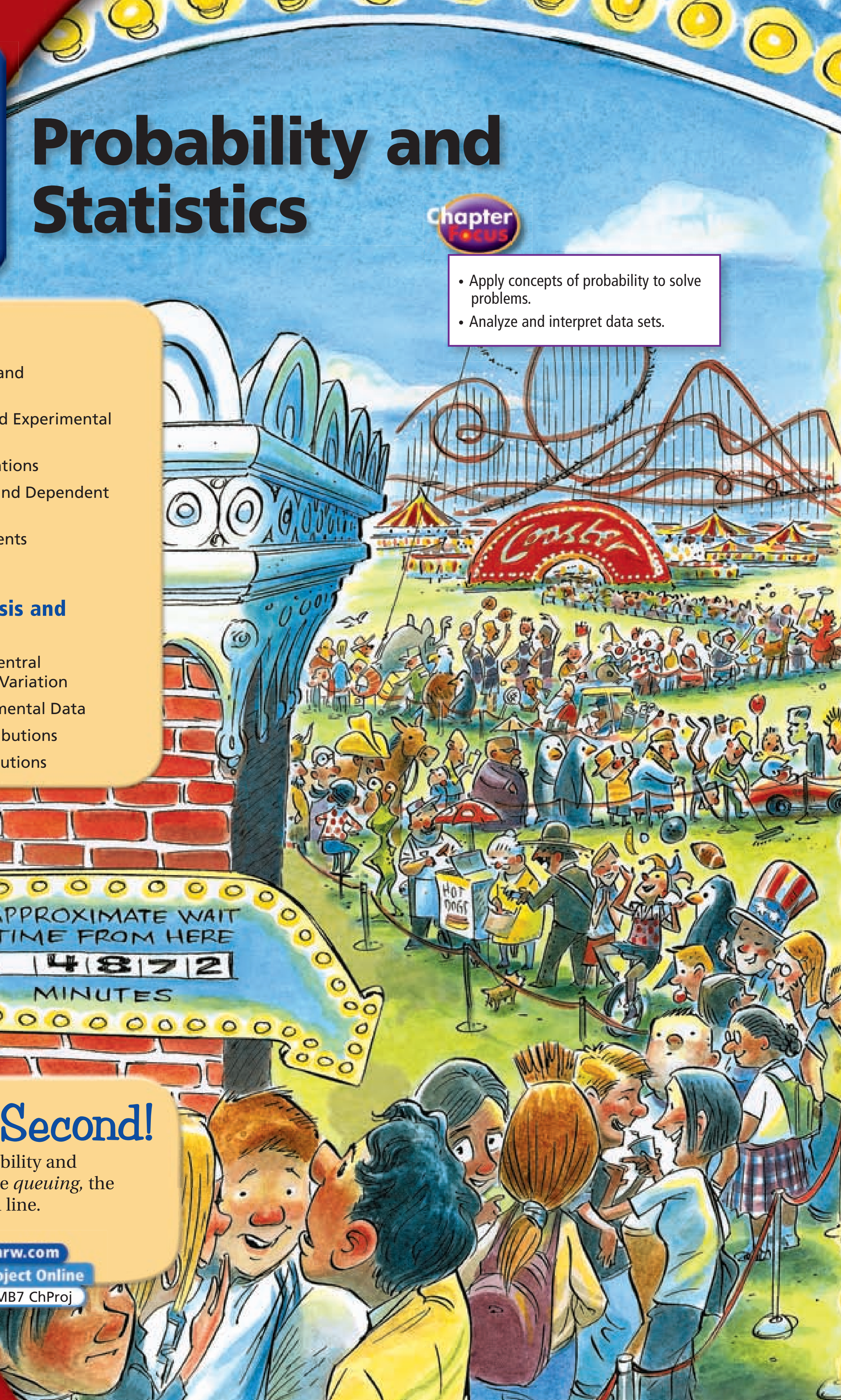
CHAPTER 11

Probability and Statistics

Chapter Focus

- Apply concepts of probability to solve problems.
- Analyze and interpret data sets.



Wait a Second!

You can use probability and statistics to analyze *queuing,* the study of waiting in line.

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ARE YOU READY?

Vocabulary

Match each term on the left with a definition on the right.

1. mean
2. median
3. ratio
4. mode

A. a comparison of two quantities by division
B. the sum of the values in a set divided by the number of values
C. the value, or values, that occur most often
D. the result of addition
E. the middle value, or mean of the two middle values, of a set when the set is ordered numerically

Tree Diagrams

5. Natalie has three colors of wrapping paper (purple, blue, and yellow) and three colors of ribbon (gold, white, and red). Make a tree diagram showing all possible ways that she can wrap a present using one color of paper and one color of ribbon.

Add and Subtract Fractions

Add or subtract.

6. $1 - \frac{14}{20}$
7. $\frac{3}{8} + \frac{5}{6}$
8. $\frac{8}{15} - \frac{2}{5}$
9. $\frac{1}{12} + \frac{1}{10}$

Multiply and Divide Fractions

Multiply or divide.

10. $\frac{1}{2} \cdot \frac{3}{7}$
11. $2\frac{1}{3} \cdot \frac{1}{4}$
12. $\frac{4}{5} \div \frac{1}{2}$
13. $5\frac{1}{3} \div \frac{1}{4}$

Percent Problems

Solve.

14. What number is 7% of 150?
15. 90% of what number is 45?
16. A $24 item receives a price increase of 12%. How much was the price increased?
17. Twenty percent of the water in a large aquarium should be changed weekly. How much water should be changed each week if an aquarium holds 65 gallons of water?

Find Measures of Central Tendency

Find the mean, median, and mode of each data set.

18. $\{9, 4, 2, 6, 4\}$
19. $\{1, 1, 1, 2, 2, 2\}$
20. $\{1, 2, 3, 4, 5, 6\}$
21. $\{18, 14, 20, 18, 14, 3, 18\}$

CHAPTER 11

Study Guide: Preview

Where You've Been

Previously, you

- made tree diagrams to find the number of possible combinations of a group of objects.
- made lists to count and arrange objects.
- calculated measures of central tendency.

In This Chapter

You will study

- solving problems involving counting and arranging.
- finding theoretical, experimental, and binomial probabilities.
- analyzing data to include expected value and standard deviation.

Where You're Going

You can use the skills in this chapter

- to find probabilities involved in games and events involving chance.
- to calculate and report appropriate measures when analyzing data.
- to form a solid foundation for studies in advanced statistics.

Key Vocabulary/Vocabulario

binomial experiment	experimento binomial
combination	combinación
conditional probability	probabilidad condicional
dependent events	sucesos dependientes
experimental probability	probabilidad experimental
factorial	factorial
independent events	sucesos independientes
outcome	resultado
permutation	permutación
theoretical probability	probabilidad teórica

Vocabulary Connections

To become familiar with some of the vocabulary terms in the chapter, consider the following. You may refer to the chapter, the glossary, or a dictionary if you like.

1. A number is the product of its *factors*. What operation do you think is involved in finding a **factorial** ?
2. A *theory* can be described as a sound and rational explanation. An *experiment* can be described as a procedure carried out in a controlled environment. Knowing this, how do you think **theoretical probability** differs from **experimental probability** ?
3. A *conditional* is used to describe something that will be done only if another thing is done. Do you think **conditional probability** is used with **independent events** or **dependent events** ? Why?
4. Each possible result of an experiment is an **outcome** . How many possible outcomes do you think a **binomial experiment** has? Why?

Reading and Writing Math

CHAPTER 11

Writing Strategy: Translate Between Words and Math

It is important to correctly interpret the type of math being described by a verbal or written description. Listen/look for key words to help you translate between the words and the math.

15. In 1626, the Dutch bought Manhattan Island for \$24 worth of merchandise. Suppose that, instead, \$24 had been invested in an account that paid 3.5% interest **compounded** annually. Find the balance in 2008.

compounded: *Compounding indicates an exponential function.*

31. Gardeners check the pH level of soil to ensure a **pH** of 6 or 7. Soil is usually more acidic in areas where rainfall is high, whereas soil in dry areas is usually more alkaline. The pH level of a certain soil sample is 5.5. What is the difference in **hydrogen ion concentration**, or $[H^+]$, between the sample and an acceptable level?

pH

hydrogen ion concentration: *These terms indicate a logarithmic function.*

parabola: *A parabola indicates a quadratic function.*

27. You are given a **parabola** with two points that have the same y-value, $(-7, 11)$ and $(3, 11)$. Explain how to find the equation for the axis of symmetry of this parabola.

Try This

Identify the key word and the type of function being described.

1. Kelly invested \$2000 in a savings account at a simple interest rate of 2.5%. How much money will she have in 8 months?

2. The diameter d in inches of a chain needed to move p pounds is given by the square root of $85p$, divided by pi. How much more can be lifted with a chain 2.5 inches in diameter than by a rope 0.5 inch in diameter?

3. A technician took a blood sample from a patient and detected a toxin concentration of 0.01006 mg/cm^3. Two hours later, the technician took another sample and detected a concentration of 0.00881 mg/cm^3. Assume that the concentration varies exponentially with time. Write a function to model the data.

4. Students found that the number of mosquitoes per acre of wetland grows by about 10 to the power $\frac{1}{2}d + 2$, where d is the number of days since the last frost. Write and graph the function representing the number of mosquitoes on each day.

11-1 Permutations and Combinations

Objectives
Solve problems involving the Fundamental Counting Principle.

Solve problems involving permutations and combinations.

Vocabulary
Fundamental Counting Principle
permutation
factorial
combination

Why learn this?
Permutations can be used to determine the number of ways to select and arrange artwork so as to give a new look each day. (See Example 2B.)

You have previously used tree diagrams to find the number of possible combinations of a group of objects. In this lesson, you will learn to use the **Fundamental Counting Principle**.

Know it! Note

Fundamental Counting Principle

If there are n items and m_1 ways to choose a first item, m_2 ways to choose a second item after the first item has been chosen, and so on, then there are $m_1 \cdot m_2 \cdot \ldots \cdot m_n$ ways to choose n items.

EXAMPLE 1 Using the Fundamental Counting Principle

A **For the lunch special, you can choose an entrée, a drink, and one side dish. How many meal choices are there?**

number of main dishes	times	number of beverages	times	number of sides	equals	number of choices
3	$\times$	4	$\times$	3	$=$	36

There are 36 meal choices.

B **In Utah, a license plate consists of 3 digits followed by 3 letters. The letters *I*, *O*, and *Q* are not used, and each digit or letter may be used more than once. How many different license plates are possible?**

digit		*digit*		*digit*		*letter*		*letter*		*letter*	
10	$\times$	10	$\times$	10	$\times$	23	$\times$	23	$\times$	23	$= 12{,}167{,}000$

There are 12,167,000 possible license plates.

Helpful Hint

In Example 1B, there are 10 possible digits and $26 - 3 = 23$ possible letters.

CHECK IT OUT!

1a. A "make-your-own-adventure" story lets you choose 6 starting points, gives 4 plot choices, and then has 5 possible endings. How many adventures are there?

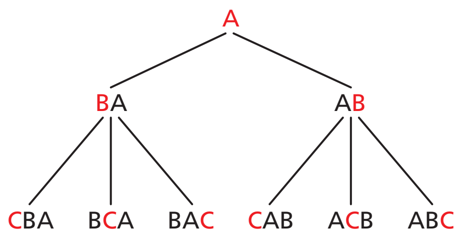
1b. A password is 4 letters followed by 1 digit. Uppercase letters (A) and lowercase letters (a) may be used and are considered different. How many passwords are possible?

A **permutation** is a selection of a group of objects in which order is important.

There is one way to arrange one item A.

A second item B can be placed first or second.

A third item C can be first, second, or third for each order above.



1 permutation

2 • 1 permutations

3 • 2 • 1 permutations

You can see that the number of permutations of 3 items is $3 \cdot 2 \cdot 1$. You can extend this to permutations of n items, which is $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$. This expression is called *n factorial*, and is written as $n!$.

Know it! Note

n Factorial

For any whole number n,

WORDS	NUMBERS	ALGEBRA
The **factorial** of a number is the product of the natural numbers less than or equal to the number. 0! is defined as 1.	$6! =$ $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$	$n! =$ $n \cdot (n-1) \cdot (n-2) \cdot (n-3) \cdot \ldots \cdot 1$

Sometimes you may not want to order an entire set of items. Suppose that you want to select and order 3 people from a group of 7. One way to find possible permutations is to use the Fundamental Counting Principle.

First Person		Second Person		Third Person		
7 choices	•	6 choices	•	5 choices	=	210 permutations

There are 7 people. You are choosing 3 of them in order.

Another way to find the possible permutations is to use factorials. You can divide the total number of arrangements by the number of arrangements that are not used. In the example above, there are 7 total people and 4 whose arrangements do not matter.

$$\frac{\text{arrangements of 7 people}}{\text{arrangements of 4 people}} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 210$$

This can be generalized as a formula, which is useful for large numbers of items.

Know it! Note

Permutations

NUMBERS	ALGEBRA
The number of permutations of 7 items taken 3 at a time is $_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!}$.	The number of permutations of n items taken r at a time is $_nP_r = \frac{n!}{(n-r)!}$.

EXAMPLE 2 Finding Permutations

A **How many ways can a club select a president, a vice president, and a secretary from a group of 5 people?**

This is the equivalent of selecting and arranging 3 items from 5.

$$_5P_3 = \frac{5!}{(5-3)!} = \frac{5!}{2!} \qquad \textit{Substitute 5 for n and 3 for r in } \frac{n!}{(n-r)!}.$$

$$= \frac{5 \cdot 4 \cdot 3 \cdot \cancel{2 \cdot 1}}{\cancel{2 \cdot 1}} \qquad \textit{Divide out common factors.}$$

$$= 5 \cdot 4 \cdot 3 = 60$$

There are 60 ways to select the 3 people.

B **An art gallery has 9 fine-art photographs from an artist and will display 4 from left to right along a wall. In how many ways can the gallery select and display the 4 photographs?**

$$_9P_4 = \frac{9!}{(9-4)!} = \frac{9!}{5!} = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} \qquad \textit{Divide out common factors.}$$

$$= 9 \cdot 8 \cdot 7 \cdot 6$$

$$= 3024$$

There are 3024 ways that the gallery can select and display the photographs.

Helpful Hint

The number of factors left after dividing is the number of items selected. In Example 2B, there are 4 photographs and 4 factors in 9 • 8 • 7 • 6.

CHECK IT OUT!

2a. Awards are given out at a costume party. How many ways can "most creative," "silliest," and "best" costume be awarded to 8 contestants if no one gets more than one award?

2b. How many ways can a 2-digit number be formed by using only the digits 5–9 and by each digit being used only once?

A **combination** is a grouping of items in which order does not matter. There are generally fewer ways to select items when order does not matter. For example, there are 6 ways to order 3 items, but they are all the same combination:

6 permutations → {ABC, ACB, BAC, BCA, CAB, CBA}

1 combination → {ABC}

To find the number of combinations, the formula for permutations can be modified.

$$\textbf{number of permutations} = \frac{\textbf{ways to arrange all items}}{\textbf{ways to arrange items not selected}}$$

Because order does not matter, divide the number of permutations by the number of ways to arrange the selected items.

$$\textbf{number of combinations} = \frac{\textbf{ways to arrange all items}}{(\textbf{ways to arrange selected items})(\textbf{ways to arrange items not selected})}$$

Combinations

NUMBERS	ALGEBRA
The number of combinations of 7 items taken 3 at a time is $_7C_3 = \dfrac{7!}{3!(7-3)!}$.	The number of combinations of n items taken r at a time is $_nC_r = \dfrac{n!}{r!(n-r)!}$.

When deciding whether to use permutations or combinations, first decide whether order is important. Use a permutation if order matters and a combination if order does not matter.

EXAMPLE 3 Pet Adoption Application

Katie is going to adopt kittens from a litter of 11. How many ways can she choose a group of 3 kittens?

Step 1 Determine whether the problem represents a permutation or combination.

The order does not matter. The group Kitty, Smoky, and Tigger is the same as Tigger, Kitty, and Smoky. It is a combination.

Step 2 Use the formula for combinations.

$$_{11}C_3 = \frac{11!}{3!(11-3)!} = \frac{11!}{3!(8!)} \qquad n = 11 \text{ and } r = 3$$

$$= \frac{11 \cdot 10 \cdot 9 \cdot \cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{3 \cdot 2 \cdot 1(\cancel{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1})} \qquad \text{Divide out common factors.}$$

$$= \frac{11 \cdot 10 \cdot 9}{3 \cdot 2 \cdot 1} = \frac{11 \cdot \cancel{10}^{5} \cdot \cancel{9}^{3}}{{}_{1}\cancel{3} \cdot {}_{1}\cancel{2} \cdot 1} = 165$$

There are 165 ways to select a group of 3 kittens from 11.

Helpful Hint

You can find permutations and combinations by using **nPr** and **nCr**, respectively, on scientific and graphing calculators.

CHECK IT OUT!

3. The swim team has 8 swimmers. Two swimmers will be selected to swim in the first heat. How many ways can the swimmers be selected?

THINK AND DISCUSS

1. Give a situation in which order matters and one in which order does not matter.

2. Give the value of $_nC_n$, where n is any integer. Explain your answer.

3. Tell what $_3C_4$ would mean in the real world and why it is not possible.

4. GET ORGANIZED Copy and complete the graphic organizer.

	Fundamental Counting Principle	Permutation	Combination
Formula			
Examples			

11-1 Exercises

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GUIDED PRACTICE

1. **Vocabulary** When you open a rotating combination lock, order is __?__ (*important* or *not important*), so this is a __?__ (*permutation* or *combination*).

SEE EXAMPLE 1 p. 794

2. Jamie purchased 3 blouses, 3 jackets, and 2 skirts. How many different outfits using a blouse, a jacket, and a skirt are possible?
3. An Internet code consists of one digit followed by one letter. The number zero and the letter *O* are excluded. How many codes are possible?

SEE EXAMPLE 2 p. 796

4. Nate is on a 7-day vacation. He plans to spend one day jet skiing and one day golfing. How many ways can Nate schedule the 2 activities?
5. How many ways can you listen to 3 songs from a CD that has 12 selections?
6. Members from 6 different school organizations decorated floats for the homecoming parade. How many different ways can first, second, and third prize be awarded?

SEE EXAMPLE 3 p. 797

7. A teacher wants to send 4 students to the library each day. There are 21 students in the class. How many ways can he choose 4 students to go to the library on the first day?
8. Gregory has a coupon for $1 off the purchase of 3 boxes of Munchie brand cereal. The store has 5 different varieties of Munchie brand cereal. How many ways can Gregory choose 3 boxes of cereal so that each box is a different variety?

PRACTICE AND PROBLEM SOLVING

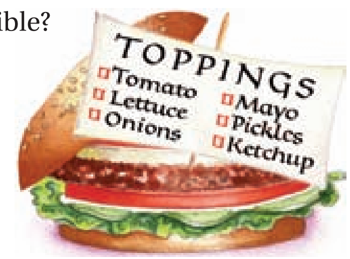
Independent Practice

For Exercises	See Example
9–10	1
11–13	2
14	3

Extra Practice

Skills Practice p. S24
Application Practice p. S42

9. **Hiking** A hiker can take 4 trails to the lake and then 3 trails from the lake to the cabins. How many routes are there from the lake to the cabins?
10. The cheerleading squad is making posters. They have 3 different colors of poster board and 4 different colors of markers. How many different posters can be made by using one poster board and one marker?
11. How many ways can you choose a manager and assistant from a 9-person task force?
12. How many identification codes are possible by using 3 letters if no letter may be repeated?
13. There are 5 airplanes ready to depart. Runway A and runway D are available. How many ways can 2 planes be assigned to runways without using the same runway?
14. **Food** How many choices of 3 hamburger toppings are possible?



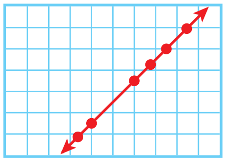
15. **What if...?** In the United Kingdom's National Lottery, you must correctly select a group of 6 numbers from 49. Suppose that the contest were changed to selecting 7 numbers. How many more ways would there be to select the numbers?

Evaluate.

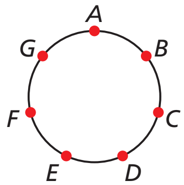
16. ${}_6P_6$
17. ${}_5C_5$
18. ${}_9P_1$
19. ${}_6C_1$
20. $\dfrac{2!}{6!}$
21. $\dfrac{4!3!}{2!}$
22. $\dfrac{9!}{7!}$
23. $\dfrac{8! - 5!}{(8-5)!}$

Geometry **Find the number of ways that each selection can be made.**

24. two marked points to determine slope



25. four points to form a quadrilateral



Compare. Write >, <, or =.

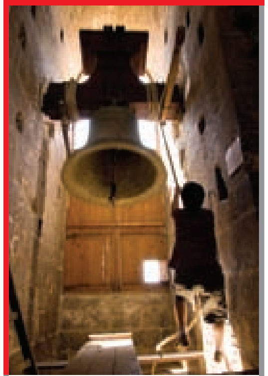
26. $_7P_3$ ■ $_7C_4$ **27.** $_7P_4$ ■ $_7P_3$ **28.** $_7C_3$ ■ $_7C_4$ **29.** $_{10}C_{10}$ ■ $_{10}P_{10}$

30. Copy and complete the table. Use the table to explain why 0! is defined as 1.

$n!$	4!	3!	2!	1!
$n(n-1)!$	$4(3!) = 24$			

LINK

Music



There are many change-ringing societies and groups, especially in the United Kingdom. Bell ringers work together to follow patterns and called changes to avoid repeating sequences.

