17}{3}} < t + 1$
$t > \approx 464158$
$464158 \div 12 \approx 38680$
The average score will drop to 0 after 38,680 years, and it is not a reasonable answer.

THINK AND DISCUSS

1. Possible answer: $\{x \mid x < 0\}$
2. Possible answer: The transformations of x and $f(x)$ are the same for the functions $f(x) + k$, $f(x - h)$, $af(x)$, $f\left(\frac{1}{b}x\right)$, $-f(x)$, and $f(-x)$.
3. Possible answer: $f(x) = a^x$: vertical translations and reflections across the x-axis change the range; $f(x) = \log_b x$: horizontal translations and reflections across the y-axis change the domain; no.
4.

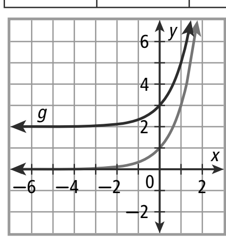
Transformation	$f(x) = 5^x$ $f(x) = e^x$	$f(x) = \log_b x$ $f(x) = \ln x$
Vertical translation	$5^x + 2$ $e^x + 2$	$\log_2 x + 2$ $\ln x + 2$
Horizontal translation	5^{x+2} e^{x+2}	$\log_2(x + 2)$ $\ln(x + 2)$
Reflection	-5^x $-e^x$	$-\log_2 x$ $-\ln x$
Vertical stretch	$2e^x$ $2(5^x)$	$2 \log_2 x$ $2 \ln x$
Vertical compression	$\frac{1}{2}(5^x)$ $\frac{1}{2}e^x$	$\frac{1}{2}\log_2 x$ $\frac{1}{2}\ln x$

EXERCISES

GUIDED PRACTICE

1.

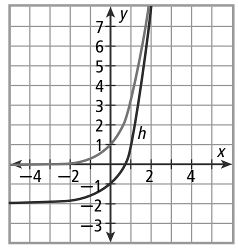
x	–2	–1	0	1	2
$g(x)$	2.1	2.3	3	5	11



$y = 2$; translation 2 units down; R: $\{y \mid y > 2\}$

2.

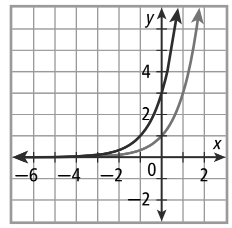
x	–2	–1	0	1	2
$h(x)$	–1.9	–1.7	–1	1	7



$y = -2$; translation 2 units down; R: $\{y \mid y > -2\}$

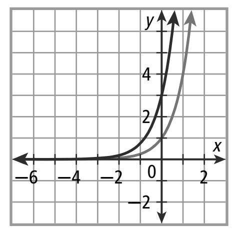
3.

x	–3	–2	–1	0	1
$j(x)$	0.11	0.33	1	3	9



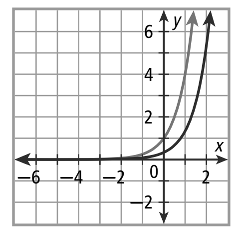
$y = 0$; translation 1 unit left

4.



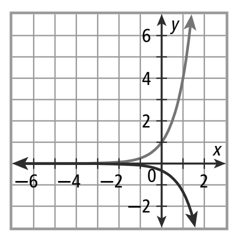
3; $y = 0$; vertical stretch by a factor of 3

5.



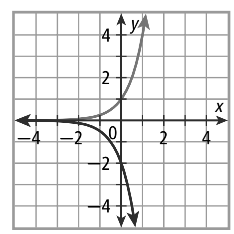
$\frac{1}{3}$; $y = 0$; vertical compression by a factor of $\frac{1}{3}$

6.



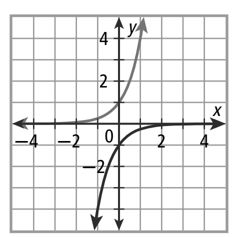
$-\frac{1}{3}$; $y = 0$; vertical compression by a factor of $\frac{1}{3}$ and reflection across the x-axis; R:$\{y \mid y < 0\}$

7.



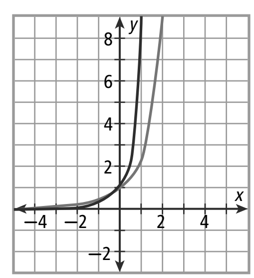
–2; $y = 0$; vertical stretch by a factor of 2 and reflection across the x-axis; R: $\{y \mid y < 0\}$

8.



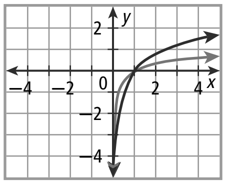
–1; $y = 0$; reflection across both axes; R: $\{y \mid y < 0]$

9.



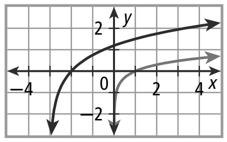
1; $y = 0$; horizontal compression by a factor of $\frac{1}{2}$

10.



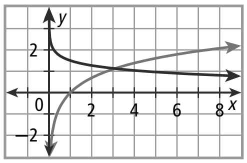
$x = 0$; vertical stretch by a factor of 2.5

11.



$x = 0$; translation 3 units left and vertical stretch by a factor of 2.5; D: $\{x \mid x > -3\}$

12.



