# Solutions Key

**Chapter 7**

## Similarity

### 7-1 Ratio and Proportion, Pages 454–459

**CHECK IT OUT! Pages 454–456**

1. Let \( m = 4 \), \( n = 5 \), \( r = 3 \), \( s = 2 \).

2. Let \( \angle \) measures be \( x \), \( 6x \), and \( 13x \). Then \( x + 6x + 13x = 180 \). After like terms are combined, \( 20x = 180 \). So \( x = 9 \). The \( \angle \) measures are \( x = 9^\circ \), \( 6x = 6(9) = 54^\circ \), and \( 13x = 13(9) = 117^\circ \).

3a. \( \frac{3}{8} = \frac{x}{56} \)

3b. \( \frac{2y}{8} = \frac{8}{4y} \)

3c. \( \frac{d}{3} = \frac{6}{2} \)

3d. \( \frac{x + 3}{4} = \frac{9}{x + 3} \)

4. \( 16s = 20t \)

5. 1 Understand the Problem

   Answer will be height of new tower.

   2 Make a Plan

   Let \( y \) be height of new tower. Write a proportion that compares the ratios of model height to actual height.

   \[
   \frac{\text{height of 1st tower}}{\text{height of 1st model}} = \frac{\text{height of new tower}}{\text{height of new model}}
   \]

   \[
   \frac{1328}{8} = \frac{y}{9.2}
   \]

   3 Solve

   \[
   1328 = \frac{1328(9.2)}{8}
   \]

   \[
   1328(9.2) = 8(y)
   \]

   \[
   12,217.6 = 8y
   \]

   \[
   y = 1527.2 \text{ m}
   \]

   4 Look Back

   Check answer in original problem. Ratio of actual height to model height is 1328 : 8 or 166 : 1. Ratio of actual height to model height for new tower is 1527.2 : 9.2 In simplest form, this ratio is also 166 : 1. So ratios are equal, and answer is correct.

### THINK AND DISCUSS, Page 457

1. No; ratio 6 : 7 is less than 1, but ratio 7 : 6 is greater than 1.

2. She can see if cross products are equal. Since \( 3(28) = 7(12) \), ratios do form a proportion. Therefore ratios are equal and fractions are equivalent.

3. **Definition:**

   A proportion is an eqn. stating that two ratios are equal.

   **Properties:**

   If \( \frac{a}{b} = \frac{c}{d} \) then \( ad = bc \).

   \[ \frac{a}{b} = \frac{c}{d} \text{ and } \frac{b}{d} = \frac{c}{a} \]

   **Example:**

   Possible answer: \( \frac{1}{3} = \frac{\frac{1}{2}}{\frac{1}{2}} \).

   **Nonexample:**

   Possible answer: \( \frac{1}{3} = \frac{\frac{1}{2}}{\frac{1}{1}} \) is not a proportion.

**EXERCISES, Pages 457–459**

**GUIDED PRACTICE, Page 457**

1. Means: 3 and 2; extremes: 1 and 6

2. \( sv, tv \)

3. \( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \)

   \[
   = \frac{9}{4 - 3} = \frac{3}{1 - (-1)} = \frac{1}{2}
   \]

   \[
   = \frac{2}{4} - \frac{(-2)}{4}
   \]

   \[
   = \frac{1}{4} - \frac{(-2)}{4}
   \]

   \[
   = \frac{1}{4} + \frac{1}{1}
   \]

4. \( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \)

   \[
   = \frac{4}{1} - \frac{(-1)}{2}
   \]

   \[
   = \frac{2}{4} - \frac{(-2)}{4}
   \]

   \[
   = \frac{1}{4} - \frac{(-2)}{4}
   \]

   \[
   = \frac{1}{4} + \frac{1}{1}
   \]

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Holt Geometry
5. \( \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \)
   \[ = \frac{-1 - 1}{2 - (-1)} = \frac{-2}{3} \]

6. Let side lengths be 2x, 4x, 5x, and 7x. Then 2x + 4x + 5x + 7x = 36. After like terms are combined, 18x = 36. So \( x = 2 \). The shortest side measures 2x = 2(2) = 4 m.

7. Let \( \angle \) measures be 5x, 12x, and 19x. Then 5x + 12x + 19x = 180. After like terms are combined, 36x = 180. So \( x = 5 \). The largest \( \angle \) measures 19x = 19(5) = 95°.

8. \( \frac{x}{2} = \frac{16}{2} \)
   \[ \Rightarrow x = 16 \]
   \[ \Rightarrow y = \frac{21}{27} = \frac{7}{9} \]

9. \( \frac{y}{3} = \frac{27}{y} \)
   \[ \Rightarrow y = 3 \]
   \[ \Rightarrow x = 5 \]

10. \( \frac{6}{58} = \frac{t}{29} \)
    \[ \Rightarrow t = 3 \]
    \[ \Rightarrow y = \pm 9 \]

11. \( \frac{2}{16} = \frac{x - 1}{x} \)
    \[ \Rightarrow x = \frac{18}{6} \]
    \[ \Rightarrow y = x - 9 \]

12. \( \frac{2a}{b} = \frac{8}{2} \)
    \[ \Rightarrow a = 4 \]
    \[ \Rightarrow b = 1 \]

13. \( \frac{3}{5} = \frac{4}{1} \)
    \[ \Rightarrow y = \frac{27}{x} \]

16. **1 Understand the Problem**
   Answer will be height of Arkansas State Capitol.

**2 Make a Plan**
Let \( x \) be height of Arkansas State Capitol. Write a proportion that compares the ratios of height to width.

height of U.S. Capitol \( \quad \) height of Arkansas Capitol
\[ \frac{\text{height of U.S. Capitol}}{\text{width of U.S. Capitol}} = \frac{\text{height of Arkansas Capitol}}{\text{width of Arkansas Capitol}} \]

\[ \frac{288}{752} = \frac{x}{y} \]

**3 Solve**
\[ \frac{288}{752} = \frac{x}{y} \]
\[ \Rightarrow x = \frac{288y}{752} \]
\[ \Rightarrow y = \frac{564}{528} \]

\[ \Rightarrow x = 216 \text{ ft} \]

**4 Look Back**
Check answer in original problem. Ratio of height to width for U.S. Capitol is 288 : 752, or 18 : 47. Ratio of height to width for Arkansas State Capitol is 216 : 564 In simplest form, this ratio is also 18 : 47. So ratios are equal, and answer is correct.
33. \( \frac{a}{b} = \frac{5}{7} \)  
7a = 5b  
\( \frac{7a}{b} = \frac{5}{7} \)

34. Cowboys lost 16 – 10 = 6 games.  
Wins: losses = 10:6 
4 : \( \frac{20}{3} \)  
= \( \frac{2}{2} \)  
= \( \frac{5}{3} \)

35. slope = \( \frac{5 + 4}{21 + 6} \)  
= \( \frac{9}{27} \)  
= \( \frac{1}{3} \)

36. slope = \( \frac{1 + 5}{6 - 16} \)  
= \( \frac{6}{-10} \)  
= \( \frac{-3}{5} \)

37. slope = \( \frac{5 + 2}{4 - 6.5} \)  
= \( \frac{7.5}{-3} \)  
= \( -2.5 \)

38. slope = \( \frac{0 - 1}{-2 + 6} \)  
= \( \frac{-1}{4} \)

39a. 1.25 in. = \( \frac{x}{9600} \) in.  
15 in. = \( \frac{x}{9600} \)

39b. 1.25(9600) = 15x  
12,000 = 15x  
x = 800 in.  
= 66 ft 8 in.

40. Quad. is a rect. because opp. sides are \( \cong \) and diagonals are \( \cong \).

41. Areas are \( 6^2 = 36 \text{ cm}^2 \) and \( 9^2 = 81 \text{ cm}^2 \).

42. \( \frac{5}{3.5} = \frac{20}{w} \)  
w = 3.5(20) = 70  
w = 14 in.

43. A ratio is a comparison of 2 numbers by div.  
A proportion is an eqn. stating that 2 ratios are \( \cong \).

44. B  
\( x + 4x + 5x = 18 \)  
\( 10x = 18 \)  
x = 1.8 in.

45. H  
\( 4x = 4(1.8) = 7.2 \) in.,  
\( 5x = 5(1.8) = 9 \) in.

46. A  
\( \frac{5}{2} = 1.25 \)  
\( \frac{5v}{2} = 2.125 \)  
v = 2.5

47. First, cross multiply:  
\( 36x = 15(72) \) = 1080  
Then divide both sides by 36:  
\( \frac{36x}{36} = \frac{1080}{36} \)  
Finally, simplify:  
x = 30

You must assume that \( x \neq 0 \).

48. Perimeters are \( 2(3) + 2(5) = 16 \) and \( 2x + 2(4) = 2x + 8 \).  
16  
\( \frac{4}{7} = \frac{2x + 8}{2(16)} \)  
8x + 32 = 112  
x = 10

49. Given \( \frac{a}{b} = \frac{c}{d} \) add 1 to both sides of eqn:  
\( \frac{a + b}{b} = \frac{c + d}{d} \)  
Adding fractions on both sides of eqn. gives  
\( \frac{a}{b} = \frac{c}{d} \)

50. Possible proportions are \( \frac{1}{2} = \frac{3}{6} = \frac{1}{2} = \frac{2}{4} = \frac{6}{12}, \)  
\( \frac{2}{6} = \frac{3}{3} = \frac{3}{2} = \frac{1}{2} \)  
and \( \frac{6}{3} = \frac{2}{1} \)  
There are 8 possible proportions. Total number of outcomes \( = 4! = 24 \).

Probability\( \frac{8}{24} = \frac{1}{3} \)

51. \( x^2 + 9x + 18 = \frac{(x + 3)(x + 6)}{} \)  
\( x^2 - 36 = \frac{(x + 3)(x - 6)}{} \)  
\( x^2 - 36 = \frac{x + 3}{x - 6}, \) where \( x \neq \pm 6 \)

52. \( y - 6(0) = -3 \)  
\( y = -3 \)  
\( -6x = -6 \)  
x = 1

53. \( 3) - 6x = -3 \)  
\( y = -3 \)  
\( -6x = -6 \)  
x = 1

54. \( y - 6(-4) = -3 \)  
\( y + 24 = -3 \)  
\( y = -27 \)  
\( y = -7 \)  
Ext. \( \triangle \) Thm. to find \( y \), then use Vert. \( \triangle \) Thm.  
\( 3y + 2y + 20 = 180 \)  
\( 5y = 160 \)  
\( y = 32 \)

55. Think: Use Same-Side  
\( y = 27 \)  
\( y = 32 \)

56. Think: Use Vert. \( \triangle \) Thm.  
\( 9^2 + 5^2 + 8^2 \)  
m\( \angle CDB = 2y + 20 \)  
81 + 25 + 49  
\( = 2(32) + 20 \)  
81 < 89  
\( \Delta = 84^\circ \)  
\( \Delta \) is acute.

57. \( 20^2 + 8^2 + 15^2 \)  
\( 400 + 64 + 225 \)  
\( 625 \)  
\( \Delta = 92 + 24^2 \)  
\( 625 \)  
\( \Delta = 625 \)

\( \Delta \) is obtuse.  
\( \Delta \) is a right triangle.

TECHNOLOGY LAB: EXPLORE THE GOLDEN RATIO, PAGES 460–461

ACTIVITY 1

1. Check students’ work. The equal ratios have the approximate value of \( 1.62 \).

2. The ratios have the same value as the ratios in Step 1.
TRY THIS, PAGE 461

1. If side length of square is 2 units, then $MB = 1$ unit and $BC = 2$ units. MC is hyp. of rt. $\triangle$ formed by $MB$ and $BC$. By Pyth. Thm., $MC = \sqrt{5}$ units

\[
AE = \sqrt{5} + 1 \text{ units}
\]

\[
AE = \frac{\sqrt{5} + 1}{2} \approx 1.618
\]

2. $BE = \sqrt{5} - 1$ units

\[
BE = \frac{\sqrt{5} - 1}{2} \approx 0.618
\]

The sign of the numerator in this fraction is different from that of the fraction in Try This Problem 1.

3. Quotients have values that approach 1.618.

4. There are $1 + 2 = 3$ rabbits.

5. There are $8 + 13 = 21$ petals on the daisy.

6. No; $\frac{5.4}{4} \approx 1.4$ 7. Yes; $\frac{4.5}{2.8} \approx 1.6$

7-2 RATIOS IN SIMILAR POLYGONS, PAGES 462–467

CHECK IT OUT! PAGES 462–464

1. $\angle C \cong \angle H$. By Rl. $\angle \equiv$ Thm., $\angle B \cong \angle G$.

By 3rd $\angle $ Thm., $\angle A \cong \angle J$.

\[
\frac{AB}{JG} = \frac{10}{5} = 2, \quad \frac{BC}{GH} = \frac{6}{3} = 2, \quad \frac{AC}{JH} = \frac{11.6}{5.8} = 2
\]

2. Step 1 Identify pairs of $\cong$.

$L \cong P$ (Given)

$L \cong N$ (Rl. $\angle \equiv$ Thm.)

$J \cong S$ (3rd $\angle$ Thm.)

Step 2 Compare corr. sides.

\[
\frac{JL}{SP} = \frac{75}{30} = 2, \quad \frac{LM}{PN} = \frac{60}{24} = 2, \quad \frac{JM}{SN} = \frac{45}{18} = 2
\]

Yes; similarity ratio is $\frac{5}{2}$ and $\triangle LMJ \sim \triangle PNS$.

3. Let $x$ be length of the model boxcar in inches. Rect. model of boxcar is $\sim$ to rect. boxcar, so corr. lengths are proportional.

\[
\frac{\text{length of boxcar}}{\text{length of model}} = \frac{\text{width of boxcar}}{\text{width of model}}
\]

\[
36.25 = \frac{1.25}{x}
\]

\[
36.25(1.25) = 9x = 45.3125 \Rightarrow x = \frac{45.3125}{9} \approx 5 \text{ in.}
\]

THINK AND DISCUSS, PAGE 464

1. $\cong$ symbol is formed.

2. Sides of rect. $EFGH$ are 9 times as long as corr. sides of rect. $ABCD$.

3. Possible answers: reg. polygons of same type; $\odot$

EXERCISES, PAGES 465–467

GUIDED PRACTICE, PAGE 465

1. Possible answer: students’ desks

2. $\angle M \cong \angle U$ and $\angle N \cong \angle V$. By 3rd $\triangle$ Thm., $\angle P \cong \angle W$.

\[
\frac{MN}{UV} = \frac{4}{8} = \frac{1}{2}, \quad \frac{MP}{UW} = \frac{3}{6} = \frac{1}{2}, \quad \frac{NP}{WV} = \frac{2}{4} = \frac{1}{2}
\]

3. $\angle A \cong \angle H$ and $\angle C \cong \angle K$. By def. of $\cong$, and taking vertices clockwise in both figures, $\angle B \cong \angle J$ and $\angle D \cong \angle L$.

\[
\frac{AB}{HJ} = \frac{8}{12} = \frac{2}{3}, \quad \frac{BC}{JK} = \frac{6}{3} = 2, \quad \frac{CD}{KL} = \frac{4}{6} = \frac{2}{3}
\]

4. Step 1 Identify pairs of $\cong$. Think: All $\cong$ of a rect. are rt. $\cong$ and are $\cong$.

$L \cong E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$, and $\angle D \cong \angle H$.