31. Critical Thinking Why are there more unique permutations of the letters in YOUNG than in GEESE?

32. Music In change ringing, a *peal* is the ringing of all possible sequences of a number of bells. Suppose that 8 bells are used and it takes 0.25 second to ring each bell. How long would it take to ring a complete peal?

33. Multi-Step Amy, Bob, Charles, Dena, and Esther are club officers.

a. Copy and complete the table to show the ways that a president, a vice president, and a secretary can be chosen if Amy is chosen president. (Use first initials for names.)

President	A	A	A	A	A	A	A	A	A	A	A	A
Vice President	B	B	B	C	C	C						
Secretary	C	D	E									

b. Extend the table to show the number of ways that the three officers can be chosen if Bob is chosen president. Make a conjecture as to the number of ways that a president, a vice president, and a secretary can be chosen.

c. Use a formula to find the number of different ways that a president, a vice president, and a secretary can be chosen. Compare your result with part **b.**

d. How many different ways can 3 club officers be chosen to form a committee? Compare this with the answer to part **c.** Which answer is a number of permutations? Which answer is a number of combinations?

34. Critical Thinking Use the formulas to divide $_nP_r$ by $_nC_r$. Predict the result of dividing $_6P_3$ by $_6C_3$. Check your prediction. What meaning does the result have?

35. Write About It Find $_9C_2$ and $_9C_7$. Find $_{10}C_6$ and $_{10}C_4$. Explain the results.

MULTI-STEP TEST PREP

36. This problem will prepare you for the Multi-Step Test Prep on page 826.

While playing the game of Yahtzee, Jen rolls 5 dice and gets the result shown at right.

a. How many different ways can she arrange the dice from left to right?

b. How many different ways can she choose 3 of the dice to reroll?

37. **ERROR ANALYSIS** Below are two solutions for “How many Internet codes can be made by using 3 digits if 0 is excluded and digits may not be repeated?” Which is incorrect? Explain the error.

A

$$\frac{9!}{3!(9-3)!}$$
$$= \frac{9!}{3!6!}$$
$$= 84 \text{ codes}$$

B

$$\frac{9!}{(9-3)!}$$
$$= \frac{9!}{6!}$$
$$= 504 \text{ codes}$$

38. **Critical Thinking** Explain how to use the Fundamental Counting Principle to answer the question in Exercise 37.

TEST PREP

39. There are 14 players on the team. Which of the following expressions models the number of ways that the coach can choose 5 players to start the game?

Ⓐ 5!　　Ⓑ $\frac{14!}{5!}$　　Ⓒ $\frac{14!}{9!}$　　Ⓓ $\frac{14!}{5!9!}$

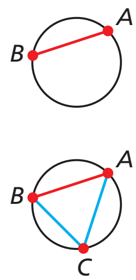
40. Which of the following has the same value as ${}_9C_4$?

Ⓕ ${}_9P_4$　　Ⓖ ${}_4C_9$　　Ⓗ ${}_9P_5$　　Ⓙ ${}_9C_5$

41. **Short Response** Rene can choose 1 elective each of the 4 years that she is in high school. There are 15 electives. How many ways can Rene choose her electives?

CHALLENGE AND EXTEND

42. **Geometry** Consider a circle with two points, A and B. You can form exactly 1 segment, $\overline{AB}$. If there are 3 points, you can form 3 segments as shown in the diagram.



a. How many segments can be formed from 4 points, 5 points, 6 points, and n points? Write your answer for n points as a permutation or combination.

b. How many segments can be formed from 20 points?

43. **Government** How many ways can a jury of 12 and 2 alternate jurors be selected from a pool of 30 potential jurors? (*Hint:* Consider how order is both important and unimportant in selection.) Leave your answer in unexpanded notation.

SPIRAL REVIEW

44. **Money** The cost to rent a boat increased from \$0.15 per mile to \$0.45 per mile. Write a function $p(x)$ for the initial cost and a function $P(x)$ for the cost after the price increase. Graph both functions on the same coordinate plane. Describe the transformation. *(Lesson 1-8)*

Solve each proportion. *(Lesson 2-2)*

45. $\frac{17}{n} = \frac{11}{77}$　　46. $\frac{2.9}{3.7} = \frac{x}{23.31}$　　47. $\frac{2.2}{n} = \frac{1.6}{9.5}$　　48. $\frac{x}{36} = \frac{98}{18}$

Identify the conic section that each equation represents. *(Lesson 10-6)*

49. $6x^2 + 3xy - 9y^2 + 5x - 2y - 16 = 0$　　50. $8x^2 + 8y^2 - 6x + 7y - 9 = 0$

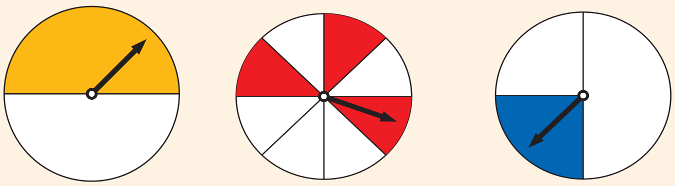
Connecting Algebra to **Geometry**

Relative Area

In *geometric probability*, the probability of an event corresponds to ratios of the areas (or lengths or volumes) or parts of one or more figures.

In the spinners shown, the probability of landing on a color is based on relative area.

$\frac{1}{2}$ shaded $\qquad$ $\frac{3}{8}$ shaded $\qquad$ $\frac{1}{4}$ shaded



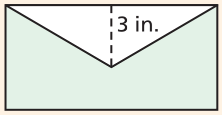
Use the area formulas at right to help you determine relative area.

Area Formulas

Figure	Formula
Rectangle	$A = bh$
Square	$A = s^2$
Triangle	$A = \frac{1}{2}bh$
Trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$
Circle	$A = \pi r^2$

Example

What portion of the rectangle is shaded? Write the relative area as a fraction, a decimal, and a percent.



Find the ratio of the area of the shaded region to the area of the rectangle.

$A = 10(5) = 50 \text{ in}^2$ *Area of the rectangle: $A = bh$*

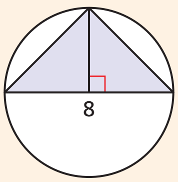
$A = \frac{1}{2}(3)(10) = 15 \text{ in}^2$ *Area of the unshaded triangle: $A = \frac{1}{2}bh$*

$$\frac{\text{area of shaded region}}{\text{area of the rectangle}} = \frac{50 - 15}{50} = \frac{35}{50} = \frac{7}{10} = 0.7, \text{ or } 70\%$$

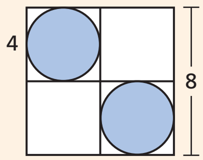
Try This

What portion of each figure is shaded? Write the relative area as a fraction, a decimal, and a percent.

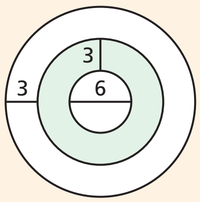
1.



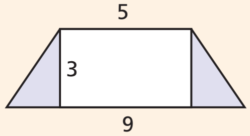
2.



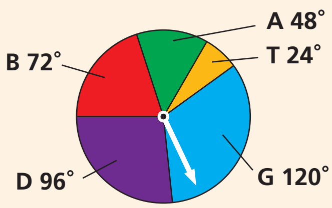
3.



4.



5. Write the relative area of each sector of the spinner as a fraction, decimal, and percent.



11-2 Theoretical and Experimental Probability

Objectives

Find the theoretical probability of an event.

Find the experimental probability of an event.

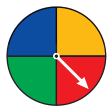
Vocabulary

probability
outcome
sample space
event
equally likely outcomes
favorable outcomes
theoretical probability
complement
geometric probability
experiment
trial
experimental probability

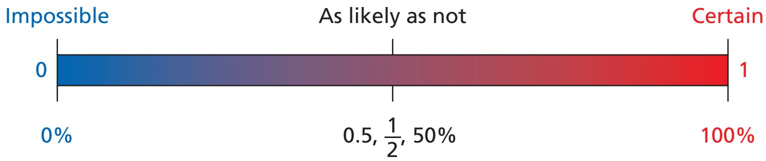
Why learn this?

You can use probability to find the chances of hitting or missing a target in the game Battleship. (See Example 2.)

Probability is the measure of how likely an event is to occur. Each possible result of a probability experiment or situation is an **outcome**. The **sample space** is the set of all possible outcomes. An **event** is an outcome or set of outcomes.

Experiment or Situation	Rolling a number cube 	Spinning a spinner 
Sample Space	{1, 2, 3, 4, 5, 6}	{red, blue, green, yellow}

Probabilities are written as fractions or decimals from 0 to 1, or as percents from 0% to 100%.



Equally likely outcomes have the same chance of occurring. When you toss a fair coin, heads and tails are equally likely outcomes. **Favorable outcomes** are outcomes in a specified event. For equally likely outcomes, the **theoretical probability** of an event is the ratio of the number of favorable outcomes to the total number of outcomes.

Know it! Note

Theoretical Probability

For equally likely outcomes,

$$P(\text{event}) = \frac{\text{number of favorable outcomes}}{\text{number of outcomes in the sample space}}.$$

EXAMPLE 1 **Finding Theoretical Probability**

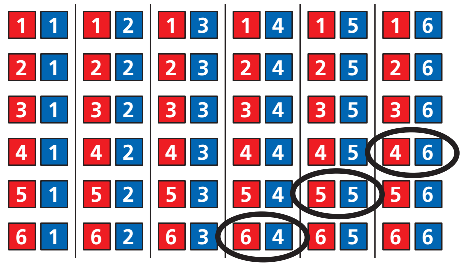
A **A CD has 5 upbeat dance songs and 7 slow ballads. What is the probability that a randomly selected song is an upbeat dance song?**

There are 12 possible outcomes and 5 favorable outcomes.

$$P(\text{upbeat dance song}) = \frac{5}{12} \approx 41.7\%$$

B **A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability that the sum is 10?**

There are 36 possible outcomes.



$$P(\text{sum is } 10) = \frac{\text{number of outcomes with sum of 10}}{36}$$

$$P(\text{sum is } 10) = \frac{3}{36} = \frac{1}{12}$$ *3 outcomes with a sum of 10: (4, 6) (5, 5), and (6, 4)*

CHECK IT OUT! **A red number cube and a blue number cube are rolled. If all numbers are equally likely, what is the probability of each event?**

1a. The sum is 6.

1b. The difference is 6.

1c. The red cube is greater.

The sum of all probabilities in the sample space is 1. The **complement** of an event E is the set of all outcomes in the sample space that are not in E.

Know it! Note

Complement

The probability of the complement of event E is

$$P(\text{not } E) = 1 - P(E).$$

EXAMPLE 2 *Entertainment Application*

The game Battleship is played with 5 ships on a 100-hole grid. Players try to guess the locations of their opponent's ships and sink them. At the start of the game, what is the probability that the first shot misses all targets?

$P(\text{miss}) = 1 - P(\text{hit})$ *Use the complement.*

$P(\text{miss}) = 1 - \frac{17}{100}$ *There are 17 total holes covered by game pieces.*

$= \frac{83}{100}$, or 83%

There is an 83% chance of the first shot missing all targets.

Battleship Pieces

Game Piece	Number of Holes Covered
Destroyer	2
Cruiser	3
Submarine	3
Battleship	4
Carrier	5

CHECK IT OUT! **2.** Two integers from 1 to 10 are randomly selected. The same number may be chosen twice. What is the probability that both numbers are less than 9?

EXAMPLE 3 Finding Probability with Permutations or Combinations

Each student received a 4-digit code to use the library computers, with no digit repeated. Manu received the code 7654. What was the probability that he would receive a code of consecutive numbers?

Step 1 Determine whether the code is a permutation or a combination.

Order is important, so it is a permutation.

Step 2 Find the number of outcomes in the sample space.

The sample space is the number of permutations of 4 of 10 digits.

$$_{10}P_4 = \frac{10!}{6!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot \cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} = 5040$$

Step 3 Find the favorable outcomes.

The favorable outcomes are the codes 0123, 1234, 2345, 3456, 4567, 5678, 6789, and the reverse of each of these numbers. There are 14 favorable outcomes.

Step 4 Find the probability.

$$P(\text{consecutive numbers}) = \frac{14}{5040} = \frac{1}{360}$$

The probability that Manu would receive a code of consecutive numbers was $\frac{1}{360}$.

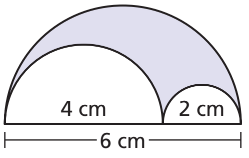
CHECK IT OUT!

3. A DJ randomly selects 2 of 8 ads to play before her show. Two of the ads are by a local retailer. What is the probability that she will play both of the retailer's ads before her show?

Geometric probability is a form of theoretical probability determined by a ratio of lengths, areas, or volumes.

EXAMPLE 4 Finding Geometric Probability

Three semicircles with diameters 2, 4, and 6 cm are arranged as shown in the figure. If a point inside the figure is chosen at random, what is the probability that the point is inside the shaded region?



Find the ratio of the area of the shaded region to the area of the entire semicircle. The area of a semicircle is $\frac{1}{2}\pi r^2$.

First, find the area of the entire semicircle.

$$A_t = \frac{1}{2}\pi(3^2) = 4.5\pi$$ *Total area of largest semicircle*

Next, find the unshaded area.

$$A_u = \left[\frac{1}{2}\pi(2^2)\right] + \left[\frac{1}{2}\pi(1^2)\right] = 2\pi + 0.5\pi = 2.5\pi$$ *Sum of areas of the unshaded semicircles*

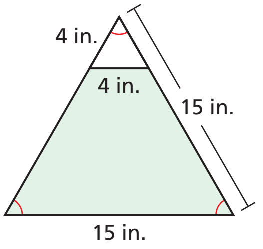
Subtract to find the shaded area.

$$A_s = 4.5\pi - 2.5\pi = 2\pi$$ *Area of shaded region*

$$\frac{A_s}{A_t} = \frac{2\pi}{4.5\pi} = \frac{2}{4.5} = \frac{4}{9}$$ *Ratio of shaded region to total area*

The probability that the point is in the shaded region is $\frac{4}{9}$.

4. Find the probability that a point chosen at random inside the large triangle is in the small triangle.



You can estimate the probability of an event by using data, or by **experiment**. For example, if a doctor states that an operation "has an 80% probability of success," 80% is an estimate of probability based on similar case histories.

Each repetition of an experiment is a **trial**. The sample space of an experiment is the set of all possible outcomes. The **experimental probability** of an event is the ratio of the number of times that the event occurs, the *frequency*, to the number of trials.

Know it! Note

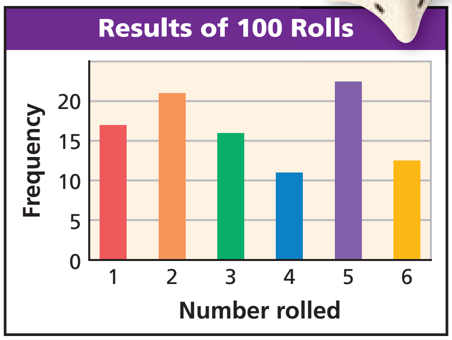
Experimental Probability

$$\text{experimental probability} = \frac{\text{number of times the event occurs}}{\text{number of trials}}$$

Experimental probability is often used to estimate theoretical probability and to make predictions.

EXAMPLE 5 Finding Experimental Probability

The bar graph shows the results of 100 tosses of an oddly shaped number cube. Find each experimental probability.



A **rolling a 3**

The outcome 3 occurred 16 times out of 100 trials.

$$P(3) = \frac{16}{100} = \frac{4}{25} = 0.16$$

B **rolling a perfect square**

$$P\left(\text{perfect square}\right) = \frac{17 + 11}{100}$$

$$= \frac{28}{100} = \frac{7}{25} = 0.28$$

The numbers 1 and 4 are perfect squares. 1 occurred 17 times and 4 occurred 11 times.

C **rolling a number other than 5**

Use the complement.

$$P(5) = \frac{22}{100}$$

5 occurred 22 times out of 100 trials.

$$1 - P(5) = 1 - \frac{22}{100} = \frac{78}{100} = \frac{39}{50} = 0.78$$

Helpful Hint

Frequencies must be whole numbers, so they can be easily read from the graph in Example 5.

5. The table shows the results of choosing one card from a deck of cards, recording the suit, and then replacing the card.

Card Suit	Hearts	Diamonds	Clubs	Spades
Number	5	9	7	5

5a. Find the experimental probability of choosing a diamond.

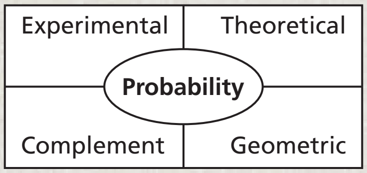
5b. Find the experimental probability of choosing a card that is not a club.

THINK AND DISCUSS

1. Explain whether the probability of an event can be 1.5.
2. Tell which events have the same probability when two number cubes are tossed: sum of 7, sum of 5, sum of 9, and sum of 11.
3. Compare the theoretical and experimental probabilities of getting heads when tossing a coin if Joe got heads 8 times in 20 tosses of the coin.

Know it! Note

4. **GET ORGANIZED** Copy and complete the graphic organizer. Give an example of each probability concept.



11-2 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 11-1
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A fair coin is tossed 8 times and lands heads up 3 times. The __?__ of landing heads is $\frac{1}{2}$.(*theoretical probability* or *experimental probability*)

SEE EXAMPLE 1 p. 802

A quarter, a nickel, and a penny are flipped. Find the probability of each of the following.

2. The quarter shows heads.
3. The penny and nickel show heads.
4. One coin shows heads.
5. All three coins land the same way.

SEE EXAMPLE 2 p. 803

6. What is the probability that a random 2-digit number (00-99) does not end in 5?
7. What is the probability that a randomly selected date in one year is not in the month of December or January?

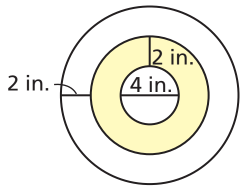
SEE EXAMPLE 3 p. 804

8. A clerk has 4 different letters that need to go in 4 different envelopes. What is the probability that all 4 letters are placed in the correct envelopes?
9. There are 12 balloons in a bag: 3 each of blue, green, red, and yellow. Three balloons are chosen at random. Find the probability that all 3 of the balloons are green.

SEE EXAMPLE 4 p. 804

Use the diagram for Exercises 10 and 11. Find each probability.

10. that a point chosen at random is in the shaded area
11. that a point chosen at random is in the smallest circle



SEE EXAMPLE 5 p. 805

Use the table for Exercises 12 and 13.

12. Find the experimental probability of spinning red.
13. Find the experimental probability of spinning red or blue.

Spinner Experiment

Color	Red	Green	Blue
Spins	5	8	7

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
14–15	1
16	2
17–18	3
19	4
20	5

Extra Practice

Skills Practice p. S24
Application Practice p. S42

There are 3 green marbles, 7 red marbles, and 5 white marbles in a bag. Find the probability of each of the following.

14. The chosen marble is white.

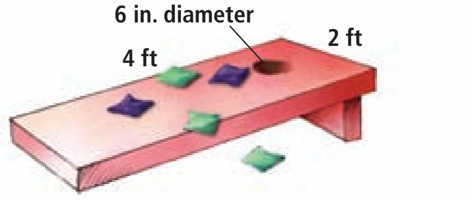
15. The chosen marble is red or white.

16. Two integers from 1 to 8 are randomly selected. The same number can be chosen both times. What is the probability that both numbers are greater than 2?

17. **Swimming** The coach randomly selects 3 swimmers from a team of 8 to swim in a heat. What is the probability that she will choose the three strongest swimmers?

18. **Books** There are 7 books numbered 1–7 on the summer reading list. Peter randomly chooses 2 books. What is the probability that Peter chooses books numbered 1 and 2?

19. **Games** In the game of corntoss, players throw corn-filled bags at a hole in a wooden platform. If a bag that hits the platform can hit any location with an equal likelihood, find the probability that a tossed bag lands in the hole.

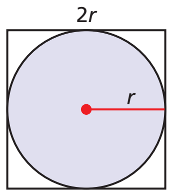


20. **Cards** An experiment consists of choosing one card from a standard deck and then replacing it. The experiment was done several times, and the results are: 8 hearts, 8 diamonds, 6 spades, and 6 clubs. Find the experimental probability that a card is red.

21. **Critical Thinking** Explain whether the experimental probability of tossing tails when a fair coin is tossed 25 times is always, sometimes, or never equal to the theoretical probability.

22. **Games** A radio station in Mississippi is giving away a trip to the Mississippi coast from any other state in the United States. Assuming an equally likely chance for a winner from any other state, what is the probability that the winner will be from a state that does not border Mississippi?

23. **Geometry** **Use the figure.**



a. A circle with radius r is inscribed in a square with side length $2r$. What is the ratio of the area of the circle to the area of the square?

b. A square board has an inscribed circle with a 15 in. radius. A small button is dropped 10,000 times on the board, landing inside the circle 7852 times. How can you use this experiment to estimate a value for π?

24. **Games** The sides of a backgammon die are marked with the numbers 2, 4, 8, 16, 32, and 64. Describe an outcome that has a probability of $\frac{2}{3}$.

25. **Computer** A player in a computer basketball program has a constant probability of making each free throw. Jack notes the success rate over a period of time.

a. Find the experimental probability for each set of 25 attempts as a decimal.