$x = 0$; reflection across the y-axis, vertical compression by a factor of $\frac{1}{3}$, and translation 1.5 units up

13. $g(x) = -0.7^{\left(\frac{x}{3} + 2\right)}$

14. $g(x) = \frac{1}{2}\log(x - 12) + 25$

15. $17 < 6 + 3\ln(t + 1)$

$11 < 3\ln(t + 1)$

$\frac{11}{3} < \ln(t + 1)$

$e^{\frac{11}{3}} < t + 1$

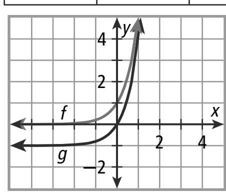
$t > e^{\frac{11}{3}} - 1 > \approx 38.1$

The model is translated 1 unit left, stretched by a factor of 3, and translated 6 units up; the height will exceed 17 feet after about 39 years.

PRACTICE AND PROBLEM SOLVING

16.

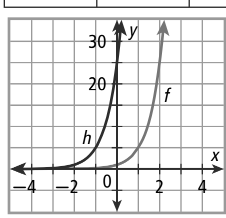
x	-2	-1	0	1	2
$g(x)$	-0.96	-0.8	0	4	24



$y = -1$; translation 1 unit down; R: $\{y \mid y > -1\}$

17.

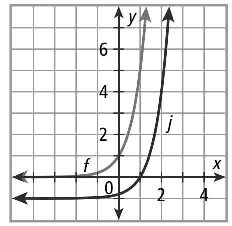
x	-2	-1	0	1	2
$h(x)$	1	5	25	125	625



$y = 0$; translation 2 units left

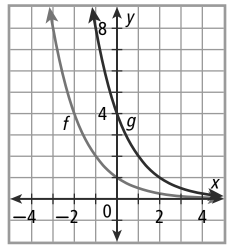
18.

x	$j(x)$
-2	-0.992
-1	-0.96
0	-0.8
1	23
2	124



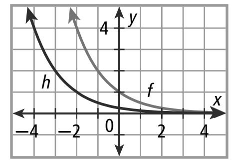
$y = -1$; translation 1 unit right and 1 unit down; R: $\{y \mid y > -1\}$

19.



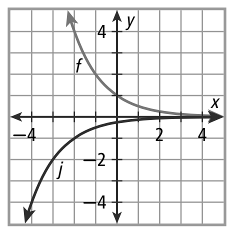
4; $y = 0$; vertical stretch by a factor of 4

20.



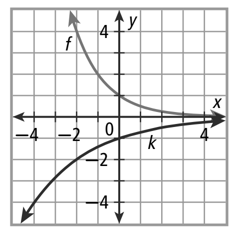
0.25; $y = 0$; vertical compression by a factor of 0.25

21.



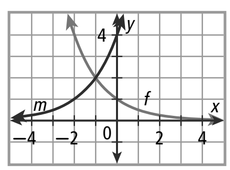
-0.25; $y = 0$; vertical compression by a factor of 0.25 and reflection across the x-axis; R: $\{y \mid y < 0\}$

22.



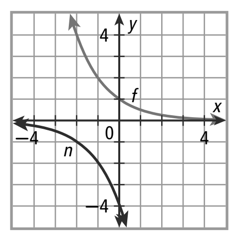
-1; $y = 0$; horizontal stretch by a factor of 2 and reflection across the x-axis; R: $\{y \mid y < 0\}$

23.



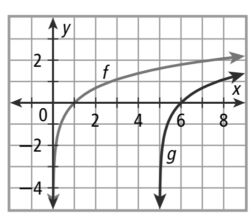
4; $y = 0$; vertical stretch by a factor of 4 and reflection across the y-axis

24.

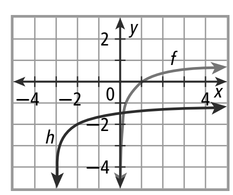


-4; $y = 0$; vertical stretch by a factor of 4 and reflection across both axes; R: $\{y \mid y < 0\}$

25.

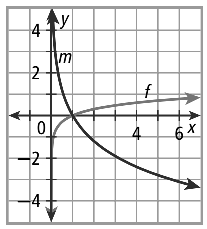


26.



$x = 5$; translation 5 units right; D: $\{x \mid x > 5\}$

$x = -3$; translation 3 units left; vertical compression by a factor of $\frac{4}{5}$, and translation 2 units down; D: $\{x \mid x > -3\}$

27. 

$x = 0$; vertical stretch by a factor of 4 and reflection across the x-axis

$x = 0$; vertical stretch by a factor of 4 and reflection in the x-axis.

28. $f(x) = -1.5\left(\frac{1}{2}\right)^{x-4}$

29. $f(x) = \ln(4x + 3) - 0.5$

30. $f(x) = e^{\left(1 - \frac{x}{3}\right)}$

31.
$$600 \le 870e^{-\frac{t}{127}}$$
$$\frac{600}{870} \le e^{-\frac{t}{127}}$$
$$\ln\frac{60}{87} \le -\frac{t}{127}$$
$$t \le -127\ln\frac{60}{87} \le \approx 47.2$$

The model is horizontally stretched by a factor of 127, reflected across the y-axis, and vertically stretched by a factor of 870. The instruments will function properly for about 47 years.

Possible answer:

Use $P(t) \approx 900(3)^{\frac{-t}{100}}$ to approximate the function. Round the predicted value of t to 50 yr.

$P(50) \approx 900(3)^{\frac{-50}{100}} \approx 300\sqrt{3} \approx 600$

The estimate confirms that after ≈ 47 yr, the power output will be about 600 W.