Step 2 Compare corr. sides.

\[
\frac{AB}{EF} = \frac{135}{90} = \frac{3}{2}, \quad \frac{AD}{EH} = \frac{45}{30} = \frac{3}{2}
\]

Yes; since opp. sides of a rect. are $\cong$, corr. sides are proportional. Similarity ratio is $\frac{3}{2}$ and $\triangle ABC \sim \triangle EFG$.

5. Step 1 Identify pairs of $\cong$.

$L \cong W$, $\angle P \cong \angle U$ (Given)

$L \cong Z$ (3rd $\angle$ Thm.)

Step 2 Compare corr. sides.

\[
\frac{RM}{XW} = \frac{8}{12} = \frac{2}{3}, \quad \frac{MP}{WU} = \frac{10}{15} = \frac{2}{3}, \quad \frac{RP}{XU} = \frac{4}{6} = \frac{2}{3}
\]

Yes; similarity ratio is $\frac{2}{3}$ and $\triangle RMP \sim \triangle XWU$.

6. Let $x$ be height of reproduction in feet. Reproduction is $\sim$ to original, so corr. lengths are proportional.

\[
\frac{\text{height of reproduction}}{\text{height of original}} = \frac{\text{width of reproduction}}{\text{width of original}}
\]

\[
\frac{x}{73} = \frac{58}{58x = 73(24) = 1752, \quad x = \frac{1752}{58} \approx 30 \text{ ft}}
\]

PRACTICE AND PROBLEM SOLVING, PAGES 465–466

7. $\angle K \cong \angle T$, $\angle L \cong \angle U$ (Given)

$L \cong S$, $\angle M \cong \angle V$ (Rt. $\angle \equiv$ Thm.)

\[
\frac{JK}{ST} = \frac{20}{24} = \frac{5}{6}, \quad \frac{KL}{TU} = \frac{14}{16.8} = \frac{5}{6}, \quad \frac{LM}{UV} = \frac{30}{36} = \frac{5}{6}
\]

$\triangle JLM \sim \triangle STU$.

8. $\angle A \cong \angle X$, $\angle C \cong \angle Z$ (Given)

$L \cong Y$ (3rd $\angle$ Thm.)

\[
\frac{AB}{XY} = \frac{8}{4} = 2, \quad \frac{BC}{YZ} = \frac{6}{3} = 2, \quad \frac{CA}{ZX} = \frac{12}{6} = 2
\]
9. Step 1  Identify pairs of \(\cong\) \(\triangle\).
\[ m\angle R = 90 - 53 = 37\,^\circ \]
\[ \angle R \cong \angle U \text{ (Def. of } \cong \text{)} \]
\[ \angle S \cong \angle Z \text{ (Rt. } \angle \cong \text{ Thm.)} \]
\[ \angle Q \cong \angle X \text{ (3rd \(\cong\) Thm.)} \]

Step 2  Compare corr. sides.
\[ \frac{QR}{XU} = \frac{35}{7} = \frac{5}{1} \quad \frac{QS}{ZU} = \frac{21}{8} = \frac{7}{4} \quad \frac{RS}{UZ} = \frac{28}{32} = \frac{7}{8} \]
Yes; similarity ratio \(= \frac{7}{8}\) \(\triangle RSQ \sim \triangle UZX\)

10. Step 1  Identify pairs of \(\cong\) \(\triangle\).
\[ \angle A \cong \angle M, \angle B \cong \angle J, \angle C \cong \angle K, \angle D \cong \angle L \]
(Rt. \(\angle \cong \text{ Thm.)})

Step 2  Compare corr. sides.
\[ \frac{AB}{MJ} = \frac{\sqrt{8}}{\sqrt{24}} = \frac{1}{3} \quad \frac{AD}{ML} = \frac{\sqrt{36}}{\sqrt{54}} = \frac{2}{3} \]
No; the rectangles are not similar.

11. model length \(= \frac{1}{56} \quad \text{car length} \quad \ell = \frac{56}{3}\)
\[ \ell = 186 \text{ in.} = 14 \text{ ft} \]

12. Let \(x, y\) be side lengths of squares \(ABCD\) and \(PQRS\). Areas are \(x^2\) and \(y^2\), so
\[ x^2 = \frac{4}{1} = 4 \quad y^2 = \frac{36}{9} = 4 \]
\[ \frac{x}{y} = \frac{1}{3} \]
\[ \sim \text{ ratio of } ABCD \text{ to } PQRS = \frac{x}{y} = \frac{1}{3} \]
\[ \sim \text{ ratio of } PQRS \text{ to } ABCD = \frac{y}{x} = \frac{3}{1} \]

13. Sometimes (iff acute \(\angle\) are \(\cong\))
14. Always (all rt. \(\angle\) are \(\cong\), all side-length ratios are \(=\))
15. Never (in trap., 1 pair sides are \(\|\), so opp. pairs of \(\angle\) cannot be \(\cong\); but in \(\square\), they are \(\cong\))
16. Always (by CPCTC, all corr. \(\angle\) are \(\cong\), and since corr. sides \(\cong\), \(\sim\) ratio = 1)
17. Sometimes (similar polygons are \(\cong\) iff \(\sim\) ratio = 1)
18. By def. of reg. polygons, corr. int. \(\angle\) are \(\cong\), and side lengths are \(\cong\) and thus proportional. So any 2 reg. polygons with same number of sides are \(\sim\).

19. \[ \frac{EF}{FG} = \frac{2}{3} \quad \frac{AB}{BC} = \frac{3}{2} \quad \frac{x + 3}{2x - 4} = \frac{4}{3} \quad \frac{3x + 9}{8x - 16} = \frac{3}{2} \]
20. \[ \frac{MP}{NP} = \frac{1}{2} \quad \frac{XZ}{YZ} = \frac{4}{3} \quad \frac{x + 5}{XZ} = \frac{30}{75} \quad \frac{5x + 25}{8x - 20} = \frac{5}{3} \]
\[ x = 15 \quad 45 = 3x \]

21. Possible answer:
Statue of Liberty’s nose \(\approx\) your nose
Statue of Liberty’s hand \(\approx\) your hand
\[ \approx \frac{2}{1} \text{ in.} \quad \approx \frac{16.4}{7} \text{ in.} \]
\[ 7 \approx 2(16.4) = 32.8 \]
\[ x \approx 4.7 \]

Estimated length of Statue of Liberty’s nose is 4.7 ft (or between 4.5 ft and 5 ft).

22. If 2 polygons are \(\sim\), then their corr. \(\triangle\) are \(\cong\) and their corr. sides are proportional. If corr. \(\triangle\) of 2 polygons are \(\cong\) and their corr. sides are proportional, then polygons are \(\sim\).

23. \(\square\)JKLM \(\sim\) \(\square\)NOPQ \(\rightarrow\) \(\angle Q \cong \angle K \rightarrow m\angle Q = 75\,^\circ\)
\(\triangle\)NOP a \(\square\) \(\rightarrow\) \(\angle Q \cong \angle O \rightarrow m\angle Q = 75\,^\circ\)
\(\angle K\) and \(\angle Q\) are 75\,^\circ. \(\square\)

24. width on blueprint = length on blueprint
\[ \frac{w}{w} = \frac{3.5}{14} = 0.25 \quad \frac{18w}{18(3.5)} = 49 \quad \frac{w}{18} \approx 2.7 \text{ in.} \]

25. Polygons must be \(\cong\). Since polygons are \(\sim\), their corr. \(\triangle\) must be \(\cong\). Since \(\sim\) ratio is 1, corr. sides must have same length.

26a. width of tree on backdrop \(\frac{1}{10}\) width of tree on flat \(\frac{9}{10}\) width of actual tree \(\frac{1}{2}\) width of actual tree
\[ \frac{9}{10} \cdot \frac{1}{2} \]
\[ \frac{9(2)}{H} \]
\[ H = 18 \text{ ft} \]

26b. height of tree on flat \(\frac{1}{10}\) height of actual tree \(\frac{9}{10}\) height of actual tree
\[ \frac{9(2)}{H} \]
\[ H = 18 \text{ ft} \]

26c. \(\sim\) ratio \(= \frac{1}{10}\) height of tree on backdrop \(\frac{9}{10}\) height of actual tree
\[ \frac{9}{18} = \frac{1}{2} \]

27. C
28. F

29. Ratios of sides are not the same: 12 _ 35 = 24 _ 7
10 _ 4 = 2.5 _ 1.5 = 4

CHALLENGE AND EXTEND, PAGE 467

30. model length \(\frac{1}{500}\)
building length \(\ell = 1\)
\[ \ell = \frac{0.4 \text{ ft}}{4.8 \text{ in.}} \]
model width \(\frac{1}{500}\)
building width \(\frac{1}{140}\)
\[ \frac{1}{500} \quad \frac{1}{140} \quad \frac{1}{500} \quad \frac{1}{140} \]

w = 0.28 ft = 3.36 in.
31. Since \( \overline{QR} \parallel \overline{ST} \), \( \angle PQR \equiv \angle PST \) and \( \angle PRQ \equiv \angle PRT \) by Alt. Int. \( \triangle \) Thm. \( \angle P \equiv \angle P \) by Reflex. Prop. of \( \equiv \). Thus corr. \( \triangle \) of \( \triangle PQR \) and \( \triangle PST \) are \( \equiv \). Since \( PS = 6 \) and \( PT = 8 \), \( \frac{PQ}{PR} = \frac{QR}{ST} = \frac{1}{2} \). Therefore \( \triangle PQR \sim \triangle PST \) by def. of \( \sim \) polygons.

32a. By HL, \( \triangle ABD \equiv \triangle CBD \), so \( \angle A \equiv \angle C \), and \( \overline{mCA} = \overline{mC} = 45^\circ \). So \( \triangle ABC \) is a \( 45^\circ-45^\circ-90^\circ \) \( \triangle \). \( \overline{AC} = \overline{AB}/2 = 1\overline{\sqrt{2}} = \overline{\sqrt{2}} \)
\( \overline{mCBD} = 90 - \angle C = 45^\circ \), so \( \triangle CDB \) is also a \( 45^\circ-45^\circ-90^\circ \) \( \triangle \). So
\( \overline{BC} = 1 = \overline{DC}/\overline{\sqrt{2}} = \overline{DB}/\overline{\sqrt{2}} \)
\( \overline{\sqrt{2}} = 2\overline{DC} = 2\overline{DB} \)
\( \overline{DC} = \overline{DB} = \overline{\sqrt{2}}/2 \)

b. From part a., corr. \( \triangle \) of \( \triangle ABC \) and \( \triangle CDB \).
\( \frac{\overline{AB}}{\overline{BD}} = \frac{\overline{AC}}{\overline{DC}} = \overline{\sqrt{2}}. \) By def. of \( \sim \), \( \triangle ABC \sim \triangle CDB \).

33a. rect. \( \square ABCD \sim \) rect. \( \square BCFE \)

b. \( \ell \) = \( \frac{1}{\ell - 1} \)
c. \( \ell(\ell - 1) = 1 \)
\( \ell^2 - \ell = 1 \)
\( \ell^2 - \ell - 1 = 0 \)
\( \ell = \frac{1 \pm \sqrt{1^2 - 4(1)(-1)}}{2} \)
\( = \frac{1 \pm \sqrt{5}}{2} \)
Think: \( \ell > 0 \), so take positive sq. root.
\( \ell = \frac{1 + \sqrt{5}}{2} \)
d. \( \ell \approx 1.6 \)

SPIRAL REVIEW, PAGE 467

34. \( \# \) of orders = \# of permutations of 4 things = \( 4! = 24 \)

35. Think: Kite \( \rightarrow \) diag. are \( \perp \). So \( \angle QTR \) is a rt. \( \angle \).
\( \overline{mQTR} = 90^\circ \)

36. Think: \( \triangle PST \equiv \triangle RST \). By CPCTC,
\( \angle PST \equiv \angle RST \)
\( \overline{mPST} = \overline{mRST} = 20^\circ \)

37. Think: \( \triangle PST \) is a rt. \( \triangle \). So \( \angle PST \) and \( \angle TPS \) are comp.
\( \overline{mTPS} = 90 - \overline{mPST} = 90 - 20 = 70^\circ \)

38. \( \frac{x}{4} = \frac{y}{10} \)
\( 10x = 4y \)
\( 10x = 4y \)
\( 10x = 4y \)

39. \( \frac{x}{4} = \frac{y}{10} \)
\( 10x = 4y \)
\( 10x = 4y \)
\( 10x = 4y \)
\( 10x = 4y \)
\( \frac{y}{4} = \frac{x}{10} \)
\( \frac{y}{4} = \frac{x}{10} \)

40. \( \frac{x}{4} = \frac{y}{10} \)
\( x = 4y \)
\( x = 4y \)
\( \frac{x}{y} = \frac{4}{10} \) or \( \frac{2}{5} \)

TECHNOLOGY LAB: PREDICT TRIANGLE SIMILARITY RELATIONSHIPS, PAGES 468–469

ACTIVITY 1, PAGE 468
3. The ratios of cor. side lengths are \( \sim \).

TRY THIS, PAGE 468
1. \( \triangle \) Sum Thm.
2. Yes; in \( \sim \), corr. sides are proportional.

ACTIVITY 2, PAGE 468
3. corr. \( \triangle \) are \( \equiv \).

TRY THIS, PAGE 469
3. Yes; if 2 \( \triangle \) have their corr. sides in same ratio, then they are \( \sim \).
4. They are similar in that both allow you to conclude that corr. \( \triangle \) are \( \equiv \). They are different in that the conjecture suggests that \( \triangle \) with corr. sides in same ratio have same shape, but SSS \( \equiv \) Thm. allows you to conclude that the \( \triangle \) have both same shape and same size.

ACTIVITY 3, PAGE 469
3. The ratio of the corr. sides of \( \triangle ABC \) and \( \triangle DEF \) are proportional.
4. The corr. \( \triangle \) of the \( \triangle \) are \( \equiv \).

TRY THIS, PAGE 469
5. Yes; corr. sides are proportional and corr. \( \triangle \) are \( \equiv \).
6. If \( \triangle \) have 2 pairs of corr. sides in same proportion and included \( \triangle \) are \( \equiv \), then \( \triangle \) are \( \sim \). This is related to the SAS \( \equiv \) Thm.

7-3 TRIANGLE SIMILARITY: AA, SSS, AND SAS, PAGES 470–477

CHECK IT OUT! PAGES 470–473
1. By \( \triangle \) Sum Thm., \( \overline{mC} = 47^\circ \), so \( \angle C \equiv \angle F \). \( \angle B \equiv \angle E \) by Rt. \( \angle \equiv \) Thm. Therefore \( \triangle ABC \sim \triangle DEF \) by AA \( \sim \).
2. \( \triangle TXU \equiv \triangle VXW \) by Vert. \( \triangle \) Thm.
\( \frac{TX}{VX} = \frac{12}{3} \) \( \frac{XU}{WX} = \frac{15}{20} \)
Therefore \( \triangle TXU \sim \triangle VXW \) by SAS \( \sim \).