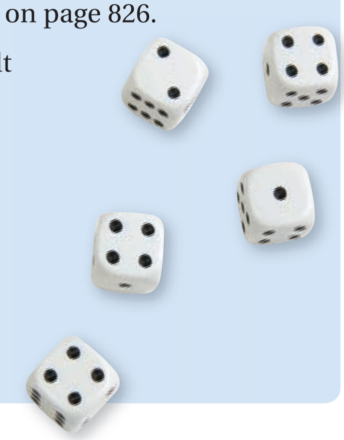
b. Find the experimental probability for the entire experiment.

c. What is the best estimate of the theoretical probability? Justify your answer.

Free Throw Shooting

Attempts	Free Throws Made
1–25	17
26–50	21
51–75	19
76–100	16

MULTI-STEP TEST PREP



26. This problem will prepare you for the Multi-Step Test Prep on page 826.

While playing Yahtzee and rolling 5 dice, Mei gets the result shown at right. Mei decides to keep the three 4's and reroll the other 2 dice.

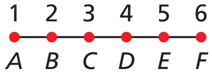
a. What is the probability that Mei will have 5 of a kind?

b. What is the probability that she will have 4 of a kind (four 4's plus something else)?

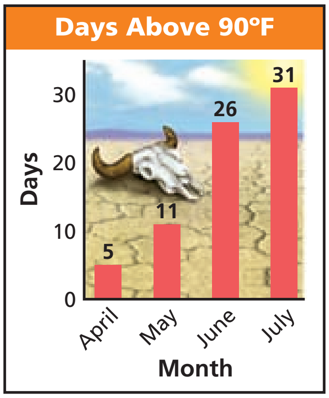
c. What is the probability that she will have exactly three 4's?

d. How are the answers to parts **a, b,** and **c** related?

27. **Geometry** The points along $\overline{AF}$ are evenly spaced. A point is randomly chosen. Find the probability that the point lies on $\overline{BD}$.



Weather **Use the graph and the following information for Exercises 28–30.** The table shows the number of days that the maximum temperature was above 90°F in Death Valley National Park in 2002.



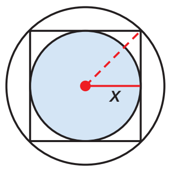
28. What is the experimental probability that the maximum temperature will be greater than 90°F on a given day in April?

29. For what month would you estimate the theoretical probability of a maximum temperature no greater than 90°F to be about 0.13? Explain.

30. May has 31 days. How would the experimental probability be affected if someone mistakenly used 30 days to calculate the experimental probability that the maximum temperature will not be greater than 90°F on a given day in May?

31. **Critical Thinking** Is it possible for the experimental probability of an event to be 0 if the theoretical probability is 1? Is it possible for the experimental probability of an event to be 0 if the theoretical probability is 0.99? Explain.

32 **Geometry** The two circles circumscribe and inscribe the square. Find the probability that a random point in the large circle is within the inner circle. (*Hint:* Use the Pythagorean Theorem.)



33. **Critical Thinking** Lexi tossed a fair coin 20 times, resulting in 12 heads and 8 tails. What is the theoretical probability that Lexi will get heads on the next toss? Explain.

34. **Athletics** Do male or female high school basketball players have a better chance of playing on college teams? on professional teams? Explain.

U.S. Basketball Players

	Men	Women
High School Players	549,500	456,900
College Players	4,500	4,100
College Players Drafted by Pro Leagues	44	32

Source: www.ncaa.org

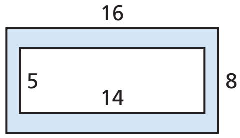
35. **Write About It** Describe the difference between theoretical probability and experimental probability. Give an example in which they may differ.

TEST PREP

36. A fair coin is tossed 25 times, landing tails up 14 times. What is the experimental probability of heads?

Ⓐ 0.44 Ⓑ 0.50 Ⓒ 0.56 Ⓓ 0.79

37. **Geometry** Find the probability that a point chosen at random in the large rectangle at right will lie in the shaded area, to the nearest percent.



Ⓕ 18% Ⓖ 45% Ⓗ 55% Ⓙ 71%

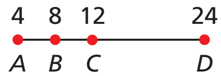
38. How many outcomes are in the sample space when a quarter, a dime, and a nickel are tossed?

Ⓐ 3 Ⓑ 6 Ⓒ 8 Ⓓ 12

39. Two number cubes are rolled. What is the theoretical probability that the sum is 5?

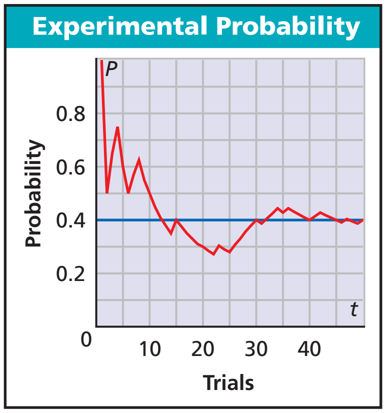
Ⓕ $\frac{1}{3}$ Ⓖ $\frac{1}{6}$ Ⓗ $\frac{1}{9}$ Ⓙ $\frac{1}{12}$

40. **Short Response** Find the probability that a point chosen at random on the part of the number line shown will lie between points *B* and *C*.



CHALLENGE AND EXTEND

41. The graph illustrates a statistical property known as the *law of large numbers*. Make a conjecture about the effect on probability as the number of trials gets very large. Give an example of how the probability might be affected for a real-world situation.



42. Four trumpet players' instruments are mixed up, and the trumpets are given to the players just before a concert. What is the probability that *no one* gets his or her trumpet back?

43. The table shows the data from a spinner experiment. Draw a reasonable spinner with 6 regions that may have been used for this experiment.

Spinner Experiment

Color	Red	Blue	Green	Yellow
Occurrences	23	44	7	26

SPIRAL REVIEW

Find the minimum or maximum value of each function. *(Lesson 5-2)*

44. $f(x) = 0.25x^2 - 0.85x + 1$

45. $f(x) = -2x^2 + 20x - 34$

Write the equation in standard form for each parabola. *(Lesson 10-5)*

46. vertex $(0, 0)$, directrix $x = -3$

47. vertex $(0, 0)$, directrix $y = 5$

48. A coach chooses 5 players for a basketball team from a group of 11. *(Lesson 11-1)*

a. How many ways can she choose 5 players?

b. How many ways can she choose 5 players to play different positions?

11-2 Technology LAB

Use with Lesson 11-2

Explore Simulations

A *simulation* is a model that uses random numbers to approximate experimental probability. You can use a spreadsheet to perform simulations. The **RAND( )** function generates random decimal values greater than or equal to 0 and less than 1. The **INT** function gives the greatest integer less than or equal to the input value. The functions can be used together to generate random integers as shown in the table

Random Numbers		
Formula	**Output**	**Example**
=RAND()	Decimal values $0 \le n < 1$	0.279606096
=100*RAND()	Decimal values $0 \le n < 100$	27.9606096
=INT(100*RAND())	Integers $0 \le n \le 99$	27
=INT(100*RAND())+1	Integers $1 \le n \le 100$	28

Activity

Use a simulation to find the experimental probability that a 65% free throw shooter will make at least 4 of his next 5 attempts.

1. To represent a percent, enter the formula for random integers from 1 to 100 into cell A1.

A1 =INT(100*RAND())+1

	A	B	C	D	E
1	38				

2. Let each row represent a trial of 5 attempts. Copy the formula from cell A1 into cells B1 through E1. Each time you copy the formula, the random values will change. To represent 10 trials, copy the formulas from row 1 into rows 2 through 10.

A1 =INT(100*RAND())+1

	A	B	C	D	E
1	72	98	34	74	87

3. Because the shooter makes 65% of his attempts, let the numbers 1 through 65 represent a successful attempt.

	A	B	C	D	E
1	✓ 25	✓ 2	✓ 62	✓ 26	✓ 38
2	✓ 30	✓ 32	66	88	✓ 9
3	✓ 27	✓ 18	✓ 9	✓ 9	93
4	98	✓ 34	✓ 10	86	99
5	87	✓ 64	✓ 4	74	✓ 36
6	✓ 5	97	69	83	✓ 51
7	✓ 39	✓ 39	80	95	97
8	✓ 32	✓ 64	✓ 51	✓ 64	✓ 46
9	✓ 52	81	✓ 39	✓ 5	✓ 36
10	✓ 48	✓ 46	✓ 45	69	✓ 21

Identify the number of successful attempts in each row, or trial. There were 4 or more successes in trials 1, 3, 8, 9, and 10. So there is about a $\frac{5}{10}$, or 50%, experimental probability that the shooter will make at least 4 of his next 5 attempts.

Note that each time you run the simulation, you may get a different probability. The more trials you perform, the more reliable your estimate will be.

Try This

Use a simulation to find each experimental probability.

1. An energy drink game advertises a 25% chance of winning with each bottle cap. Find the experimental probability that a 6-pack will contain at least 3 winners.
2. In a game with a 40% chance of winning, your friend challenges you to win 4 times in a row. Find the experimental probability of this happening in the next 4 games.
3. **Critical Thinking** How would you design a simulation to find the probability that a baseball player with a .285 batting average will get a hit in 5 of his next 10 at bats?

11-3 Independent and Dependent Events

Objectives

Determine whether events are independent or dependent.

Find the probability of independent and dependent events.

Vocabulary

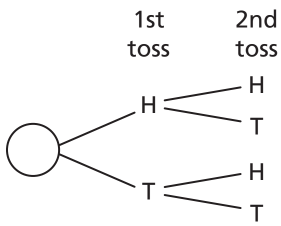
independent events
dependent events
conditional probability

Who uses this?

Political analysts can use demographic information and probabilities to predict the results of elections. (See Example 3.)

Events are **independent events** if the occurrence of one event does not affect the probability of the other.

If a coin is tossed twice, its landing heads up on the first toss and landing heads up on the second toss are independent events. The outcome of one toss does not affect the probability of heads on the other toss. To find the probability of tossing heads twice, multiply the individual probabilities, $\frac{1}{2} \cdot \frac{1}{2}$, or $\frac{1}{4}$.



Know it! Note

Probability of Independent Events

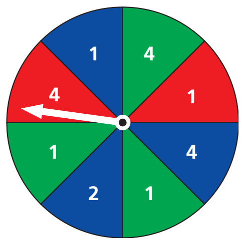
If A and B are independent events, then $P(A \text{ and } B) = P(A) \cdot P(B)$.

EXAMPLE 1 Finding the Probability of Independent Events

Find each probability.

A **spinning 4 and then 4 again on the spinner**

Spinning a 4 once does not affect the probability of spinning a 4 again, so the events are independent.



$P(4 \text{ and then } 4) = P(4) \cdot P(4)$

$\frac{3}{8} \cdot \frac{3}{8} = \frac{9}{64}$ *3 of the 8 equal sectors are labeled 4.*

B **spinning red, then green, and then red on the spinner**

The result of any spin does not affect the probability of any other outcome.

$P(\text{red, then green, and then red}) = P(\text{red}) \cdot P(\text{green}) \cdot P(\text{red})$

$= \frac{1}{4} \cdot \frac{3}{8} \cdot \frac{1}{4} = \frac{3}{128}$ *2 of the 8 equal sectors are red; 3 are green.*

CHECK IT OUT!

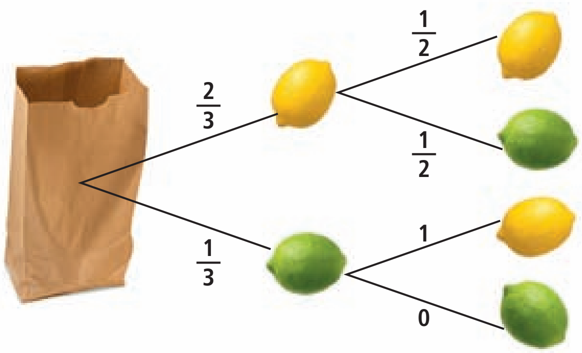
Find each probability.

1a. rolling a 6 on one number cube and a 6 on another number cube

1b. tossing heads, then heads, and then tails when tossing a coin 3 times

Events are **dependent events** if the occurrence of one event affects the probability of the other. For example, suppose that there are 2 lemons and 1 lime in a bag. If you pull out two pieces of fruit, the probabilities change depending on the outcome of the first.

The tree diagram shows the probabilities for choosing two pieces of fruit from a bag containing 2 lemons and 1 lime.



The probability of a specific event can be found by multiplying the probabilities on the branches that make up the event. For example, the probability of drawing two lemons is $\frac{2}{3} \cdot \frac{1}{2} = \frac{1}{3}$.

To find the probability of dependent events, you can use **conditional probability** $P(B \mid A)$, the probability of event B, given that event A has occurred.

Know it! Note

Probability of Dependent Events

If A and B are dependent events, then $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$, where $P(B \mid A)$ is the probability of B, given that A has occurred.

EXAMPLE 2

Finding the Probability of Dependent Events

Two number cubes are rolled—one red and one blue. Explain why the events are dependent. Then find the indicated probability.

1 1	1 2	1 3	1 4	1 5	1 6
2 1	2 2	2 3	2 4	2 5	2 6
3 1	3 2	3 3	3 4	3 5	3 6
4 1	4 2	4 3	4 4	4 5	4 6
5 1	5 2	5 3	5 4	5 5	5 6
6 1	6 2	6 3	6 4	6 5	6 6

A **The red cube shows a 1, and the sum is less than 4.**

Step 1 Explain why the events are dependent.

$P(\text{red } 1) = \frac{6}{36} = \frac{1}{6}$ *Of 36 outcomes, 6 have a red 1.*

$P(\text{sum} < 4 \mid \text{red } 1) = \frac{2}{6} = \frac{1}{3}$ *Of 6 outcomes with a red 1, 2 have a sum less than 4.*

The events "the red cube shows a 1" and "the sum is less than 4" are dependent because $P(\text{sum} < 4)$ is different when it is known that a red 1 has occurred.

Step 2 Find the probability.

$$P(A \text{ and } B) = P(A) \cdot P(B \mid A)$$

$$P(\text{red } 1 \text{ and sum} < 4) = P(\text{red } 1) \cdot P(\text{sum} < 4 \mid \text{red } 1)$$

$$= \frac{1}{6} \cdot \frac{2}{3} = \frac{1}{18}$$

Helpful Hint

In Example 2A, you can check to see that 2 of the 36 outcomes, or $\frac{1}{18}$, have a red 1 and a sum less than 4: (1, 1) and (1, 2).

Explain why the events are dependent. Then find the indicated probability.

B **The blue cube shows a multiple of 3, and the sum is 8.**

The events are dependent because $P(\text{sum is } 8)$ is different when the blue cube shows a multiple of 3.

$P(\text{blue multiple of } 3) = \frac{2}{6} = \frac{1}{3}$ — *Of 6 outcomes for blue, 2 have a multiple of 3.*

$P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \frac{2}{12} = \frac{1}{6}$ — *Of 12 outcomes that have a blue multiple of 3, 2 have a sum 8.*

$P(\text{blue multiple of 3 and sum is } 8) =$

$P(\text{blue multiple of } 3) \cdot P(\text{sum is } 8 \mid \text{blue multiple of } 3) = \left(\frac{1}{3}\right)\left(\frac{1}{6}\right) = \frac{1}{18}$

CHECK IT OUT! **Two number cubes are rolled—one red and one black. Explain why the events are dependent, and then find the indicated probability.**

2. The red cube shows a number greater than 4, and the sum is greater than 9.

Conditional probability often applies when data fall into categories.

EXAMPLE 3 Using a Table to Find Conditional Probability

Largest Texas Counties' Votes for President 2004 (thousands)

County	Bush	Kerry	Other
Harris	581	472	5
Dallas	345	336	4
Tarrant	349	207	3
Bexar	260	210	3
Travis	148	197	5

The table shows the approximate distribution of votes in Texas' five largest counties in the 2004 presidential election. Find each probability.

A **that a voter from Tarrant County voted for George Bush**

$P(\text{Bush} \mid \text{Tarrant}) = \frac{349}{559} \approx 0.624$ — *Use the Tarrant row. Of 559,000 Tarrant voters, 349,000 voted for Bush.*

B **that a voter voted for John Kerry and was from Dallas County**

$P(\text{Dallas} \mid \text{Kerry}) = \frac{336}{1422}$ — *Of 1,422,000 who voted for Kerry, 336,000 were from Dallas County.*

$P(\text{Kerry and Dallas} \mid \text{Kerry}) = \frac{1422}{3125} \cdot \frac{336}{1422}$ — *There were 3,125,000 total voters.*

≈ 0.108

CHECK IT OUT! **Find each probability.**

3a. that a voter from Travis county voted for someone other than George Bush or John Kerry

3b. that a voter was from Harris county and voted for George Bush

In many cases involving random selection, events are independent when there is replacement and dependent when there is not replacement.

EXAMPLE 4 Determining Whether Events Are Independent or Dependent

Two cards are drawn from a deck of 52. Determine whether the events are independent or dependent. Find the probability.

A **selecting two aces when the first card is replaced**

Replacing the first card means that the occurrence of the first selection will not affect the probability of the second selection, so the events are independent.

$P(\text{ace} \mid \text{ace on first draw}) = P(\text{ace}) \cdot P(\text{ace})$

$= \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{169}$ *4 of the 52 cards are aces.*

B **selecting a face card and then a 7 when the first card is not replaced**

Not replacing the first card means that there will be fewer cards to choose from, affecting the probability of the second selection, so the events are dependent.

$P(\text{face card}) \cdot P(7 \mid \text{first card was a face card})$

$= \frac{12}{52} \cdot \frac{4}{51} = \frac{4}{221}$ *There are 12 face cards, four 7's and 51 cards available for the second selection.*

Remember!

A standard card deck contains 4 suits of 13 cards each. The face cards are the jacks, queens, and kings.

CHECK IT OUT! **A bag contains 10 beads—2 black, 3 white, and 5 red. A bead is selected at random. Determine whether the events are independent or dependent. Find the indicated probability.**

4a. selecting a white bead, replacing it, and then selecting a red bead

4b. selecting a white bead, not replacing it, and then selecting a red bead

4c. selecting 3 nonred beads without replacement

THINK AND DISCUSS

1. Describe some independent events.

2. Extend the rule for the probability of independent events to more than two independent events. When might this be used?

Know it! Note

3. GET ORGANIZED Copy and complete the graphic organizer. In each box, compare independent and dependent events and their related probabilities.

Probability of Independent Events vs. Probability of Dependent Events

Similarities | Differences

11-3 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 11-3
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** Two events are __?__ if the occurrence of one event does not affect the probability of the other event. (*independent* or *dependent*)

SEE EXAMPLE 1
p. 811

Find each probability.

2. rolling a 1 and then another 1 when a number cube is rolled twice
3. a coin landing heads up on every toss when it is tossed 3 times

SEE EXAMPLE 2
p. 812

Two number cubes are rolled—one blue and one yellow. Explain why the events are dependent. Then find the indicated probability.

4. The blue cube shows a 4 and the product is less than 20.
5. The yellow cube shows a multiple of 3, given that the product is 6.

SEE EXAMPLE 3
p. 813

The table shows the results of a quality-control study of a lightbulb factory. A lightbulb from the factory is selected at random. Find each probability.

6. that a shipped bulb is not defective
7. that a bulb is defective and shipped

Lightbulb Quality

	Shipped	Not Shipped
Defective	10	45
Not Defective	942	3

SEE EXAMPLE 4
p. 814

A bag contains 20 checkers—10 red and 10 black. Determine whether the events are independent or dependent. Find the indicated probability.

8. selecting 2 black checkers when they are chosen at random with replacement
9. selecting 2 black checkers when they are chosen at random without replacement

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
10–11	1
12–14	2
15–16	3
17–18	4

Extra Practice

Skills Practice p. S24
Skills Practice p. S42

Find each probability.

10. choosing the same activity when two friends each randomly choose 1 of 4 extracurricular activities to participate in
11. rolling an even number and then rolling a 6 when a number cube is rolled twice

Two number cubes are rolled—one blue and one yellow. Explain why the events are dependent. Then find the indicated probability.

12. The yellow cube is greater than 5 and the product is greater than 24.
13. The blue cube is less than 3 and the product is 8.
14. The table shows immigration to the United States from three countries in three different years. A person is randomly selected. Find each probability.
    a. that a selected person is from Cuba, given that the person immigrated in 1990
    b. that a person came from Spain and immigrated in 2000
    c. that a selected person immigrated in 1995, given that the person was from Ghana.

Immigration to the United States

Country	1990	1995	2000
Cuba	10,645	17,937	20,831
Ghana	4,466	3,152	4,344
Spain	1,886	1,321	1,264

Employment Find each probability.

15. that a person with an advanced degree is employed

16. that a person is not a high school graduate and is not employed

Employment by Education Level, Ages 21–24		
Education Level	Employed (millions)	Not employed (millions)
Not a high school graduate	1.060	0.834
High school graduate	2.793	1.157
Some college	4.172	1.634
Bachelor's degree	1.53	0.372
Advanced degree	0.104	0.041

A bag contains number slips numbered 1 to 9. Determine whether the events are independent or dependent, and find the indicated probability.

17. selecting 2 even numbers when 2 slips are chosen without replacement

18. selecting 2 even numbers when 2 slips are chosen with replacement

Determine whether the events are independent or dependent.

19. A coin comes up heads, and a number cube rolled at the same time comes up 6.

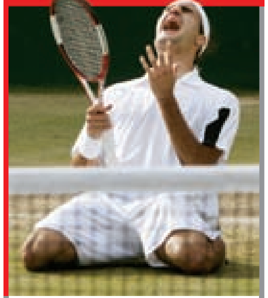
20. A 4 is drawn from a deck of cards, set aside, and then an ace is drawn.