32. vertical stretch by e^2; $g(x) = e^2e^x$

33. A

34. E

35. D

36. C

37. F

38. The graph of $y = 2^{x-2} + 4$ is a translation 2 units right and 4 units up of the graph of $y = 2^x$. The asymptote is $y = 4$, and the graph approaches this line as the value of x decreases. The domain is still all real numbers, the range changes to values greater than 4, and the intercept is $2^{0-2} + 4 = 4.25$.
D: $\mathbb{R}$; R: $\{y \mid y > 4\}$;
x-intercept: none;
y-intercept: (o, 4.25)

39. The graph of $y = 5\log(x + 3)$ is a vertical stretch by a factor of 5 and a translation 3 units left of the graph of $y = \log x$. The vertical asymptote changes to $x = -3$. The domain changes to numbers greater than −3 and the range is still all real numbers, and the intercept is $5\log(0 + 3) \approx 2.39$.
D: $\{x \mid x > -3\}$; R: $\mathbb{R}$;
x-intercept: −2;
y-intercept: (o, 2.39)

40. never

41. always

42. never

43. sometimes

44a.
$$2(1000) = 1000\left(1 + \frac{r}{4}\right)^{4(5)}$$
$$2 = \left(1 + \frac{r}{4}\right)^{20}$$
$$\ln 2 = 20\ln\left(1 + \frac{r}{4}\right)$$
$$\frac{\ln 2}{20} = \ln\left(1 + \frac{r}{4}\right)$$
$$e^{\frac{\ln 2}{20}} = 1 + \frac{r}{4}$$
$$r = 4\left(e^{\frac{\ln 2}{20}} - 1\right) \approx 0.141$$

A rate of about 14.1% will double the investment in 5 years.

b.
$$2(1000) = 1000\left(1 + \frac{0.035}{4}\right)^{4t}$$
$$2 = 1.00875^{4t}$$
$$\ln 2 = 4t\ln 1.00875$$
$$t = \frac{\ln 2}{4\ln 1.00875} \approx 20$$

It will take about 20 years to double the investment at a rate of 3.5%.

c.
$$A(10) = 1000\left(1 + \frac{0.035}{4}\right)^{4(10)} \approx 1416.91$$

The total amount will be about $1416.91 after 10 years.

45. C

46. A

47. B

48a. vertical stretch by a factor of 2

b. horizontal translation 5 units right; D: $\{x \mid x > 5\}$

c. horizontal stretch by a factor of 2

d. Possible answer: horizontal compression since $0.95^t = 0.97^{kt}$, where $k =$ the constant $\frac{\log 0.95}{\log 0.97}$, which is > 1

49. Changing h translates the graph right (+) or left (−), and changing k translates the graph up (+) or down (−).

50. Possible answer: Translation left or right: Replace x with $x - h$. Translation up or down: Add k. Reflect across x-axis: Multiply b^x by −1. Reflect across y-axis: Replace x with $-x$. Vertical stretch or compression: Multiply b^x by $a \neq \pm 1$. Horizontal stretch or compression: Divide x by $c \neq \pm 1$.

51a. $N(t) = \frac{1}{3}(1257)(0.99)^t = 419(0.99)^t$

b. $N(m) = 419(0.99)^{\frac{m}{12}}$

c. $m = 12 + 5 = 17$

$N(17) = 419(0.99)^{\frac{17}{12}} \approx 413$

There were about 413 soybean farms at the end of May 1991.

TEST PREP

52. A

53. H

54. B

CHALLENGE AND EXTEND

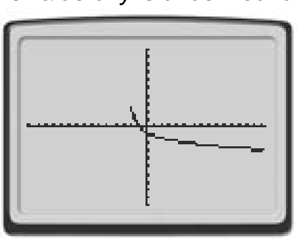
55a. translation 1 unit up

b. horizontal compression by $\frac{1}{10}$

c. They are equivalent.

d. $\log(10x) = \log 10 + \log x = 1 + \log x$

56. The value of y is undefined for $x \leq -2$. ed: s/b 2a? 3?



57. For $(x - h) > 1$, $f(x) > 0$. For $(x - h) = 1$, $f(x) = 0$. For $0 < (x - h) < 1$, $f(x) < 0$. For $(x - h) < 0$, $f(x) < 0$ is undefined.

SPIRAL REVIEW

58.
$$\begin{aligned} f(x) &= x^2 - 2x + 5 \\ &= \left(x^2 - 2x\right) + 5 \\ &= \left(x^2 - 2x\right) + 1 + 5 - 1 \\ &= (x - 1)^2 + 4 \end{aligned}$$
min: (1, 4);
D: $\mathbb{R}$;
R: $\{y \mid y > 4\}$;

59.
$$\begin{aligned} f(x) &= 4x^2 + x - 5 \\ &= 4\left(x^2 + \frac{1}{4}x\right) - 5 \\ &= 4\left(x^2 + \frac{1}{4}x + \frac{1}{64}\right) - 5 - 4\left(\frac{1}{64}\right) \\ &= 4\left(x + \frac{1}{8}\right)^2 - \frac{81}{16} \end{aligned}$$
min: $\left(-\frac{1}{8}, -\frac{81}{16}\right)$;
D: $\mathbb{R}$;
R: $\{y \mid y > -\frac{81}{16}\}$;