3. Step 1 Prove \( \triangle \) are \( \sim \).
It is given that \( \angle RSV \equiv \angle T \). By the Reflex. Prop. of \( \equiv \), \( \angle R \equiv \angle R \). Therefore \( \triangle RSV \sim \triangle RTU \) by AA \( \sim \).

Step 2 Find \( RT \).
\( \frac{RT}{TU} = \frac{RU}{SV} \)
\( RT = 12 \)
\( 10 \)
\( 8RT = 10(12) = 120 \)
\( RT = 15 \)
4. **Statements** | **Reasons**
---|---
1. $M$ is mdpt. of $JK$, $N$ is mdpt. of $KL$, and $P$ is mdpt. of $JL$. | 1. Given
2. $MP = \frac{1}{2}KL$, $MN = \frac{1}{2}JL$, $NP = \frac{1}{2}KJ$. | 2. $\triangle$ Midsegs. Thm.
3. $MP = \frac{MN}{JL} = \frac{NP}{KJ} = \frac{1}{2}$. | 3. Div. Prop. of $\cong$.
4. $\triangle JKL \sim \triangle NPM$. | 4. SSS $\sim$ Step 3

5. $FG = BF$
$AC = AB$
$FG = 4$
$5x = 4x$
$FG(4x) = 4(5x)$
$4FG = 20$
$FG = 5$

**THINK AND DISCUSS, PAGE 473**

1. $\angle A \cong \angle D$ or $\angle C \cong \angle F$.
2. $BA = \frac{3}{5}$
3. No; corr. sides need to be proportional but not necessarily $\cong$ for $\triangle$ to be $\sim$.

4. **Congruence** |
**Similarity** |
---|---
$SSS$ | If 3 sides of $\triangle$ are $\cong$ to 3 sides of another $\triangle$, then the $\triangle$ are $\cong$.
$SAS$ | If 2 sides and the included $\angle$ of $\triangle$ are $\cong$ to 2 sides and the included $\angle$ of another $\triangle$, then the $\triangle$ are $\cong$.
$AA$ | If 2 $\triangle$ of 1 $\triangle$ are $\cong$ to 2 $\triangle$ of another $\triangle$, then the $\triangle$ are $\cong$.

**EXERCISES, PAGES 474–477**

**GUIDED PRACTICE, PAGE 474**

1. By def. of $\cong$, $\angle C \cong \angle H$. By $\triangle$ Sum Thm., $m\angle A = 47^\circ$, so $\angle A \cong \angle F$. Therefore $\triangle ABC \sim \triangle FGH$ by AA $\sim$.
2. $\angle P \cong \angle T$ (given). $\angle QST$ is a rt. $\angle$ by the Lin. Pair Thm., so $\angle QST \cong \angle RSP$. Therefore $\triangle QST \sim \triangle RSP$ by AA $\sim$.
3. $DE = \frac{8}{16} = \frac{1}{2}$, $DF = \frac{6}{12} = \frac{1}{2}$, $EF = \frac{10}{20} = \frac{1}{2}$
Therefore $\triangle DEF \sim \triangle JKL$ by SSS $\sim$.
4. $\angle NMP \cong \angle RMQ$ (given)
$\frac{MN}{MR} = \frac{4}{6} = \frac{2}{3}$, $\frac{MP}{MQ} = \frac{8}{4} + \frac{12}{8} = \frac{2}{3}$
Therefore $\triangle MNP \sim \triangle MRQ$ by SAS $\sim$.

5. **Step 1** Prove $\triangle$ are $\sim$.
It is given that $\angle C \cong \angle E$, $\angle A \cong \angle A$ by Reflex. Prop. of $\cong$. Therefore $\triangle AED \cong \triangle ACB$ by AA $\sim$.

**Step 2** Find $AB$.
$AB = BC$
$AD \parallel DE$
$AB = 15$
$6 = 9$
$9AB = 15(6) = 90$
$AB = 10$

6. **Step 1** Prove $\triangle$ are $\sim$.
Since $UV \parallel WY$, by Alt. Int. $\angle$ Thm., $\angle U \cong \angle Y$ and $\angle V \cong \angle X$. Therefore $\triangle UVW \sim \triangle YXW$ by AA $\sim$.

**Step 2** Find $WY$.
$\frac{WY}{WX} = \frac{WU}{WV}$
$WY = 8.75$
$9 = 7$
$7WY = 9(8.75) = 78.75$
$WY = 11.25$

7. **Statements** | **Reasons**
---|---
1. $MN \parallel KL$
2. $\angle JMN \cong \angle JKL$, $\angle JNM \cong \angle JKL$
3. $\triangle JMN \sim \triangle JKL$
4. $\triangle PQR \sim \triangle PST$
5. $\triangle ACB \cong \triangle RPQ$
6. $\triangle SP \cong \triangle TP$, $\triangle RP = 3$
7. $\angle QST \cong \angle QST$

8. **Statements** | **Reasons**
---|---
1. $SQ = 2QO$, $TR = 2RP$
2. $SP = SQ + QP$, $TP = TR + RP$
3. $SP = 2QO + QP$, $TP = 2RP + RP$
4. $SP = SQ + QP$, $TP = 3RP$
5. $SP = 3QO$, $QP = 3$
6. $\angle P \cong \angle B$
7. $\triangle P \cong \triangle B$
8. $\triangle ABC \sim \triangle DEF$ by AA $\sim$.

9. SAS or SSS $\sim$ Thm.

10. **Step 1** Prove $\triangle$ are $\sim$.
$\angle S \cong \angle S$ by Reflex. Prop. of $\cong$.
$\frac{SA}{SC} = \frac{733 + 586}{586} = 2.25$, $\frac{SB}{SD} = \frac{800 + 644}{644} = 2.24$
Therefore $\triangle SAB \sim \triangle SCD$ by SAS $\sim$.

**Step 2** Find $AB$.
$\frac{AB}{SA} = \frac{533}{2.25} = \frac{SC}{SD}$
$AB = 2.25(533)$
$= 1200\text{ m or } 1.2\text{ km}$

**PRACTICE AND PROBLEM SOLVING, PAGES 475–476**

11. $\angle G \cong \angle G$ by Reflex. Prop. of $\cong$. $\angle GLH \cong \angle K$ by Rt. $\angle$ Thm. Therefore $\triangle HLG \sim \triangle JKG$ by AA $\sim$.

12. By Isosc. $\triangle$ Thm., $\angle B \cong \angle C$ and $\angle E \cong \angle F$. By $\triangle$ Sum Thm.,
$32 + 2m\angle B = 180$
$2m\angle B = 148^\circ$
$m\angle B = 74^\circ$
By def. of $\cong$, $\angle B \cong \angle E$ and $\angle C \cong \angle F$.
Therefore $\triangle ABC \sim \triangle DEF$ by AA $\sim$. 
13. $\angle K \cong \angle K$ by Reflex. Prop. of $\cong$.
\[KL = \frac{6}{3} \quad KM = \frac{5 + 4}{2} \quad KL = \frac{6}{2} \]
Therefore $\triangle KLM \sim \triangle KNL$ by SAS $\cong$.

14. $\frac{UV}{VW} = \frac{WU}{YZ} = \frac{4}{8}$
\[\frac{XY}{XZ} = \frac{5.5}{11} \]
Therefore $\triangle UVW \sim \triangle XYZ$ by SSS $\cong$.

15. **Step 1** Prove $\triangle$ are $\sim$.
It is given that $\angle ABD \cong \angle C$, $\angle A \cong \angle A$ by Reflex.
Prop. of $\cong$. Therefore $\triangle ABD \cong \triangle ACB$ by AA $\sim$.

**Step 2** Find $AB$.
\[\frac{AB}{AC} = \frac{AD}{AB} \]
\[\frac{4 + 12}{AB} = \frac{4}{AB} \]
\[AB^2 = 4(16) = 64 \]
\[AB = +\sqrt{64} = 8 \]

16. **Step 1** Prove $\triangle$ are $\sim$.
Since $ST \parallel VW$, $\angle PST \equiv \angle V$ by corr. $\triangle$ post. $\angle P \equiv \angle P$ by Reflex. Prop. of $\cong$. Therefore $\triangle PST \sim \triangle PVW$ by AA $\sim$.

**Step 2** Find $PS$.
\[PS = ST \]
\[PV = VW \]
\[PS = 10 \]
\[PS + 6 = 17.5 \]
\[7PS = 4(PS + 6) \]
\[3PS = 24 \]
\[PS = 8 \]

17. **Statements** | **Reasons**
--- | ---
1. $CD = 3AC$, $CE = 3BC$ | 1. Given
2. $\frac{CD}{AC} \cong \frac{CE}{BC}$ | 2. Div. Prop. of $\cong$
3. $\triangle ACG \cong \triangle DCE$ | 3. Vert. $\triangle$ Thm.
4. $\triangle ABC \sim \triangle DEC$ | 4. SAS $\sim$ Steps 2, 3

18. **Statements** | **Reasons**
--- | ---
1. $\frac{PR}{QR} = \frac{MR}{NR}$ | 1. Given
2. $\angle R \cong \angle R$ | 2. Reflex. Prop. of $\cong$
3. $\triangle POR \sim \triangle MNR$ | 3. SAS $\sim$ Steps 1, 2
4. $\angle 1 \cong \angle 2$ | 4. Def. of $\sim$ $\triangle$

19. By Vert. $\triangle$ Thm., $\angle 1 \cong \angle 2$. Since vert. sides are $\parallel$, $\angle 3 \cong \angle 4$ by corr. $\triangle$ post., so marked $\triangle$ are $\sim$.

Therefore,
\[\frac{h}{2} = \frac{32.5}{54} \]
\[h \approx 1.5 \text{ in.} \]

20. yes; SAS $\sim$

21. yes; SSS $\sim$

22. no

23. Think: $\triangle POR \cong \triangle PST$ by AA $\sim$.
\[\frac{PS}{ST} = \frac{QR}{PO} \]
\[\frac{x + 3}{x + 5} = \frac{3}{4} \]
\[4(x + 3) = 3(x + 5) \]
\[4x + 12 = 3x + 15 \]
\[x = 3 \]

24. Think: $\triangle EFG \cong \triangle HJG$ by AA $\cong$.
\[\frac{EG}{FG} = \frac{GH}{GJ} \]
\[\frac{2x - 2}{x + 9} = \frac{15}{20} \]
\[20(2x - 2) = 15(x + 9) \]
\[40x - 40 = 15x + 135 \]
\[25x = 175 \]
\[x = 7 \]

25a. Think: Calculate slant edge lengths for each base edge length pyramid.

Pyramid A: \[\frac{12}{10} = \frac{6}{5} \]

Pyramid B: \[\frac{9}{7.2} = \frac{5}{4} \]

Pyramid C: \[\frac{9.6}{8} = \frac{6}{5} \]

Since slant edges of each pyramid are $\cong$,
Pyramids A and C are $\sim$ by SSS $\cong$.

Lengths are $\cong$.

b. base of A = \[\frac{10}{8} = \frac{5}{4} \]

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26. Possible answer: Yes; if corr. Δ are ≅ and corr. sides are prop., \( \triangle ABC \sim \triangle XYZ \).

27. Think: Since all horiz. lines are ||, 3 Δ with horiz. bases are ~ by AA ~.

\[
\begin{align*}
\frac{JK}{MN} &= \frac{3}{6} \quad \frac{6}{9} \\
9JK &= 6(3) = 18 \\
9MN &= 6(6) = 36 \\
JK &= 2 \text{ ft} \\
MN &= 4 \text{ ft}
\end{align*}
\]

28. Since \( \triangle ABC \sim \triangle DEF \), by def. of ~, \( \angle A \equiv \angle D \) and \( \angle B \equiv \angle E \). Similarly, since \( \triangle DEF \sim \triangle XYZ \), \( \angle D \equiv \angle X \) and \( \angle E \equiv \angle Y \). Thus by Trans. Prop. of ~, \( \angle A \equiv \angle X \) and \( \angle B \equiv \angle Y \). So \( \triangle ABC \sim \triangle XYZ \) by AA ~.

29. Possible answer:

30. Since \( \triangle KNJ \) is isosc. with vertex \( \angle N \), \( \overline{KN} \equiv \overline{NJ} \) by def. of an isosc. Δ. \( \angle N \equiv \angle NJK \) by Isosc. Δ Thm. It is given that \( \angle H \equiv \angle L \), so \( \angle GHL \equiv \angle MLK \) by AA ~.

31a. The Δ are ~ by AA ~ if you assume that camera is || to hurricane (that is, \( YX \parallel AB \)).

b. \( \triangle YWZ \sim \triangle BCZ \) and \( \triangle XWZ \sim \triangle ACZ \), also by AA ~.

c. \[
\begin{align*}
\frac{XW}{WZ} &= \frac{50}{150} \\
\frac{150XW}{50AC} &= \frac{50}{150} \\
\frac{YW}{WZ} &= \frac{50}{150} \\
\frac{BC}{AC} &= \frac{50BC}{150} \\
\frac{150XW + 150YW}{50AC + 50BC} &= \frac{5250}{150} \\
\frac{50AB}{50AB} &= \frac{150(35)}{5250} \\
AB &= 105 \text{ mi}
\end{align*}
\]

32. Solution B is incorrect. The proportion should be \[
\frac{8}{10} = \frac{8 + y}{14}.
\]

33. Let measure of vertex Δ be \( x \)°. Then by Isosc. Δ Thm., base Δ in each will measure \( \left( \frac{180 - x}{2} \right) \)°. So Δ are ~ by AA ~.

TEST PREP, PAGE 477

34. C

35. J

36. C

\[\text{Rects.} \sim \frac{BC}{\overline{FG}}, \angle C \sim \angle G, \text{and} \overline{CD} \sim \overline{GH}, \text{which are conditions for SAS ~.}\]

37. \[\frac{x}{12} = \frac{8}{8} \quad 8x = 12(20) = 240 \\
x = 30\]

CHALLENGE AND EXTEND, PAGE 477

38. Assume that \( AB < DE \) and choose \( X \) on \( \overline{DE} \) so that \( AB \parallel DX \). Then choose \( Y \) on \( \overline{DF} \) so that \( XY \parallel EF \).

39. Assume that \( AB < DE \) and choose \( X \) on \( \overline{DE} \) so that \( XE = AB \). Then choose \( Y \) on \( \overline{EF} \) so that \( XY \parallel DF \).