21. A 1 is rolled on a number cube, and then a 4 is rolled on the same number cube.

22. A dart hits the bull's-eye, and a second dart also hits the bull's eye.

LINK

Tennis

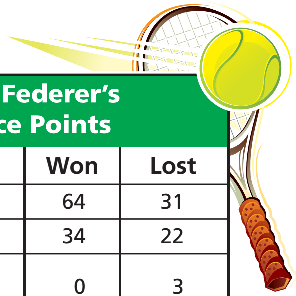


Wimbledon has been played annually since 1877 at the All England Lawn Tennis and Croquet Club.

23. Tennis In the 2004 Wimbledon Men's Tennis Championship final, Roger Federer defeated Andy Roddick in three sets.

a. What was the probability that Federer won the point when his second serve was in?

b. When Federer lost a point, what was the probability that he *double faulted*?



Roger Federer's Service Points	Won	Lost
First Serve In	64	31
Second Serve In	34	22
Second Serve Out (Double Fault)	0	3

24. Multi-Step At one high school, the probability that a student is absent today, given that the student was absent yesterday, is 0.12. The probability that a student is absent today, given that the student was present yesterday, is 0.05. The probability that a student was absent yesterday is 0.1. Draw a tree diagram to represent the situation. What is the probability that a randomly selected student was present yesterday and today?

MULTI-STEP TEST PREP

25. This problem will prepare you for the Multi-Step Test Prep on page 826.

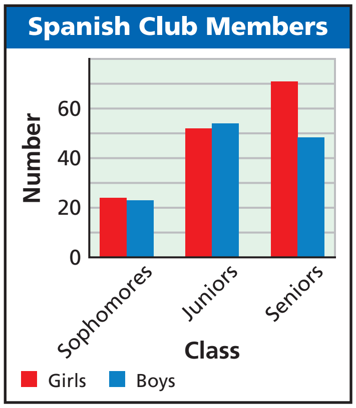
While playing Yahtzee, Jake rolls 5 dice and gets the result shown at right. The rules allow him to reroll these dice 2 times. Jake decides to try for all 5's, so he rerolls the 2 and the 3.

a. What is the probability that Jake gets no additional 5's in either of the 2 rolls?

b. What is the probability that he gets all 5's on his first reroll of the 2 and the 3?

c. What is the probability that he gets all 5's on his first reroll, given that at least one of the dice is a 5?

Estimation **Use the graph to estimate each probability.**



26. that a Spanish club member is a girl

27. that a senior Spanish club member is a girl

28. that a male Spanish club member is a senior

29. **Critical Thinking** A box contains 100 balloons. Eighty are yellow, and 20 are green. Fifty are marked "Happy Birthday!" and 50 are not. A balloon is randomly chosen from the box. How many yellow "Happy Birthday!" balloons must be in the box if the event "a balloon is yellow" and the event "a balloon is marked 'Happy Birthday!'" are independent?

30. **Travel** Airline information for three years is given in the table.

a. Complete the table.

b. What was the probability that a scheduled flight in 2004 was canceled?

c. An on-time flight is selected randomly for study. What is the probability that it was a flight from 2005?

Scheduled Flights (thousands) January to July

	2003	2004	2005	Total
On Time		3197	3237	
Delayed	598		877	2321
Canceled	61	68		
Total	3761		4196	

Source: Bureau of Transportation Statistics

31. **Write About It** The "law of averages" is a nonmathematical term that means that events eventually "average out." So, if a coin comes up heads 10 tosses in a row, there is a greater probability that it will come up tails on the eleventh toss. Explain the error in this thinking.

TEST PREP

32. What is the probability that a person's birthday falls on a Saturday next year, given that it falls on a Saturday this year?

(A) 0 (B) $\frac{1}{7}$ (C) $\frac{1}{2}$ (D) 1

33. Which of the following has the same probability as rolling doubles on 2 number cubes 3 times in a row?

(F) A single number cube is rolled 3 times. The cube shows 5 each time.

(G) Two number cubes are rolled 3 times. Each time the sum is 6.

(H) Two number cubes are rolled 3 times. Each time the sum is greater than 2.

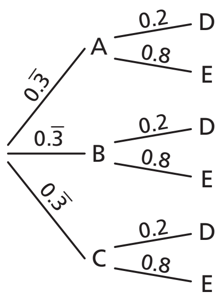
(J) Three number cubes are rolled twice. Each time all cubes show the same number (triples).

34. **Extended Response** **Use the tree diagram.**

a. Find $P(D \mid A)$, $P(D \mid B)$, and $P(D \mid C)$.

b. Does the tree diagram represent independent or dependent events? Explain your answer.

c. Describe a scenario for which the tree diagram could be used to find probabilities.



CHALLENGE AND EXTEND

35. Two number cubes are rolled in succession and the numbers that they show are added together. What is the only sum for which the probability of the sum is independent of the number shown on the first roll? Explain.

36. **Birthdays** People born on February 29 have a birthday once every 4 years.

a. What is the smallest group of people in which there is a greater than 50% chance that 2 people share a birthday? (Do not include February 29.)

b. What is the probability that in a group of 150 people, none are born on February 29?

c. What is the least number of people such that there is a greater than 50% chance that one of the people in the group has a birthday on February 29?

37. There are 150 people at a play. Ninety are women, and 60 are men. Half are sitting in the lower level, and half are sitting in the upper level. There are 35 women sitting in the upper level. A person is selected at random for a prize. What is the probability that the person is sitting in the lower level, given that the person is a woman? Is the event "person is sitting in the lower level" independent of the event "person is a woman"? Explain.

38. **Medicine** Suppose that strep throat affects 2% of the population and a test to detect it produces an accurate result 99% of the time.

a. Complete the table.

b. What is the probability that someone who tests positive actually has strep throat?

Per 10,000 People Tested

	Have strep	Do not have strep	Total
Test Positive	■	■	■
Test Negative	■	■	■
Total	■	■	10,000

SPIRAL REVIEW

39. **Sports** A basketball player averaged 18.3 points per game in the month of December. In January, the same basketball player averaged 32.5 points per game. *(Lesson 2-6)*

a. Write the average number of points scored as a function of games played for both months, $p(d)$ and $p(j)$.

b. Graph $p(d)$ and $p(j)$ on the same coordinate plane.

c. Describe the transformation that occurred.

Solve each system of equations by graphing. Round your answer to the nearest tenth. *(Lesson 10-7)*

40. $\begin{cases} 2x^2 - 4y^2 = 12 \\ y = 2 \end{cases}$

41. $\begin{cases} 4x^2 - 2y^2 = 18 \\ -x^2 + 6y^2 = 22 \end{cases}$

42. $\begin{cases} x^2 + y^2 = 16 \\ 2y + 5x^2 = -3 \end{cases}$

Two number cubes are rolled. Find each probability. *(Lesson 11-2)*

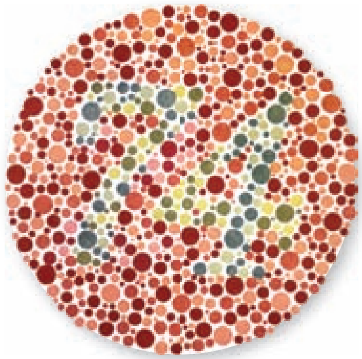
43. The sum is 12.

44. The sum is less than 5.

45. At least one number is odd.

46. At least one number is less than 3.

11-4 Compound Events



Objectives
Find the probability of mutually exclusive events.

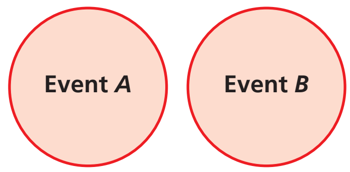
Find the probability of inclusive events.

Vocabulary
simple event
compound event
mutually exclusive events
inclusive events

Why learn this?
You can use the probability of compound events to determine the likelihood that a person of a specific gender is color-blind. (See Example 3.)

A **simple event** is an event that describes a single outcome. A **compound event** is an event made up of two or more simple events. **Mutually exclusive events** are events that cannot both occur in the same trial of an experiment. Rolling a 1 and rolling a 2 on the same roll of a number cube are mutually exclusive events.

Mutually Exclusive Events



Know it! Note

Remember!
Recall that the union symbol ∪ means "or."

Mutually Exclusive Events

WORDS	ALGEBRA	EXAMPLE
The probability of two mutually exclusive events *A or B* occurring is the sum of their individual probabilities.	For two mutually exclusive events *A* and *B*, $P(A \cup B) = P(A) + P(B)$.	When a number cube is rolled, $P(\text{less than } 3) =$ $P(1 \text{ or } 2) =$ $P(1) + P(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$.

EXAMPLE 1 Finding Probabilities of Mutually Exclusive Events

A drink company applies one label to each bottle cap: "free drink," "free meal," or "try again." A bottle cap has a $\frac{1}{10}$ probability of being labeled "free drink" and a $\frac{1}{25}$ probability of being labeled "free meal."

a. Explain why the events "free drink" and "free meal" are mutually exclusive.

Each bottle cap has only one label applied to it.

b. What is the probability that a bottle cap is labeled "free drink" or "free meal"?

$$P(\text{free drink} \cup \text{free meal}) = P(\text{free drink}) + P(\text{free meal})$$
$$= \frac{1}{10} + \frac{1}{25} = \frac{5}{50} + \frac{2}{50} = \frac{7}{50}$$

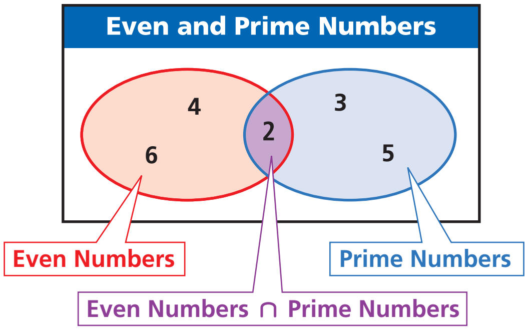
CHECK IT OUT!

1. Each student cast one vote for senior class president. Of the students, 25% voted for Hunt, 20% for Kline, and 55% for Vila. A student from the senior class is selected at random.

a. Explain why the events "voted for Hunt," "voted for Kline," and "voted for Vila" are mutually exclusive.

b. What is the probability that a student voted for Kline or Vila?

Inclusive events are events that have one or more outcomes in common. When you roll a number cube, the outcomes "rolling an even number" and "rolling a prime number" are not mutually exclusive. The number 2 is both prime and even, so the events are inclusive.



Remember!

Recall that the intersection symbol $\cap$ means "and."

There are 3 ways to roll an even number, $\{2, 4, 6\}$.

There are 3 ways to roll a prime number, $\{2, 3, 5\}$.

The outcome "2" is counted twice when outcomes are added $(3 + 3)$. The actual number of ways to roll an even number or a prime is $3 + 3 - 1 = 5$. The concept of subtracting the outcomes that are counted twice leads to the following probability formula.

Know it! Note

Inclusive Events

WORDS	The probability of two inclusive events A or B occurring is the sum of their individual probabilities minus the probability of *both* occurring.
ALGEBRA	For two inclusive events A and B, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
EXAMPLE	When you roll a number cube, $P(\text{even number or prime}) =$ $P(\text{even or prime}) = P(\text{even}) + P(\text{prime}) - P(\text{even and prime})$ $= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \frac{5}{6}$.

EXAMPLE 2 Finding Probabilities of Inclusive Events

Find each probability on a die.

A rolling a 5 or an odd number

$$P(5 \text{ or odd}) = P(5) + P(\text{odd}) - P(5 \text{ and odd})$$

$$= \frac{1}{6} + \frac{3}{6} - \frac{1}{6} \qquad \textit{5 is also an odd number.}$$

$$= \frac{1}{2}$$

B rolling at least one 4 when rolling 2 dice

$$P(4 \text{ or } 4) = P(4) + P(4) - P(4 \text{ and } 4)$$

$$= \frac{1}{6} + \frac{1}{6} - \frac{1}{36} \qquad \textit{There is 1 outcome in 36 where both dice show 4.}$$

$$= \frac{11}{36}$$

CHECK IT OUT! **A card is drawn from a deck of 52. Find the probability of each.**

2a. drawing a king or a heart

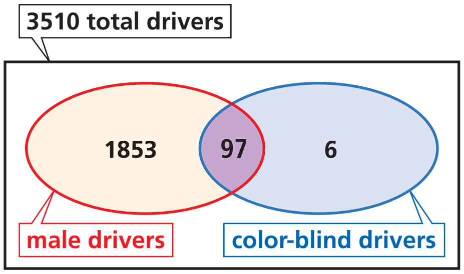
2b. drawing a red card (hearts or diamonds) or a face card (jack, queen, or king)

EXAMPLE 3 *Health Application*

Of 3510 drivers surveyed, 1950 were male and 103 were color-blind. Only 6 of the color-blind drivers were female. What is the probability that a driver was male or was color-blind?

Step 1 Use a Venn diagram.

Label as much information as you know. Being male and being color-blind are inclusive events.



Step 2 Find the number in the overlapping region.

Subtract 6 from 103. This is the number of color-blind males, 97.

Step 3 Find the probability.

$$= P(\text{male} \cup \text{color-blind}) =$$

$$= P(\textbf{male}) + P(\textbf{color-blind}) - P(\textbf{male} \cap \textbf{color-blind})$$

$$= \frac{\mathbf{1950}}{3510} + \frac{\mathbf{103}}{3510} - \frac{\mathbf{97}}{3510} = \frac{1956}{3510} \approx 0.557$$

The probability that a driver was male or was color-blind is about 55.7%.

Helpful Hint

As you work through Example 3, fill in the Venn diagram with information as you find it.

CHECK IT OUT!

3. Of 160 beauty spa customers, 96 had a hair styling and 61 had a manicure. There were 28 customers who had only a manicure. What is the probability that a customer had a hair styling or a manicure?

Recall from Lesson 11-2 that the complement of an event with probability p, all outcomes that are not in the event, has a probability of $1 - p$. You can use the complement to find the probability of a compound event.

EXAMPLE 4 *Book Club Application*

There are 5 students in a book club. Each student randomly chooses a book from a list of 10 titles. What is the probability that at least 2 students in the group choose the same book?

$P(\text{at least 2 students choose same}) = 1 - P(\text{all choose different})$ *Use the complement.*

$$P(\text{all choose different}) = \frac{\textbf{number of ways 5 students can choose different books}}{\textbf{total number of ways 5 students can choose books}}$$

$$= \frac{{}_{10}P_5}{10^5}$$

$$= \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6}{10 \cdot 10 \cdot 10 \cdot 10 \cdot 10} = \frac{30{,}240}{100{,}000} = 0.3024$$

$$P(\text{at least 2 students choose same}) = 1 - 0.3024 = 0.6976$$

The probability that at least 2 students choose the same book is 0.6976, or 69.76%.

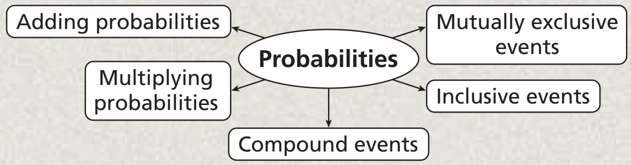
CHECK IT OUT!

4. In one day, 5 different customers bought earrings from the same jewelry store. The store offers 62 different styles. Find the probability that at least 2 customers bought the same style.

THINK AND DISCUSS

1. Explain why the formula for inclusive events, $P(A \cup B) = P(A) + P(B) - P(A \cap B)$, also applies to mutually exclusive events.
2. Tell whether the probability of sharing a birthday with someone else in the room is the same whether your birthday is March 13 or February 29. Explain.
3. **GET ORGANIZED** Copy and complete the graphic organizer. Give at least one example for each.

Know it! Note



11-4 Exercises

go.hrw.com
Homework Help Online
KEYWORD: MB7 11-4
Parent Resources Online
KEYWORD: MB7 Parent

GUIDED PRACTICE

1. **Vocabulary** A compound event where one outcome overlaps with another is made up of two __?__. (*inclusive event* or *mutually exclusive events*)

A bag contains 25 marbles: 10 black, 13 red, and 2 blue. A marble is drawn from the bag at random.

SEE EXAMPLE 1
p. 819

2. Explain why the events “getting a black marble” and “getting a red marble” are mutually exclusive.
3. What is the probability of getting a red or a blue marble?
4. A car approaching an intersection has a 0.1 probability of turning left and a 0.2 probability of turning right. Explain why the events are mutually exclusive. What is the probability that the car will turn?

SEE EXAMPLE 2
p. 820

Numbers 1–10 are written on cards and placed in a bag. Find each probability.

5. choosing a number greater than 5 or choosing an odd number
6. choosing an 8 or choosing a number less than 5
7. choosing at least one even number when selecting 2 cards from the bag

SEE EXAMPLE 3
p. 821

Five years after 650 high school seniors graduated, 400 had a college degree and 310 were married. Half of the students with a college degree were married.

8. What is the probability that a student has a college degree or is married?
9. What is the probability that a student has a college degree or is not married?
10. What is the probability that a student does not have a college degree or is married?

SEE EXAMPLE 4
p. 821

11. A vending machine offers 8 different drinks. One day, 6 employees each purchased a drink from the vending machine. Find the probability that at least 2 employees purchased the same drink.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
12–13	1
14–15	2
16–18	3
19	4

Extra Practice

Skills Practice p. S24
Application Practice p. S42

Jump ropes are given out during gym class. A student has a $\frac{1}{6}$ chance of getting a red jump rope and a $\frac{1}{3}$ chance of getting a green jump rope. Meg is given a jump rope.

12. Explain why the events "getting a red jump rope" and "getting a green jump rope" are mutually exclusive.

13. What is the probability that Meg gets a red or green jump rope?

The letters *A*–*P* are written on cards and placed in a bag. Find the probability of each outcome.

14. choosing an *E* or choosing a *G*

15. choosing an *E* or choosing a vowel

Lincoln High School has 98 teachers. Of the 42 female teachers, 8 teach math. One-seventh of all of the teachers teach math.

16. What is the probability that a teacher is a woman or teaches math?

17. What is the probability that a teacher is a man or teaches math?

18. What is the probability that a teacher is a man or does not teach math?

19. A card is drawn from a deck of 52 and recorded. Then the card is replaced, and the deck is shuffled. This process is repeated 13 times. What is the probability that at least one of the cards drawn is a heart?

20. Critical Thinking Events *A* and *B* are mutually exclusive. Must the complements of events *A* and *B* be mutually exclusive? Explain by example.

LINK

Television

In 2004, about 109.6 million U.S. households had televisions. Nielsen's *rating points*, such as those for *CSI*, represent the percent of these households tuned to a show.

21. Television According to Nielsen Media Research, on June 21, 2005, from 9 to 10 P.M., the NBA Finals Game 7 between San Antonio and Detroit had a 22 *share* (was watched by 22% of television viewers), while *CSI* had a 15 share. What is the probability that someone who was watching television during this time watched the NBA Finals or *CSI*? Do you think that this is theoretical or experimental probability? Explain.

School Arts **Use the table for Exercises 22 and 23.**

22. What would you need to know to find the probability that a U.S. public school offers music or dance classes?

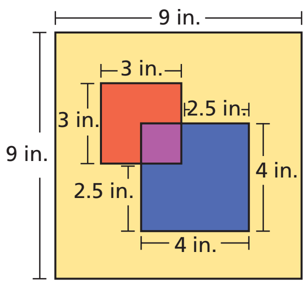
23. What is the minimum probability that a U.S. public school offers visual arts or drama? What is the maximum probability?

Arts Offered by U.S. Public Schools

Class Type	Music	Visual arts	Dance	Drama and theater
Percent of Schools	94%	87%	20%	19%

24. Geometry A square dartboard contains a red square and a blue square that overlap. A dart hits a random point on the board.

a. Find $P(\text{red} \cap \text{blue})$.
b. Find $P(\text{red})$.
c. Find $P(\text{red} \cup \text{blue})$.
d. Find $P(\text{yellow})$.



25. Genetics One study found that 8% of men and 0.5% of women are born color-blind. Of the study participants, 52% were men.

a. Which probability would you expect to be greater: that a study participant is male *and* born color-blind or that a participant is male *or* born color-blind? Explain.

b. What is the probability that a study participant is male and born color-blind? What is the probability that a study participant is male or born color-blind?

MULTI-STEP TEST PREP

26. This problem will prepare you for the Multi-Step Test Prep on page 826.

While playing Yahtzee, Amanda rolls five dice and gets the result shown. She decides to keep the 1, 2, and 4, and reroll the 5 and 6.

a. After rerolling the 5 and 6, what is the probability that Amanda will have a "large straight" (1-2-3-4-5) or three 4's?

b. After rerolling the 5 and 6, what is the probability that Amanda will have a "small straight" (1-2-3-4 plus anything else) or a pair of 3's?

27. Public Safety In a study of canine attacks, the probability that the victim was under 18 years of age was 0.8. The probability that the attack occurred on the dog owner's property was 0.64. The probability that the victim was under 18 years of age or the attack occurred on the owner's property was 0.95. What was the probability that the victim was under 18 years of age and the attack occurred on the owner's property?

28. Politics A 4-person leadership committee is randomly chosen from a group of 24 candidates. Ten of the candidates are men, and 14 are women.

a. What is the probability that the committee is all male or all female?

b. What is the probability that the committee has at least 1 man or at least 1 woman?