60.
$$\begin{aligned} f(x) &= -x^2 - x + 1 \\ &= -\left(x^2 + x\right) + 1 \\ &= -\left(x^2 + x + \frac{1}{4}\right) + 1 + \frac{1}{4} \\ &= -\left(x + \frac{1}{2}\right)^2 + \frac{5}{4} \end{aligned}$$
max: $\left(-\frac{1}{2}, \frac{5}{4}\right)$;
D: $\mathbb{R}$;
R: $\{y \mid y < \frac{5}{4}\}$;

61. $f(x) \approx 0.032x^3 - 0.0076x^2 + 0.073x + 1.30$

62. $\ln e^{x+2} = x + 2$

63. $\ln e^{-5x} = -5x$

64. $e^{\ln(x-1)} = x - 1$

65. $e^{\ln \frac{x}{4}} = \frac{x}{4}$

7-8 CURVE FITTING WITH EXPONENTIAL AND LOGARITHMIC MODELS, PAGES 545–551

CHECK IT OUT!

1a. yes; 1.5

2. no

2. $B(t) \approx 199(1.25)^t$; ed: s/b 2a?

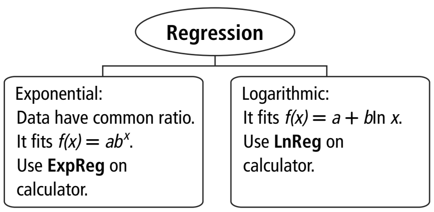
$$\begin{aligned} 2000 &\approx 199(1.25)^t \\ \frac{2000}{199} &\approx 1.25^t \\ \ln \frac{2000}{199} &\approx t \ln 1.25 \\ t &\approx \frac{\ln \frac{2000}{199}}{\ln 1.25} \approx 10.3 \end{aligned}$$

The number of bacteria will reach 2000 in about 10.3 min.

3. $S(t) \approx 0.59 + 2.64 \ln t$;
$8.0 \approx 0.59 + 2.64 \ln t$
$7.41 \approx 2.64 \ln t$
$\frac{7.41}{2.64} \approx \ln t$
$t \approx e^{\frac{7.41}{2.64}} \approx 16.6$
The speed will reach 8.0 m/s in about 16.6 min.

THINK AND DISCUSS

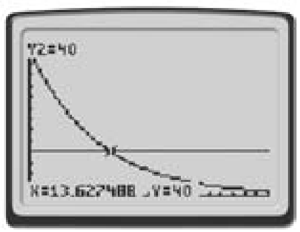
1. if there is a common ratio between the data values
2. Possible answer: There is only 1 ratio between data values, so there is no way to determine if that ratio is constant.
3.



EXERCISES

GUIDED PRACTICE

1. exponential regression
2. no 3. yes; $\frac{2}{3}$ 4. no 5. yes; $\frac{4}{3}$
6. $T(t) \approx 131.4(0.92)^t$; Enter the data and use an exponential regression. Graph the function and find where the value falls below 40.



It will take about 13.6 min for the tea to reach the temperature.

7. $P(t) \approx 621.6 + 1221 \ln t$;
$8000 \approx 621.6 + 1221 \ln t$
$7378.4 \approx 1221 \ln t$
$\frac{7378.4}{1221} \approx \ln t$
$t \approx e^{\frac{7378.4}{1221}} \approx 421$
It will take about 421 mo for the population to reach 8000.

PRACTICE AND PROBLEM SOLVING

8. no 9. no
10. yes; 1.5 11. yes; $\frac{1}{2}$
12. $P(t)^a \approx 9.35(1.045)^t$;
$120 < 14.6(1.045)^t$
$\frac{120}{14.6} < 1.045^t$
$\ln \frac{120}{14.6} < t \ln 1.045$
$t > \frac{\ln \frac{120}{14.6}}{\ln 1.045} > \approx 47.8$
$1970 + 47 = 2027$
The population will exceed 120 million in 2027.
13. $T(t) \approx 4.45(1.17)^t$;
$100 < 4.45(1.17)^t$
$\frac{100}{4.45} < 1.17^t$
$\ln \frac{100}{4.45} < t \ln 1.17$
$t > \frac{\ln \frac{100}{4.45}}{\ln 1.17} > \approx 19.8$
$1990 + 20 + 1 = 2011$
The number of telecommuters will exceed 100 million in 2011.
14. $t(p) \approx -60 + 22.4 \ln p$;
$t(500) \approx -60 + 22.4 \ln 500 \approx 79.2$
$1940 + 80 = 2020$
The population will reach 500 in year 2020.
15. yes; $f(x) = 1.55(7.54)^x$ 16. yes; $f(x) = 2(0.5)^x$
17. Possible answer: a third data point because 2 points can be fit by many different functions
18. $r(d) \approx 10.99(0.9995)^d$;
$r(4000) \approx 10.99(0.9995)^{4000} \approx 1.40$
The calf survival rate is 1.40 per 100 cows at snow depths of 4000 mm.
19. Use year 2002 as the starting year ($t = 0$).
$s(t) \approx 68.24(3.6878)^t$;
$t = 2 + 3 = 5$
$s(5) \approx 68.24(3.6878)^5 \approx 46{,}545{,}300$
The sales will be about 46,545,300 in three years.
20. Possible answer: an exponential decay model; the ratios are nearly constant.
21. Possible answer: nth differences have the same common ratio as first differences.

22a. $F(t) = 2011.6(0.984)^t$

b. $1 - 0.984 = 0.016 = 1.6\%$

c. $t = 2010 - 1970 = 40$
$F(40) = 2011.6(0.984)^{40} \approx 1055$
There will be about 1,055,000 acres of farmland in 2010.