40. Think: Use \( \Delta \) Sum Thm. and def. of ~.

\[
\frac{m\angle X + m\angle Y + m\angle Z}{700} = 180 \quad 2x + 5y + 102 = x + 5x + y = 180 \\
6x + 6y = 78 \\
x + y = 13 \\
y = 13 - x
\]

Think: Use def. of ~.

\[\angle A \equiv \angle X \]

\[m\angle A = m\angle X \]

\[50 = 2x + 5y \]

\[50 = 2x + 5(13 - x) \]

\[50 = 65 - 3x \]

\[3x = 15 \]

\[x = 5 \]

\[y = 13 - 5 = 8 \]

\[m\angle Z = 5(5) + 8 = 33^\circ\]

SPIRAL REVIEW, PAGE 477

41. \[100 = \frac{96 + 99 + 105 + 105 + 94 + 107 + x}{700} = 606 + x \]

\[x = 94\]

42. Possible answer: (0, 4), (0, 0), (2, 0)

43. Possible answer: (0, k), (2k, k), (2k, 0), (0, 0)
44. \[ 2x = \frac{35}{10} \quad 25 \quad 50x = 350 \quad x = 7 \]

45. \[ 5y = \frac{25}{450} \quad 10y = 450(25) \quad y^2 = 12250 \quad y = \pm 115 \]

46. \[ \frac{b - 5}{28} = \frac{7}{b - 5} \]
\[(b - 5)^2 = 28(7) = 196 \]
\[b - 5 = \pm 14 \]
\[b = 5 \pm 14 = 19 \text{ or } -9 \]

7A MULTI-STEP TEST PREP, PAGE 478

1. \[ \frac{\text{height of model}}{\text{height of real engine}} = \frac{1}{2.5(87)} \]
\[x = \frac{1}{87} \]
\[x = 217.5 \text{ in. } \approx 18 \text{ ft} \]

2. \[ \frac{\text{height of model}}{\text{height of real station}} = \frac{1}{2.5(87)} \]
\[y = \frac{1}{87} \]
\[87y = 20 \]
\[y \approx 0.23 \text{ ft } \approx 2^{3/4} \text{ in.} \]

3. \[ \frac{\text{height of model}}{\text{height of actual restaurant}} = \frac{1}{2.5(87)} \]
\[z = \frac{1}{24} \]
\[87z = 24 \]
\[z \approx 0.28 \text{ ft } \approx 3 \text{ in.} \]

4. \[ \text{base of B} = \frac{8}{14} = \frac{5}{7} \]
\[\text{slant of B} = \frac{6}{10} = \frac{3}{5} \quad ; \quad \text{not } \sim \]
\[\text{base of G} = \frac{14}{14} = \frac{7}{7} \quad ; \quad \text{slant of G} = \frac{10}{20} = \frac{5}{5} \quad ; \quad \text{not } \sim \]
\[\text{base of H} = \frac{6}{6} = \frac{3}{3} \quad ; \quad \text{slant of H} = \frac{4.5}{9} = \frac{4.5}{9} \quad ; \quad \text{sim} \]
\[\text{base of B} = \frac{8}{6} = \frac{4}{3} \quad ; \quad \text{slant of B} = \frac{6}{10} = \frac{3}{5} \quad ; \quad \text{not } \sim \]

5. Bank's and hotel's roofs are \( \sim \), by SSS \( \sim \).

13. 

1. \( ABCD \) is a \( \square \).
2. \( AD \parallel BC \)
3. \( \triangle EFG \equiv \triangle FBG \)
4. \( \triangle EGD \equiv \triangle FGB \)
5. \( \triangle EDG \sim \triangle FGB \)

14. 

1. \( MQ = \frac{1}{3} \quad MN, \quad MR = \frac{1}{3} \quad MP \quad 1 \quad \text{Given} \)
2. \( MQ = \frac{1}{3} \quad MR = \frac{1}{3} \quad MP \)
3. \( MN = \frac{1}{3} \quad MP \quad 3 \quad \text{Trans. Prop of } = \)
4. \( \angle LM = \angle M \quad 4 \quad \text{Reflex. Prop of } = \)
5. \( \triangle MQR \sim \triangle MNP \quad 5 \quad \text{SAS } \sim \text{ Steps 3, 4} \)

15. Think: \( \triangle XYZ \sim \triangle VUZ \) with ratio of proportion \( \frac{5}{2} \)

READY TO GO? PAGE 479

1. \[ \text{slope } = \frac{-1 + 2}{4 + 1} = \frac{1}{5} \]

2. \[ \text{slope } = \frac{-3 - 3}{2 + 1} = \frac{-6}{3} = \frac{-2}{1} \]

3. \[ \text{slope } = \frac{1 - 3}{4 + 4} = \frac{-2}{8} = \frac{-1}{4} \]

4. \[ \text{slope } = 0 \]

5. \[ \frac{y}{6} = \frac{12}{9} \]
\[9y = 6(12) = 72 \]
\[y = 8 \]

6. \[ \frac{16}{24} = \frac{20}{30} \]
\[16t = 20(24) = 480 \]
\[t = 30 \]

7. \[ \frac{x - 2}{4} = \frac{9}{x - 2} \]
\[\frac{(x - 2)^2}{4(9)} = \frac{36}{2(24)} = \frac{3(y)}{y^2} \]
\[x - 2 = \pm 6 \]
\[x = 2 \pm 6 \]
\[= -4 \text{ or } 8 \]
\[y = \pm 4 \]

TECHNOLOGY LAB: INVESTIGATE ANGLE BISECTORs OF A TRIANGLE, PAGE 480

TRY THIS, PAGE 480

1. \( BD = \frac{CD}{AC} \quad \text{or} \quad BD = \frac{AB}{AC} \)
2. \( BD = \frac{AB}{AC} \quad \text{or} \quad BD = \frac{CD}{AC} \)

ACTIVITY 2:

2. Check students' work.
3. \[ DI = \frac{DE + DF}{\text{perimeter } \triangle DEF} \]
4. \( \frac{DI}{DG} = \frac{DE + DF}{DE + DF + EF} \) the length of the seg. from the vertex of the bisected \( \angle \) to the incenter divided by the length of the seg. from the vertex to the opp. side is \( = \) to the sum of the sides of the bisected \( \angle \) divided by the perimeter of the \( \triangle \).

TRY THIS, PAGE 480

3. Check students' work.

4. Check students' work.

7-4 APPLYING PROPERTIES OF SIMILAR TRIANGLES, PAGES 481–487

CHECK IT OUT! PAGES 482–483

1. It is given that \( \overline{PQ} \parallel \overline{LM} \), so \( \frac{PL}{PN} = \frac{QM}{QN} \) by \( \triangle \) Prop. Thm.
\[
\begin{align*}
3 & = 2 \\
PQ & = 2PN \\
PN & = 7.5
\end{align*}
\]

2. \( AD = 36 - 20 = 16 \) and \( BE = 27 - 15 = 12 \), so
\[
\begin{align*}
ds & = 20 \\
AD & = 16 \\
BE & = 12 \\
DC & = \frac{EC}{BE} \parallel \overline{AB} \text{ by Conv. of } \triangle \text{ Prop. Thm.}
\end{align*}
\]

3. \( \overline{AK} \parallel \overline{BL} \parallel \overline{CM} \parallel \overline{DN} \)
\[
\begin{align*}
LM & = \frac{2.6}{2.4} \\
KL & = \frac{2.6}{2.4} \\
MN & = \frac{2.4}{2.2} \\
2.4(MN) & = 2.6(2.2) \\
MN & = 2.4 \text{ cm}
\end{align*}
\]

4. \( \triangle \) Proportionality Thm.: If \( \overline{EF} \parallel \overline{BC} \), then \( \frac{AE}{EC} = \frac{AF}{FC} \).

EXERCISES, PAGES 484–487

GUIDED PRACTICE, PAGES 484–485

1. It is given that \( \overline{CD} \parallel \overline{FG} \), so \( \frac{CE}{CF} = \frac{DE}{DG} \) by \( \triangle \) Prop. Thm.
\[
\begin{align*}
24 & = 40 \\
DG & = 960 \\
\frac{RM}{RN} & = 30
\end{align*}
\]

2. It is given that \( \overline{QR} \parallel \overline{PN} \), so \( \frac{QM}{QP} = \frac{RM}{RN} \) by \( \triangle \) Prop. Thm.
\[
\begin{align*}
8 & = 10 \\
RN & = 6.25
\end{align*}
\]

3. \( \frac{EC}{AC} = \frac{1.5}{1.5} = 1; \frac{ED}{DB} = \frac{1.5}{1.5} = 1 \)
Since \( \frac{EC}{AC} = \frac{ED}{DB} \), \( \overline{AB} \parallel \overline{CD} \) by Conv. of \( \triangle \) Prop. Thm.

4. \( \frac{VU}{US} = \frac{67.5}{54} = 5; \frac{VT}{TR} = \frac{90}{72} = \frac{5}{4} \)
Since \( \frac{VU}{US} = \frac{VT}{TR} \), \( \overline{TU} \parallel \overline{RS} \) by Conv. of \( \triangle \) Prop. Thm.

5. Let \( \ell \) represent length of Broadway between 34th and 35th Streets.
\[
\begin{align*}
\ell & = 250 \\
275 & = 240 \\
240\ell & = 275(250) \\
\ell & \approx 286 \text{ ft}
\end{align*}
\]

6. \( \frac{QR}{RS} = \frac{PQ}{PS} \) by \( \triangle \) Bis. Thm.
\[
\begin{align*}
x - 2 & = 12 \\
x + 1 & = 16 \\
16(x - 2) & = 12(x + 1) \\
16x - 32 & = 12x + 12 \\
4x & = 44 \\
x & = 11 \\
QR & = 11 - 2 = 9; RS = 11 + 1 = 12
\end{align*}
\]
7. \(BC = \frac{AB}{AD}\) by \(\triangle \angle \text{Bis. Thm.}\)
\[\frac{6}{2} = \frac{9}{2y - 4}\]
\(6(2y - 4) = 9(2)\)
\(12y - 24 = 18\)
\(12y = 42\)
\(y = 3\)
\(CD = 5 - 1 = 4; AD = 2(5) - 4 = 6\)

**PRACTICE AND PROBLEM SOLVING, PAGES 485–486**

8. \(\frac{GJ}{HK}\) by Conv. of \(\triangle\) Prop. Thm.
\[\frac{6}{8} = \frac{8}{8}\]
\(6\) is 3, 4, 5.

9. \(\frac{XY}{XZ}\) by Conv. of \(\triangle\) Prop. Thm.
\[\frac{30 - 18}{30} = \frac{12}{30}\]
\(XZ = 20\)

10. \(\frac{EC}{CA} = \frac{12}{4} = \frac{3}{1}\)
\(\frac{ED}{DB} = \frac{14}{2} = \frac{7}{3}\)
So \(AB \parallel CD\) by Conv. of \(\triangle\) Prop. Thm.

11. \(\frac{PM}{MQ} = \frac{9 - 2.7}{2.7} = \frac{21}{3}\)
\(\frac{PN}{NR} = \frac{10 - 3}{3} = \frac{7}{3}\)
So \(MN \parallel QR\) by Conv. of \(\triangle\) Prop. Thm.

12. \(\frac{LM}{GL} = \frac{HJ}{JH}\)
\(\frac{10.4}{2.6} = \frac{11.3}{10.4}\)
\(LM = \frac{2.6}{11.3} \approx 2.39\)
\(\approx 2.83\)

13. \(\frac{BC}{CD} = \frac{AD}{BD}\)
\(\frac{z - 4}{10} = \frac{12}{10}\)
\(z = 10\)
\(BC = 10 - 4 = 6; CD = 10\)

14. \(\frac{TU}{ST}\)
\(\frac{2y}{4y - 2} = \frac{14.4}{24}\)
\(24(2y) = 14.4(4y - 2)\)
\(48y = 57.6y - 28.8\)
\(28.8 = 9.6y\)
\(y = 3\)
\(ST = 4(3) - 2 = 10; TU = 2(3) = 6\)

15. \(\frac{AB}{BD} = \frac{AC}{CE}\)
\(\frac{16}{4} = \frac{AE}{EG}\)

16. \(\frac{AD}{DF} = \frac{AE}{EG}\)
\(\frac{18}{10} = \frac{AF}{AG}\)
\(\frac{18}{10} = \frac{AB}{AC}\)

17. \(\frac{BD}{CE} = \frac{EG}{DF}\)
\(\frac{20}{14} = \frac{AB}{AC}\)

18. \(\frac{BD}{CE} = \frac{DF}{EG}\)
\(\frac{20}{14} = \frac{AB}{AC}\)

19. \(\frac{AB}{AC} = \frac{EF}{CG}\)

20. \(\frac{AB}{AC} = \frac{EF}{CG}\)

21. Let \(x\) represent length of 3rd side.
\(\frac{x}{2} = \frac{x - 2}{1}\)
\(2x = x + 2\)
\(x = 6\)
\(x = 6 (since x > 0)\)

22a. \(\frac{AC}{BD} = \frac{CE}{DF}\)
\(\frac{81.6}{80} = \frac{70}{80}\)
\(\frac{81.6}{80} = \frac{10}{8}\)

23. **Statements**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\triangle ABC)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. (\angle A \angle A)</td>
<td>2. Reflex. Prop. of (\equiv)</td>
</tr>
<tr>
<td>3. (\triangle AEF \cong \triangle ABC)</td>
<td>3. SAS &amp; Step, 1, 2</td>
</tr>
<tr>
<td>4. (\triangle AEF \cong \angle ABC)</td>
<td>4. Def. of (\triangle)</td>
</tr>
<tr>
<td>5. (\triangle AEF \cong \triangle ABC)</td>
<td>5. Conv. of Corr. (\triangle) Post.</td>
</tr>
</tbody>
</table>

24. **Statements**

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. (\triangle ABC) (\parallel \triangle DEF)</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. Draw (\triangle DEF) intersecting (\triangle ABC) at (X)</td>
<td>2. 2 pts. determine a line</td>
</tr>
<tr>
<td>3. (\triangle DEF)</td>
<td>3. (\triangle) Prop. Thm.</td>
</tr>
<tr>
<td>4. (\triangle DEF)</td>
<td>4. (\triangle) Prop. Thm.</td>
</tr>
<tr>
<td>5. (\triangle DEF)</td>
<td>5. Trans. Prop. of (\triangle)</td>
</tr>
</tbody>
</table>

25a. \(\frac{PR}{RT} = \frac{QS}{SU}\)
\(\frac{x}{2} = \frac{x - 2}{1}\)
\(2x = x + 2\)
\(x = 6\)
\(x = 6 (since x > 0)\)

26. Think: Use \(\triangle\) Prop. Thm. and \(\triangle \angle \text{Bis. Thm.} \)
\(\frac{EF}{CD} = \frac{AB}{AD}\)
\(\frac{BE}{BC} = \frac{AB}{AD}\)
\(\frac{EF}{24} = \frac{4}{10}\)
\(\frac{3}{} = \frac{3}{}\)
\(3EF = 40\)
\(EF = 13\frac{1}{3}\)
27. \[ \frac{ST}{TQ} = \frac{SR}{RO} = \frac{PN}{NM} \]

28. Total length along Chavez St. is

\[ 150 + 200 + 75 = 425 \text{ ft.} \]

29. Draw a seg. on tracing paper whose length is \( = \) to

Think: Use Proportionality Corollary.

30. Possible answer: \( \frac{BD}{CD} = \frac{AB}{AC}, \triangle \angle \text{ Bis. Thm.} \)

TEST PREP, PAGE 487

32. C

\[ \frac{US}{SR} = \frac{20}{35} = \frac{4}{7}, \frac{VT}{TR} = \frac{16}{28} = \frac{4}{7} \]

33. J

\[ \frac{AB}{25} = \frac{16}{20} \]

34. C

Let \( x \) be dist. to 1st St.

\[ \frac{2.4}{2.8} = \frac{3}{4} \]

35. \[ \frac{x}{24} = \frac{5}{16}, \frac{y}{15} = \frac{4}{20}, \frac{z}{17} = \frac{20}{24}, \frac{y}{15} = \frac{20}{24} \]

CHALLENGE AND EXTEND, PAGE 487

36. \[ P = AB + BC + AC \]

37. Given: \( \triangle ABC \sim \triangle XYZ \), \( \overline{AD} \) bisects \( \angle BAC \), and \( \overline{XY} \) bisects \( \angle YXZ \).