29. Multi-Step The game Scrabble contains letter tiles that occur in different numbers. Suppose that one tile is selected.

a. What is the probability of choosing a vowel if *Y* is not included?

b. What is the probability of choosing a *Y*?

c. What is the probability of choosing a vowel if *Y* is included? How does this relate to the answer to parts **a** and **b**?

Distribution of Scrabble Tiles

Tiles	Frequency
J, K, Q, X, Z	1
B, C, F, H, M, P, V, W, Y, blank	2
G	3
D, L, S, U	4
N, R, T	6
O	8
A, I	9
E	12

30. Write About It Demonstrate two ways to find the probability of a coin's landing heads up at least once in 2 tosses of a coin.

TEST PREP

31. For a quilt raffle, 2500 tickets numbered 0001–2500 are sold. Jamie has number 1527. The winning raffle number is read one digit at a time. The first winning number begins "One...". After the first digit is called, Jamie's chances of winning do which of the following?

Ⓐ Go to 0

Ⓑ Stay the same

Ⓒ Increase from $\frac{1}{2500}$ to $\frac{1}{1527}$

Ⓓ Increase from $\frac{1}{2500}$ to $\frac{1}{1000}$

32. A fair coin is tossed 4 times. Given that each of the first 3 tosses land tails up, what is the probability that all 4 tosses land tails up?

Ⓕ 0.5

Ⓖ Greater than 0.5

Ⓗ 0.5^4

Ⓙ Between 0.5^4 and 0.5

33. If Travis rolls a 5 on a number cube, he lands on "roll again." If Travis rolls a number greater than 3, he'll pass "start" and collect \$100. What is the probability that Travis rolls again or collects \$100?

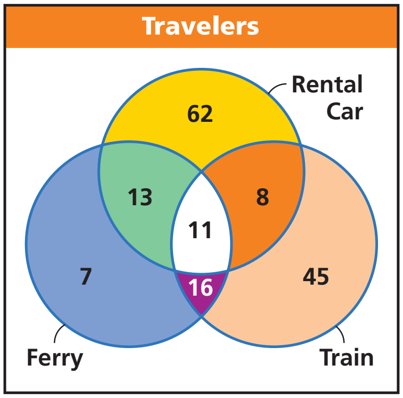
Ⓐ $\frac{1}{6}$ Ⓑ $\frac{1}{5}$ Ⓒ $\frac{1}{4}$ Ⓓ $\frac{1}{2}$

34. **Short Response** What is the probability of an event or its complement? Explain.

CHALLENGE AND EXTEND

35. What is the probability that at least 2 people in a group of 10 people have the same birthday? (Assume no one in the group was born on February 29th.)

Travel **For Exercises 36–38, use the Venn diagram, which shows the transportation methods used by 162 travelers. Find each probability if a traveler is selected at random.**



36. $P(\text{ferry or train})$

37. $P(\text{ferry or rental car})$

38. $P(\text{train and ferry, or train and rental car})$

Use the table of probabilities and the following information for Exercises 39–41. Hint: Draw a Venn diagram.

For any three events A, B, and C, $P(A \text{ or } B \text{ or } C) =$
$P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

Event	$P(A)$	$P(B)$	$P(C)$	$P(A \cap B)$	$P(A \cap C)$	$P(B \cap C)$	$P(A \cap B \cap C)$
Probability	0.5	0.3	0.7	0.2	0.3	0.1	0.1

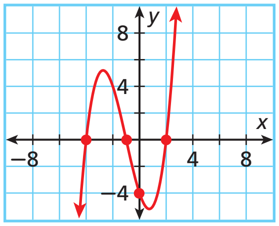
39. Find $P(B \cup C)$. **40.** Find $P(A \cup B \cup C)$. **41.** Find $P(B \cap (A \cup C))$.

SPIRAL REVIEW

Write a cubic function for each graph. *(Lesson 6-9)*

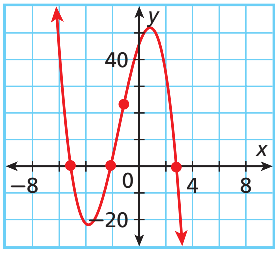
42.

x	y
−4	0
−1	0
0	−4
2	0



43.

x	y
−5	0
−2	0
−1	24
3	0



Graph each function. *(Lesson 9-2)*

44. $f(x) = \begin{cases} 2 & \text{if } x < -1 \\ 2x + 4 & \text{if } x \geq -1 \end{cases}$

45. $g(x) = \begin{cases} 1 - x^2 & \text{if } x < 1 \\ x^2 - 1 & \text{if } x \geq 1 \end{cases}$

Find each probability. *(Lesson 11-3)*

46. A coin is tossed twice and it lands heads up both times.

47. A coin is tossed 4 times and it lands heads up, heads up, tails up, and then tails up.

48. Two number cubes are rolled. The sum is greater than 10. The first number cube is 6.

CHAPTER 11

SECTION 11A

MULTI-STEP TEST PREP

Probability

Roll Call Yahtzee is played with 5 dice. A player rolls all 5 dice and may choose to roll any or all of the dice a second time and then a third time. At that point, the player scores points for various combinations of dice, such as 3 of a kind, 4 of a kind, or 5 of a kind.

1. How many possible rolls of 5 dice are there?
2. What is the probability of rolling five 6's on the first roll of the dice?
3. What is the probability of rolling 5 of any one number on the first roll?
4. Miguel's first roll is shown at right. He decides to reroll the 6's. What is the probability that he has a 1, 2, 3, 4, and 5 after this roll?
5. What is the probability that Miguel has a 1, 2, 3, 4, and 5 after the roll, given that at least one of the dice comes up a 4?
6. What is the probability that Miguel has a 1, 2, 3, 4, and 5 or a pair of 2's after the roll in Problem 4?
7. What is the probability that Miguel has a 1, 2, 3, 4, and any other number or a pair of 4's after the roll in Problem 4?

READY TO GO ON?

CHAPTER 11

SECTION 11A

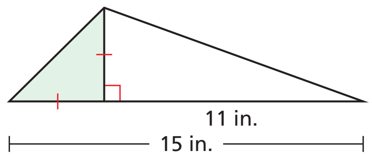
Quiz for Lessons 11-1 Through 11-4

11-1 Permutations and Combinations

1. A security code consists of 5 digits (0–9), and a digit may not be used more than once. How many possible security codes are there?
2. Adric owns 8 pairs of shoes. How many ways can he choose 4 pairs of shoes to pack into his luggage?
3. A plumber received calls from 5 customers. There are 6 open slots on today's schedule. How many ways can the plumber schedule the customers?

11-2 Theoretical and Experimental Probability

4. A cooler contains 18 cans: 9 of lemonade, 3 of iced tea, and 6 of cola. Dee selects a can without looking. What is the probability that Dee selects iced tea?
5. Jordan has 9 pens in his desk; 2 are out of ink. If his mom selects 2 pens from his desk, what is the probability that both are out of ink?
6. Find the probability that a point chosen at random inside the figure shown is in the shaded area.



7. A number cube is tossed 50 times, and a 2 is rolled 12 times. Find the experimental probability of not rolling a 2.

11-3 Independent and Dependent Events

8. Explain why the events "getting tails, then tails, then tails, then tails, then heads when tossing a coin 5 times" are independent, and find the probability.
9. Two number cubes are rolled—one red and one black. Explain why the events "the red cube shows a 6" and "the sum is greater than or equal to 10" are dependent, and find the probability.
10. The table shows the breakdown of math students for one school year. Find the probability that a Geometry student is in the 11th grade.
11. A bag contains 25 checkers—15 red and 10 black. Determine whether the events "a red checker is selected, not replaced, and then a black checker is selected" are independent or dependent, and find the probability.

Math Students by Grade

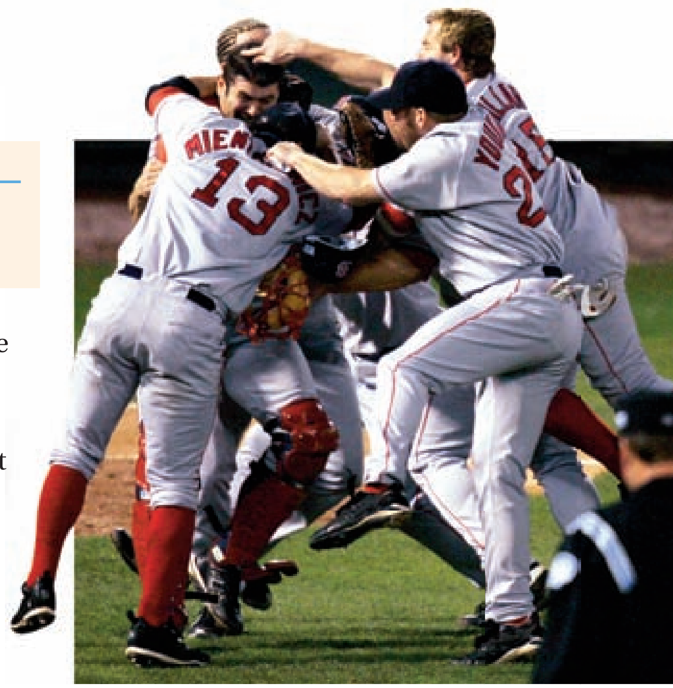
	Geometry	Algebra 2
9th Grade	26	0
10th Grade	68	24
11th Grade	33	94

11-4 Compound Events

Numbers 1–30 are written on cards and placed in a bag. One card is drawn. Find each probability.

12. drawing an even number or a 1
13. drawing an even number or a multiple of 7
14. Of a company's 85 employees, 60 work full time and 40 are married. Half of the full-time workers are married. What is the probability that an employee works part time or is not married?

11-5 Measures of Central Tendency and Variation



Objectives

Find measures of central tendency and measures of variation for statistical data.

Examine the effects of outliers on statistical data.

Vocabulary

expected value
probability distribution
variance
standard deviation
outlier

Who uses this?

Statisticians can use measures of central tendency and variation to analyze World Series results. (See Example 2.)

Recall that the *mean*, *median*, and *mode* are measures of central tendency—values that describe the center of a data set.

The *mean* is the sum of the values in the set divided by the number of values. It is often represented as $\bar{x}$. The *median* is the middle value or the mean of the two middle values when the set is ordered numerically. The *mode* is the value or values that occur most often. A data set may have one mode, no mode, or several modes.

EXAMPLE 1 Finding Measures of Central Tendency

Reading Math

A set of *univariate* data involves a single variable. The data set in Example 1 involves only one variable—number of days—so it is univariate.

Find the mean, median, and mode of the data.

Number of days from mailing to delivery: 6, 4, 3, 4, 2, 5, 3, 4, 5, 2, 3, 4

Mean: $\dfrac{6+4+3+4+2+5+3+4+5+2+3+4}{12} = \dfrac{45}{12} = 3.75$ days

Median: 2 2 3 3 3 4 ¦ 4 4 4 5 5 6 $\quad \dfrac{4+4}{2} = 4$ days

Mode: The most common result is 4 days.

CHECK IT OUT! **Find the mean, median, and mode of each data set.**

1a. $\{6, 9, 3, 8\}$ **1b.** $\{2, 5, 6, 2, 6\}$

A *weighted average* of a data set gives greater importance, or weight, to some values in the set than to others. To find a weighted average, multiply each value by its weight. Then divide the sum of these products by the sum of the weights.

Suppose a teacher grades students' work in a class by using a weighted average in which homework has a weight of 30%, tests have a weight of 40%, and the final exam has a weight of 30%. Mia has a homework score of 84, a test score of 88, and a final exam score of 91.

$$\text{Mia's weighted average} = \frac{84(0.30) + 88(0.40) + 91(0.30)}{0.30 + 0.40 + 0.30} = \frac{87.7}{1.00} = 87.7$$

For an experiment with numerical outcomes, the **expected value** is the weighted average of the possible outcomes. The weight for each outcome is its probability.

EXAMPLE 2 Finding Expected Value

The probability distribution for the number of games played in each World Series for the years 1923–2004 is given below. Find the expected number of games in a World Series.

Helpful Hint

The sum of all of the probabilities in a probability distribution is 1. In Example 2,

$\frac{5}{27} + \frac{5}{27} + \frac{6}{27} + \frac{11}{27} = 1$

World Series Games				
Number of Games n in World Series	4	5	6	7
Probability of n Games	$\frac{5}{27}$	$\frac{5}{27}$	$\frac{6}{27}$	$\frac{11}{27}$

$$\text{expected value} = 4\left(\frac{5}{27}\right) + 5\left(\frac{5}{27}\right) + 6\left(\frac{6}{27}\right) + 7\left(\frac{11}{27}\right)$$ *Use the weighted average.*

$$= \frac{20}{27} + \frac{25}{27} + \frac{36}{27} + \frac{77}{27} = \frac{158}{27} \approx 5.85$$ *Simplify.*

The expected number of games in a World Series is about 5.85.

CHECK IT OUT!

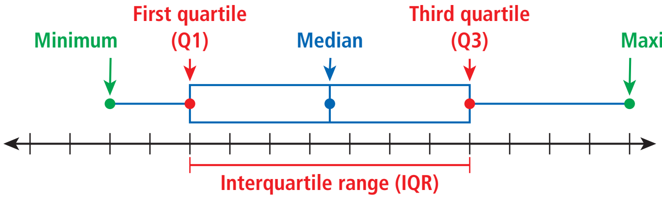
2. The probability distribution of the number of accidents in a week at an intersection, based on past data, is given below. Find the expected number of accidents for one week.

Number of accidents n	0	1	2	3
Probability of n accidents	0.75	0.15	0.08	0.02

A *box-and-whisker plot* shows the spread of a data set. It displays 5 key points: the **minimum** and **maximum** values, the **median**, and the **first** and **third quartiles**.

Reading Math

The first quartile is sometimes called the 25th percentile, and the third quartile is sometimes called the 75th percentile.

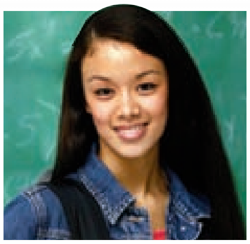


The quartiles are the medians of the lower and upper halves of the data set. If there are an odd number of data values, do not include the median in either half.

The *interquartile range*, or IQR, is the difference between the 1st and 3rd quartiles, or Q3 – Q1. It represents the middle 50% of the data.

Student to Student

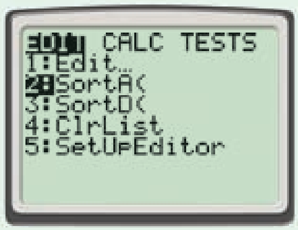
Box-and-Whisker Plots



Jenny Rivera
Lincoln High School

I know I have to sort the data before I can make a box-and-whisker plot. I enter the data into my graphing calculator list. Then I use the STAT sort feature to sort the list in ascending order.

Other times, I'll use a spreadsheet and sort the data there.



EXAMPLE 3 Making a Box-and-Whisker Plot and Finding the Interquartile Range

Make a box-and-whisker plot of the data. Find the interquartile range.

$\{5, 3, 9, 2, 14, 6, 8, 9, 5, 8, 13, 3, 15, 7, 4, 2, 12, 8\}$

Step 1 Order the data from least to greatest.

2, 2, 3, 3, 4, 5, 5, 6, 7, 8, 8, 8, 9, 9, 12, 13, 14, 15

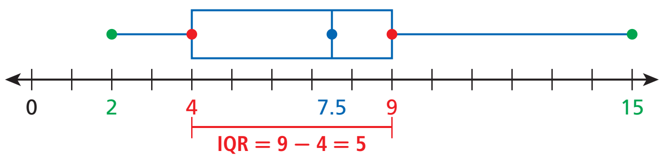
Step 2 Find the minimum, maximum, median, and quartiles.

$\boxed{2, 2, 3, 3, 4, 5, 5, 6, 7}, \boxed{8, 8, 8, 9, 9, 12, 13, 14, 15}$

Minimum | First quartile | Median 7.5 | Third quartile | Maximum

Step 3 Draw a box-and-whisker plot.

Draw a number line, and plot a point above each of the five values. Then draw the box from the first quartile to the third quartile with a line segment through the median. Draw whiskers from the box to the minimum and maximum.



The interquartile range is 5, the length of the box in the diagram.

CHECK IT OUT!

3. Make a box-and-whisker plot of the data. Find the interquartile range.
$\{13, 14, 18, 13, 12, 17, 15, 12, 13, 19, 11, 14, 14, 18, 22, 23\}$

The data sets $\{19, 20, 21\}$ and $\{0, 20, 40\}$ have the same mean and median, but the sets are very different. The way that data are spread out from the mean or median is important in the study of statistics.

Reading Math

The symbol commonly used to represent the mean is $\bar{x}$, or "*x* bar." The symbol for standard deviation is the lowercase Greek letter *sigma*, σ.

A *measure of variation* is a value that describes the spread of a data set. The most commonly used measures of variation are the *range*, the interquartile range, the *variance*, and the *standard deviation*.

The **variance**, denoted by σ^2, is the average of the squared differences from the mean. **Standard deviation**, denoted by σ, is the square root of the variance and is one of the most common and useful measures of variation.

Low standard deviations indicate data that are clustered near the measures of central tendency, whereas high standard deviations indicate data that are spread out from the center.

Finding Variance and Standard Deviation
Step 1. Find the mean of the data, $\bar{x}$.
Step 2. Find the difference between the mean and each data value, and square it.
Step 3. Find the variance, σ^2, by adding the squares of all of the differences from the mean and dividing by the number of data values.
Step 4. Find the standard deviation, σ, by taking the square root of the variance.

EXAMPLE 4 Finding the Mean and Standard Deviation

The data represent the number of milligrams of a substance in a patient's blood, found on consecutive doctor visits. Find the mean and the standard deviation of the data.

$\{14, 13, 16, 9, 3, 7, 11, 12, 11, 4\}$

Step 1 Find the mean.

$$\bar{x} = \frac{14 + 13 + 16 + 9 + 3 + 7 + 11 + 12 + 11 + 4}{10} = 10$$

Step 2 Find the difference between the mean and each data value, and square it.

Data Value x	14	13	16	9	3	7	11	12	11	4
$x - \bar{x}$	4	3	6	−1	−7	−3	1	2	1	−6
$(x - \bar{x})^2$	16	9	36	1	49	9	1	4	1	36

Step 3 Find the variance.

$$\sigma^2 = \frac{16 + 9 + 36 + 1 + 49 + 9 + 1 + 4 + 1 + 36}{10} = 16.2$$

Find the average of the last row of the table.

Step 4 Find the standard deviation.

$$\sigma = \sqrt{16.2} \approx 4.02$$

The standard deviation is the square root of the variance.

The mean is 10 mg, the standard deviation is about 4.02 mg.

CHECK IT OUT!

4. Find the mean and standard deviation for the data set of the number of elevator stops for several rides.

$\{0, 3, 1, 1, 0, 5, 1, 0, 3, 0\}$

Remember!

Enter lists in the graphing calculator by pressing STAT and choosing **1:Edit...**

An **outlier** is an extreme value that is much less than or much greater than the other data values. Outliers have a strong effect on the mean and standard deviation. If an outlier is the result of measurement error or represents data from the wrong population, it is usually removed. There are different ways to determine whether a value is an outlier. One is to look for data values that are more than 3 standard deviations from the mean.

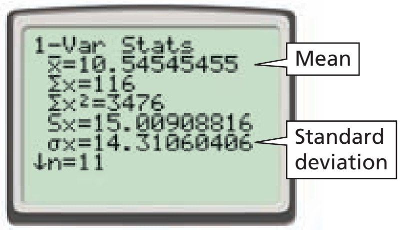
EXAMPLE 5 Examining Outliers

The number of electoral votes in 2004 for 11 western states are shown. Find the mean and the standard deviation of the data. Identify any outliers, and describe how they affect the mean and the standard deviation.

Step 1 Enter the data values into list **L1** on a graphing calculator.

Step 2 Find the mean and standard deviation.

On the graphing calculator, press STAT, scroll to the **CALC** menu, and select **1:1-Var Stats**.

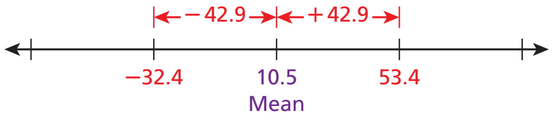


The mean is about 10.5, and the standard deviation is about 14.3.

Step 3 Identify the outliers.

Look for data values that are more than 3 standard deviations away from the mean in either direction.

Three standard deviations is about $3(14.3) = 42.9$.



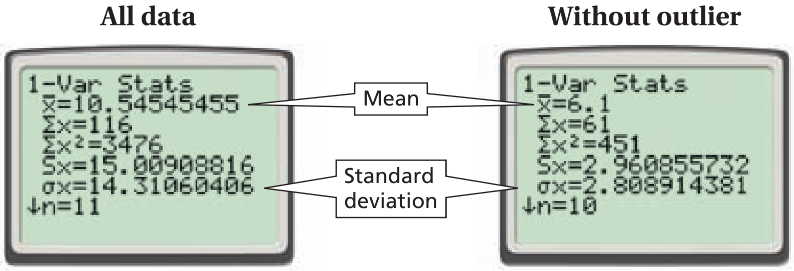
Values 42.9 units below the mean are negative and would not make sense in the problem (a state cannot have a negative number of electoral votes).

Values greater than 53.4 are outliers, so 55, the number of California electoral votes, is an outlier.