23a. $t = 1$;
$100 - S = 100(0.795)^1$
$100 - S = 79.5$
$S = 20.5$
$t = 2$;
$100 - S = 100(0.795)^2$
$100 - S = 63.20$
$S = 36.8$
$t = 8$
$100 - S = 100(0.795)^8$
$100 - S = 16.0$
$S = 84.0$
The speed is 20.5 mi/h at 1 s, 36.8 mi/h at 2 s, and 84.0 mi/h at 8 s.

b. $s = 100\left(0.8^t\right)$; the value of a is accurate to the nearest whole number; the value of b is accurate to the nearest thousandth.

24. Possible answer: There will be a constant ratio between consecutive values rather than constant differences.

25a. exponential

b. Linear; the log of an exponential function of x is linear.

TEST PREP

26. C

27. F

28. $3.5 \times (3.5 \div 2) = 6.125$

CHALLENGE AND EXTEND

29. Solve the system $\begin{cases} 48 = ab^2 \\ 300 = ab^4 \end{cases}$ for a and b.

Substitute for a: $300 = \left(\frac{48}{b^2}\right)b^4$.
Solve for b: $6.25 = b^2 \rightarrow b = 2.5$ $(b > 0)$.
Solve for a: $a = 7.68$.
So $f(x) = 7.68(2.5)^x$.

30a. $f(t) \approx 0.014\left(0.936^t\right)$

b. The initial concentration was 0.014 mg/cm^3, and it was not above the health risk level.

c.
$$0.00010 > 0.014\left(0.936^t\right)$$
$$\frac{0.00010}{0.014} > 0.936^t$$
$$\ln\frac{0.00010}{0.014} > t\ln 0.936$$
$$t > \frac{\ln\frac{0.00010}{0.014}}{\ln 0.936} > \approx 74.7$$
This will be about 75 hours later.

31. Exponential: $\{y \mid y \le 0\}$ causes an error.
Logarithmic: $\{x \mid x \le 0\}$ or $\{y \mid y \le 0\}$ causes an error.

SPIRAL REVIEW

32. $|-5x| = 45$
$-5x = \pm 45$
$x = \pm 9$

33. $|x + 4| = 0$
$x + 4 = 0$
$x = -4$

34. $|2x - 4| = 3$
$2x - 4 = \pm 3$
$2x = 4 \pm 3$
$x = \frac{7}{2}$ or $\frac{1}{2}$

35. $2|2x| + 1 = 10$
$2|2x| = 9$
$4x = \pm 9$
$x = \pm\frac{9}{4}$

36. $x^2 + 2x - 3 = 0$
$(x - 1)(x + 3) = 0$
$x - 1 = 0$ or $x + 3 = 0$
$x = 1$ or $x = -3$

37. $3x^2 + 24x = 0$
$3x(x + 8) = 0$
$x = 0$ or $x + 8 = 0$
$x = 0$ or $x = -8$

38. $2x^2 + 10x + 12 = 0$
$2\left(x^2 + 5x + 6\right) = 0$
$2(x + 2)(x + 3) = 0$
$x + 2 = 0$ or $x + 3 = 0$
$x = -2$ or $x = -3$

39. $x^2 + 9x - 36 = 0$
$(x - 3)(x + 12) = 0$
$x - 3 = 0$ or $x + 12 = 0$
$x = 3$ or $x = -12$

40. $\frac{1}{64} = 4^{x+5}$
$2^{-6} = \left(2^2\right)^{x+5}$
$-6 = 2(x + 5)$
$-3 = x + 5$
$x = -8$

41. $81^x = 3^{x+4}$
$\left(3^4\right)^x = 3^{x+4}$
$4x = x + 4$
$3x = 4$
$x = \frac{4}{3}$

42. $8^{\frac{x}{3}} = \left(\frac{1}{2}\right)^{x+2}$
$\left(2^3\right)^{\frac{x}{3}} = \left(2^{-1}\right)^{x+2}$
$2^x = 2^{-(x+2)}$
$x = -x - 2$
$2x = -2$
$x = -1$

43. $216^x = 6^{2x}$
$\left(6^3\right)^x = 6^{2x}$
$3x = 2x$
$x = 0$

READY TO GO ON? PAGE 553

1. $3^x = \frac{1}{27}$
$3^x = 3^{-3}$
$x = -3$

2. $49^{x+4} < 7^{\frac{x}{2}}$
$\left(7^2\right)^{x+4} < 7^{\frac{x}{2}}$
$2(x + 4) < \frac{x}{2}$
$\frac{3}{2}x < -8$
$x < -\frac{16}{3}$

3. $13^{3x-1} = 91$
$(3x - 1)\ln 13 = \ln 91$
$3x - 1 = \frac{\ln 91}{\ln 13}$
$x = \frac{1 + \frac{\ln 91}{\ln 13}}{3}$
$x \approx 0.92$

4. $2^{x+4} = 20$
$(x + 4)\ln 2 = \ln 20$
$x + 4 = \frac{\ln 20}{\ln 2}$
$x = \frac{\ln 20}{\ln 2} - 4$
$x \approx 0.32$

5. $\log_4(x - 1) \ge 3$
$x - 1 \ge 4^3$
$x \ge 65$

6. $\log_2 x^{\frac{1}{3}} = 5$
$x^{\frac{1}{3}} = 2^5$
$\left(x^{\frac{1}{3}}\right)^3 = \left(2^5\right)^3$
$x = 2^{15} = 32{,}768$

7. $\log 16x - \log 4 = 2$

$\log \frac{16x}{4} = 2$

$\log 4x = 2$

$4x = 10^2$

$x = 25$

8. $\log x + \log (x + 3) = 1$

$\log x(x + 3) = 1$

$x(x + 3) = 10^1$

$x^2 + 3x - 10 = 0$

$(x - 2)(x + 5) = 0$

$x - 2 = 0$ or $x + 5 = 0$

$x = 2$ or $x = -5$

$x = 2$ since x must be positive.