Prove: \( \frac{AD}{XY} = \frac{AB}{XY} \)

<table>
<thead>
<tr>
<th>Statements</th>
<th>Reasons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ( \triangle ABC \sim \triangle XYZ )</td>
<td>1. Given</td>
</tr>
<tr>
<td>2. ( \angle B \cong \angle Y )</td>
<td>2. Def. of ( \sim ) polygons</td>
</tr>
<tr>
<td>3. ( m\angle BAC = m\angle YXZ )</td>
<td>3. Def. of ( \sim ) polygons</td>
</tr>
<tr>
<td>4. ( AD ) bisects ( \angle BAC ) and ( XY ) bisects ( \angle YXZ )</td>
<td>4. Given</td>
</tr>
<tr>
<td>5. ( m\angle BAC = 2m\angle BAD ), ( m\angle YXZ = 2m\angle YXW )</td>
<td>5. Def. of ( \angle ) bis.</td>
</tr>
<tr>
<td>6. ( 2m\angle BAD = 2m\angle YXW )</td>
<td>6. Trans. Prop. of =</td>
</tr>
<tr>
<td>7. ( m\angle BAD = m\angle YXW )</td>
<td>7. Div. Prop. of =</td>
</tr>
<tr>
<td>8. ( \triangle ABD \sim \triangle XYW )</td>
<td>8. AA ( \sim ) Steps 2, 7</td>
</tr>
<tr>
<td>9. ( \frac{AD}{XY} = \frac{AB}{XY} )</td>
<td>9. ( \triangle ) Prop. Thm.</td>
</tr>
</tbody>
</table>
38. \[ X \] 

SPIRAL REVIEW, PAGE 487

40. \( 5 = 1 + 4, 6 = 2 + 4, \ldots \) nth term is \( n + 4 \)
41. \( 3 = 3(1), 6 = 3(2), \ldots \) nth term is \( 3n \)
42. \( 1 = 1^2, 4 = 2^2, 9 = 3^2, \ldots \) nth term is \( n^2 \)
43. Let \( C = (x, y) \).
\[
\begin{align*}
3 &= 1 + x \\
6 &= 1 + x \\
x &= 5
\end{align*}
\]
\[
\begin{align*}
-7 &= 4 + y \\
-14 &= 4 + y \\
y &= -18
\end{align*}
\]
\( C = (5, -18) \)
44. \( \angle A \cong \angle A \) (Reflex. Prop. of \( \cong \) )
\[
\begin{align*}
\frac{AB}{AD} &= \frac{8}{12} = \frac{2}{3} \\
\frac{AC}{AE} &= \frac{6}{9} = \frac{2}{3}
\end{align*}
\]
Therefore \( \triangle ABC \sim \triangle ADE \) by SAS \( \sim \).
45. \( \angle KLM \cong \angle NLM \) (Vert. \( \angle \) Thm.)
\( \angle K \cong \angle N \) (\( \angle \) Sum Thm. \( \rightarrow m\angle N = 68^\circ \))
Therefore \( \triangle KLM \sim \triangle MNL \) by AA \( \sim \).

39. Possible answer: Check students' work.

7-5 USING PROPORTIONAL RELATIONSHIPS, PAGES 488–494

CHECK IT OUT! PAGES 488–490

1. Step 1 Convert measurements to inches.
\( GH = 5 \text{ ft} 6 \text{ in.} = 5(12) \text{ in.} + 6 \text{ in.} = 66 \text{ in.} \)
\( JH = 5 \text{ ft} 5(12) \text{ in.} = 60 \text{ in.} \)
\( NM = 14 \text{ ft} 2 \text{ in.} = 14(12) \text{ in.} + 2 \text{ in.} = 170 \text{ in.} \)
Step 2 Find \( \triangle \).
Because sun's rays are \( \parallel \), \( \angle J \cong \angle N \). Therefore \( \triangle GHJ \cong \triangle LMN \) by AA \( \sim \).
Step 3 Find LM.
\[
\begin{align*}
GH &= JH \\
LM &= NM \\
LM &= 60 \\
LM &= 170
\end{align*}
\]
\( 60LM = 66(170) \)
\( LM = 187 \text{ in.} = 15 \text{ ft} 7 \text{ in.} \)

2. Use a ruler to measure dist. between City Hall and El Centro College. Dist. is 4.5 cm.
To find actual dist. \( y \), write a proportion comparing map dist. to actual dist.
\[
\begin{align*}
\frac{4.5}{1.5} &= \frac{300}{y} \\
1.5y &= 4.5(300) \\
1.5y &= 1350 \\
y &= 900
\end{align*}
\]
Actual dist. is 900 m, or 0.9 km.

3. Step 1 Set up proportions to find length \( \ell \) and width \( w \) of of scale drawing.
\[
\begin{align*}
\ell &= \frac{1}{74} = \frac{1}{20} \\
w &= \frac{1}{60} = \frac{1}{20}
\end{align*}
\]
\( 20\ell = 74 \\
20w = 60
\]
\( \ell = 3.7 \text{ in.} \\
w = 3 \text{ in.} \)
Step 2 Use a ruler to draw a rect. with new dimensions. (Check students' work.)

4. Similarity ratio of \( \triangle ABC \) to \( \triangle DEF \) is \( \frac{4}{12} \) or \( \frac{1}{3} \).
By Proportional Perimeters and Areas Thm., ratio of \( \triangle ' \) perimeters is also \( \frac{1}{3} \) and ratio of \( \triangle ' \) areas is \( \left( \frac{1}{3} \right)^2 \), or \( \frac{1}{9} \).

<table>
<thead>
<tr>
<th>Perimeter</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P = 1 )</td>
<td>( A = \frac{1}{9} )</td>
</tr>
<tr>
<td>( 42 \text{ in.} )</td>
<td>( 96 \text{ in.} )</td>
</tr>
<tr>
<td>( 3P = 42 )</td>
<td>( 9A = 96 )</td>
</tr>
<tr>
<td>( P = 14 \text{ mm} )</td>
<td>( A = 10\frac{2}{3} \text{ mm}^2 )</td>
</tr>
</tbody>
</table>

Perimeter of \( \triangle ABC \) is 14 mm, and area is \( 10\frac{2}{3} \text{ mm}^2 \).

THINK AND DISCUSS, PAGE 490

1. Set up a proportion: \( \frac{5.5}{x} = \frac{1}{25} \). Then solve for \( x \) to find actual dist.: \( x = 5.5(25) = 137.5 \text{ mi.} \)

2.

<table>
<thead>
<tr>
<th>Similar Figures</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Similarity ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} ) or ( \frac{3}{4} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of perimeters</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} ) or ( \frac{3}{4} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Ratio of areas</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{1}{2} ) or ( \frac{3}{4} )</td>
</tr>
</tbody>
</table>
1. indirect measurement

2. Step 1 Convert measurements to inches.
5 ft 6 in. = 5(12) in. + 6 in. = 66 in.
4 ft = 4(12) in. = 48 in.
40 ft = 40(12) in. = 480 in.
Step 2 Find ~ Δ.
Since marked ∆ are ≅, ∆ are ~ by AA ∼.
Step 3 Find height of dinosaur, h.

3. Use a ruler to measure to-scale length of AB. Length is 0.25 in.
To find actual length AB, write a proportion comparing to-scale length to actual length.
\[
\frac{0.25}{1} = \frac{AB}{48}
\]
\[
AB = 0.25(48) = 12 \text{ ft}
\]

4. Use a ruler to measure to-scale length of CD. Length is 0.75 in.
To find actual length CD, write a proportion comparing to-scale length to actual length.
\[
\frac{0.75}{1} = \frac{CD}{48}
\]
\[
CD = 0.75(48) = 36 \text{ ft}
\]

5. Use a ruler to measure to-scale length of EF. Length is 1.25 in.
To find actual length EF, write a proportion comparing to-scale length to actual length.
\[
\frac{1.25}{1} = \frac{EF}{124}
\]
\[
EF = 1.25(124) = 60 \text{ ft}
\]

6. Use a ruler to measure to-scale length of FG. Length is 0.5 in.
To find actual length FG, write a proportion comparing to-scale length to actual length.
\[
\frac{0.5}{1} = \frac{FG}{48}
\]
\[
FG = 0.5(48) = 24 \text{ ft}
\]

7. Step 1 Set up proportions to find length ℓ and width w of scale drawing.
\[
\frac{10}{\ell} = \frac{1}{1}
\]
\[
10 \text{ cm} = \ell \text{ cm}
\]
\[
\frac{4.6}{w} = \frac{1}{1}
\]
\[
w = 4.6 \text{ cm}
\]
Step 2 Use a ruler to draw a rect. with new dimensions. (Check students' drawings.)

8. Step 1 Set up proportions to find length ℓ and width w of scale drawing.
\[
\frac{2}{\ell} = \frac{1}{1}
\]
\[
2 \ell = 10 \text{ cm}
\]
\[
\frac{4.6}{w} = \frac{2}{1}
\]
\[
2w = 4.6 \text{ cm}
\]
\[
\ell = 5 \text{ cm}
\]
\[
w = 2.3 \text{ cm}
\]
Step 2 Use a ruler to draw a rect. with new dimensions. (Check students' drawings.)

9. Step 1 Set up proportions to find length ℓ and width w of scale drawing.
\[
\frac{b}{10} = \frac{2.3}{4.6}
\]
\[
2.3b = 10 \text{ in.}
\]
\[
\frac{b}{4.3} = \frac{2.3}{2}
\]
\[
w = 4.6 \text{ cm}
\]
Step 2 Use a ruler to draw a rect. with new dimensions. (Check students' drawings.)

11. Ratio of areas is \(\left(\frac{2}{3}\right)^2\), or \(\frac{4}{9}\).
12. \(\frac{b}{12} = \frac{A}{9}\)
\[2A = 12(9) = 108\]
\[A = 27\]
Area of RSTU is 27 cm².

15. Step 1 Set up proportions to find base b and height h of scale drawing.
\[
\frac{b}{1.5} = \frac{100}{150}
\]
\[
b = 10 \text{ ft}
\]
\[
\frac{h}{1.5} = \frac{200}{225}
\]
\[
h = 1.5 \text{ in.}
\]
Step 2 Use a ruler to draw a rt. ∆ with new dimensions. (Check students' drawings.)

16. Step 1 Set up proportions to find base b and height h of scale drawing.
\[
\frac{b}{1} = \frac{300}{150}
\]
\[
b = 6 \text{ in.}
\]
\[
\frac{h}{1} = \frac{300}{150}
\]
\[
h = 1 \text{ in.}
\]
Step 2 Use a ruler to draw a rt. ∆ with new dimensions. (Check students' drawings.)

17. Step 1 Set up proportions to find base b and height h of scale drawing.
\[
\frac{b}{1} = \frac{150}{150}
\]
\[
b = 1 \text{ in.}
\]
\[
\frac{h}{1} = \frac{150}{150}
\]
\[
h = 1.3 \text{ in.}
\]
Step 2 Use a ruler to draw a rt. ∆ with new dimensions. (Check students' drawings.)
18. \( \frac{P}{P} = \frac{2}{2} = \frac{3}{3} \)  
\( \frac{2}{2} \)  
\( \frac{381 - 3}{3} = \frac{762}{25} \)  
\( 19. \frac{A}{A} = \left( \frac{2}{3} \right)^2 = \frac{4}{9} \)  
\( \frac{1944}{9A} = 7776 \)  
\( A = 864 \text{ m}^2 \)  
\( 20. \text{ scale factor} = \frac{10 \text{ ft}}{0.5 \text{ in.}} = \frac{20}{\frac{10}{8}} = \frac{20}{x} \)  
\( \text{map dist.} = \frac{20}{16} \)  
\( x = \frac{30}{20} \)  
\( \approx 38 \text{ ft} \)  
\( 21. \text{ map dist.} = \frac{10 \text{ in.}}{8 \text{ in.}} = \frac{x}{16} \)  
\( x = \frac{20}{16} \)  
\( \approx 32 \text{ ft} \)  
\( 22. \text{ map dist.} = \frac{25 \text{ in.}}{16} = \frac{25}{16} \)  
\( x = \frac{30}{20} \)  
\( \approx 32 \text{ ft} \)  
\( 23. \text{ map dist.} = \frac{32 \text{ in.}}{16} = \frac{32}{16} \)  
\( x = \frac{30}{20} \)  
\( \approx 39 \text{ ft} \)  
\( 24. \text{ By Proportional Perimeters and Areas Thm.,} \)  
\( \sim \text{ ratio} = \text{ratio of perimeters} = \frac{8}{9} \)  
\( 25. \text{ By Proportional Perimeters and Areas Thm.,} \)  
\( \frac{16}{25} = (\sim \text{ ratio})^2 \)  
\( \sim \text{ ratio} = \sqrt{16} = 4 \)  
\( \frac{5}{25} = \frac{4}{5} \)  
\( 26. \text{ ratio of areas} = (\sim \text{ ratio})^2 \)  
\( \frac{4}{81} = (\text{ratio of perims.})^2 \)  
\( \text{ratio of perims.} = \sqrt{\frac{4}{81}} = \frac{2}{9} \)  
\( 27. \text{ scale width} = \frac{1}{50} \)  
\( \text{model width} = \frac{15}{50} \)  
\( w = \frac{15}{50} = 0.3 \text{ ft} \)  
\( \text{scale length} = \frac{1}{50} \)  
\( \text{model length} = \frac{1}{50} \)  
\( \ell = \frac{1}{50} \)  
\( \ell = \frac{60}{50} = 1.2 \text{ ft} \)  
\( 28a. \)  
\( \text{hyp. of } \triangle POR = \sqrt{3^2 + 4^2} = 5 \text{ in.} \)  
\( \text{hyp. of } \triangle WXY = \sqrt{6^2 + 8^2} = 10 \text{ in.} \)  
\( \text{perimeter of } \triangle POR = 3 + 4 + 5 \)  
\( \text{perimeter of } \triangle WXY = 6 + 8 + 10 \)  
\( = 12 = 1 \)  
\( 2 \)  
\( b. \text{ area of } \triangle POR = \frac{1}{2}(4)(4) \)  
\( \text{area of } \triangle WXY = \frac{1}{2}(8)(6) \)  
\( = \frac{6}{24} = \frac{1}{4} \)  
\( c. \text{ The ratio of areas is square of ratio of perimeters.} \)  
\( 29. \text{ Let } \ell_1 \text{ and } w_1 \text{ be dimensions of rect. } \triangle ABCD; \text{ let } \ell_2 \)  
\( \text{ and } w_2 \text{ be dimensions of rect. } \triangle EFGH. \)  
\( A_1 = \ell_1 \cdot w_1 \)  
\( 135 = \ell_1(9) \)  
\( \ell_1 = 15 \text{ in.} \)  
\( 30. \text{ Check students’ work.} \)  
\( \text{scale length} = \ell = \frac{2}{5} \)  
\( \text{actual length} = 94 = \frac{2}{10} \)  
\( 10 \ell = 23.5 \)  
\( 60 \ell = 2.35 \text{ in.} \)  
\( \text{scale width} = w = \frac{1}{5} \)  
\( \text{actual width} = 50 = \frac{1}{10} \)  
\( 10w = 12.5 \)  
\( w = 1.25 \text{ in.} \)  
\( 31a. \)  
\( \sim \text{ ratio} = 1 \)  
\( \frac{1}{2} \)  
\( \text{ft} = \frac{1}{24} \text{ in.} \)  
\( 2 \)  
\( b. \text{ actual dimensions are } 24(2) = 48 \text{ in. and } 24(3) = 72 \text{ in.} \)  
\( \text{actual area} = (48)(72) = 3456 \text{ in.} \)  
\( \text{model area} = (2)(3) = 6 \text{ in.} \)  
\( 6 \text{ in.} = 576 \)  
\( c. \text{ actual area} = (4 \text{ ft})(6 \text{ ft}) = 24 \text{ ft}^2 \)  
\( 32. \text{ In photo, height of person } = \frac{1}{2} \text{ in. and height of} \)  
\( \text{statue } \approx 1\frac{5}{8} \text{ in.} \)  
\( \text{actual height of statue} \)  
\( \text{height of statue in photo} \)  
\( \text{actual height of person} \)  
\( h \approx \frac{5}{1.625} \approx 0.5 \)  
\( 0.5h \approx 8 \)  
\( h \approx 16 \text{ ft} \)  
\( 33. \)  
\( \text{map length} = \text{scale factor} \)  
\( \frac{1}{2} \)  
\( \text{cm} = \frac{1}{9} \text{ cm} \)  
\( \text{1 km} = \frac{900,000 \text{ cm}}{9 \text{ km}} \)  
\( \ell = \frac{1}{9} \text{ cm} \)
34. By Δ Midseg. Thm., def. of mdpt., and SSS ≅, ΔXYZ ≅ Δ ZJX; so Δ have same height h.
Therefore height of ΔJKL = h + h = 2h.
Since KL = 2ZX,
area of ΔJKL = \frac{1}{2}(2ZX)(2h)
= 2(ZX)h
= 4\left(\frac{1}{2}(ZX)(h)\right)
= 4(area of ΔXYZ)