Check $\dfrac{|\text{value} - \text{mean}|}{\text{standard deviation}} = \dfrac{|55 - 10.5|}{14.3} \approx 3.1$

55 is about 3.1 standard deviations from the mean, so it is an outlier.

Step 4 Remove the outlier to see the effect that it has on the mean and standard deviation.



The outlier in the data set causes the mean to increase from 6.1 to $\approx$ 10.5 and the standard deviation to increase from $\approx$ 2.8 to $\approx$ 14.3.

CHECK IT OUT!

5. In the 2003 and 2004 American League Championship Series, the New York Yankees scored the following numbers of runs against the Boston Red Sox: 2, 6, 4, 2, 4, 6, 6, 10, 3, 19, 4, 4, 2, 3. Identify the outlier, and describe how it affects the mean and standard deviation.

THINK AND DISCUSS

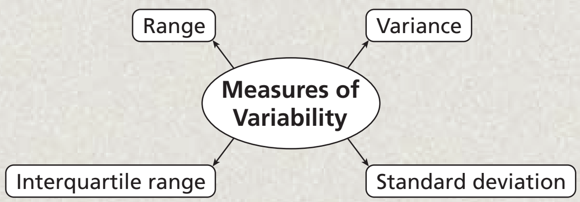
1. Describe the effect of adding a constant to each data value on the mean.

2. Describe the effect of adding a constant to each data value on the standard deviation.

3. What effect does doubling the variance have on the standard deviation?

Know it! Note

4. GET ORGANIZED Copy and complete the graphic organizer. In each box, define and give an example of each measure.



11-5 Exercises

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GUIDED PRACTICE

1. **Vocabulary** A measure of variation, or spread of a data set, is the __?__. (*variance* or *expected value*)

SEE EXAMPLE 1 p. 828

Find the mean, median, and mode of each data set.

2. $\{5, 7, 4, 7, 6, 7\}$
3. $\{2, 4, 4, 6, 6, 6, 7, 8\}$
4. $\{10, 14, 18, 22, 26\}$

SEE EXAMPLE 2 p. 829

5. Find the expected value of the prize.

Prize Giveaway						
Value	\$0	\$1	\$5	\$20	\$100	\$1000
Probability	0.9359	0.05	0.01	0.003	0.001	0.0001

SEE EXAMPLE 3 p. 830

Make a box-and-whisker plot of the data. Find the interquartile range.

6. $\{3, 5, 2, 2, 8, 9, 1, 11\}$
7. $\{2, 4, 1, 4, 2, 2, 7, 4\}$
8. $\{33, 34, 31, 27, 22\}$

SEE EXAMPLE 4 p. 831

Find the variance and standard deviation.

9. $\{3, 3, 4, 5, 5\}$
10. $\{10, 12, 14, 15, 18, 20, 23\}$
11. $\{7, 14, 21, 28, 35, 42\}$

SEE EXAMPLE 5 p. 831

12. **Measurement** Students in a fourth-grade class were asked to measure the widths of their desks in centimeters. They recorded the following measures: 49, 50, 49, 48, 49, 19, 50, 49, 48, 50, 49, and 50. Identify the outlier, and describe how it affects the mean and the standard deviation.

PRACTICE AND PROBLEM SOLVING

Independent Practice

For Exercises	See Example
13–15	1
16	2
17–19	3
20–22	4
23	5

Extra Practice
Skills Practice p. S25
Application Practice p. S42

Find the mean, median, and mode of each data set.

13. $\{4, 16, 25, 9, 36, 49\}$
14. $\{1, 7, 7, 2, 3, 14, 127, 8\}$
15. $\{5, 10, 15, 20, 25\}$

16. Find the expected number of heads.

Three Coins Are Tossed				
Number of Heads	0	1	2	3
Probability	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

Make a box-and-whisker plot of the data. Find the interquartile range.

17. $\{12, 15, 12, 6, 18, 29\}$
18. $\{2, 2, 3, 8, 2, 8, 2, 42\}$
19. $\{3, 4, 3, 1, 2\}$

Find the variance and standard deviation.

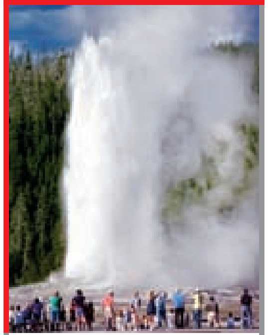
20. $\{4, 4, 4, 4, 5\}$
21. $\{8, 12, 30, 35, 48, 50, 62\}$
22. $\{14, 26, 40, 52\}$

23. **Football** The 2004 Cincinnati Bengals scored 24, 16, 9, 17, 17, 23, 20, 26, 17, 14, 58, 27, and 28 points in their first 13 games. Find the mean and the standard deviation of the data. Identify the outlier, and describe how it affects the mean and the standard deviation.

24. **Critical Thinking** Write a set of data in which neither the mean nor the median are data values.

25. **Shopping** You are at a store and want to purchase an accurate room thermometer. One says 73°F, six say 75°F, eight say 76°F, and one says 37°F. Which measure of central tendency would you be least likely to use to pick a thermometer? Explain.

Geology

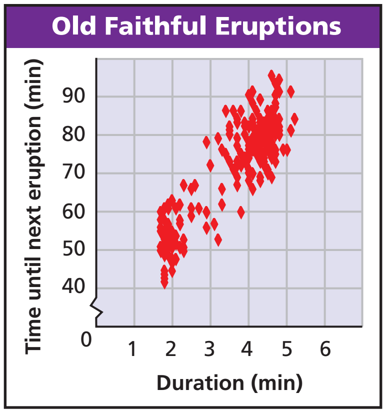


The Old Faithful Geyser at Yellowstone National Park can send 8500 gallons of boiling water to a height of 185 feet.

For a data set with a first quartile of Q1 and a third quartile of Q3, a value less than Q1 – 1.5(IQR) or greater than Q3 + 1.5(IQR) may be considered to be an outlier. Use this rule to identify any outliers in each data set. Show your work.

26. $\{2, 3, 4, 5, 5, 25\}$ **27.** $\{91, 90, 79, 15, 82, 90, 88\}$ **28.** $\{1, 36, 34, 33, 35, 92\}$

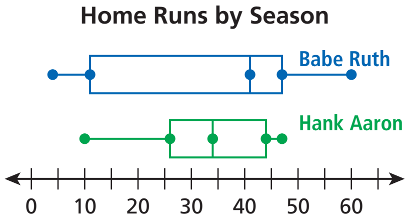
Geology **Use the graph of 222 eruptions of the Old Faithful Geyser for Exercises 29 and 30.**



29. The duration has a mean of 3.6 min and a standard deviation of 1.1 min. What duration time intervals would be outliers? Describe any outliers for duration on the graph.

30. The time between eruptions has a mean of 71 min and a standard deviation of 12.8 min. What time intervals would be outliers? Describe any outliers for time intervals on the graph.

Estimation **Use the box-and-whisker plots for Exercises 31–34.**



31. Which player hit the most home runs in a season? By approximately how many home runs did he do so?

32. Which player had the greater median number of home runs? Estimate how much greater.

33. Estimate the interquartile range for both players.

34. Which data set has the smaller standard deviation? Explain.

35. You have a 0.1% chance of winning \$500 and a 99.9% chance of losing \$1. What is the expected value of your gain? (*Hint*: The two possible outcomes for this "experiment" are + 500 and −1.)

36. Suppose that you have a 10% chance of winning \$100, a 30% chance of losing \$2, and a 60% chance of breaking even. What is the expected value?

37. **ERROR ANALYSIS** Two students attempt to find the standard deviation of 4, 6, 8, and 10. Which is incorrect? Explain the error.

A

$7 - 4 = 3 \rightarrow 9$
$7 - 6 = 1 \rightarrow 1$
$7 - 8 = -1 \rightarrow 1$
$7 - 10 = -3 \rightarrow \underline{9}$
$20 \div 4 = 5$
$\sqrt{5} \approx 2.24$

B

$7 - 4 = 3 \rightarrow 3$
$7 - 6 = 1 \rightarrow 1$
$7 - 8 = -1 \rightarrow 1$
$7 - 10 = -3 \rightarrow \underline{3}$
$8 \div 4 = 2$
$\sqrt{2} \approx 1.41$

38. **Write About It** Is an expected value always, sometimes, or never a value in the data set? Give an example to justify your answer.

39. **Games** In a game, you multiply the values of two number cubes.

a. What is the expected value of this product?

b. What is the probability that a product is greater than the expected value?

c. What is the probability that a product is less than the expected value?

d. Are the answers to parts **b** and **c** equal? Explain.

MULTI-STEP TEST PREP

40. This problem will prepare you for the Multi-Step Test Prep on page 844. The table shows the total annual precipitation for San Diego, California.

Year	1994	1995	1996	1997	1998
Precipitation (in.)	9.4	17.0	7.3	7.0	16.1
Year	1999	2000	2001	2002	2003
Precipitation (in.)	5.4	6.9	8.5	4.2	9.2

a. Find the mean annual precipitation and the standard deviation.

b. In what years was the precipitation more than one standard deviation from the mean?

c. Find the median and interquartile range for the data.

TEST PREP

41. Which data set would give the smallest standard deviation?

Ⓐ $\{1, 5, 7, 50\}$

Ⓑ $\{2, 10, 102, 110\}$

Ⓒ $\{100, 200, 300, 400\}$

Ⓓ $\{100, 101, 102, 105\}$

42. Which of the following is NOT true about the data sets $\{0, 48, 49, 50, 51, 52, 100\}$ and $\{0, 1, 2, 50, 98, 99, 100\}$?

Ⓕ The means are equal.

Ⓖ The ranges are equal.

Ⓗ The variances are equal.

Ⓙ The medians are equal.

43. The mean score on a test is 50. Which cannot be true?

Ⓐ Half the scores are 0, and half the scores are 100.

Ⓑ The range is 50.

Ⓒ Half the scores are 25, and half the scores are 50.

Ⓓ Every score is 50.

CHALLENGE AND EXTEND

44. A data set has a mean of 4, a median of 3, and a standard deviation of 1.6.

a. Suppose that every value of the data set is multiplied by 5. What is the mean, median, and standard deviation of the new data set?

b. Suppose that 5 is added to every value of the original data set. What is the mean, median, and standard deviation of the new data set?

45. A deck of cards is shuffled. What is the expected number of cards that will be in the same position that they were in originally? (*Hint:* Look at decks of 1, 2, 3, and 4 cards.)

SPIRAL REVIEW

46. **Business** Li was paid \$725 a month plus \$1.75 for every magazine she sold. Li earned \$1425 one month. How many magazines did Li sell? *(Lesson 2-1)*

Find each product. *(Lesson 6-2)*

47. $(2 - x^2)(2x^2 + 5x - 3)$

48. $4xy^2(x^2y + 3x^2 - 2y)$

A number cube is rolled. Find each probability. *(Lesson 11-4)*

49. an even number or a 1

50. an odd number or a 4

51. a number divisible by 2 or 6

11-5 Algebra LAB

Collect Experimental Data

You can perform an experiment to generate, collect, organize, and analyze data in order to form mathematical conjectures.

Use with Lesson 11-5

Activity

Make a table of the sum of two number cubes.

Blue cube

Red cube	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Try This

1. Describe any symmetry you notice in the table.
2. Make a probability distribution by using theoretical probabilities.

Sums	2	3	4	5	6	7	8	9	10	11	12
Probability											

3. Find the expected value by using the theoretical probability distribution.
4. Which sum is most likely? least likely?
5. Do any two different sums have the same probability? If so, what are those sums?
6. Roll two number cubes 36 times. Record the results in a table.
7. Make a probability distribution of your data.
8. Find the expected value by using your probability distribution.

Answer the following questions based on your experiment.

9. Which sum was most likely? least likely?
10. Did any two different sums have the same probability? If so, what are those sums?
11. Compare your results with the theoretical results.
12. Combine the results of your experiment with those of other students. How do the experimental results of the group compare with your results? with the theoretical results?

11-6 Binomial Distributions

Objectives

Use the Binomial Theorem to expand a binomial raised to a power.

Find binomial probabilities and test hypotheses.

Vocabulary

Binomial Theorem
binomial experiment
binomial probability

Why learn this?

You can use binomial distributions to determine your chances of winning a marketing contest. (See Example 3.)

You used Pascal's triangle to find binomial expansions in Lesson 6-2. The coefficients of the expansion of $(x + y)^n$ are the numbers in Pascal's triangle, which are actually combinations.

Pascal's Triangle	Combinations (Binomial Coefficients)	Binomial Expansion
1	${}_0C_0$	$(x+y)^0 = 1$
1 1	${}_1C_0 \quad {}_1C_1$	$(x+y)^1 = x + y$
1 2 1	${}_2C_0 \quad {}_2C_1 \quad {}_2C_2$	$(x+y)^2 = x^2 + 2xy + y^2$
1 3 3 1	${}_3C_0 \quad {}_3C_1 \quad {}_3C_2 \quad {}_3C_3$	$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

The pattern in the table can help you expand any binomial by using the **Binomial Theorem**.

Know it! Note

Binomial Theorem

For any whole number n,

$$(x+y)^n = {}_nC_0x^ny^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \cdots + {}_nC_{n-1}x^1y^{n-1} + {}_nC_nx^0y^n$$

EXAMPLE 1

Expanding Binomials

Use the Binomial Theorem to expand each binomial.

A $(x+y)^4$

The sum of the exponents for each term is 4.

$$\begin{aligned}(x+y)^4 &= {}_4C_0x^4y^0 + {}_4C_1x^3y^1 + {}_4C_2x^2y^2 + {}_4C_3x^1y^3 + {}_4C_4x^0y^4\\ &= 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4\\ &= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4\end{aligned}$$

B $(3p+q)^3$

$$\begin{aligned}(3p+q)^3 &= {}_3C_0(3p)^3q^0 + {}_3C_1(3p)^2q^1 + {}_3C_2(3p)^1q^2 + {}_3C_3(3p)^0q^3\\ &= 1 \cdot 27p^3 \cdot 1 + 3 \cdot 9p^2q + 3 \cdot 3pq^2 + 1 \cdot 1q^3\\ &= 27p^3 + 27p^2q + 9pq^2 + q^3\end{aligned}$$

Remember!

In the expansion of $(x+y)^n$, the powers of x decrease from n to 0 and the powers of y increase from 0 to n. Also, the sum of the exponents is n for each term. (Lesson 6-2)

CHECK IT OUT! **Use the Binomial Theorem to expand each binomial.**

1a. $(x-y)^5$

1b. $(a+2b)^3$

A **binomial experiment** consists of n independent trials whose outcomes are either successes or failures; the probability of success p is the same for each trial, and the probability of failure q is the same for each trial. Because there are only two outcomes, $p + q = 1$, *or* $q = 1 - p$. Below are some examples of binomial experiments:

Experiment	Success	Failure	P(success)	P(failure)
10 flips of a coin	Heads	Tails	$p = 0.5$	$q = 1 - p = 0.5$
100 rolls of a number cube	Roll a 3.	Roll any other number.	$p = \frac{1}{6}$	$q = \frac{5}{6}$

Suppose the probability of being left-handed is 0.1 and you want to find the probability that 2 out of 3 people will be left-handed. There are ${}_3C_2$ ways to choose the two left-handed people: LLR, LRL, and RLL. The probability of each of these occurring is 0.1(0.1)(0.9). This leads to the following formula.

Know it! Note

Binomial Probability

If a binomial experiment has n trials in which p is the probability of success and q is the probability of failure in any given trial, then the **binomial probability** that there will be exactly r successes is:

$$P(r) = {}_nC_r\, p^r q^{n-r}$$

EXAMPLE 2 Finding Binomial Probabilities

One in 5 boats going through a *slough* at midday will bypass the harbor and head out to sea. Four boats are going through the slough.

A What is the probability that exactly 2 boats will head out to sea?

The probability that a boat will head out to sea is $\frac{1}{5}$, or 0.2.

$P(r) = {}_nC_r p^r q^{n-r}$

$P(2) = {}_4C_2(0.2)^2(0.8)^{4-2}$ *Substitute 4 for n, 2 for r, 0.2 for p, and 0.8 for q.*

$= 6(0.04)(0.64) = 0.1536$

The probability that exactly 2 of the boats will head out to sea is about 15.4%.

B What is the probability that at least 2 boats will head out to sea?

At least 2 boats is the same as exactly 2, 3, or 4 boats heading out to sea.

$P(2) + P(3) + P(4)$

$0.1536 + {}_4C_3(0.2)^3(0.8)^{4-3} + {}_4C_4(0.2)^4(0.8)^{4-4}$

$0.1536 + 0.0256 + 0.0016 = 0.1808$

The probability that at least 2 boats will head out to sea is about 18.1%.

CHECK IT OUT!

2a. Students are assigned randomly to 1 of 3 guidance counselors. What is the probability that Counselor Jenkins will get 2 of the next 3 students assigned?

2b. Ellen takes a multiple-choice quiz that has 5 questions, with 4 answer choices for each question. What is the probability that she will get at least 2 answers correct by guessing?

EXAMPLE 3 Problem-Solving Application

PROBLEM SOLVING

Vince buys 10 juice drinks. What is the probability that he will get at least 2 prizes?

Sweepstakes Prizes Chances of Winning	
Free drink	1 in 5
Water bottle	1 in 22
T-shirt	1 in 250
Music player	1 in 100,000
Car	1 in 20,000,000
Any prize	1 in 4

1 Understand the Problem

The **answer** will be the probability that Vince will get at least 2 prizes.

List the important information:

- Vince buys 10 juice drinks.
- The binomial probability that each bottle wins a prize is $\frac{1}{4}$.

2 Make a Plan

The direct way to solve the problem is to calculate $P(2) + P(3) + P(4) + \cdots + P(10)$.

An easier way is to use the complement. "Getting 0 or 1 prize" is the complement of "getting at least 2 prizes." Find this probability, and then subtract the result from 1.

3 Solve

Step 1 Find $P(0 \text{ or } 1 \text{ prize})$.

$$\begin{aligned} & \quad\; P(0) \qquad\qquad + \qquad\qquad P(1) \\ &= {}_{10}C_0\,(0.25)^0(0.75)^{10-0} + {}_{10}C_1\,(0.25)^1(0.75)^{10-1} \\ &= 1(1)(0.75)^{10} + 10(0.25)(0.75)^9 \\ &\approx 0.0563 + 0.1877 \\ &\approx 0.2440 \end{aligned}$$

Step 2 Use the complement to find the probability.

$1 - 0.2440$ *Subtract from 1.*

≈ 0.7560

The probability that Vince will get at least 2 prizes is about 0.76.

4 Look Back

The answer is reasonable, as the expected number of winners is $\frac{1}{4}$ of 10, $= 2.5$, which is greater than 2. So the probability that Vince will get at least 2 prizes should be greater than 0.5.

CHECK IT OUT!

3a. Wendy takes a multiple-choice quiz that has 20 questions. There are 4 answer choices for each question. What is the probability that she will get at least 2 answers correct by guessing?

3b. A machine has a 98% probability of producing a part within acceptable tolerance levels. The machine makes 25 parts an hour. What is the probability that there are 23 or fewer acceptable parts?

THINK AND DISCUSS

1. Describe and explain the sum of p and q for a binomial experiment.
2. Tell what three expressions are multiplied to find the probability that there will be r successes in a binomial experiment of n trials.

Know it! Note

3. **GET ORGANIZED** Copy and complete the graphic organizer. Solve each problem that you include.

Binomial Experiments	
Probability	**Example**
Probability of r successes in n trials	
Probability of at least r successes	
Probability of at most r successes	
Probability using a complement	

11-6 Exercises

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GUIDED PRACTICE

1. **Vocabulary** There are __?__ possible outcomes in each trial of a *binomial experiment*.

SEE EXAMPLE 1 p. 837

Use the Binomial Theorem to expand each binomial.

2. $(x + 3)^4$
3. $(3x + 5)^3$
4. $(p - 2)^6$
5. $(x + y)^6$

SEE EXAMPLE 2 p. 838

6. **School** The principal will randomly choose 6 students from a large school to represent the school in a newspaper photograph. The probability that a chosen student is an athlete is 30% (assume that this doesn't change). What is the probability that 4 athletes are chosen? What is the probability that at least 4 athletes are chosen?
7. **Shopping** Wilma bought 4 boxes of Crunch-A-Lot cereal. One out of every 5 boxes has a coupon for a free box of Crunch-A-Lot. What is the probability that Wilma got 3 coupons? What is the probability that Wilma got at least 2 coupons?

SEE EXAMPLE 3 p. 839

8. **Manufacturing** In a manufacturing plant, there is a 2% chance that a stamp will be placed on a box upside down. The plant shipped 30 boxes today. What is the probability that at least 2 of the boxes have an upside-down stamp?

PRACTICE AND PROBLEM SOLVING

Use the Binomial Theorem to expand each binomial.

9. $(y + 5)^4$
10. $(2m - 1)^3$
11. $(4 + 3x)^5$
12. $(2a + 3c)^3$

13. **Civil Rights** In a survey of more than 100,000 high school students in 2004 by researchers at the University of Connecticut, 83% agreed with the statement "People should be allowed to express unpopular opinions." If 8 students are selected at random, what is the probability that at least 6 agree with the statement?

Independent Practice	
For Exercises	See Example
9–12	1
13–14	2
15–16	3

Extra Practice
Skills Practice p. S25
Application Practice p. S42

14. Five marbles are randomly selected with replacement. The probability that a black marble is chosen is 15%. What is the probability that 2 marbles are black? What is the probability that at least 2 marbles are black?