9. $A = P(1 + r)^n$

$\frac{A}{P} = (1 + r)^n$

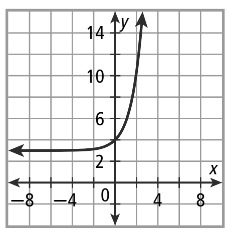
$\log \frac{A}{P} = n \log (1 + r)$

$$n = \frac{\log \frac{A}{P}}{\log (1 + r)}$$

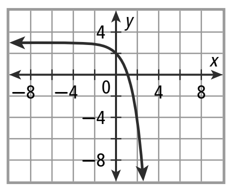
$$n \geq \frac{\log \left(\frac{2000}{500}\right)}{\log (1 + 0.035)} \geq \approx 40.3$$

It will take about 40.3 quarters, or 10.07 yrs, for the account to contain at least $2000.

10.



11.



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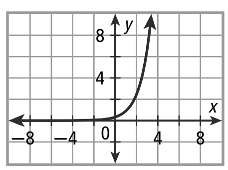
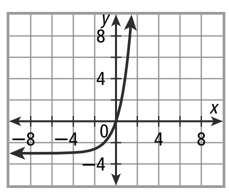


13.



14. $\ln e^2 = 2$

15. $\ln e^{\frac{x}{2}} = \frac{x}{2}$

16. $e^{\ln (1 - 3a)} = 1 - 3a$

17. $\ln e^{b + 5} = b + 5$

18a. $\frac{1}{2} = e^{-5730k}$

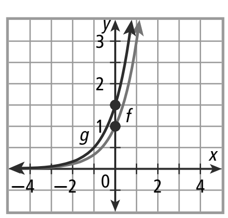
$\ln \frac{1}{2} = -5730k$

$$k = \frac{\ln \frac{1}{2}}{-5730} \approx 0.00012$$

b. $N_{1000} = 10e^{-0.00012(1000)} \approx 8.87$

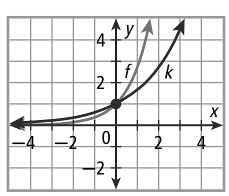
There will be about 8.87 g left after 1000 years.

19.



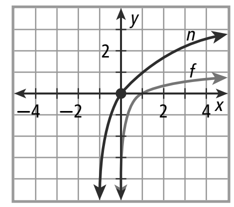
1.5; $y = 0$; The graph is stretched vertically by a factor of 1.5.

20.



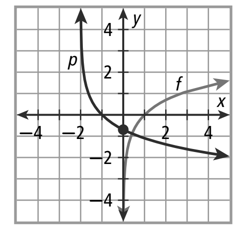
1; $y = 0$; The graph is stretched horizontally by a factor of 2.

21.



0; $x = -1$; The graph is shifted 1 unit left and stretched vertically by a factor of 3.5;
D: $\{x \mid x > -1\}$

22.



≈ -0.69; $x = -2$; The graph is shifted 2 units left and reflected across the x-axis;
D: $\{x \mid x > -2\}$

23. $f(x) = -0.5^{2x}$

24. yes; constant ratio: 2; $f(x) = (1.5)2^x$

25. linear function; $f(x) = 0.9x + 1.5$.

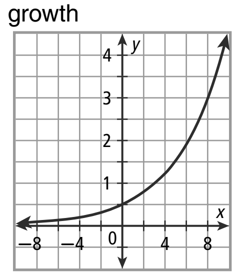
STUDY GUIDE: REVIEW, PAGES 554–557

1. natural logarithmic function **2.** asymptote

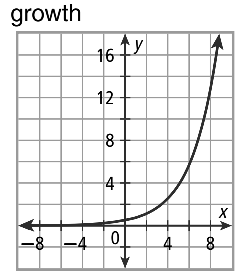
3. inverse relation

LESSON 7-1

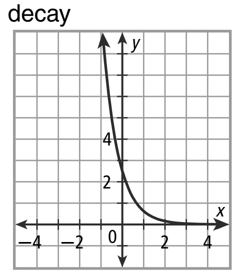
4. growth



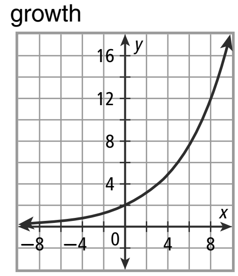
5. growth



6. decay



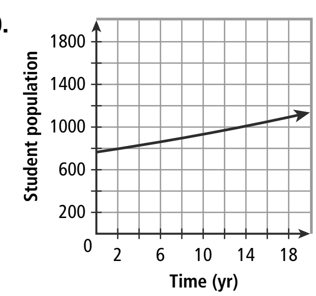
7. growth



8. growth

9. $P(t) = 765(1.02)^t$

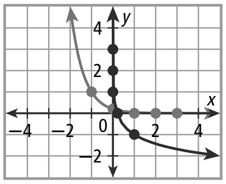
10.