\text{area of ΔJKL} = \frac{4}{1} \text{area of ΔXYZ}

35. 1 cm : 5 m; Since each cm will represent 5 m, this drawing will be \frac{1}{5} size of the 1 cm : 1 m drawing.

36. \frac{4(x - 2)}{4(2x)} = \frac{x - 2}{2x} = \frac{4}{9}
9(x - 2) = 8x
9x - 18 = 8x
x = 18
AB = 18 - 2 = 16 units
HE = 2(18) = 36 units

37. With a scale of 1 : 1, drawing is same size as actual object.

38. Suppose x and y are whole-number side lengths of smaller square and larger square. Then 2x^2 = y^2.
Thus x\sqrt{2} = y. A whole number that is multiplied by \sqrt{2} cannot equal a whole number, since \sqrt{2} is irrational.

TEST PREP, PAGE 493

39. D
area of ΔRST = (scale factor)^2(area of ΔABC)
= \left(\frac{15}{10}\right)^2(24) = \frac{9}{4}(24) = 54 \text{ m}^2

40. G
\frac{3.75}{\ell} = 0.25
\ell = 15 \text{ ft}

41. C. Ratio of perimeters = \sim \text{ ratio} = \frac{4}{9}

42. F
area of Δ2 = \left(\sim \text{ ratio}\right)^2(area of Δ1)
= \left(\frac{x}{2}\right)^2(16) = 4 \text{ ft}^2

CHALLENGE AND EXTEND, PAGE 494

43a. \frac{x}{1.5 \times 10^8 \text{ km}} = \frac{1 \text{ km}}{10^3 \text{ km}} = 10^3 \text{ m}
x = 10^3 \text{ m} \left(1.5 \times 10^8 \text{ km}\right)
= 1.5 \times 10^2 \text{ m or 150 m}

43b. \frac{d}{1.28 \times 10^4 \text{ km}} = \frac{10^3 \text{ m}}{10^9 \text{ km}}
d = \frac{10^3 \text{ m}}{10^9 \text{ km}} \left(1.28 \times 10^4 \text{ km}\right)
= 1.28 \times 10^{-2} \text{ m or 1.28 cm}

44. It is given that ΔABC \sim ΔDEF. Let \frac{AB}{DE} = x. Then
AB = DEx by Mult. Prop. of \sim. Similarly, BC = EFx and AC = DFx. By Add. Prop. of \sim, AB + BC + AC = DEx + EFx + DFx. Thus AB + BC + AC
= x(DF + EF + DF). By Div. Prop. of \sim,
AB + BC + AC
= \frac{AB}{DE} + \frac{BC}{EF} + \frac{AC}{DF}. By subst., AB + BC + AC
= \frac{AB}{DE} + \frac{BC}{EF} + \frac{AC}{DF}

45. It is given that ΔPOR \sim ΔWXY, Draw \perp s from Q and X to meet PR at S and WY at Z. By def. of \sim polygons, \frac{PQ}{QR} = \frac{RQ}{WY} = \frac{QR}{XY} = \frac{SP}{ZW} and \perp P \equiv \perp W.
In ΔPQS and ΔWXZ, \perp PSQ \equiv \perp WZX. Thus
ΔPQS \sim ΔWXZ by AA \sim.
\frac{PQ}{WZ} = \frac{QS}{XZ} = \frac{PS}{WZ}
by def. of \sim polygons.
\frac{QR}{XY} = \frac{SP}{ZW} by subst.
Area of ΔPQR = \frac{PR^2}{WY^2}.
Area of ΔWXZ = \frac{PQ^2}{WZ^2}.

46a. \frac{6}{WX} = \frac{7}{XY} = \frac{1}{2}
WX = 12
XY = 14
10
\frac{1}{2}
YW = \frac{1}{2}
YZ = 20
WZ = 24

b. Quad. PQRS
<table>
<thead>
<tr>
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<th>Length (m)</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>QR</td>
<td>7</td>
</tr>
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<td>PS</td>
<td>12</td>
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</table>

Quad. WXYZ
<table>
<thead>
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<th>Side</th>
<th>Length (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WXY</td>
<td>12</td>
</tr>
<tr>
<td>YZ</td>
<td>20</td>
</tr>
</tbody>
</table>

47. (x - 3)^2 = 49
x - 3 = \pm 7
x = 3 \pm 7
10 or -4
x = -1 \pm 2
= -3 or 1

48. (x + 1)^2 = 4
(x + 1)^2 = 4
x + 1 = \pm 2
= 10 or -4
x = -1 \pm 2
= -3 or 1

49. 4(x + 2)^2 - 28 = 0
4(x + 2)^2 = 28
(x + 2)^2 = 7
x + 2 = \pm \sqrt{7}
x = -2 \pm \sqrt{7}
= 0.65 or -4.65

50. slope of \overrightarrow{AB} = \frac{2}{3}; slope of \overrightarrow{CD} = \frac{-2}{3}
slope of \overrightarrow{BC} = slope of \overrightarrow{AD} = 0
\overrightarrow{AB} \parallel \overrightarrow{CD} and \overrightarrow{BC} \parallel \overrightarrow{AD}, so ABCD is a \square.
7-6 Dilations and Similarity in the Coordinate Plane, Pages 495–500

CHECK IT OUT! Pages 495–497

1. Step 1 Multiply vertices of photo \(A(0, 0), B(0, 4), C(3, 4), D(3, 0)\) by \(\frac{1}{2}\).

   Rect. \(ABCD\) Rect. \(A'B'C'D'\)

   \[\begin{align*}
   A(0, 0) &\rightarrow A'(\frac{1}{2}(0), \frac{1}{2}(0)) \rightarrow A'(0, 0) \\
   B(0, 0) &\rightarrow B'(\frac{1}{2}(0), \frac{1}{2}(4)) \rightarrow B'(0, 2) \\
   C(0, 0) &\rightarrow C'(\frac{1}{2}(3), \frac{1}{2}(4)) \rightarrow C'(1.5, 2) \\
   D(0, 0) &\rightarrow D'(\frac{1}{2}(3), \frac{1}{2}(0)) \rightarrow D'(1.5, 0)
   \end{align*}\]

   Step 2 Plot pts. \(A'(0, 0), B'(0, 2), C'(1.5, 2),\) and \(D'(1.5, 0)\). Draw the rectangle.

   Check student’s work.

2. Since \(\triangle MON \sim \triangle POQ\),

   \[
   \frac{PQ}{MO} = \frac{OO}{ON}
   \]

   \[
   \frac{-15}{10} = \frac{3}{2} \rightarrow \frac{-30}{ON} \rightarrow ON = -60
   \]

   \[ON = -20\]

   \(N\) lies on \(y\)-axis, so its \(x\)-coord. is 0. Since \(ON = -20\), its \(y\)-coord. must be -20. Coords. of \(N\) are \((0, -20)\).

   \([0, -30) \rightarrow \left(\frac{2}{3}(0), \frac{2}{3}(-30)\right) \rightarrow (0, -20)\), so scale factor is \(\frac{2}{3}\).

3. Step 1 Plot pts. and draw \(\triangle\).

   Step 2 Use Dist. Formula to find side lengths.

   \[
   RS = \sqrt{(-3 + 2)^2 + (1 - 0)^2} = \sqrt{2}\n   \]

   \[
   RT = \sqrt{(0 + 2)^2 + (1 - 0)^2} = \sqrt{5}\n   \]

   \[
   RU = \sqrt{(-5 + 2)^2 + (3 - 0)^2} = \sqrt{18} = 3\sqrt{2}\n   \]

   \[
   RV = \sqrt{(4 + 2)^2 + (3 - 0)^2} = \sqrt{45} = 3\sqrt{5}\n   \]

   Step 3 Find similarity ratio.

   \[
   \frac{RS}{RU} = \frac{\sqrt{2}}{3\sqrt{2}} = \frac{1}{3} \quad \frac{RT}{RV} = \frac{\sqrt{5}}{3\sqrt{5}} = \frac{1}{3}
   \]

   Since \(\frac{RS}{RU} = \frac{RT}{RV}\) and \(\angle R \cong \angle R\) by Reflex. Prop. of \(\cong\), \(\triangle RST \sim \triangle RUV\) by SAS ~.

4. Step 1 Multiply each coord. by \(3\) to find coords of vertices of \(\triangle M'N'P'\).

   \(M(-2, 1) \rightarrow M'(3(-2), 3(1)) = M'(-6, 3)\)

   \(N(2, 2) \rightarrow N'(3(2), 3(2)) = N'(6, 6)\)

   \(P(-1, -1) \rightarrow P'(3(-1), 3(-1)) = P'(-3, -3)\)

   Step 2 Graph \(\triangle M'N'P'\).

   Step 3 Use Dist. Formula to find side lengths.

   \[
   MN = \sqrt{(2 + 2)^2 + (2 - 1)^2} = \sqrt{17}\n   \]

   \[
   M'N' = \sqrt{(6 + 6)^2 + (6 - 3)^2} = \sqrt{153} = 3\sqrt{17}\n   \]

   \[
   NP = \sqrt{(-1 - 2)^2 + (-1 - 2)^2} = \sqrt{18} = 3\sqrt{2}\n   \]

   \[
   N'P' = \sqrt{(-3 - 6)^2 + (-3 - 6)^2} = \sqrt{162} = 9\sqrt{2}\n   \]

   \[
   MP = \sqrt{(1 + 2)^2 + (-1 - 1)^2} = \sqrt{5}\n   \]

   \[
   M'P' = \sqrt{(-3 + 6)^2 + (-3 + 6)^2} = \sqrt{45} = 3\sqrt{5}\n   \]

   Step 4 Find similarity ratio.

   \[
   \frac{MN}{M'N'} = \frac{3\sqrt{17}}{3\sqrt{17}} = 1 \quad \frac{NP}{N'P'} = \frac{9\sqrt{2}}{9\sqrt{2}} = 1 \quad \frac{MP}{M'P'} = \frac{3\sqrt{5}}{3\sqrt{5}} = 1
   \]

   Since \(\frac{MN}{M'N'} = \frac{NP}{N'P'} = \frac{MP}{M'P'}\) \(\triangle MNP \sim \triangle M'N'P'\) by SSS ~.

THINK AND DISCUSS, Page 497

1. The scale factor is \(4\), since each coord. of preimage is multiplied by \(4\) in order to get coords. of image.
6. Step 1 Plot pts. and draw Δ.

Step 2 Use Dist.
Formula to find side lengths.

\[ AB = \sqrt{(1 - 0)^2 + (1 - 0)^2} = \sqrt{2} \]
\[ AC = \sqrt{(3 - 0)^2 + (2 - 0)^2} = \sqrt{13} \]
\[ AD = \sqrt{(2 - 0)^2 + (2 - 0)^2} = \sqrt{8} = 2\sqrt{2} \]
\[ AE = \sqrt{(6 - 0)^2 + (4 - 0)^2} = \sqrt{52} = 2\sqrt{13} \]

Step 3 Find similarity ratio.

\[ \frac{AB}{AD} = \frac{2}{\sqrt{2}} = \frac{1}{2} \quad \frac{AC}{AE} = \frac{\sqrt{13}}{2\sqrt{13}} = \frac{1}{2} \]

Since \( \frac{AB}{AD} \) and \( \angle A \equiv \angle A \) by Reflex. Prop. of \( \cong \), \( \triangle ABC \sim \triangle ADE \) by SAS \( \sim \).

7. Step 1 Plot pts. and draw Δ.

Step 2 Use Dist.
Formula to find side lengths.

\[ JK = \sqrt{(3 + 1)^2 + (4 - 0)^2} = \sqrt{20} = 2\sqrt{5} \]
\[ JL = \sqrt{(3 + 1)^2 + (2 - 0)^2} = \sqrt{20} = 2\sqrt{5} \]
\[ JM = \sqrt{(4 + 1)^2 + (6 - 0)^2} = \sqrt{45} = 3\sqrt{5} \]
\[ JN = \sqrt{(5 + 1)^2 + (3 - 0)^2} = \sqrt{45} = 3\sqrt{5} \]

Step 3 Find similarity ratio.

\[ \frac{JK}{JM} = \frac{2\sqrt{5}}{\sqrt{3\sqrt{5}}} = \frac{2}{\sqrt{3}} \quad \frac{JL}{JN} = \frac{2\sqrt{5}}{3\sqrt{3\sqrt{5}}} = \frac{2}{3} \]

Since \( \frac{JK}{JM} \) and \( \angle J \equiv \angle J \) by Reflex. Prop. of \( \cong \), \( \triangle JKL \sim \triangle JMN \) by SAS \( \sim \).
8. Step 1 Multiply each coord. by 2 to find coords of vertices of $\triangle A'B'C'$.
   $A(1, 4) \rightarrow A'(2(1), 2(4)) = A'(2, 8)$
   $B(1, 1) \rightarrow B'(2(1), 2(1)) = B'(2, 2)$
   $C(3, 1) \rightarrow C'(2(3), 2(1)) = C'(6, 2)$

Step 2 Graph $\triangle A'B'C'$.