15. Genetics A woman is expecting triplets. What is the probability that there are 2 girls and 1 boy? What is the probability that all 3 babies are girls?

16. Botany A tree has a 25% chance of flowering. In a random sample of 15 trees, what is the probability that at least 4 develop flowers?

Use the Binomial Theorem to expand each binomial.

17. $(x - y)^5$ **18.** $(c + 6)^3$ **19.** $(4k - 1)^4$ **20.** $(p + q)^7$

Evaluate $P(r) = {}_nC_r\, p^r q^{n-r}$, where $q = 1 - p$.

21. $p = 0.8, n = 3, r = 2$ **22.** $p = 0.5, n = 5, r = 1$ **23.** $p = \frac{1}{3}, n = 4, r = 2$

24. Travel A small airline overbooks flights on the assumption that several passengers will not show up. Suppose that the probability that a passenger shows up is 0.91. What is the probability that a 20-seat flight with 22 tickets sold will be able to seat all passengers who arrive?

25. Genetics A hedgehog has a litter of 4. What is the probability that all 4 are male? What is the probability that at least 3 are male?

Find each probability when a fair coin is tossed 10 times.

26. more than 7 heads **27.** at least 2 heads **28.** exactly 5 heads

29. Quality Control An auto part has a 95% chance of being made within its tolerance level and a 5% chance of being pulled as defective. What is the probability that in a box of 8 parts, no more than 1 is defective?

30. Graphing Calculator The **randBin** function simulates a binomial experiment and reports the number of successes. To simulate a binomial experiment with $n = 6$ and $p = 0.3$ five times, press MATH, move to **PRB**, select **randBin(** and enter 6, 0.3, and 5, separated by commas.

a. Simulate a binomial experiment with $n = 5$ and $p = 0.8$ five times.

b. Use the formula to find the probability of at least 4 successes.

c. How do your simulation results compare?

31. Multi-Step For $P = 0.8$ and $n = 10$, use a calculator to find the binomial probabilities for $r = 0$ to $r = 10$. Round to the nearest hundredth. Construct a bar graph of the probabilities. Describe the shape of the graph. How does the graph relate to the expected value?

32. Critical Thinking Which is more likely, a family with 4 children of 2 girls and 2 boys or a family of 4 children with 3 of one gender and 1 of the other? Explain.

MULTI-STEP TEST PREP

33. This problem will prepare you for the Multi-Step Test Prep on page 844.

Based on historical data, the expected number of rainy days in San Antonio, Texas, during a calendar year is 82. Assume that rainy days are independent events.

a. What is the probability that there will be rain on any given day in San Antonio?

b. What is the probability that there will be exactly 3 rainy days during any given week?

c. What is the probability that there will be at least 3 rainy days during any given week?

34. There are 10 marbles in a bag. Half are striped, and half are not striped. Explain why choosing 3 marbles without replacement and noting whether they are striped does not fit the definition of a binomial experiment.

LINK

Math History

The Granger Collection, New York

The trials in a binomial experiment are often called Bernoulli trials after Italian mathematician Jacob Bernoulli (1654–1705).

35. **Air Travel** In 2003, 20.46% of all direct flights from Dallas/Fort Worth to Los Angeles International Airport were delayed. Kelly flew that route 4 times and was on a delayed flight 3 times. What is the probability that she would have been on a delayed flight at least 3 times?

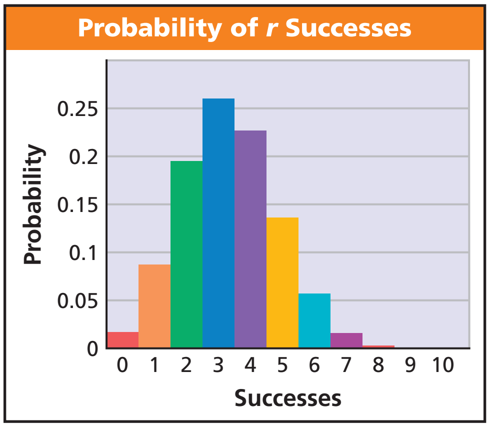
36. **Games** As the ball drops, it has an equal chance of making a left turn or right turn at each peg.

   a. What is the probability of a home run?
   b. What is the probability of an out?
   c. What is the probability of a hit (a single, double, triple, or home run)?
   d. How are the answers to parts **b** and **c** related?

37. **Pets** A survey showed that 45% of dog owners take their dog with them on vacation. If 5 dog owners go on vacation, what is the probability that fewer than 3 take their dog?

38. **Write About It** Describe a situation for which it would be beneficial to use the complement to find binomial probabilities.

Estimation **Use the graph for Exercises 39 and 40. The graph shows the probability of *r* successes in 10 trials of a binomial experiment.**



39. Estimate the probability of 2 or fewer successes.

40. Estimate the binomial probability p. Explain how you arrived at your answer.

TEST PREP

41. Which of the following is NOT true about a binomial experiment?
   (A) The outcomes are either successes or failures.
   (B) The trials are dependent.
   (C) The probability of success is constant.
   (D) The trials are identical.

42. In a binomial experiment with 2 trials and a probability of success on each trial of 40%, what is the probability of exactly 1 success?
   (F) 16% (G) 36% (H) 48% (J) 52%

43. In a binomial experiment, the probability of success is 20%. Which gives the probability of 3 successes in 5 trials?
   (A) $3(0.2)^3(0.8)^2$ (B) $10(0.2)^3(0.8)^2$ (C) $3(0.2)^2(0.8)^3$ (D) $10(0.2)^2(0.8)^3$

44. **Gridded Response** A part has a 4% chance of being discarded for imperfections. Out of 10 randomly selected parts, what is the probability that no more than 1 has an imperfection? Round to the nearest whole percent.

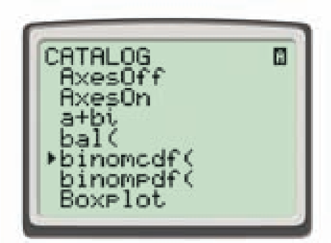
45. **Short Response** About 18.8% of the people in the United States have one of the 100 most common last names. What is the probability that in a group of 10 randomly-selected people, 3 or more have one of these names?

CHALLENGE AND EXTEND

46. **Genetics** There is about a 0.1 probability that a person is left-handed. There are 650 people in an auditorium.
    a. What is the expected number of left-handed people in the auditorium? Explain.
    b. The standard deviation for a binomial experiment with n trials is given by $\sqrt{npq}$. Describe the number of left-handed people that you would expect in the auditorium as an interval within 1 standard deviation of the expected number.

47. Find each probability. Which is greater?
    a. rolling at least one 1 in 6 rolls of a die
    b. rolling at least two 1's in 12 rolls of a die

48. **Calculator** The **binomcdf** function, found in 2nd CATALOG 0, computes the cumulative probability of r successes in a binomial experiment of n trials with a probability of success p. To compute the probability of at most 3 successes in a binomial experiment with $n = 6$ and $p = 0.3$, use **binomcdf**, enter 6, 0.3, and 3, separated by commas, and press ENTER. Use the **binomcdf** function to find the probability of *at least* 4 successes in a binomial experiment of 20 trials with probability of success 0.4.



49. Show why any number ${}_{n+1}C_{r+1}$ in Pascal's triangle is the sum of the two numbers above it, ${}_{n}C_{r}$ and ${}_{n}C_{r+1}$ where r is not equal to 0 or n, and $n > 1$.

50. **Bowling** A bowler has a 0.4 probability of making exactly 1 strike in 2 frames, either in the first frame or the second frame. Assume that the bowler's probability p of getting a strike is the same for any frame.
    a. Write an equation and solve for p.
    b. Find the probability that the bowler makes strikes in both frames.

SPIRAL REVIEW

For each function, evaluate $f(-3)$, $f(0)$, and $f(2)$. *(Lesson 1-7)*

51. $f(x) = -x^2 + 2x - 4$

52. $f(x) = (-x)^2 - 3x + 1$

Determine whether y is an exponential function of x. If so, use exponential regression to find a function that models the data. *(Lesson 7-8)*

53.

x	1	2	3	4	5
y	1.4	2.6	3.8	5.0	6.2

54.

x	1	2	3	4	5
y	10	22	36	52	70

Find the mean, median, and mode of each data set. *(Lesson 11-5)*

55. $\{2, 18, 15, 14, 18\}$

56. $\{6, 13, 9, 7, 6, 4\}$

57. $\{24, 20, 32, 24, 16, 34\}$

58. $\{10, 5, 15, 5, 8\}$

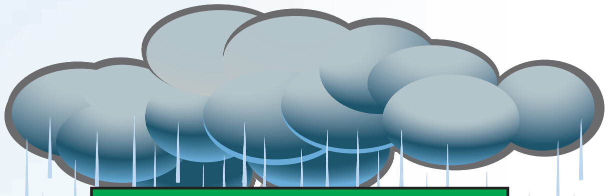
CHAPTER 11

SECTION 11B

MULTI-STEP TEST PREP

Data Analysis and Statistics

Rain Reign Many people think of Seattle, Washington, as one of the rainiest cities in the United States. The table provides precipitation data for Seattle and Atlanta, Georgia, over a 10-year period. By analyzing this data set, you can decide for yourself whether Seattle deserves its soggy reputation.



Annual Precipitation (in.)

Year	Seattle	Atlanta
1994	34.8	60.0
1995	42.6	52.8
1996	50.7	44.6
1997	43.3	51.7
1998	44.1	46.2
1999	42.1	38.9
2000	28.7	35.6
2001	37.6	38.4
2002	31.4	47.6
2003	41.5	52.9

1. Find the mean annual precipitation and the standard deviation for Seattle and for Atlanta.

2. For which city do the data cluster more closely around the mean?

3. Find the interquartile range for Seattle and for Atlanta.

4. For which city do the data cluster more closely around the median?

5. During a calendar year, the expected number of rainy days in Atlanta is 115. Find the probability that it will rain on any given day. Then find the probability that it will rain there on at least 2 days during any given week.

6. Based on your findings, why do you think Seattle, rather than Atlanta, has a reputation as a rainy city?

READY TO GO ON?

CHAPTER 11

SECTION 11B

Quiz for Lessons 11-5 Through 11-6

11-5 Measures of Central Tendency and Variation

1. Mr. Ortega took the following number of sick days per year for the last 5 years: 4, 2, 6, 3, 2. Find the mean, median, and mode of the data set.
2. The probability distribution for the number of defects in a shipment of alarm clocks, based on past data, is given below. Find the expected number of defects in a shipment of alarm clocks.

Number of Defects, *n*	0	1	2	3	4
Probability of *n* Defects	0.82	0.11	0.04	0.02	0.01

3. Make a box-and-whisker plot of the data. Find the interquartile range. Ages of employees at a movie theater: 17, 23, 18, 22, 45, 28, 21, 25
4. The lengths of fish caught, in inches, during one fishing trip are given. Find the lengths within 1 standard deviation of the mean.

   Lengths of fish caught: 14, 28, 16, 20, 22, 33, 12, 30, 30, 25

The data set shows the amount of money, rounded to the nearest dollar, spent by 20 consecutive shoppers at a home-improvement store.

35, 18, 49, 55, 280, 29, 42, 61, 19, 80, 33, 45, 67, 28, 71, 37, 48, 50, 31, 22

5. Find the mean and standard deviation of the data.
6. Identify the outlier, and describe how it affects the mean and standard deviation.

11-6 Binomial Distributions

7. Use the Binomial Theorem to expand $(m - 2n)^3$.

The spinner shown is spun 10 times.

8. What is the probability that the spinner will land in the blue area exactly 5 times?
9. What is the probability that the spinner will land in the blue area at least 3 times?

A multiple-choice quiz has 5 questions. Each question has 3 possible answers. A student guesses the answer to each question. Find each probability.

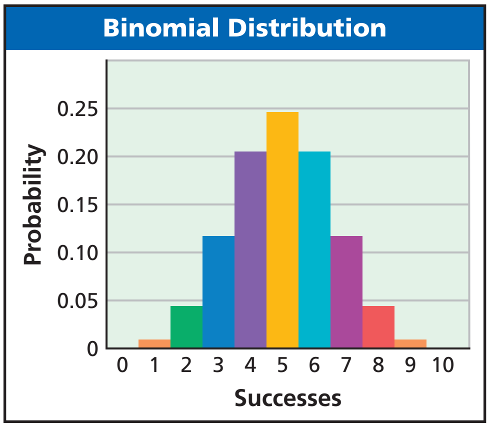
10. The student answers all 5 questions correctly.
11. The student answers exactly 1 question correctly.
12. The student answers all 5 questions incorrectly.
13. The student answers at least 1 question correctly.

EXTENSION Normal Distributions

Objectives
Recognize normally distributed data.

Use the characteristics of the normal distribution to solve problems.

A *random variable* is associated with the possible outcomes of an experiment. For example, if the random variable X is associated with the possible outcomes of rolling a number cube, the possible values of X are 1, 2, 3, 4, 5, and 6.

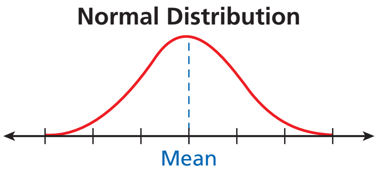


A *probability distribution* shows the probabilities that correspond to the possible values of a random variable. Probability distributions can be based on either discrete or continuous data.

The binomial distributions that you studied in Lesson 11-6 were *discrete probability distributions* because there were a finite number of possible outcomes.

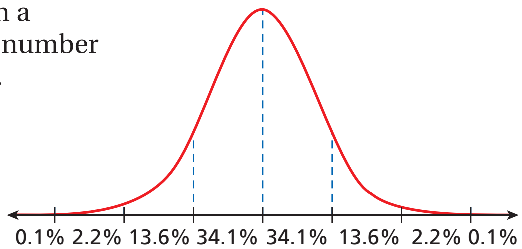
In a *continuous probability distribution*, the outcome can be any real number—for example, the time it takes to complete a task.

You may be familiar with the bell-shaped curve called the *normal curve*. A *normal distribution* is a function of the mean and standard deviation of a data set that assigns probabilities to intervals of real numbers associated with continuous random variables.



Normal Distributions
The probability assigned to a real-number interval is the area under the normal curve in that interval. Because the area under the curve represents probability, the total area under the curve is 1.
The maximum value of a normal curve occurs at the mean.
The normal curve is symmetric about a vertical line through the mean.
The normal curve has a horizontal asymptote at $y = 0$.

The figure shows the percent of data in a normal distribution that falls within a number of standard deviations from the mean.



The diagram shows the following:

- About 68% lie within 1 standard deviation of the mean.
- About 95% lie within 2 standard deviations of the mean.
- More than 99.7% lie within 3 standard deviations of the mean.

EXAMPLE 1 Finding Normal Probabilities

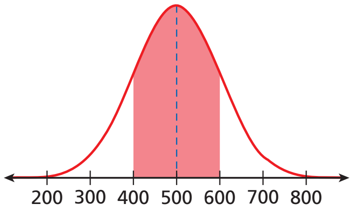
The SAT is designed so that scores are normally distributed with a mean of 500 and a standard deviation of 100.

A **What percent of SAT scores are between 400 and 600?**

Both 400 and 600 are 1 standard deviation from the mean. Use the percents from the figure on the previous page.

$34.1\% + 34.1\% = 68.2\%$

About 68.2% of the scores are between 400 and 600.



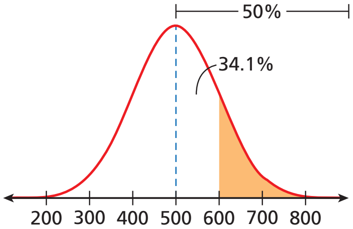
> **Reading Math**
> Each end of a normal distribution is called a *tail*.

B **What is the probability that an SAT score is above 600?**

Because the graph is symmetric, the right side of the graph shows 50% of the data.

$50\% - 34.1\% = 15.9\%$

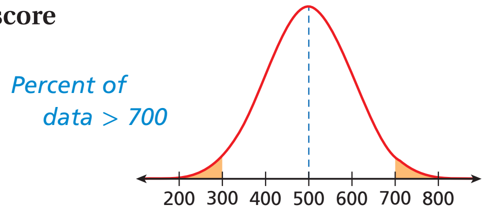
The probability that an SAT score is above 600 is about 0.159, or 15.9%.



C **What is the probability that an SAT score is less than 300 or greater than 700?**

$50\% - (34.1\% + 13.6\%) = 2.3\%$

Because the curve is symmetric, the probability that an SAT score is less than 300 or greater than 700 is about $2(2.3\%)$, or 4.6%.



CHECK IT OUT! **Use the information above to answer the following.**

1. What is the probability that an SAT score is above 300?

EXTENSION Exercises

A standardized test has a mean of 50 and a standard deviation of 4. Find the probability of test scores in the following ranges.

1. between 42 and 58 **2.** below 46 **3.** between 46 and 54

The amount of coffee in a can has a mean of 350 g and a standard deviation of 4 g.

4. What percent of cans have less than 338 g of coffee?

5. What is the probability that a can has between 342 g and 350 g of coffee?

6. What is the probability that a can has less than 342 g or more than 346 g of coffee?

Flight 202's arrival time is normally distributed with a mean arrival time of 4:30 P.M. and a standard deviation of 15 minutes.

7. Find the probability that an arrival time is after 4:45 P.M.

8. Find the probability that an arrival time is between 4:15 P.M. and 5:00 P.M.

CHAPTER 11

Study Guide: Review

Vocabulary

Complete the sentences below with vocabulary words from the list above.

1. If the occurrence of one event affects the probability of the other, then the events are ___?___.

2. A(n) ___?___ can also be called a weighted average.

3. When arranging items, order is important when using a(n) ___?___.

11-1 Permutations and Combinations *(pp. 794–800)*

EXAMPLES

- **If you have 8 vases to choose from, how many ways can you arrange 5 of them on a shelf?**

The order matters, so it is a permutation.

$$_8P_5 = \frac{8!}{(8-5)!} = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$
$$= 8 \cdot 7 \cdot 6 \cdot 5 = 6720$$

There are 6720 ways to arrange the vases.

- **If 7 pizza toppings are available, how many ways can you choose 2 toppings?**

The order does not matter, so it is a combination.

$$_7C_2 = \frac{7!}{2!(7-2)!} = \frac{7 \cdot 6 \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{2 \cdot 1(\cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1})}$$
$$= \frac{42}{2} = 21$$

There are 21 ways to choose the toppings.

EXERCISES

4. How many different 7-digit telephone numbers can be made if the first digit cannot be 7, 8, or 9?

5. From a group of 12 volunteers, a surveyor must choose 5 to complete an advanced survey. How many groups of 5 people can be chosen?

6. In one day, a salesman plans to visit 6 out of 14 companies that are in the neighborhood. How many ways can he plan the visits?

7. How many ways can 7 people arrange themselves inside a van that has 10 seats?

8. The caterer told Kathy that she can choose 3 entrées from the 6 listed on the menu. How many groups of 3 entrées can she choose?

11-2 Theoretical and Experimental Probability *(pp. 802–809)*

EXAMPLES

A paper clip holder has 100 paper clips: 30 are red, 20 are yellow, 25 are green, 15 are pink, and 10 are black. A paper clip is randomly chosen. Find each probability.

- The paper clip is green.

$$P(\text{green}) = \frac{\text{number of green paper clips}}{\text{total number of paper clips}}$$

$$= \frac{25}{100} = \frac{1}{4}$$

- The paper clip is not pink.

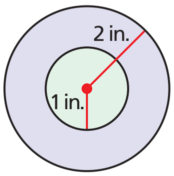
$$P(\text{not pink}) = 1 - P(\text{pink}) = 1 - \frac{15}{100} = \frac{17}{20}$$

- **Carl and Pedro each put their names in a hat for a door prize. Two names will be selected, and there are a total of 40 names in the hat. What is the probability that Carl wins the first prize and Pedro wins the second?**

  The number of outcomes in the sample space is the number of ways that 2 people can be selected from 40 and then ordered.

$$P(\text{Carl, then Pedro}) = \frac{1}{{}_{40}P_2} = \frac{1}{1560}$$

- **A dart is randomly thrown at the dartboard. What is the probability that it lands in the outer ring?**



$$P(\text{outer ring}) = \frac{\text{area of outer ring}}{\text{area of dart board}}$$

$$= \frac{\text{area of large circle} - \text{area of inner circle}}{\text{area of large circle}}$$

$$= \frac{\pi(3)^2 - \pi(1)^2}{\pi(3)^2} = \frac{9\pi - 1\pi}{9\pi} = \frac{8\pi}{9\pi} = \frac{8}{9}$$

- **The table shows the results of 75 tosses of a number cube. Find the experimental probability of rolling a 4.**

1	2	3	4	5	6
10	12	16	15	9	13

$$P(4) = \frac{\text{number of times 4 occurred}}{\text{number of trials}} = \frac{15}{75}$$

$$= \frac{1}{5} = 0.2$$

EXERCISES

Two number cubes are rolled. What is the probability of each event?

9. Sum is 8.

10. Difference is 1.

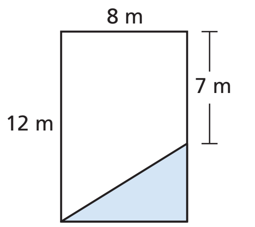
11. Sum is even.

12. Product is less than 30.

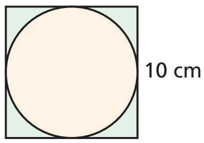
13. The 10-member math team randomly selects 4 representatives to send to a meet. What is the probability that the 4 members chosen are the 4 with the lowest math grades?