11. ≈ 845 **12.** ≈ 13.5 yr

LESSON 7-2

13.



14. $P_T = P_L(1 - 0.03)$

15. $P_L = \dfrac{P_T}{0.97}$

16. $K = \dfrac{8}{5}M$;

$25 \text{ mi} = \dfrac{8}{5}(25) \text{ km} = 40 \text{ km}$

LESSON 7-3

17. $\log_3 243 = 5$

18. $\log_9 1 = 0$

19. $\log_{\frac{1}{3}} 27 = -3$

20. $2^4 = 16$

21. $10^1 = 10$

22. $0.6^2 = 0.36$

23. 2

24. 2

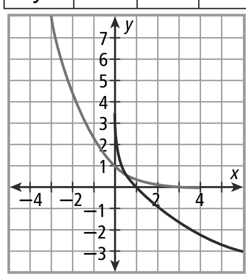
25. −1

26. −2

27. 0

28.

x	−2	−1	0	1	2
y	4	2	1	0.5	0.25



D: $\{x \mid x > 0\}$; R: $\mathbb{R}$

LESSON 7-4

29. $\log_2 8 + \log_2 16$
$\log_2 (8 \cdot 16)$
$\log_2 128$
7

30. $\log 100 + \log 10{,}000$
$\log (100 \cdot 10{,}000)$
$\log 1{,}000{,}000$
6

31. $\log_2 128 - \log_2 2$
$\log_2 \dfrac{128}{2}$
$\log_2 64$
6

32. $\log 10 - \log 0.1$
$\log \dfrac{10}{0.1}$
$\log 100$
2

33. $\log_5 25^2$
$\log_5 \left(5^2\right)^2$
$\log_5 5^4$
4

34. $\log 10^5 + \log 10^4$
$\log \left(10^5 \cdot 10^4\right)$
$9 \log 10$
9

35.

$$L = 10 \log \frac{I}{I_0}$$
$$\frac{L}{10} = \log \frac{I}{I_0}$$
$$10^{\frac{L}{10}} = \frac{I}{I_0}$$
$$I = 10^{\frac{L}{10}} I_0$$
$$L + 10 = 10 \log \frac{I}{I_0}$$
$$\frac{L + 10}{10} = \log \frac{I}{I_0}$$
$$10^{\frac{L+10}{10}} = \frac{I}{I_0}$$
$$I = 10^{\frac{L+10}{10}} I_0$$
$$10^{\frac{L+10}{10}} I_0 \div 10^{\frac{L}{10}} I_0 = 10^{\frac{L+10}{10} - \frac{L}{10}} = 10$$

The sound today was 10 times more intense than the sound yesterday.

LESSON 7-5

36. $3^{x-1} = \dfrac{1}{9}$
$3^{x-1} = 3^{-2}$
$x - 1 = -2$
$x = -1$

37. $\left(\dfrac{1}{2}\right)^x \le 64$
$\left(2^{-1}\right)^x \le 2^6$
$-x \le 6$
$x \ge -6$

38. $\log x^{\frac{5}{2}} > 2.5$
$2.5 \log x > 2.5$
$\log x > 1$
$x > 10$

39. $500 = 250(1 + 0.04)^n$
$2 = 1.04^n$
$\ln 2 = n \ln 1.04$
$n = \dfrac{\ln 2}{\ln 1.04} \approx 17.67$

It will take about 17.67 years to double the money.

LESSON 7-6

40a. $t = 2003 - 1940 = 63$
$194 = 22e^{63k}$
$\dfrac{194}{22} = e^{63k}$
$\ln \dfrac{194}{22} = 63k$
$k = \dfrac{\ln \frac{194}{22}}{63} \approx 0.0346$

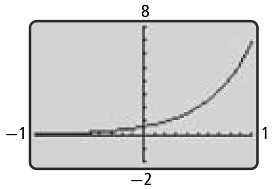
b. $t = 2020 - 1940 = 80$
$P(80) = 22e^{0.0346(80)} \approx 349$

The population will be about 349 in 2020.

LESSON 7-7

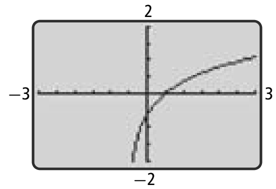
41. $g(x) = -3e^x - 2$

42.



y-intercept: 0.6; $y = 0$; vertically compressed by a factor of $\frac{3}{5}$ and horizontally compressed by a factor of $\frac{1}{6}$

43.



x-intercept: 0.5; $x = -0.5$; translated $\frac{1}{2}$ unit left and vertically stretched by a factor of 2

44. $V(t) = 5300(1 - 0.35)^t$

45. vertically stretched by a factor of 5300

LESSON 7-8

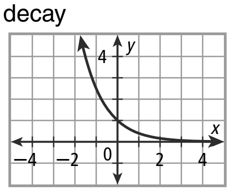
46. $f(x) \approx 11.26(1.05)^x$

47. $f(x) \approx -97.8 + 56.4 \ln x$

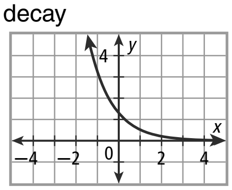
48. the exponential function; $r^2 \approx 0.94$ versus $r^2 \approx 0.60$ for the logarithmic function