Step 3 Use Dist. Formula to find side lengths.
   $AB = \sqrt{(1 - 1)^2 + (4 - 2)^2} = 3$
   $A'B' = \sqrt{(2 - 2)^2 + (8 - 2)^2} = 6$
   $BC = \sqrt{(3 - 1)^2 + (1 - 1)^2} = 2$
   $B'C' = \sqrt{(6 - 2)^2 + (2 - 2)^2} = 4$
   $AC = \sqrt{(3 - 1)^2 + (1 - 4)^2} = \sqrt{13}$
   $A'C' = \sqrt{(6 - 2)^2 + (2 - 8)^2} = 2\sqrt{13}$

Step 4 Find similarity ratio.
   $\frac{A'B'}{AB} = \frac{6}{3} = 2, \frac{B'C'}{BC} = \frac{4}{2} = 2, \frac{A'C'}{AC} = \frac{2\sqrt{13}}{\sqrt{13}} = 2$

Since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC} \triangle ABC \sim \triangle A'B'C'$ by SSS $\sim$.

9. Step 1 Multiply each coord. by $\frac{3}{2}$ to find coords of vertices of $\triangle R'S'T'$.
   $R(-2, 2) \rightarrow R'\left(\frac{3}{2}(-2), \frac{3}{2}(2)\right) = R'(-3, 3)$
   $S(2, 4) \rightarrow S'\left(\frac{3}{2}(2), \frac{3}{2}(4)\right) = S'(3, 6)$
   $T(0, -2) \rightarrow T'\left(\frac{3}{2}(0), \frac{3}{2}(-2)\right) = T'(0, -3)$

Step 2 Graph $\triangle R'S'T'$.

Step 3 Use Dist. Formula to find side lengths.
   $RS = \sqrt{(2 + 2)^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$
   $R'S' = \sqrt{(3 + 3)^2 + (6 - 3)^2} = \sqrt{45} = 3\sqrt{5}$
   $ST = \sqrt{(0 - 2)^2 + (-2 - 4)^2} = \sqrt{40} = 2\sqrt{10}$
   $S'T' = \sqrt{(0 - 3)^2 + (-3 - 6)^2} = \sqrt{90} = 3\sqrt{10}$
   $RT = \sqrt{(0 + 2)^2 + (-2 - 2)^2} = \sqrt{20} = 2\sqrt{5}$
   $R'T' = \sqrt{(0 + 3)^2 + (-3 - 3)^2} = \sqrt{45} = 3\sqrt{5}$

Step 4 Find similarity ratio.
   $\frac{R'S'}{RS} = \frac{3\sqrt{5}}{2\sqrt{5}} = \frac{3}{2}, \frac{S'T'}{ST} = \frac{3\sqrt{10}}{2\sqrt{10}} = \frac{3}{2}$
   $\frac{R'T'}{RT} = \frac{3\sqrt{5}}{2\sqrt{5}} = \frac{3}{2}$

Since $\frac{R'S'}{RS} = \frac{S'T'}{ST} = \frac{R'T'}{RT}$, $\triangle RST \sim \triangle R'S'T'$ by SSS $\sim$.

PRACTICE AND PROBLEM SOLVING, PAGE 499

10. Coords. of kite are $A(4, 5)$, $B(9, 7)$, $C(10, 11)$, and $D(6, 10)$.
    Coords. of image are $A(2, 2.5)$, $B(4.5, 3.5)$, $C(5, 5.5)$, and $D(3, 5)$.

11. $\frac{UO}{XO} = \frac{OV}{OY}$
    $-9 = -3$
    $XO = -8$
    $TZ = -3XO$
    $XO = -24$
    $X$ on x-axis $\rightarrow X = (-24, 0)$
    $(-9, 0) \rightarrow \left(\frac{8}{3}(-9), \frac{8}{3}(0)\right) = (-24, 0)$, so scale factor is $\frac{8}{3}$. 

12. \[ \frac{MO}{ON} = \frac{KO}{OL} \]
\[ \frac{16}{-24} = \frac{-15}{10} \]
\[ -240 = -240 KO \]
\[ KO = 10 \]

K on y-axis \( \iff \) \( K = (0, 10) \)

(0, 16) \( \rightarrow \) \( \left( \frac{5}{8}, \frac{5}{8} \right) (16) = (0, 10) \), so scale factor is \( \frac{5}{8} \).

13. \[ DE = \sqrt{2^2 + 4^2} = 2\sqrt{5}, \]
\[ DF = \sqrt{4^2 + 4^2} = 4\sqrt{2} \]
\[ DG = \sqrt{3^2 + 6^2} = 3\sqrt{5}, \]
\[ DH = \sqrt{6^2 + 6^2} = 6\sqrt{2} \]
\[ \frac{DE}{DG} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}, \]
\[ \frac{DF}{DH} = \frac{4\sqrt{2}}{6\sqrt{2}} = \frac{1}{3} \]
\[ \angle D \equiv \angle D \text{ by Reflex. Prop. of } \equiv. \]

So \( \triangle DEF \sim \triangle DGH \) by SAS ~.

14. \[ \frac{MN}{MQ} = \frac{5\sqrt{2}}{10\sqrt{5}} = \frac{1}{2}, \]
\[ \frac{MP}{MR} = \frac{5\sqrt{10}}{10\sqrt{10}} = \frac{1}{2} \]
\[ \angle M \equiv \angle M \text{ by Reflex. Prop. of } \equiv. \]

So \( \triangle MNP \sim \triangle MPQ \) by SAS ~.

15. **Step 1**

Multiply each coord. by 3 to find coords of \( \triangle JK'K' \).

\( J(-2,0) \rightarrow J'(3(-2), 3(0)) = J'(-6, 0) \)

\( K(-1, -1) \rightarrow K'(3(-1), 3(-1)) = K'(-3, -3) \)

\( L(-3, -2) \rightarrow L'(3(-3), 3(-2)) = L'(-9, -6) \)

**Step 2**

Graph \( \triangle JK'K' \).

**Step 3**

Find side lengths.

\( JK = \sqrt{1^2 + 3^2} = \sqrt{10}, \]
\( J'K' = \sqrt{3^2 + 3^2} = 3\sqrt{2} \)

\( KL = \sqrt{2^2 + 1^2} = \sqrt{5}, \]
\( K'L' = \sqrt{3^2 + 3^2} = 3\sqrt{5} \)

\( JL = \sqrt{1^2 + 2^2} = \sqrt{5}, \]
\( J'L' = \sqrt{3^2 + 6^2} = 3\sqrt{5} \)

**Step 4**

Verify similarity.

Since \( \frac{JK'}{JK} = \frac{KL'}{KL} = \frac{J'L'}{JL} = 3, \quad \triangle JKL \sim \triangle J'K'L' \) by SSS ~.

16. **Step 1**

Multiply each coord. by \( \frac{1}{2} \) to find coords of \( \triangle MNP' \).

\( M(0, 4) \rightarrow M'(\frac{1}{2}(0), \frac{1}{2}(4)) = M'(0, 2) \)

\( N(4, 2) \rightarrow N'(\frac{1}{2}(4), \frac{1}{2}(2)) = N'(2, 1) \)

\( P(2, -2) \rightarrow P'(\frac{1}{2}(2), \frac{1}{2}(-2)) = P'(1, -1) \)

**Step 2**

Graph \( \triangle MNP' \).

**Step 3**

Find side lengths.

\( MN = \sqrt{4^2 + 2^2} = 2\sqrt{5}, \]
\( MN' = \sqrt{2^2 + 1^2} = \sqrt{5} \)

\( NP = \sqrt{2^2 + 2^2} = \sqrt{5}, \]
\( NP' = \sqrt{1^2 + 2^2} = \sqrt{5} \)

\( MP = \sqrt{2^2 + 2^2} = 2\sqrt{2}, \]
\( MP' = \sqrt{1^2 + 3^2} = \sqrt{10} \)

**Step 4**

Verify similarity.

Since \( \frac{MN}{MN'} = \frac{NP}{NP'} = \frac{MP}{MP'} = \frac{1}{2}, \quad \triangle MNP \sim \triangle MNP' \) by SSS ~.

17. It is not a dilation; it changes shape of transformed figure.

18. Solution B is incorrect. Scale factor is ratio of a lin. measure of image to corr. lin. measure of preimage, so scale factor is \( \frac{UW}{RT} = \frac{3}{2} \).

19. They are reciprocals. Similarity ratio of \( \triangle ABC \) to \( \triangle A'B'C' \) is \( \frac{AB}{A'B'} \). Scale factor is \( \frac{A'B'}{AB} \).

20a. Should use origin as vertex of rt. \( \angle \); 1 unit reps. 60 cm ~ 3 units rep. 180 cm; so coords. are \( J(0, 1), K(0, 0), L(3, 0) \).

b. \( J \rightarrow J'(3(0), 3(1)) = J'(0, 3) \)

\( K \rightarrow K'(3(0), 3(0)) = K'(0, 0) \)

\( L \rightarrow L'(3(3), 3(0)) = L'(9, 0) \)

**TEST PREP, PAGE 500**

21. A

Check similarity ratio: \( \frac{2.4}{4} = \frac{3}{5} = \frac{-6}{-10} \)

22. H

Perimeter is a lin. measure. So \( P' = 2P = 2(60) = 120 \).

23. A

\[ AB = 4, \quad AC = BC = \sqrt{2^2 + 4^2} = 2\sqrt{5} \]

\[ DE = |3 - 1| = 2, \quad DF = EF = \sqrt{1^2 + 2^2} = \sqrt{5} \]

\[ \frac{DE}{AB} = \frac{EF}{BC} = \frac{DF}{AC} = \frac{1}{2} \]
24. 15
   \[ A \to A'(3, 3), 3(2)) = A'(9, 6) \]
   \[ B \to B'(3(7), 3(5)) = B'(21, 15) \]
   \[ A'B' = \sqrt{12^2 + 9^2} = \sqrt{225} = 15 \]

**CHALLENGE AND EXTEND, PAGE 500**

25. Possible ~ statements: \( \triangle XYZ \sim \triangle MNP, \triangle MPN, \triangle NMP, \triangle PNM, \triangle PMN, \) or \( \triangle PN. \) For each ~ statement, \( Z \) could lie either above or below \( XY. \)

So there are \( 2(6) = 12 \) different \( \triangle. \) They are all different, since \( MN, NP, \) and \( MP \) are all \( \neq. \)

26. scale factor \( = \frac{XY}{MP} = 2 \frac{4}{3} = 2 \)

From \( M \) to \( N \) is rise of 2 and run of 1. So from \( X \) to \( Z \)

is either rise of 1 and run of \( \frac{1}{2} \) or rise of \(-1\) and run of \( \frac{1}{2}. \)

Therefore \( Z = \left(1 \pm \frac{1}{2}, -2 \pm 1\right) = \left(\frac{1}{2}, 1\right)-3 \)

or \( \left(\frac{1}{2}, -3\right). \)

27. All corr. \( \triangle \) of rects. are \( \equiv \) because they are all rt. \( \triangle. \)

Suppose 1st rect. has vertex on line \( y = 2x \) at \((a, b).\)

This pt. is a solution to the eqn., so \( b = 2a, \) and coords. of vertex are \((a, 2a).\) Similarly, for 2nd rect.,

 coords. of vertex on line \( y = 2x \) must be \((c, 2c).\)

1st rect. has dimensions \( a \) and \( 2a, \) and 2nd rect. has

dimensions \( c \) and \( 2c. \) So all ratios of corr. sides \( = \frac{c}{a}\)

Therefore rects. are ~ by def.

28. scale factor \( = \frac{DE}{AB} = \frac{6}{3} = 2 \)

From \( A \) to \( C \) is rise of 2 and run of 1.

2 positions for \( F \) are reflections in horiz. line \( DE. \) So from

\( D \) to \( F \) is rise of \( \pm 1 \) and run of \( 2. \) Therefore

\( F = (1 + 2, -1 \pm 1) = (3, 3) \) or \( (3, -5). \)

**SPIRAL REVIEW, PAGE 500**

29. Possible answer: \( 2(50) + 5 + w \geq 250 \)

\[ 105 + w \geq 250 \]

30. Think: \( \triangle DEH \equiv \triangle FEH \) by HL. So by CPCTC,

\( HF = DF \)

\( HF = DF = 6.71 \)

31. Think: By Isosc. \( \triangle \) Thm., \( \angle EDH \equiv \angle EFH, \) so by Rt.

\( \angle \equiv \text{Thm.}, \) 3rd \( \triangle \) Thm., and ASA, \( \triangle DFG \equiv \triangle FDJ. \)

So by CPCTC,

\( JF \equiv GD \)

\( JF = GD = 5 \)


\[
CF = \sqrt{CH^2 + HF^2} = \sqrt{2^2 + 6.71^2} \approx 7.00
\]

33. \[ \frac{RT}{RS} = \frac{UV}{US} \]

\[ \frac{RT}{9} = \frac{6 + 2}{6} = \frac{4}{3} \]

\[ 3RT = 36 \]

\[ RT = 12 \]

34. \[ \frac{VT}{RU} = \frac{US}{US} \]

\[ \frac{VT}{x} = \frac{2}{6} = \frac{1}{3} \]

\[ 3x = x + 3 \]

\[ 2x = 3 \]

\[ x = 1.5 \]

\[ VT = x = 1.5 \]

**DIRECT VARIATION, PAGE 501**

TRY THIS, PAGE 501

1. Step 1 Make a table to record data.

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<th>Perimeter</th>
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<td>18</td>
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<td>( \frac{1}{2} )</td>
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<td>72</td>
</tr>
<tr>
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<td>18</td>
<td>108</td>
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<td>24</td>
<td>144</td>
</tr>
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<td>4</td>
<td>30</td>
<td>180</td>
</tr>
</tbody>
</table>

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<td>144</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
<td>180</td>
</tr>
</tbody>
</table>

Step 2 Graph pts.

Since pts. are collinear and line that contains them includes origin, relationship is a direct variation.

Step 3 Find eqn. of direct variation.

\[ y = kx \]

180 = k(5)

36 = k

Thus constant of variation is 36.
2. **Step 1** Make a table to record data. 

<table>
<thead>
<tr>
<th>Scale Factor $x$</th>
<th>Side Lengths $a = x(3)$</th>
<th>$b = x(6)$</th>
<th>$c = x(7)$</th>
<th>Perimeter $P = a + b + c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$1\frac{1}{2}$</td>
<td>3</td>
<td>$3\frac{1}{2}$</td>
<td>8</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>12</td>
<td>14</td>
<td>32</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>18</td>
<td>21</td>
<td>48</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>24</td>
<td>28</td>
<td>64</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>30</td>
<td>35</td>
<td>80</td>
</tr>
</tbody>
</table>

**Step 2** Graph pts. 
Since pts. are collinear and line that contains them includes origin, relationship is a direct variation.

**Step 3** Find eqn. of direct variation. 
$y = kx$ 
$80 = k(5)$ 
$k = 16$ 
Thus constant of variation is 16.

3. **Step 1** Make a table to record data. 