14. A 5-digit code is given to all cashiers at a store to let them log onto the cash register. What is the probability that an employee receives a code with all 5 numbers the same?

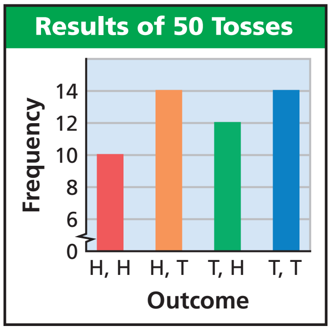
15. Find the probability that a point chosen at random inside the rectangle is in the shaded area.



16. Find the probability that a point chosen at random inside the square is not inside the circle.



The bar graph shows the results of tossing two pennies 50 times. Find the experimental probability of each of the following.



17. tossing 2 heads

18. tossing at least 1 tail

19. not tossing a head

20. tossing exactly 1 tail

Two pennies are tossed. Find the theoretical probability of each of the following.

21. tossing 2 heads

22. tossing at least 1 tail

23. not tossing a head

24. tossing exactly 1 tail

11-3 Independent and Dependent Events *(pp. 811–818)*

EXAMPLES

A bag contains slips of papers with the following numbers: 2, 2, 3, 3, 4, 5, 6. Determine whether the events are independent or dependent, and find the indicated probability.

- You select a 3, keep the paper, and then your friend selects a 3.

  Keeping the paper with the first 3 changes the number of 3's left in the bag for your friend to choose from, so the events are dependent.

  $$P(3, \text{ then } 3) = P(3) \cdot P(3 \mid 3).$$
  $$= \frac{2}{7} \cdot \frac{1}{6} = \frac{2}{42} = \frac{1}{21}$$

- You select a number greater than 3, replace the paper, and then your friend selects a number less than 3.

  Replacing the paper with the number greater than 3 means that your friend will also select from the same papers, so the occurrence of the first selection does not affect the probability of the second selection. The events are independent.

  $$P(>3, \text{ then } <3) = P(>3) \cdot P(<3)$$
  $$= \frac{3}{7} \cdot \frac{2}{7} = \frac{6}{49}$$

EXERCISES

Explain why the events are independent, and find the probability.

25. rolling "doubles" 3 times in a row when rolling 2 number cubes

26. selecting a red pen and then a blue pen, when selecting 2 pens from a bag of 10 red and 15 blue pens with replacement

The table shows the age and marital status of the members of an environmental group. One person from the group is randomly selected. Find each probability.

Marital Status by Age

	18–34	35–50	51–65	66+
Married	6	20	22	4
Single	14	22	11	0

27. that the selected person is single, given that he or she is in the 35–50 age group

28. that a married person is 66 or older

29. that a person aged 18–50 is married

30. that a person in the group is single and in the 18–34 age group

11-4 Compound Events *(pp. 819–825)*

EXAMPLES

Andy is using his calculator to obtain a random number from 10 to 20. Find the probability that

- Andy gets a 15 or a multiple of 2.

  10 11 **12** 13 **14** **15** **16** 17 **18** 19 **20**

  $$\frac{1}{11} + \frac{6}{11} = \frac{7}{11}$$ *The events are mutually exclusive.*

- Andy gets a multiple of 3 or a multiple of 5.

  10 11 **12** 13 14 **15** 16 17 **18** 19 **20**

  $$\frac{3}{11} + \frac{3}{11} - \frac{1}{11} = \frac{5}{11}$$ *The events are inclusive.*

- Andy gets all different numbers if he has the calculator randomly select 5 numbers.

  $$\frac{{}_{11}P_5}{11^5} = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{11 \cdot 11 \cdot 11 \cdot 11 \cdot 11} = \frac{55{,}440}{161{,}051} \approx 0.3442$$

EXERCISES

A store is handing out coupons. One-third of the coupons offer a 10% discount, half offer a 15% discount, and one-sixth offer a 20% discount. A customer is handed a coupon.

31. Explain why the events "10% discount" and "15% discount" are mutually exclusive.

32. What is the probability that the coupon offers a 10% discount or a 15% discount?

A card is drawn from a deck of 52. Find the probability of each outcome.

33. drawing a red card or drawing a 5

34. drawing a club or drawing a heart

35. Of 120 males and 180 females who took an eye exam, 170 passed. One-third of the males did not pass. What is the probability that a person who took the exam passed or was male?

11-5 Measures of Central Tendency and Variation *(pp. 828–835)*

EXAMPLES

- **The probability distribution for the number of substitute teachers needed is given. Find the expected number of substitute teachers needed on any given day.**

Number of Substitutes *n*	0	1	2	3	4
Probability of *n* Substitutes	0.05	0.08	0.38	0.41	0.08

$$0(0.05)+1(0.08)+2(0.38)+3(0.41)+4(0.08)=2.39$$

The expected number of substitutes is 2.39.

- **The number of books in each box shipped from a warehouse is given. Find the number within 1 standard deviation of the mean.**

12, 10, 4, 8, 24, 16, 14, 10, 10, 8, 16

Step 1 Find the mean.

$$\frac{12+10+4+8+24+16+14+10+10+8+16}{11}=12$$

Step 2 Find the variance. Add the squares of all the differences from the mean, and divide by the number of data values.

$$\frac{0+4+64+16+144+16+4+4+4+16+16}{11}\approx 26.2$$

Step 3 Take the square root: $\sqrt{26.2}\approx 5.1$

The number within 1 standard deviation of the mean is $\approx 12 \pm 5.1$, or [6.9, 17.1].

EXERCISES

Find the mean, median, and mode of each data set.

36. 5, 8, 0, 8, 6

37. 12, 15, 13, 13, 15, 12

38. The probability distribution for the number of arrests made in a small town on one day is given below. Find the expected number of arrests on any one day.

Number of Arrests *n*	0	1	2	3
Probability of *n* Arrests	0.65	0.22	0.1	0.03

39. Make a box-and-whisker plot of the data. Then find the interquartile range.

33, 52, 65, 48, 83, 29, 33, 50, 71

40. The number of races that a runner won every year for 10 years is given. Find the number of wins within 1 standard deviation of the mean. 5, 7, 4, 11, 8, 10, 8, 6, 9, 7

41. The principal reported that the mean of a standardized test score for the school was 81.3 and the standard deviation was 4.4. Sharon scored 96. Is her score an outlier? Explain.

42. On 6 quizzes, Aaron scored 73, 88, 86, 90, 87, and 29. Find the mean and standard deviation of the data. On his seventh quiz, he scored 32. Describe how his seventh score affects the mean and standard deviation.

11-6 Binomial Distributions *(pp. 837–843)*

EXAMPLES

Sheila bought 5 energy bars. Each has a 1 in 10 chance of winning a free energy bar.

- What is the probability that Sheila will win 3 energy bars?

$$P(3) = {}_5C_3(0.1)^3(0.9)^{5-3} \qquad P(r) = {}_nC_r p^r q^{n-r}$$
$$= 10(0.001)(0.81) = 0.0081$$

- What is the probability that Sheila will win at least 1 energy bar?

$P(\text{at least } 1) = 1 - P(0)$

$P(0) = {}_5C_0(0.1)^0(0.9)^{5-0} \approx 0.5905$

$1 - 0.5905 = 0.4095$

EXERCISES

Use the Binomial Theorem to expand each binomial.

43. $(5+2x)^3$

44. $(x-2y)^4$

45. The probability of Ike making a free throw is 0.65. He shoots 75 free throws. Find the expected number of free throws made and the standard deviation.

46. A spinner is divided into 6 equal sections, numbered 1 through 6. It is spun 8 times. What is the probability that the spinner lands on 1 exactly 3 times? What is the probability that the spinner lands on 1 at least 2 times?

CHAPTER 11

CHAPTER TEST

1. A mall employee is dressing a mannequin. There are 6 pairs of shoes, 4 types of jeans, and 8 sweaters. Using 1 of each, how many ways can the mannequin be dressed?
2. How many ways can you award first, second, and third place to 8 contestants?
3. How many ways can a group of 3 students be chosen from a class of 30?
4. Four cards are randomly selected from a standard deck of 52 playing cards. What is the probability that the cards are all jacks, all queens, or all kings?
5. The table shows the results of tossing 2 coins. Find the experimental probability of tossing 2 tails.

HH	HT	TH	TT
3	6	5	6

Each letter of the alphabet is written on a card. The cards are placed into a bag. Determine whether the events are independent or dependent, and find the indicated probability.

6. The letter *D* is drawn, replaced in the bag, and then the letter *J* is drawn.
7. Three vowels are drawn without replacement.

A card is drawn from a bag containing the 9 cards shown. Find each probability.

8. selecting a *C* or an even number
9. selecting an odd number or a multiple of 3
10. The probability distribution for the number of absent students on any given day for a certain class is given. Find the expected number of absent students.

Number of Students Absent *n*	0	1	2	3	4
Probability of *n* Absent Students	$\frac{7}{20}$	$\frac{5}{20}$	$\frac{4}{20}$	$\frac{3}{20}$	$\frac{1}{20}$

The number of known satellites of the planets in the solar system (as of 2005) is given.

	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Moons	0	0	1	2	63	33	27	13	1

Source: NASA Planetary Data System, 2005

11. Make a box-and-whisker plot of the data. Find the interquartile range.
12. Is 63 an outlier? Explain.
13. Identify the outlier in the following data set: 93, 107, 110, 103, 98, 95, 12, 111, 128, 99, 114, and 90. Describe how the outlier affects the mean and the standard deviation.
14. Use the Binomial Theorem to expand $(3x + y)^4$.

The probability of winning a carnival game is 15%. Elaine plays 10 times.

15. Find the probability that Elaine will win 2 times.
16. Find the probability that Elaine will win at least 2 times.

COLLEGE ENTRANCE EXAM PRACTICE

CHAPTER 11

FOCUS ON SAT MATHEMATICS SUBJECT TESTS

The reference information at the beginning of a test is usually the same each time the test is given. Memorize this information so that you won't have to refer back to it during the test. When you take the test, note whether any information is different from what you expected.

You may want to time yourself as you take this practice test. It should take you about 6 minutes to complete.

HOT TIP! If you do not know how to solve a general problem, try working out a simple example or two to see if a general solution method becomes apparent. But do not spend too much time on examples. If you are still stuck after a while, move on to the next problem.

1. Two cards are drawn from a standard deck of 52 cards. What is the probability that a king and a queen are drawn?

(A) $\frac{1}{169}$

(B) $\frac{2}{169}$

(C) $\frac{8}{663}$

(D) $\frac{14}{663}$

(E) $\frac{4}{169}$

2. A number cube is rolled twice. What is the probability of getting a 6 at least once?

(A) $\frac{1}{36}$

(B) $\frac{1}{6}$

(C) $\frac{11}{36}$

(D) $\frac{1}{3}$

(E) $\frac{5}{6}$

3. Your CD player can hold 6 CDs. You have 10 CDs to choose from, one of which is your favorite and is always in your player. How many ways can the player be filled if order does not matter?

(A) 126

(B) 210

(C) 720

(D) 15,120

(E) 151,200

4. Of 100 students, 37 play an instrument, 45 play sports, and 11 do both. What is the probability that a student neither plays an instrument nor plays sports?

(A) 0.145

(B) 0.18

(C) 0.29

(D) 0.40

(E) 0.82

5. A student's mean score after 4 quizzes was 72. After the fifth quiz, the mean increased to 75. What was the student's score on the fifth quiz?

(A) 60

(B) 72

(C) 84

(D) 87

(E) 100

CHAPTER 11

TEST TACKLER

Standardized Test Strategies

Multiple Choice: None of the Above or All of the Above

Given a multiple-choice test item where one of the answer choices is *none of the above* or *all of the above,* the correct response is the best, most-complete answer choice available.

To answer these types of test items, compare each answer choice with the question and determine if the answer is true or false. If you determine that more than one of the choices is true, then the correct choice is likely to be *all of the above.*

If you do not know how to solve the problem and have to guess at the answer, more often than not, *all of the above* is correct and *none of the above* is incorrect.

EXAMPLE 1

There are 8 players on the chess team. Which of the following models the number of ways that the coach can choose 2 players to start the game?

Ⓐ $_8C_2$

Ⓑ $\dfrac{8!}{2!(6!)}$

Ⓒ 28

Ⓓ All of the above

LOOK at each choice separately, and determine if it is true or false.

As you consider each choice, mark it "true" or "false."

Consider Choice A: *Because order does not matter, this is a combination problem. The number of combinations of 8 players, taken 2 at a time, is given by $_nC_r$, where $n = 8$ and $r = 2$. So, $_8C_2$ is a correct model of the combination.*

Choice A is *"true."* The answer could be choice A, but you need to check if choices B and C are also correct because the answer could be *all of the above.*

Consider Choice B: *The number of combinations of 8 players, taken two at a time, is given by $_nC_r = \frac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.*

$$_nC_r = \frac{n!}{r!(n-r)!} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!}$$

Choice B is also a correct model of the combination. Choice B is *"true."* The answer is likely to be choice D, *all of the above,* but you still should check to see if choice C is true.

Consider Choice C: *The number of combinations of 8 players, taken two at a time, is given by $_nC_r = \frac{n!}{r!(n-r)!}$, where $n = 8$ and $r = 2$.*

$$_nC_r = \frac{n!}{r!(n-r)!} = \frac{8!}{2!(8-2)!} = \frac{8!}{2!(6)!} = 28$$

Choice C is also a correct model of the combination. Choice C is *"true."* Because choices A, B, and C are all *"true,"* the correct answer choice is choice D, *all of the above.*

Be careful of problems with double negatives. Read the problem statement and each answer choice twice before selecting an answer.

Read each test item and answer the questions that follow.

Item A

The mean score on a test is 68. Which can NOT be true about the scores?

(A) Every score is 68.

(B) Half are 68, and half are 0.

(C) Half are 94, and half are 38.

(D) None of these

1. What is the definition of *mean*?
2. Read the problem statement again. If an answer choice is true, is that the correct response? Explain.
3. Willie determined that both choices A and C could be true statements, so he chose choice D as his response. Do you agree? If not, what would you have done differently?

Item B

For a number cube, what is the probability of rolling a 2 or a number greater than 4?

(F) 50%

(G) $P(\text{rolling a 2} \cup \text{rolling 5 or 6}) = P(\text{rolling a 2}) + P(\text{rolling 5 or 6})$

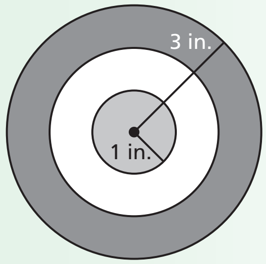
(H) $\frac{1}{6} + \frac{2}{6}$

(J) All of the above

4. Is this event mutually exclusive or inclusive? How do you know? Determine if choice G is a true or false statement.
5. If you roll a number cube, what is the probability of rolling a 2? What is the probability of rolling a 5 or 6?
6. Simplify choice H to find its value. Is this value equivalent to any other answer choices?
7. How many answer choices are correct? What is the correct response?

Item C

Suppose that a dart lands at a random point on the circular dartboard. Find the probability that the dart lands inside only the dark gray or white region. The radius of the dartboard is 3 inches.



(A) $\frac{1}{9}$

(B) 8π

(C) $\frac{8}{9}$

(D) None of these

8. A student finds that both choice A and choice B are incorrect. To save time, he chooses choice D as his answer because he figures it is likely that choice C will also be incorrect. Do you think that this student made a wise decision? Explain.
9. What is the formula for the area of a circle? What is the area of this dartboard? How can you determine the area of the dark gray and white regions?
10. Find if choice A, B, or C is true, and determine the response to the test item.

Item D

Each gym member receives a 3-digit code to use for a locker combination with no digit repeated. Grace received the code 210. What was the probability that she would receive a code of consecutive numbers?

(F) $1.\overline{6}\%$

(G) $\frac{1}{45}$

(H) $\frac{1}{{}_{10}P_3}$

(J) All of the above

11. How can you determine if choice J is correct?
12. Are the values given in choices F, G, and H equivalent? What does this tell you about choice J?

CHAPTER 11

STANDARDIZED TEST PREP

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State Test Practice Online
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CUMULATIVE ASSESSMENT, CHAPTERS 1–11

Multiple Choice

1. There were 8 dogs in a litter. How many ways can Mike choose 2 dogs?

(A) 20,160

(B) 56

(C) 28

(D) $\frac{1}{28}$

2. What is the median of the test scores given?
$\{97, 78, 61, 90, 95, 96, 80, 67, 86, 88, 90, 92\}$

(F) 85

(G) 88

(H) 89

(J) 90

3. The table shows the number of teachers, coaches, and students at a high school of each gender. What is the probability, to the nearest hundredth, that a coach is male?

School Population and Gender

	Male	Female
Teachers	12	24
Coaches	17	9
Students	429	453

(A) 0.65

(B) 0.35

(C) 0.04

(D) 0.02

4. For $f(x) = ab^x$, if x increases by 1, the value of $f(x)$ does which of the following?

(F) $f(x)$ increases by b.

(G) $f(x)$ is multiplied by b.

(H) $f(x)$ increases by a.

(J) $f(x)$ is multiplied by a.

5. A slice of an 18-inch diameter pizza that is cut into sixths sells for \$3.25. At this rate, how much should a slice that is one eighth of a 16-inch diameter pizza sell for, to the nearest \$0.05?

(A) \$1.75

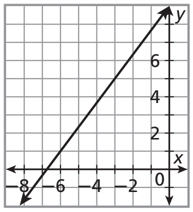
(B) \$1.95

(C) \$2.15

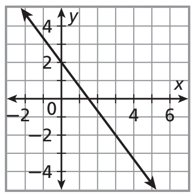
(D) \$2.45

6. Which graph shows a line with a slope of $-\frac{4}{3}$ that passes through $(5, 2)$?

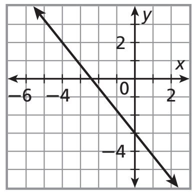
(F)



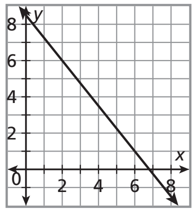
(G)



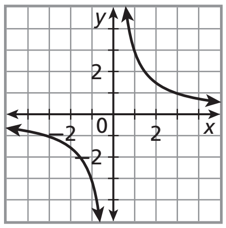
(H)



(J)



7. Which type of function is shown in the graph?



(A) exponential

(B) polynomial

(C) radical

(D) rational

8. Which conic section does the equation represent?
$2x^2 + 9xy + 10y^2 + 4x + 5y + 8 = 0$

Ⓕ Parabola

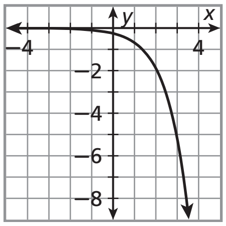
Ⓖ Hyperbola

Ⓗ Ellipse

Ⓙ Circle

HOT TIP! **In item 9, remember that a real number with a 0 exponent is 1. You can use mental math to quickly evaluate each function and compare your result to the corresponding value in the graphed function.**

9. Which is the equation of the graph below?



Ⓐ $f(x) = 0.25(2.75^x)$

Ⓑ $f(x) = -2.75(0.25^x)$

Ⓒ $f(x) = 2.75(0.25^x)$

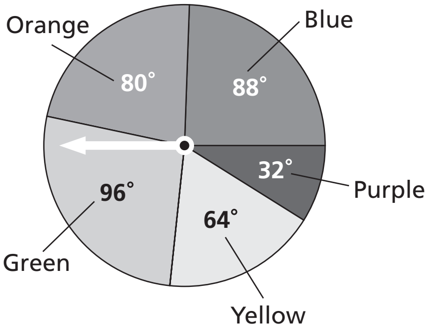
Ⓓ $f(x) = -0.25(2.75^x)$

Gridded Response

10. What value of x makes the equation true?
$6(x - i) - 2i = (4 - i)^2$

11. Use long division to find the coefficient of the x term in the quotient.
$(2x^3 + 5x^2 + 10x + 7) \div (x + 1)$

Use the spinner for Items 12 and 13.



12. What is the probability of the spinner landing on the orange or purple sector, to the nearest hundredth?

13. What is the probability that the spinner will land on green in at least 2 of the next 3 spins? Write the answer to the nearest thousandth.

Short Response

14. Find the center and the radius of a circle that has a diameter with the endpoints $(-2, 8)$ and $(4, 2)$.

a. What is the length of the diameter?

b. The endpoints $(4, 8)$ and $(-2, y)$ are located on the circle. Find the missing value of y.

15. The table below shows the number of students that graduated from a high school from 1920 to 2000, measured every 20 years.

x	1920	1940	1960	1980	2000
y	9	59	159	409	909

a. Write a polynomial function for the data. Let x be the number of years since 1920. Round your answer to the nearest thousandth.

b. At this rate how many students will graduate in 2020? Round your answer to the nearest student.

c. About how many students graduated in 1990? Round your answer to the nearest student.

16. The Badgers won 70% of their games this season. They won 5 of the 12 games they played during their last road trip. Before the road trip, the Badgers had won 75% of their games.

a. How many wins and losses did the Badgers have during the season?

b. How many wins and losses did the Badgers have before their last road trip?

Extended Response

17. Randy wants to collect 515 baseball cards. Each week Randy buys 15 baseball cards.

a. Create a table to represent this situation where t is the amount of time in weeks and c is the number of cards that Randy still wants to buy.

b. Write an equation to model the data in the table.

c. Graph the equation.

d. After how many weeks will Randy meet his goal?