CHAPTER TEST, PAGE 558

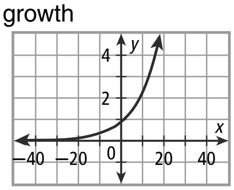
1. decay



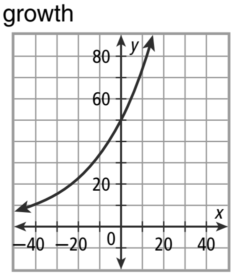
2. decay



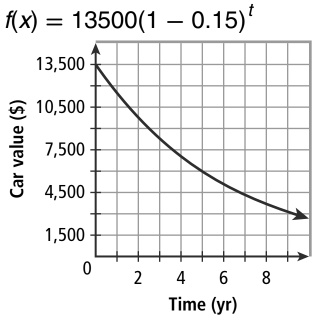
3. growth



4. growth

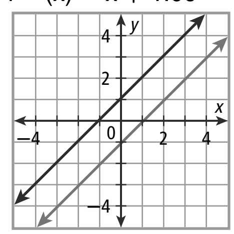


5. $f(x) = 13500(1 - 0.15)^t$

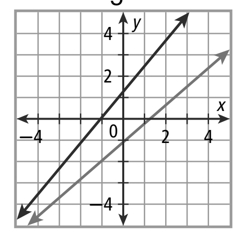


The value will fall below $3000 in the 10th year.

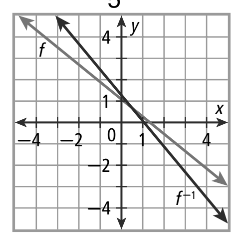
6. $f^{-1}(x) = x + 1.06$



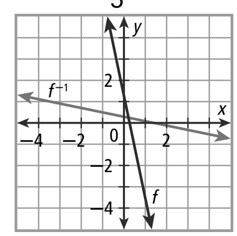
7. $f^{-1}(x) = \frac{6}{5}(x + 1.06)$



8. $f^{-1}(x) = \frac{6}{5}(1.06 - x)$



9. $f^{-1}(x) = \frac{6}{5}(1.06 - 4x)$



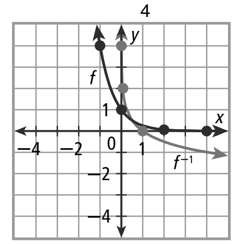
10. $\log_{16} 2 = \frac{1}{4}$

11. $\log_{16} \frac{1}{4} = -0.5$

12. $\left(\frac{1}{4}\right)^{-3} = 64$

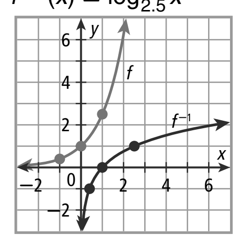
13. $81^{-\frac{1}{4}} = \frac{1}{3}$

14. $f^{-1}(x) = \log_{\frac{1}{4}} x$



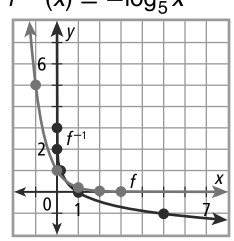
D: $\{x \mid x > 0\}$; R: $\mathbb{R}$

15. $f^{-1}(x) = \log_{2.5} x$



D: $\{x \mid x > 0\}$; R: $\mathbb{R}$

16. $f^{-1}(x) = -\log_5 x$



D: $\{x \mid x > 0\}$; R: $\mathbb{R}$

17. $\log_4 128 - \log_4 8$
$\log_4 \frac{128}{8}$
$\log_4 16$
2

18. $\log_2 12.8 + \log_2 5$
$\log_2 (12.8 \cdot 5)$
$\log_2 64$
6

19. $\log_3 243^2$
$\log_3 \left(3^5\right)^2$
$10 \log_3 3$
10

20. $5 \log_5 x = x$

21. $3^{x-1} = 729^{\frac{x}{2}}$
$3^{x-1} = \left(3^6\right)^{\frac{x}{2}}$
$x - 1 = 6\left(\frac{x}{2}\right)$
$x - 1 = 3x$
$x = -\frac{1}{2}$

22. $5^{1.5 - x} \le 25$
$5^{1.5 - x} \le 5^2$
$1.5 - x \le 2$
$x \ge -0.5$

23. $\log_4(x + 48) = 3$

$x + 48 = 4^3$

$x = 16$

24. $\log 6x^2 - \log 2x = 1$

$\log\left(\frac{6x^2}{2x}\right) = 1$

$\log 3x = 1$

$3x = 10^1$

$x = 3\frac{1}{3}$

25. $5 > 15(0.95)^x$

$\frac{1}{3} > 0.95^x$

$\ln\frac{1}{3} > x\ln 0.95$

$x > \frac{\ln\frac{1}{3}}{\ln 0.95} > \approx 21.4$

It will take about 21.4 min to reduce to less than 5 mL.

26. $\frac{1}{2} = e^{-24000k}$

$\ln\frac{1}{2} = -24000k$

$k = \frac{\ln\frac{1}{2}}{-24000} \approx 0.00002888$

$100e^{-0.00002888(5)} \approx 99.986$

There will be about 99.986 g left after 5 years.

27. $f(x) = 3\ln(x + 2) + 1$

28. $f(x) = 48.64 + 24.6\ln x$;

$100 < 48.64 + 24.6\ln x$

$51.36 < 24.6\ln x$

$\frac{51.36}{24.6} < \ln x$

$x > e^{\frac{51.36}{24.6}} > \approx 8.1$

The population will exceed 100 in year 8.