<table>
<thead>
<tr>
<th>Scale Factor $x$</th>
<th>Side Length $s = x(3)$</th>
<th>Perimeter $P = 4s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{1}{2}$</td>
<td>$1\frac{1}{2}$</td>
<td>6</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>36</td>
</tr>
<tr>
<td>4</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>5</td>
<td>15</td>
<td>60</td>
</tr>
</tbody>
</table>

**Step 2** Graph pts. 
Since pts. are collinear and line that contains them includes origin, relationship is a direct variation.

**Step 3** Find eqn. of direct variation. 
$y = kx$ 
$60 = k(5)$ 
$k = 12$ 
Thus constant of variation is 12.
7. \[
\frac{\text{plan length of } \overline{EF}}{EF} = 0.5 = \frac{1.5}{30} = \frac{1.5}{60} = 0.6 \text{ ft}
\]
8. 5 ft 3 in. = 5(12) + 3 in. = 63 in.
   5 ft 10 in. = 5(12) + 10 in. = 70 in.
   40 ft = 40(12) in. = 480 in.
   \[ h = \frac{480}{70} = 6.857 \]
   \[ h = 432(480) = 63 \text{ ft} \]
9. By the Dist. Formula:
   \[
   AD = \sqrt{1^2 + 2^2} = \sqrt{5};
   AB = \sqrt{2^2 + 4^2} = 2\sqrt{5}
   \]
   \[
   AE = \sqrt{2^2 + 1^2} = \sqrt{5};
   AC = \sqrt{4^2 + 2^2} = 2\sqrt{5}
   \]
   \[ \angle A \cong \angle A \text{ by the Reflex. Prop. of } \cong. \]
   By SAS \( \sim \), \( \triangle ADE \sim \triangle ABC \).
10. By the Dist. Formula:
    \[
    RS = \sqrt{2^2 + 1^2} = \sqrt{5};
    RU = \sqrt{4^2 + 2^2} = 2\sqrt{5}
    \]
    \[
    RT = |3 - 0| = 3;
    RV = |6 - 0| = 6
    \]
    \[
    \frac{RS}{RT} = \frac{RU}{RV} = \frac{1}{2} \triangle SRT \cong \triangle URV \text{ by the Vert. } \triangle \text{ Thm.}
    \]
    By SAS \( \sim \), \( \triangle RST \sim \triangle RUV \).
11. \[
    PQ = QR = 2;
    P'O' = Q'R' = 6
    \]
    \[
    PR = \sqrt{2^2 + 2^2} = 2\sqrt{2};
    P'R' = \sqrt{6^2 + 6^2} = 6\sqrt{2}
    \]
    \[
    \frac{P'O'}{PQ} = \frac{Q'R'}{QR} = \frac{6}{2} = 3;
    \frac{PR}{P'R'} = \frac{6\sqrt{2}}{6\sqrt{2}} = 3
    \]
    By SSS \( \sim \), \( \triangle P'Q'R' \sim \triangle PQR \).
12. \[
    AB = \sqrt{4^2 + 2^2} = 2\sqrt{5};
    A'B' = \sqrt{6^2 + 3^2} = 3\sqrt{5}
    \]
    \[
    BC = \sqrt{6^2 + 6^2} = 2\sqrt{13};
    B'C' = \sqrt{9^2 + 6^2} = 3\sqrt{13}
    \]
    \[
    \frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{3}{2}
    \]
    By SSS \( \sim \), \( \triangle A'B'C' \sim \triangle ABC \).

**STUDY GUIDE: REVIEW, PAGES 504-507**

1. proportion
2. dilation
3. means
4. ratio

**LESSON 7-1, PAGE 504**

5. slope of \( m = \frac{1}{5} \)
6. slope of \( n = \frac{-3}{6} = \frac{-1}{2} \)
7. slope of \( p = \frac{6}{4} = \frac{3}{2} \)
8. Let \( x, y \) be the largest and smallest parts respectively.
   \[
   x + y = \frac{6 + 3}{84} = \frac{9}{84(9)} = \frac{14}{42}
   \]
   \[ x + y = 54 \]
   The sum of the smallest and largest parts is 54.
9. \[ \ell = \frac{7}{12} \]
   \[ w = \frac{7}{12} \]
   \[
   P = 2\ell + 2w
   = 2\left(\frac{7}{12}\right) + 2w
   \]
   \[ 6P = 7w + 12w \]
   \[ 6(95) = 19w \]
   \[ w = 30 \]
   \[ \ell = \frac{7}{12}(30) = 17.5 \]
   Side lengths are 17.5, 30, 17.5, 30.
10. \[
    y = \frac{9}{7} \]
    \[
    3y = 63 \]
    \[
    y = 21 \]
    \[
    10s = 100 \]
    \[
    s = 10 \]
11. \[
    4x = \frac{1}{y} \]
    \[
    x^2 = 36 \]
    \[
    x = \pm 6 \]
12. \[
    z = \frac{4}{x} \]
    \[
    z - 1 = \frac{36}{144} = (z - 1)^2 \]
    \[
    z - 1 = \pm 12 \]
    \[
    z = 13 \text{ or } -11 \]
13. \[
    y + 1 = \frac{2}{24} = \frac{3}{3(y + 1)} \]
    \[
    3(y + 1)^2 = 48 \]
    \[
    (y + 1)^2 = 16 \]
    \[
    y + 1 = \pm 4 \]
    \[
    y = -1 \pm 4 \]
    \[
    y = 3 \text{ or } -5 \]

**LESSON 7-2, PAGE 505**

16. \[
    \frac{JK}{PO} = \frac{8}{4.8} = \frac{5}{3};
    \frac{JM}{PS} = \frac{5}{3} \]
   all \( \triangle \) are rt \( \triangle \), so \( \cong \)
   yes, by def. of \( \sim \); \( \sim \) ratio = \( \frac{5}{3} \); \( \triangle \) \( \cong \triangle \)
17. yes, by AA \( \sim \); \( \sim \) ratio = \( \frac{TU}{WX} = \frac{12}{6} = 2 \);
   \( \triangle \) \( \cong \triangle \)
**LESSON 7-3, PAGE 505**

18. Statements | Reasons
--- | ---
1. \( JL = \frac{1}{3} JN, JK = \frac{1}{3} JM \) | 1. Given
2. \( \frac{JL}{JN} = \frac{1}{3} \frac{JK}{JM} = \frac{1}{3} \) | 2. Div. Prop. of =
3. \( \frac{JL}{JM} = \frac{JK}{JM} \) | 3. Trans. Prop. of =
4. \( \angle J \cong \angle J \) | 4. Reflex. Prop. of =
5. \( \triangle JKL \sim \triangle JMN \) | 5. SAS \( \sim \) Steps 3, 4

19. Statements | Reasons
--- | ---
1. \( QR \parallel ST \) | 1. Given
2. \( \angle RQP \cong \angle STP \) | 2. Alt. Int. \( \triangle \) Thm.
3. \( \angle RPQ \cong \angle SPT \) | 3. Vert. \( \triangle \) Thm.
4. \( \triangle PQR \sim \triangle PTS \) | 4. AA \( \sim \) Steps 2, 3

20. Statements | Reasons
--- | ---
1. \( BC \parallel CE \) | 1. Given
2. \( \angle ABD \cong \angle C \) | 2. Corr. \( \angle \) Post.
3. \( \angle ADB \cong \angle E \) | 3. Corr. \( \angle \) Post.
4. \( \triangle ABD \sim \triangle ACE \) | 4. AA \( \sim \) Steps 2, 3
5. \( \overline{AB} = \overline{BD} \) | 5. Def. of \( \sim \) polygons
6. \( \overline{AC} = \overline{CE} \) | 6. Cross Products Prop.

**LESSON 7-4, PAGE 506**

21. \( CE = \frac{8}{15} \frac{12}{12} \)

22. \( ST = 3 \frac{9}{10} \)

23. \( JK = \frac{1}{2} \frac{JL}{JM} = \frac{JL}{JN} \)

24. \( \frac{EC}{EA} = \frac{ED}{EB} = \frac{3}{7} \)

25. \( SU = \frac{SV}{RU} \frac{y + 1}{12} \frac{2y}{y} \)

26. \( x + 6 = \frac{2x}{24} \frac{30}{24} \)

27. \( P = a + b + c \text{ where } b = a + x, c = 3 + 5 = 8 \)

28. 3 ft = 3(12) in. = 36 in.

5 ft 4 in. = 5(12) + 4 in. = 64 in.

14 ft 3 in. = 14(12) + 3 in. = 171 in.

\( x = \frac{171}{64} \)

36x = 10,944

\( x = 304 \text{ in.} = 25 \text{ ft in.} \)

29. \( \frac{6}{x} = \frac{12}{3 + x} \)

\( 6(x + 3) = 12x \)

\( 18 + 6x = 12x \)

\( 18 = 6x \)

\( x = 3 \text{ ft} \)

**LESSON 7-5, PAGE 507**

30. By the Dist. Formula:

\( RS = \sqrt{2^2 + 2^2} = 2\sqrt{2}; RU = \sqrt{4^2 + 4^2} = 4\sqrt{2} \)

\( RT = \sqrt{1^2 + 3^2} = \sqrt{10}; RV = \sqrt{2^2 + 6^2} = 2\sqrt{10} \)

\( \frac{RS}{RT} = \frac{1}{2} \frac{RU}{RV} \)

\( RS \parallel RU \) by the Reflex. Prop. of =.

So \( \triangle RST \sim \triangle RUV \) by SAS \( \sim \).

31. By the Dist. Formula:

\( JK = \sqrt{2^2 + 1^2} = \sqrt{5}; JM = \sqrt{8^2 + 4^2} = 4\sqrt{5} \)

\( JL = |2 - 4| = 2; JN = |4 - 4| = 8 \)

\( \frac{JK}{JM} = \frac{JL}{JN} = \frac{1}{4} \angle J \cong \angle J \) by the Reflex. Prop. of =.

So \( \triangle JKL \sim \triangle JMN \) by SAS \( \sim \).

32. \( AO = OB \)

\( CO = OD \)

\( 12 = OB \)

\( 18 = OB \)

\( -108 = 18OB \)

\( OB = -6 \)

Since \( x\)-coord. of \( B \) is \( 0, B = (0, -6) \).

Scale factor = \( \frac{12}{18} = \frac{2}{3} \).

33. Image vertices are \( K'(0, 9), L'(0, 0), M'(12, 0) \).

By the Dist. Formula:

\( KL = 3; KL' = 9; LM = 4; L'M' = 12 \)

\( KM = \sqrt{3^2 + 4^2} = 5; K'M' = \sqrt{9^2 + 12^2} = 15 \)

All proportions = 3, so \( \triangle KLM \sim \triangle K'L'M' \) by SSS \( \sim \).

**CHAPTER TEST, PAGE 508**

1. slope of \( \ell = \frac{-6 - 4}{10 + 6} = -\frac{5}{8} \)

2. \( \frac{5}{8} = 3.5 \)

\( w = 28 \)

\( w = 5.6 \text{ in.} \)

3. \( \angle B \cong \angle N \text{ and } \angle C \cong \angle P; \) yes, by AA \( \sim \);

\( \sim \) ratio = \( \frac{AB}{MN} = \frac{40}{60} = \frac{2}{3}; \triangle ABC \sim \triangle MNP \)
4. **DE**
   \[ \frac{HJ}{22} = \frac{5}{2} \quad \text{and therefore} \quad \sim \text{ratio} = \frac{5}{2} \]
   **DEFG** \( \sim \) **HJKL** by def.

5. ** Statements ** | ** Reasons **
   |---|---|
   1. **RSTU** is a \( \square \). | 1. Given |
   2. **RU** \( || \) **ST** | 2. Def. of \( \square \) |
   3. \( \angle VRW \equiv \angle TSW \) | 3. Alt. Int. \( \angle \) Thm. |
   4. \( \angle RWV \equiv \angle SWT \) | 4. Vert \( \angle \) Thm. |
   5. \( \triangle RWV \sim \triangle SWT \) | 5. AA \( \sim \) Steps 3, 4 |

6. **CD** = **DG**
   \[ \frac{AB}{BG} = 2.5 \]
   \[ \frac{9CD}{9} = 15 \]
   \[ CD \approx 1.7 \text{ ft} \]
   **EF** = **FG**
   \[ \frac{AB}{BG} = 3 \]
   \[ \frac{2.5}{9} = \frac{5}{18} \]
   \[ 9FG = 7.5 \]
   **FG** \( \approx \) 0.8 ft

7. **YW** = **WZ**
   \[ \frac{XY}{XZ} = \frac{t}{8} = \frac{t - 2}{12.8} \]
   \[ 12.8(t) = 8(t - 2) \]
   \[ 6.4t = 8t - 16 \]
   \[ 16t = 1.6t \]
   \[ t = 10 \]
   **YW** = \( \frac{10}{2} = 5 \)
   **WZ** = \( t - 2 = 8 \)

8. **PR** = 21
   \[ \frac{10}{21} = \frac{18}{18PR} = 210 \]
   \[ PR = 11.\overline{2} \]

9. 5 ft 8 in. = 5(12) + 8 in. = 68 in.
   3 ft = 36 in.; 27 ft = 324 in.
   \[ h = 324 \]
   \[ 68 = 36 \]
   \[ h = 68(9) = 612 \text{ in.} = 51 \text{ ft} \]

10. plan length of \( \frac{AB}{1.5} \)
    \[ \frac{30}{1.5} = \frac{37.5}{1.5AB} \]
    \[ \frac{AB}{25} = \frac{37.5}{1.5} \]

11. By the Dist. Formula:
    \[ AB = \sqrt{3^2 + 1^2} = \sqrt{10}; AD = \sqrt{9^2 + 3^2} = 3\sqrt{10} \]
    \[ AC = |3 - 5| = 2; AE = |1 - 5| = 6 \]
    \[ \frac{AB}{AD} = \frac{AC}{AE} = \frac{1}{3} \quad \angle A \equiv \angle A \text{ by the Reflex. Prop. of } \equiv \]
    So \( \triangle KJL \sim \triangle JMN \) by SAS \( \sim \).

12. ** COLLEGE ENTRANCE EXAM PRACTICE, PAGE 509 **

   1. \( \triangle ABC \)
      \[ \frac{CD}{DE} = \frac{AB}{BC} \]
      \[ \frac{CD}{DE} = \frac{21}{14} \]
      \[ \frac{BC}{DE} = \frac{1}{2} \]
      \[ 9 - BC = \frac{8}{2} \]
      \[ 3BC = 9 \]
      \[ BC = 3 \]
      \[ \text{Since } \overline{BD} \text{ is horiz., } y-\text{coord. of } C = 1; \]
      \[ \text{so } C = (1 + 3, 1) = (4, 1). \]

   3. **D**; \( x + y + z = 750,000 \text{ and } x:y:z = 4:5:6 \)
      \[ \frac{z}{750,000} = \frac{6}{4 + 5 + 6} = \frac{2}{5} \]
      \[ 5z = 1,500,000 \]
      \[ z = 300,000 \]

   5. \( \text{In any square, all } \angle \text{ are rt } \angle \text{, so } \equiv; \text{ all sides are } \equiv. \)